Hanbury Brown-Twiss (HBT) interferometry with event-by-event fluctuations

Christopher J. Plumberg
In collaboration with Chun Shen and Ulrich Heinz
(arXiv:1306.1485)

The Ohio State University

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Background and Motivation

Hanbury-Brown–Twiss (HBT) interferometry (also, 'intensity interferometry' or 'femtoscopy') relies on two-particle momentum correlations to study the geometric and flow properties of heavy-ion collisions:

- azimuthally-sensitive HBT analyses communicate important information about deformations in the structure of the freeze-out surface
- odd harmonics present in HBT radii known to open the window to the study of event-by-event fluctuations
- fulfills a vital role in constraining the initial state of the fireball and its subsequent evolution

Part 1: 3rd order HBT

HBT Basics

Two particles: $\vec{p}_1,\ \vec{p}_2 \longrightarrow \vec{q} \equiv \vec{p}_1 - \vec{p}_2,\ \vec{K} \equiv \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$

Correlation function: $C(\vec{p}_1, \vec{p}_2) \equiv \dfrac{E_{p_1} E_{p_2} \frac{dN}{d^3 p_1 d^3 p_2}}{\left(E_{p_1} \frac{dN}{d^3 p_1}\right)\left(E_{p_2} \frac{dN}{d^3 p_2}\right)}$

Ignoring final-state interactions, C may be fit to the form:

$$C(\vec{q}, \vec{K}) = 1 \pm \lambda(\vec{K}) \exp\left(-\sum_{i,j=o,s,l} R^2_{ij}(\vec{K}) q_i q_j\right),$$

$R^2_{ij} = R^2_{ij}(|\vec{K}|, \Phi_K) \to$ *measure* Φ_K *with respect to what?*

Fourier moments of $R^2_{ij}(\vec{K})$

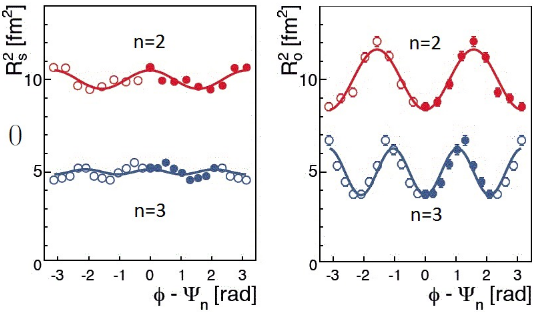
- Experimentally, one measures HBT correlations as a function of the *difference* between Φ_K and one of the flow angles Ψ_n
  $\Rightarrow$ we plot observable quantities against $\Phi_K - \Psi_n$, $(n = 1, 2, 3, \ldots)$
- The flow angle is defined by Ψ_n in $v_n e^{in\Psi_n} \equiv \left\langle e^{in\phi_p} \right\rangle$
- The v_n are the anisotropic flow coefficients and ϕ_p is the azimuthal angle of $\vec{p}_T$ of the emitted particles in the lab frame
- $\Rightarrow$ Fourier-decompose the R^2_{ij}:

$$\begin{aligned} R^2_{ij}(|\vec{K}|, \Phi_K) &= 2\sum_{n=1}^{\infty} \Big(R^{2(c)}_{ij,n}(|\vec{K}|) \cos[n(\Phi_K - \Psi_n)] \\ &+ R^{2(s)}_{ij,n}(|\vec{K}|) \sin[n(\Phi_K - \Psi_n)] \Big) + R^2_{ij,0}(|\vec{K}|) \end{aligned}$$

PHENIX data

T. Niida, (QM 2012, arXiv:1304.2876) (integrated over $K_\perp$)

$R^2_{s,n} > 0$



$R^2_{o,n} < 0$

Important features to understand:

- Different signs of Fourier coefficients in out and side directions
- Different oscillation amplitudes: $R^2_{o,n}/R^2_{s,n} \gg 1$

Emission function

We define the emission function $S(x, K)$ as the Wigner density of the fireball

$$\text{Emission function:} \quad \int d^4x \, S(x, K) = E_K \frac{dN}{d^3K}$$

Taking $\lambda(\vec{K}) = 1$, C and S may be related by

$$C(\vec{q}, \vec{K}) \approx 1 + \left| \frac{\int d^4x \, e^{iq\cdot x} S(x, K)}{\int d^4x \, S(x, K)} \right|^2$$

- For Gaussian sources $S(x, K)$, $R^2_{ij} = \langle (\tilde{x}_i - \beta_i \tilde{t})(\tilde{x}_j - \beta_j \tilde{t}) \rangle$, where
- $\tilde{x}_i \equiv x_i - \langle x_i \rangle$, $\tilde{t} \equiv t - \langle t \rangle$, $\vec{\beta} \equiv \vec{K}/K^0$ and
- $\langle f(x) \rangle \equiv \frac{\int d^4x \, f(x) S(x,K)}{\int d^4x \, S(x,K)}$

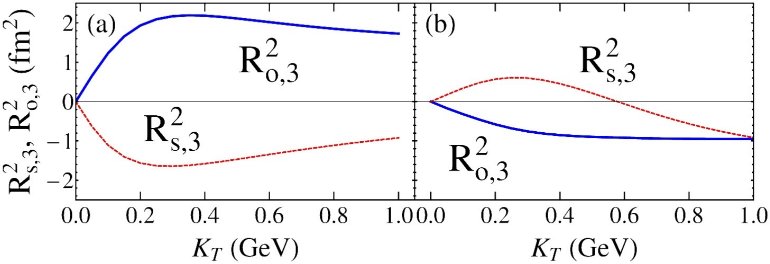
$\Rightarrow$ given $S(x, K)$, $R^2_{ij}(\vec{K})$ may be computed directly

Emission function

- Consider S with two kinds of different triangular deformations:
  - "Geometric case" - Triangular spatial deformation with radial flow, no triangular flow
  - "Flow case" - Triangular flow, no spatial deformation
- Can obtain triangular oscillations of R_{ij}^2 from
  - triangular flow deformation
  - triangular spatial deformation coupled to radial flow
  - combinations thereof

HBT oscillation amplitudes: two examples

Geometric case | Flow case



Part 1: Conclusions

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- Without radial flow, a triangular spatial deformation of the source at freeze-out leaves no measurable trace in the HBT radii oscillations
- Triangular oscillations of HBT radii may generally result from an admixture of triangular collective flow *and* triangular spatial deformation coupling to radially symmetric flow
- We can distinguish "flow domination" from "geometry domination" by the phases and K_T-dependence of the respective oscillation amplitudes; PHENIX data appear to point to "flow domination"

Part 2:

Event-by-event hydrodynamics

Next steps...

- Our analysis assumed Gaussian, ensemble-averaged ansatz for $S(x, K)$
  $\Rightarrow$ *what do we find in an event-by-event (EBE) hydrodynamic treatment?*
- Construct (viscous) hydrodynamic emission function using Cooper-Frye formula:

$$
\begin{aligned}
S_{hydro}(x, K) &= \frac{1}{(2\pi)^3} \int_{\Sigma(x_f)} K \cdot d^3\sigma(x_f)\delta^4(x - x_f) f(x_f, K), \\
f(x, K) &= f_0 + \delta f \\
&= \frac{1}{e^{(K \cdot u - \mu)/T} \pm 1} + \frac{\chi(K^2) K^\mu K^\nu \pi_{\mu\nu}}{2T^2(e + p)} f_0(1 \pm f_0)
\end{aligned}
$$

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$$

- How are our results affected by ensemble-averaging?
- How should one extract HBT radii from a non-Gaussian source?

(At least) 3 Ways of getting HBT radii

- Option 1: compute R_{ij}^2 from S_{hydro} via source variances,

$$R_{ij}^2 = \langle (\tilde{x}_i - \beta_i \tilde{t})(\tilde{x}_j - \beta_j \tilde{t}) \rangle = -\frac{1}{2(C-1)} \left. \frac{\partial^2 C}{\partial q_i \partial q_j} \right|_{q \to 0}$$

- Option 2: Construct $C(q, K)$ from $S_{hydro} \to$ extract R_{ij}^2 from Gaussian fit to correlator, $C - 1 \sim \exp\left(-\sum_{i,j} R_{ij}^2 q_i q_j\right)$
- Option 3: Construct $C(q, K)$ from $S_{hydro} \to$ extract R_{ij}^2 from "q-moments method":

$$\frac{1}{2}\left(\mathcal{R}^{-1}\right)_{ij} = \frac{\int d^3q \, q_i q_j \, (C-1)}{\int d^3q \, (C-1)}, \quad \text{where } \mathcal{R} \equiv \left(R_{ij}^2\right)(K)$$

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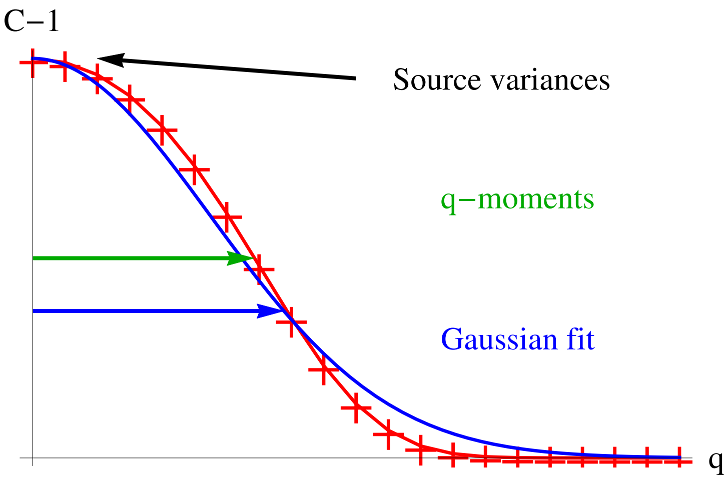
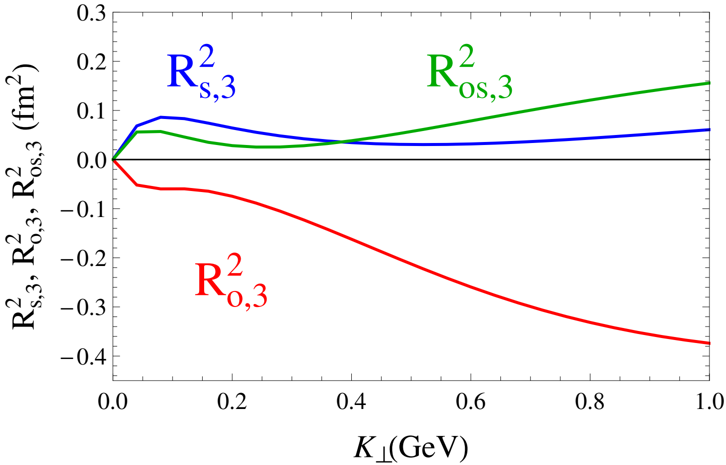
Comparison of HBT methods



Results

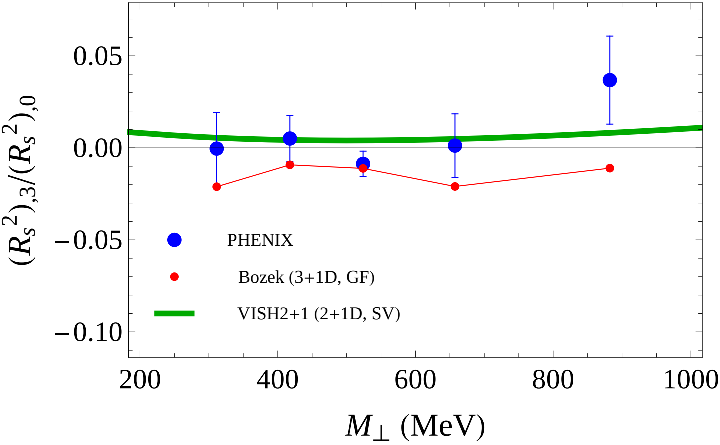
Hydrodynamic approach

$K_\perp$-dependence of $R^2_{ij,3}$ from hydrodynamics



Hydrodynamic approach

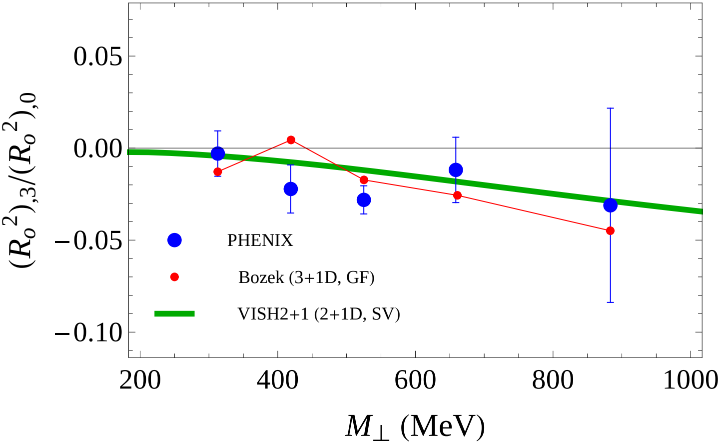
$M_\perp$-dependence of $R^2_{s,3}/R^2_{s,0}$ from hydrodynamics[1]



[1] arXiv:1401.7680, arXiv:1401.4894

Hydrodynamic approach

$M_\perp$-dependence of $R^2_{o,3}/R^2_{o,0}$ from hydrodynamics[2]



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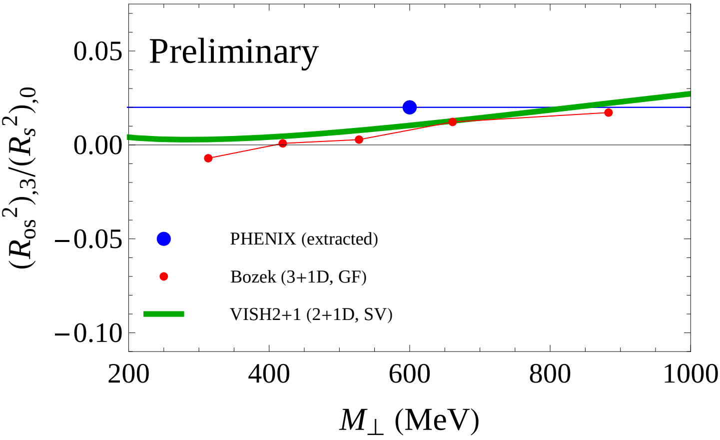
Hydrodynamic approach

$M_\perp$-dependence of $R^2_{os,3}/R^2_{s,0}$ from hydrodynamics[3]



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Part 2: Conclusions

- VISH2+1 qualitatively reproduces general trends of PHENIX data
- Qualitative features of $K_\perp$-dependence of hydrodynamic $R^2_{ij,3}$ similar to toy model for small $K_\perp$, more discrepancies at $K_\perp \gtrsim 0.3$ GeV
- Subtleties involving ensemble-averaging and the construction of the correlation function have not been addressed here

Thanks for your attention!

Thanks also to my collaborators

Ulrich Heinz and Chun Shen!

Double-Fourier formalism

Define

$$
\begin{aligned}
S_{\ell,m} &\equiv e^{-im\Psi_3}\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}e^{i\ell\phi}\int_{-\pi}^{\pi}\frac{d\Phi_K}{2\pi}e^{im\Phi_K}S(\phi,\Phi_K),\\
\longrightarrow \mathcal{Z}_\ell &\equiv e^{-i\ell\Psi_3}\sum_{m=-\infty}^{\infty}S_{\ell,m-\ell}e^{-im(\Phi_K-\Psi_3)} \equiv \mathcal{X}_\ell + i\mathcal{Y}_\ell
\end{aligned}
$$

We can show, e.g.,

$$
\begin{aligned}
\left\langle x_s^2\right\rangle &= \int_{-\infty}^{\infty}d\eta\int_0^{\infty}\tau\,d\tau\int_0^{\infty}r\,dr\pi r^2\left(\mathcal{X}_0-\mathcal{X}_2\right)\\
\left\langle x_s\right\rangle &= \int_{-\infty}^{\infty}d\eta\int_0^{\infty}\tau\,d\tau\int_0^{\infty}r\,dr2\pi r\mathcal{Y}_1
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$$

- Since $R_s^2 = \left\langle x_s^2\right\rangle - \left\langle x_s\right\rangle^2$, no dependence on $\ell \geq 3$ (similarly for other R_{ij}^2)!
- N.B.: same expression contains all orders in Φ_K

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Toy model for the source

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$$
\begin{aligned}
S(x, K) \quad = \quad & \frac{S_0(K)}{\tau} \exp\left[-\frac{(\tau-\tau_f)^2}{2\Delta\tau^2} - \frac{(\eta-\eta_0)^2}{2\Delta\eta^2}\right. \\
& -\frac{r^2}{2R^2}\left(1+2\bar{\epsilon}_3\cos(3(\phi-\bar{\psi}_3))\right) \\
- \quad & \left.\frac{M_\perp}{T_0}\cosh(\eta-Y)\cosh\eta_t + \frac{K_\perp}{T_0}\cos(\phi-\Phi_K)\sinh\eta_t\right]
\end{aligned}
$$

where

$$
\eta_t = \frac{\eta_f r}{R}\left(1+2\bar{v}_3\cos(3(\phi-\bar{\psi}_3))\right)
$$

- $\bar{\epsilon}_3$: triangular azimuthal deformation
- $\bar{v}_3$: triangular flow deformation
- η_f: collective radial flow rapidity
- $\bar{\psi}_3$: triangular flow velocity angle, points in direction of largest flow rapidity and steepest descent of spatial density profile (note: $\Psi_n \neq \bar{\psi}_n$ in general)