



U.S. DEPARTMENT OF
ENERGY

Office of Science



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Balance function as a unique probe of the quark gluon plasma: experimental overview and outlook

Jinjin(Au-Au) Pan

BNL Nuclear Physics Seminar | 02/25/2020

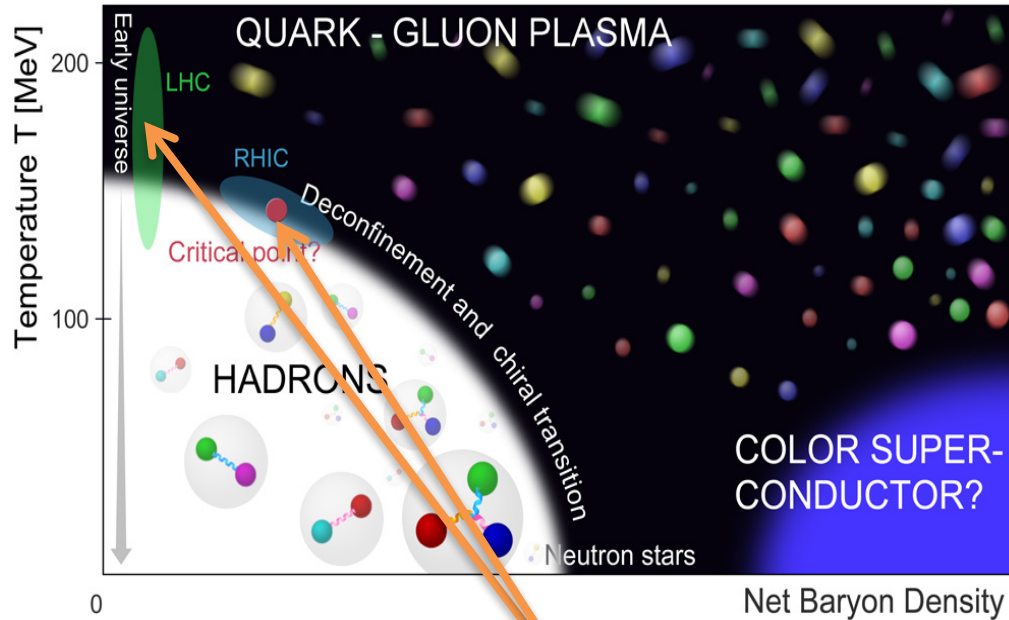
Just off shift from STAR control room this morning



Status of STAR:

- *2 days ago we successfully reached the goal of 11.5 GeV Au-Au (230M good MB events) data taking during my shift*
- *right now STAR is taking 9.2 GeV Au-Au data*

Why carry out relativistic heavy ion collisions?



Discovery of QGP:
RHIC 2005 Au-Au
LHC 2010 Pb-Pb

Quark Gluon Plasma (QGP)

- *hot and dense QCD matter*
- *strongly coupled perfect liquid*
- *deconfined quarks*
- *early universe*

Open questions:

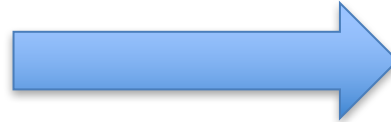
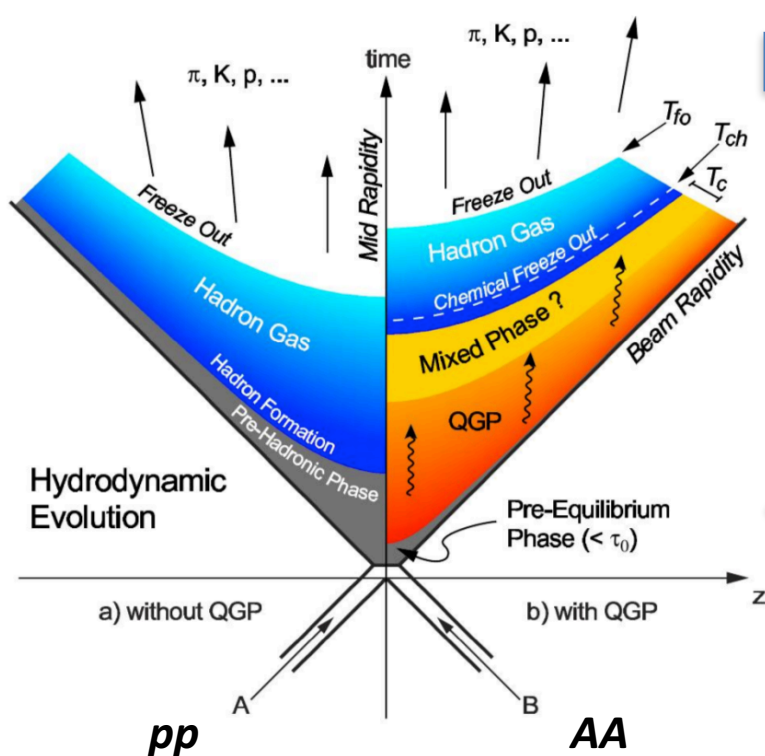
- *QCD phase diagram*
- *critical point*
- *light and strange quark production time*
- *hadronization*
- *transport*

...

Balance Function – distribution of balancing charges

$$B(\vec{p}_1, \vec{p}_1)$$

Bass, Danielewicz, Pratt PRL 85, 2689 (2000)



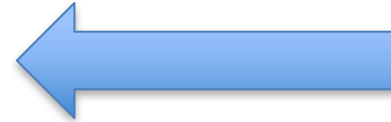
*measure BF of final state
identified hadron pairs*

\pm — General conserved charges:

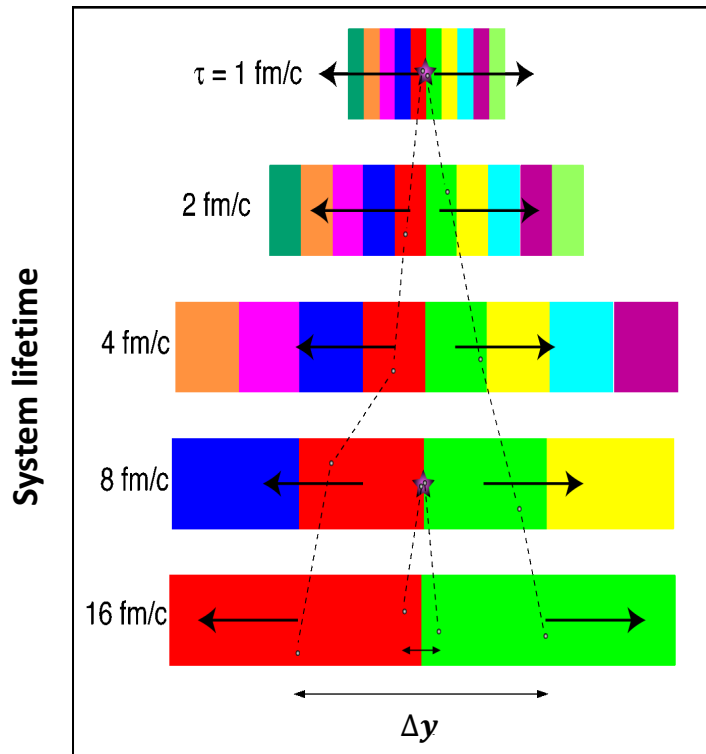
- e: electric charge
- S: strangeness
- B: baryon number



*learn about light and strange
quark production time*



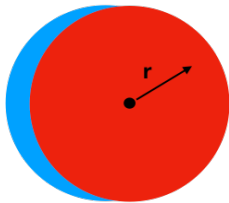
Balancing charge separation in Δy



Balancing charge separation

Bass, Danielewicz, Pratt PRL 85, 2689 (2000)

Later hadronization towards central collisions -> narrower BF of $\pi\pi$, KK , pp

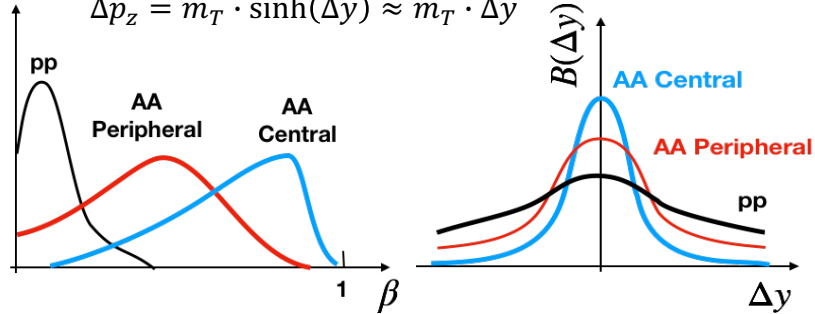


$\beta \propto r^\alpha$
Blast wave

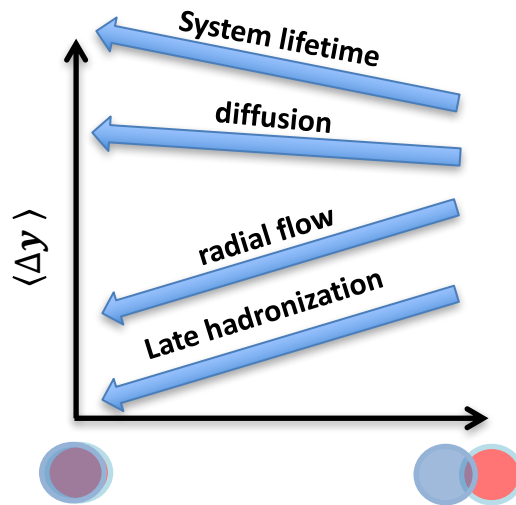
$p(\beta)$

Voloshin PLB 632 (2006) 490-494

$$\Delta p_z = m_T \cdot \sinh(\Delta y) \approx m_T \cdot \Delta y$$



Larger radial flow towards central collisions leads to smaller separation of balancing pairs in Δy





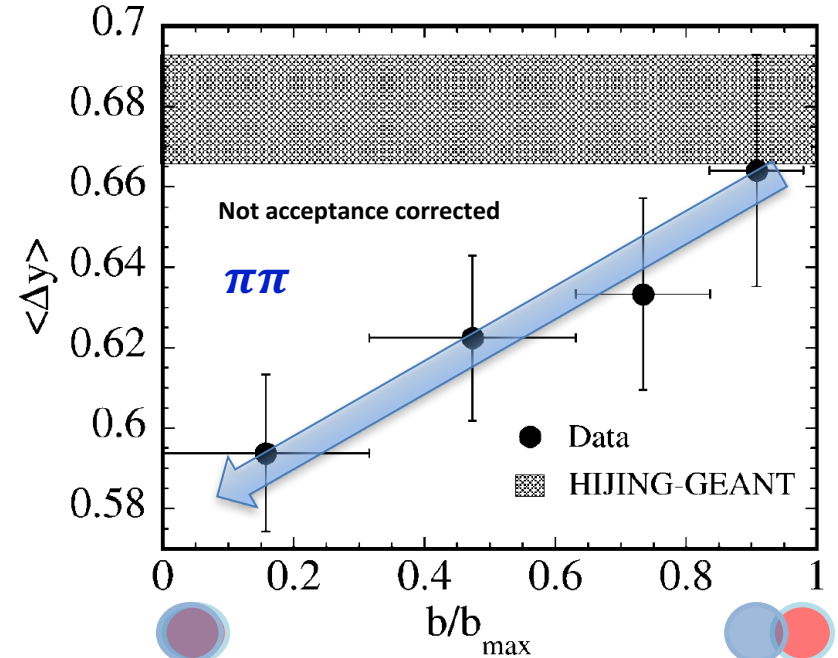
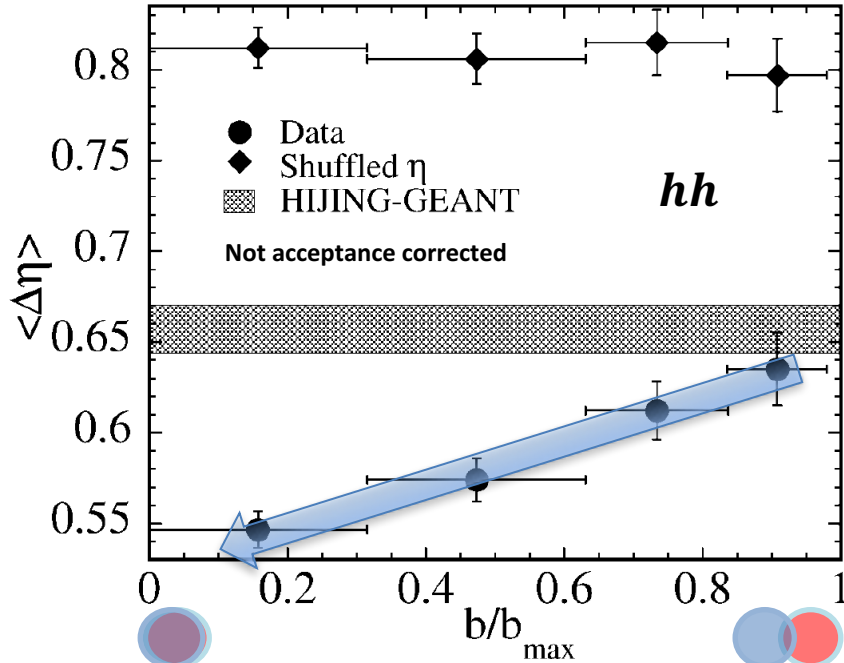
BF of unidentified hadron pairs hh and $\pi\pi$

PRL 90, 172301 (2003)

Au-Au @ 130 GeV

$0.1 < p < 2.0 \text{ GeV}/c$

$0.1 < p_T, p < 0.7 \text{ GeV}/c$



Narrowing of B^{hh} and $B^{\pi\pi}$ towards central Au-Au collisions

-> larger radial flow towards central collisions leads to smaller separation of balancing pairs in $\Delta\eta$ & Δy

-> later hadronization towards central collisions leads to narrower BF



BF of identified hadron pairs $\pi\pi$ and KK

Au-Au @ 200 GeV

PRC 82, 024905 (2010)

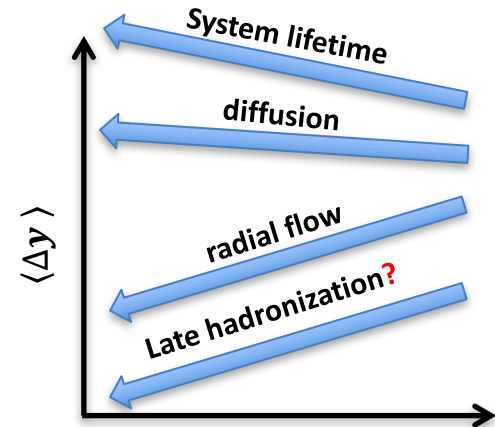
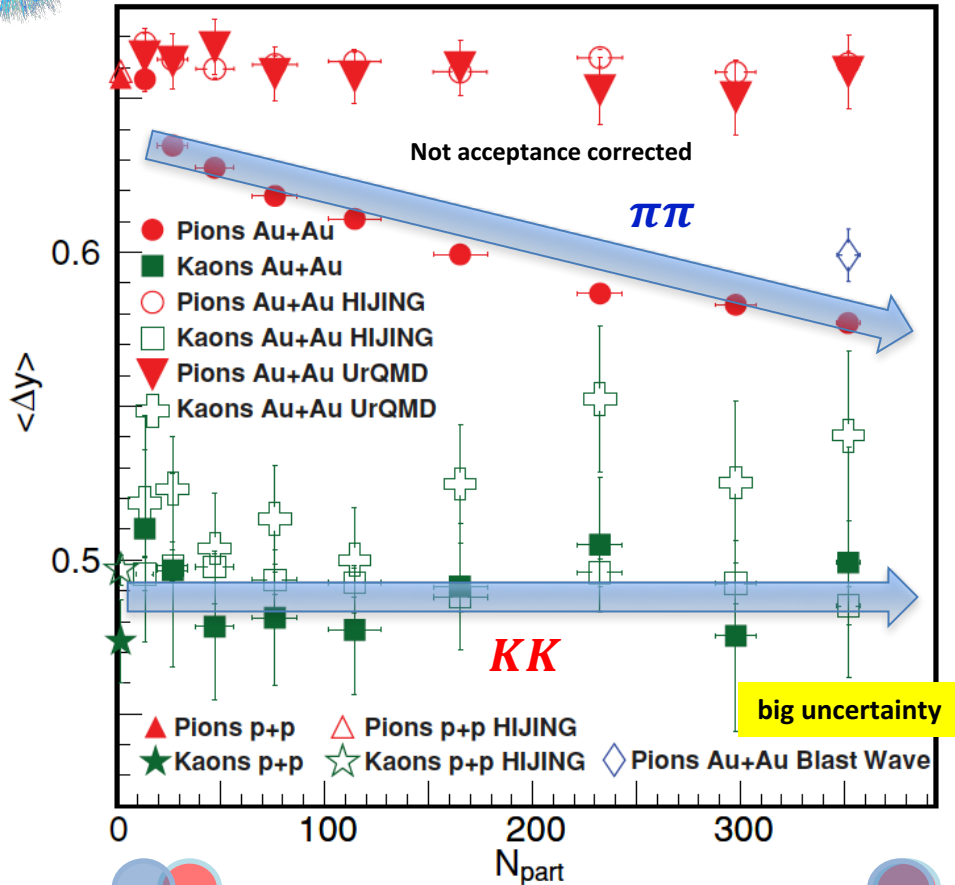
$0.2 < p_T < 0.6 \text{ GeV}/c$

$B^{\pi\pi}(\Delta y)$ narrow towards central collisions, while $B^{KK}(\Delta y)$ no centrality dependence.

-> larger radial flow towards central collisions leads to smaller separation of balancing pairs in Δy for $\pi\pi$ and KK .

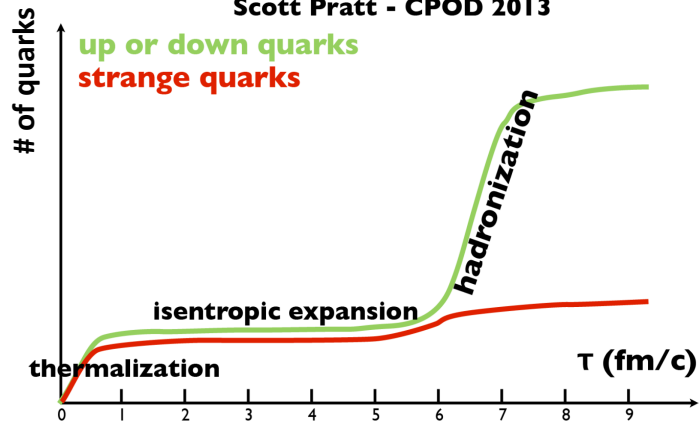
-> $\langle \Delta y \rangle$ for KK is smaller than $\pi\pi$ due to ϕ decay.

-> no late hadronization for KK ?

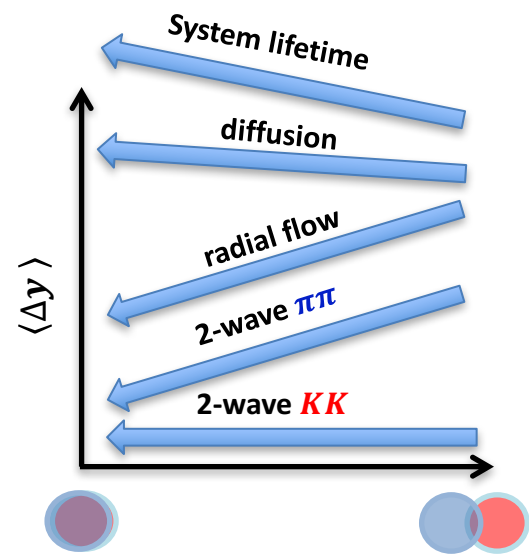


Two-wave Quark Production

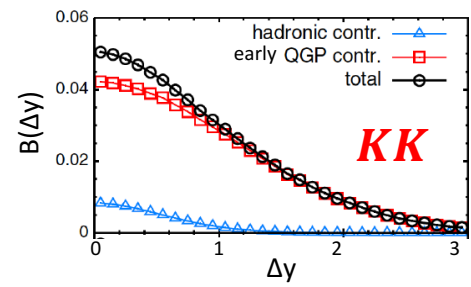
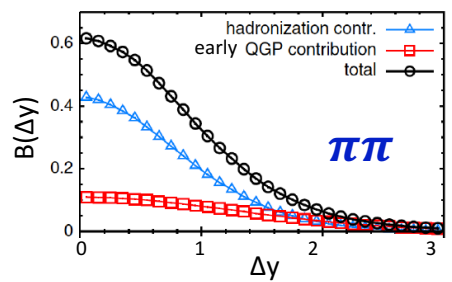
Scott Pratt - CPOD 2013



Generalized BF (between different particle species)
 -> understand balancing between quark species
 -> access to light and strange quark production time



Pratt PRL. 108, 212301 (2012)

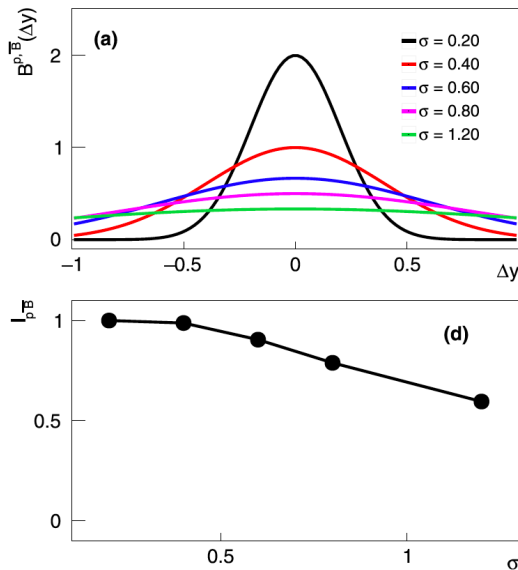
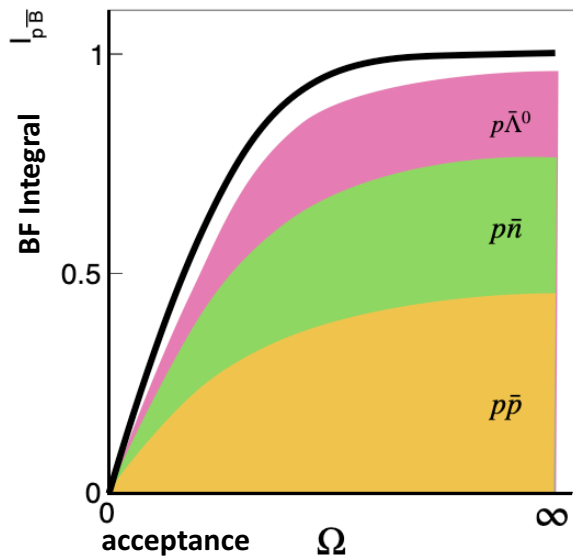


$\pi\pi$: larger 2nd wave up/down quark production -> smaller $\langle \Delta y \rangle$ -> narrow towards central collisions

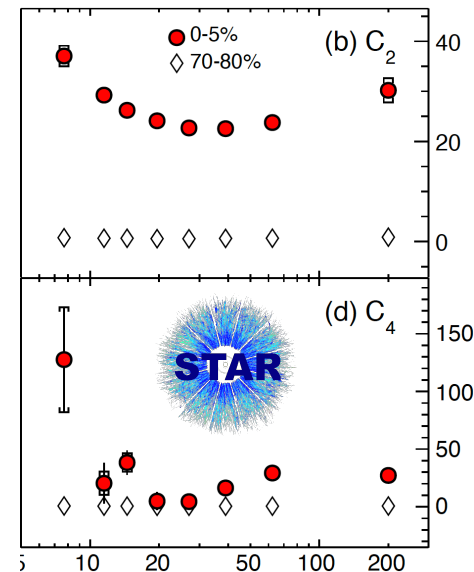
KK : dominant 1st wave strange quark production -> same $\langle \Delta y \rangle$ -> no centrality dependence

Baryon number BF & Net-baryon fluctuation

Pruneau, PRC 100, 034905 (2019)



STAR, arXiv:2001.02852
submitted to Nature Physics



BF Integral

- hadron species pairing probability (never measured before)
- interplay of pair production process, acceptance, and BF width

$$1 - \frac{C_2(\Delta N_p)}{C_2^{Skellam}(\Delta N_p)} = I_{BF}(\Omega)$$

high energy A-A collisions
in the limit $\langle N_p \rangle = \langle N_{\bar{p}} \rangle$

Differential baryon number BF in BES -> critical point search

Two-particle Number Correlation Function

2-Particle Cumulant

$$C_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) = \rho_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) - \rho_1^\alpha(\vec{p}^\alpha) \cdot \rho_1^\beta(\vec{p}^\beta)$$

$\alpha, \beta - \mathbf{h}, \pi, \mathbf{K}, \mathbf{p}, \Lambda$ and $\Xi \dots$

α – reference particle

β – associate particle

ρ_1, ρ_2 – single particle & pair number density per event

Measurement:
M – Measured

$$C_{2,M}^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) \approx \boxed{\varepsilon_1^\alpha(\vec{p}^\alpha) \cdot \varepsilon_1^\beta(\vec{p}^\beta)} \cdot C_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)$$

assuming efficiency factorizes

$$\varepsilon_2^\alpha(\vec{p}^\alpha, \vec{p}^\beta) \approx \varepsilon_1^\alpha(\vec{p}^\alpha) \cdot \varepsilon_1^\beta(\vec{p}^\beta)$$

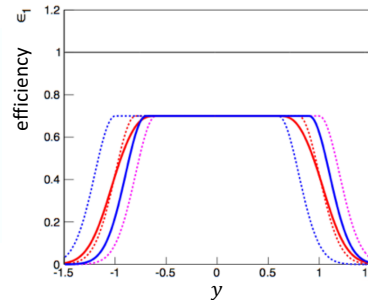
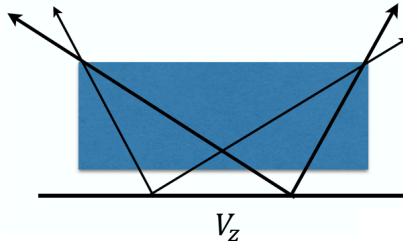
Require separate efficiency estimation for single particles

Normalized 2-Particle Cumulant

$$R_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) = \frac{C_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^\alpha(\vec{p}^\alpha) \cdot \rho_1^\beta(\vec{p}^\beta)} = \frac{\rho_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^\alpha(\vec{p}^\alpha) \cdot \rho_1^\beta(\vec{p}^\beta)} - 1$$

Efficiencies cancel in the ratio -> robust observable

Acceptance correction



Ravan et al., PRC 89, 024906 (2014)

$$\text{weight: } w_{\pm}(y, \varphi, V_z, p_T) = \frac{\rho_{avg}^{\pm}(p_T)}{\rho^{\pm}(y, \varphi, V_z, p_T)}$$

small V_z bins

Balance Function (BF) Definition

Associated particle distribution
(per trigger yield)

$$A^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) = \frac{\rho_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^\alpha(\vec{p}^\alpha)} - \rho_1^\beta(\vec{p}^\beta) = \rho_1^\beta(\vec{p}^\beta) \cdot R_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)$$

BF of positive reference particle α^+

$$B^{\alpha^+\beta}(\vec{p}^\alpha, \vec{p}^\beta) = A^{\alpha^+\beta^-} - A^{\alpha^+\beta^+} = \frac{\rho_2^{\alpha^+\beta^-}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^+}(\vec{p}^\alpha)} - \rho_1^{\beta^-}(\vec{p}^\beta) - \frac{\rho_2^{\alpha^+\beta^+}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^+}(\vec{p}^\alpha)} + \rho_1^{\beta^+}(\vec{p}^\beta)$$

BF of negative reference particle α^-

$$B^{\alpha^-\beta}(\vec{p}^\alpha, \vec{p}^\beta) = A^{\alpha^-\beta^+} - A^{\alpha^-\beta^-} = \frac{\rho_2^{\alpha^-\beta^+}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^-}(\vec{p}^\alpha)} - \rho_1^{\beta^+}(\vec{p}^\beta) - \frac{\rho_2^{\alpha^-\beta^-}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^-}(\vec{p}^\alpha)} + \rho_1^{\beta^-}(\vec{p}^\beta)$$

Balance function

$$B^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) = \frac{1}{2} [B^{\alpha^+\beta}(\vec{p}^\alpha, \vec{p}^\beta) + B^{\alpha^-\beta}(\vec{p}^\alpha, \vec{p}^\beta)]$$

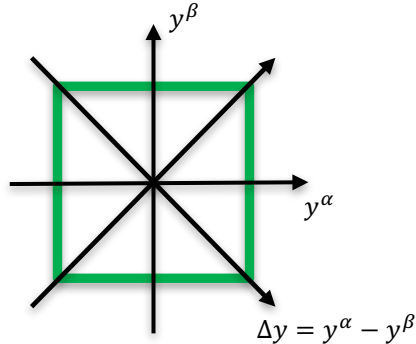
$$= \frac{1}{2} [\rho_1^{\beta^-}(\vec{p}^\beta) \cdot R_2^{\alpha^+\beta^-}(\vec{p}^\alpha, \vec{p}^\beta) - \rho_1^{\beta^+}(\vec{p}^\beta) \cdot R_2^{\alpha^+\beta^+}(\vec{p}^\alpha, \vec{p}^\beta) + \rho_1^{\beta^+}(\vec{p}^\beta) \cdot R_2^{\alpha^-\beta^+}(\vec{p}^\alpha, \vec{p}^\beta) - \rho_1^{\beta^-}(\vec{p}^\beta) \cdot R_2^{\alpha^-\beta^-}(\vec{p}^\alpha, \vec{p}^\beta)]$$

Balance Function Measurement

$$\begin{aligned}
 & B^{\alpha\beta}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) \\
 &= \frac{1}{2} [\rho_1^{\beta-}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha+\beta-}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) - \rho_1^{\beta+}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha+\beta+}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) \\
 &+ \rho_1^{\beta+}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha-\beta+}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) - \rho_1^{\beta-}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha-\beta-}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta)]
 \end{aligned}$$

- measure $R_2^{\alpha\beta}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta)$ for interested p_T range
- $\rho_1^{\beta\pm}(y^\beta, \varphi^\beta)$ (assuming constant for mid-rapidity) calculated from previous p_T spectra measurements

$$\begin{aligned}
 & 0.2 \leq p_T^{\pi, K} \leq 2.0 \text{ GeV}/c \\
 & 0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c
 \end{aligned}$$

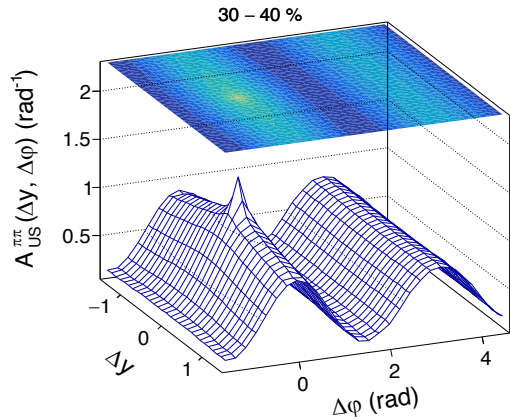


$$B^{\alpha\beta}(\Delta y, \Delta\varphi) = \int B^{\alpha\beta}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) dy^\beta d\varphi^\beta$$

average over y^β

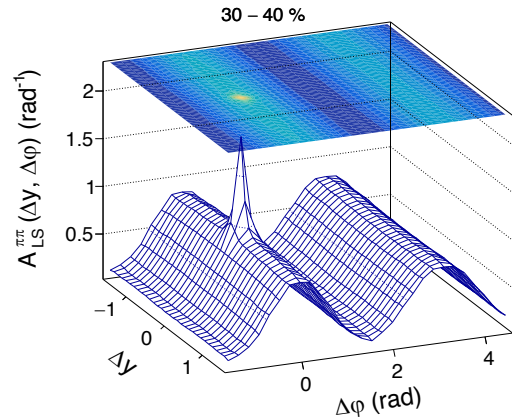
BF measures charge-dependent (CD) correlations

$$A_{US}^{\alpha\beta} = \frac{1}{2} (A^{\alpha^+\beta^-} + A^{\alpha^-\beta^+})$$



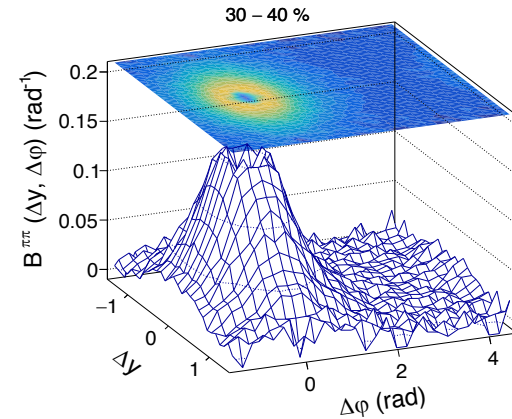
Remove charge independent effects

$$A_{LS}^{\alpha\beta} = \frac{1}{2} (A^{\alpha^+\beta^+} + A^{\alpha^-\beta^-})$$



Keep effects related to balancing pairs

$$B^{\alpha\beta}(\Delta\psi, \Delta\phi) = A_{CD}^{\alpha\beta} = A_{US}^{\alpha\beta} - A_{LS}^{\alpha\beta}$$



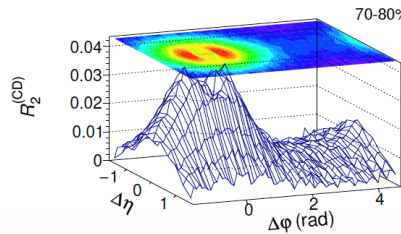
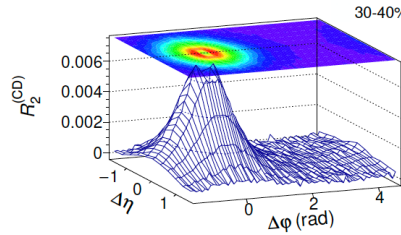
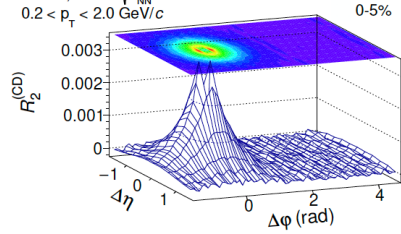


ALICE



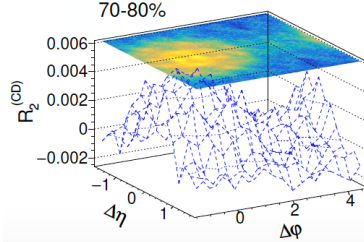
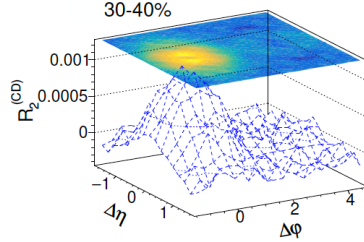
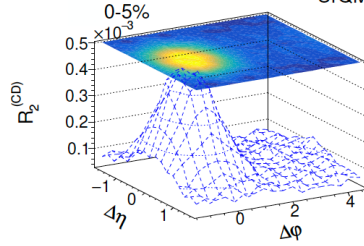
$R_2^{(CD)}$ vs. models — unidentified hadrons

ALICE, Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV
 $0.2 < p_T < 2.0$ GeV/c

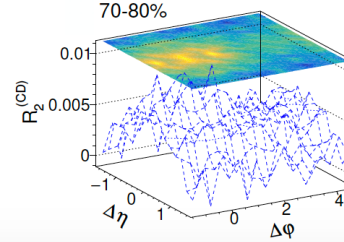
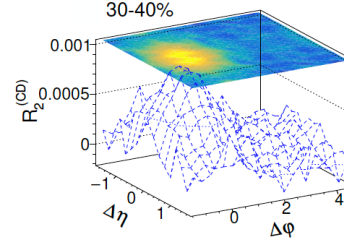
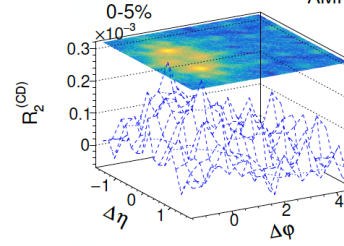


PRC 100, 044903 (2019)

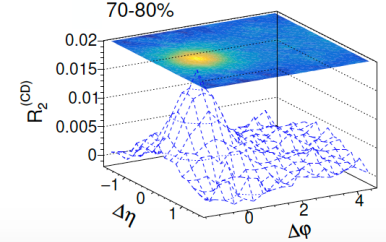
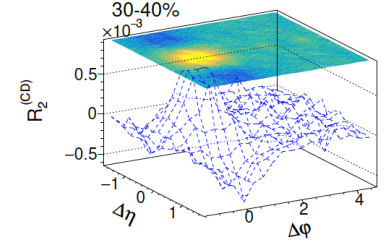
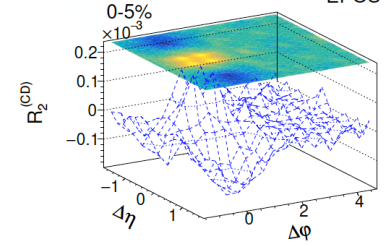
UrQMD



AMPT



EPOS



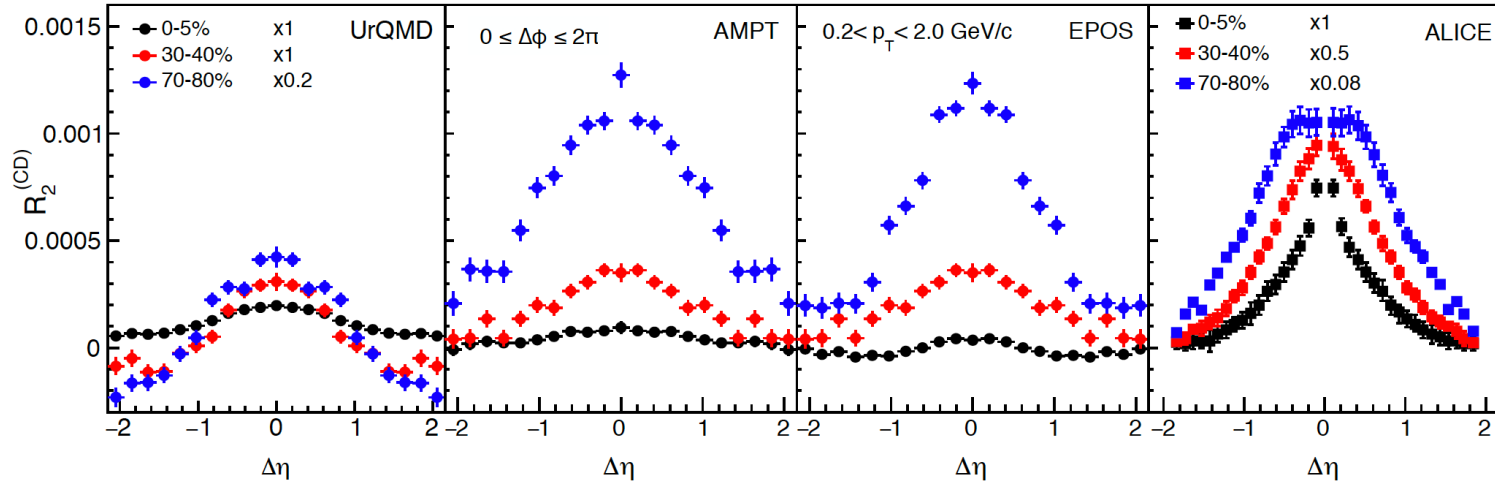
Basu, Gonzalez, JP, et al. arXiv:2001.07167, Submitted to PRC

- models qualitatively reproduce near-side peak, but **Not** its amplitude and collision centrality evolution.
- broad dip at $(\Delta y, \Delta \phi) = (0, 0)$ in data due to HBT **Not** reproduced by models -> no HBT afterburner
- models qualitatively reproduce away-side tail in peripheral and its suppression in central collisions -> resonance decays, e.g. ρ^0

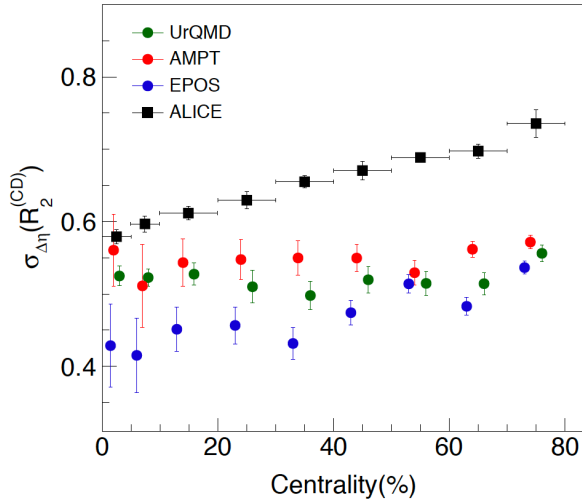


$R_2^{(CD)}$ vs. models — unidentified hadrons

ALICE, PRC 100, 044903 (2019)



Basu, Gonzalez, JP, et al. arXiv:2001.07167, Submitted to PRC



- models **Not** reproduce magnitude and centrality evolution of longitudinal rms
- EPOS: reproduces a narrowing but widths **too narrow** by $\sim 30\%$
 -> corona particle dominance since **No** event-by-event charge conservation in core
 -> average radial flow imparted to corona \gg core
- UrQMD: weak amplitude of near-side peak -> insufficient high-mass resonances
- AMPT: weak amplitude of near-side peak -> incomplete handling of charge conservation



ALICE

BF of full species matrix $(\pi^\pm, K^\pm, p/\bar{p}) \times (\pi^\pm, K^\pm, p/\bar{p})$

\pm — General conserved charges:

- **e**: electric charge
- **S**: strangeness
- **B**: baryon number

✓: previous works

✓: JP PhD dissertation 2019 (ALICE)

		$B^{\alpha\beta}(\Delta y, \Delta\varphi)$	h^\pm	π^\pm	K^\pm	p/\bar{p}
e		h^\pm	✓			
e		π^\pm		✓✓	✓	✓
e	S	K^\pm		✓	✓✓	✓
e	B	p/\bar{p}		✓	✓	✓

1st BF measurement of full species matrix of $(\pi^\pm, K^\pm, p/\bar{p}) \times (\pi^\pm, K^\pm, p/\bar{p})$.



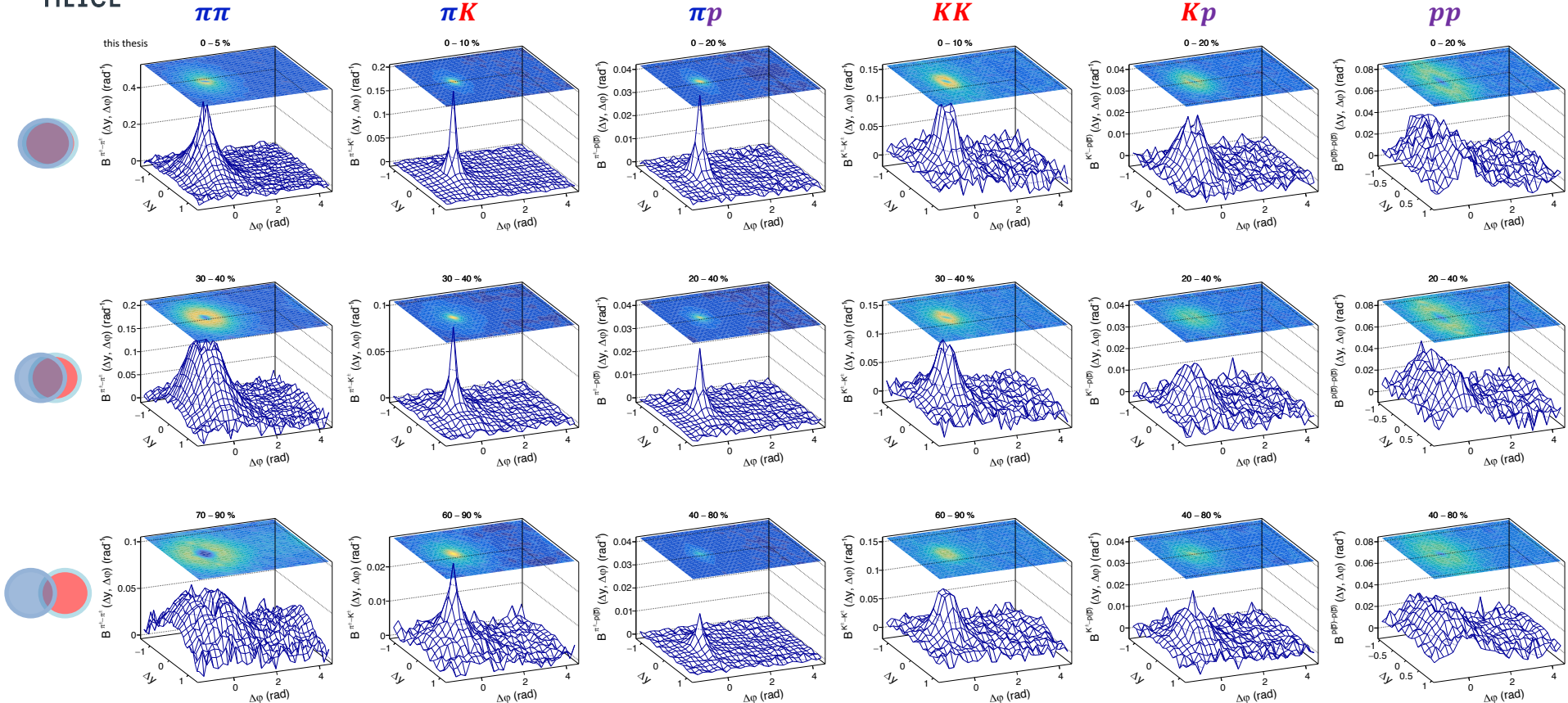
ALICE

PID BF – Full Species Matrix

$$0.2 \leq p_T^{\pi,K} \leq 2.0 \text{ GeV}/c$$

$$0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c$$

JP PhD dissertation, arXiv:1911.02234
 JP (for ALICE), NPA 982 (2019) 315–318
 JP et al. (ALICE), PRL + PRC In Preparation





ALICE

1D BF Δy Projections

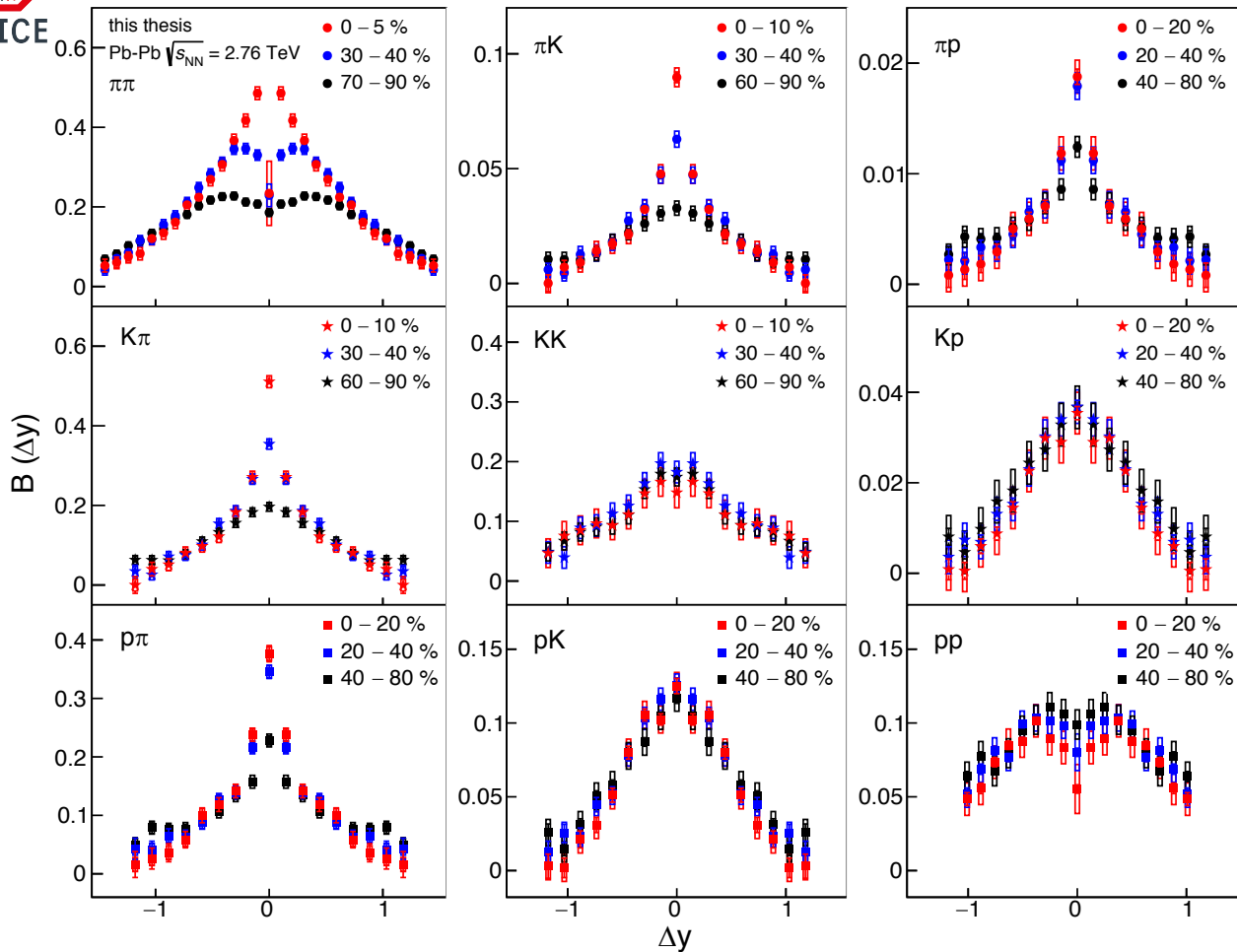
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JP (for ALICE), NPA 982 (2019) 315–318

JP et al. (ALICE), PRL + PRC In Preparation



$$|\Delta\phi| < \pi$$

Note different scale $\pi\pi$: clear centrality dependence KK : no centrality dependence

-> consistent with radial flow and two wave quark production

BF including π :

Clear centrality dependence

BF including K, p :

no / little centrality dependence

-> different production mechanisms for π, K, p



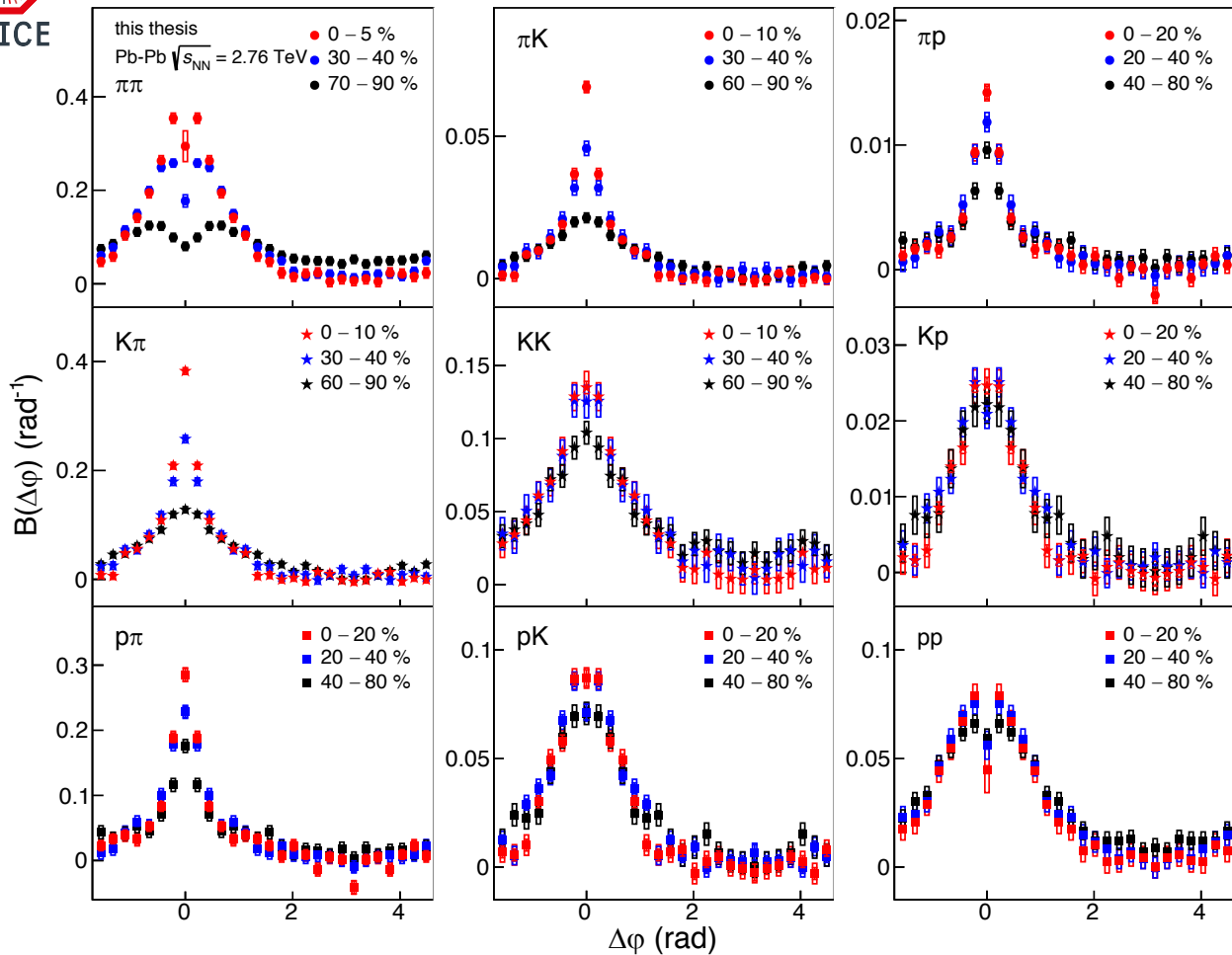
ALICE

1D BF $\Delta\varphi$ Projections

$$0.2 \leq p_T^{\pi,K} \leq 2.0 \text{ GeV}/c$$

$$0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c$$

JP PhD dissertation, arXiv:1911.02234
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 JP et al. (ALICE), PRL + PRC In Preparation



$|\Delta\gamma| < 1.4 \pi\pi$
 $|\Delta\gamma| < 1.0 pp$
 $|\Delta\gamma| < 1.2$ other pairs

Note different scale

BF including π :
 Clear centrality dependence

BF including K, p :
 no / little centrality dependence

-> different production mechanisms
 for π, K, p



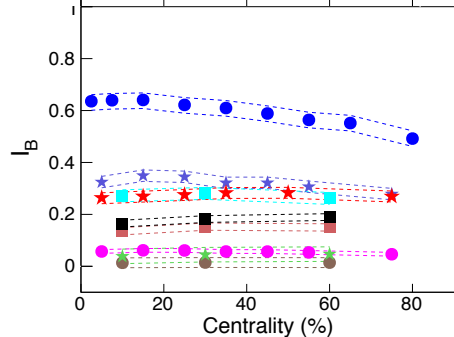
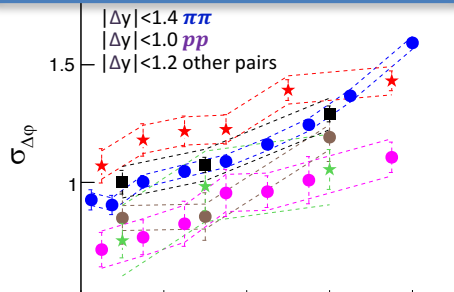
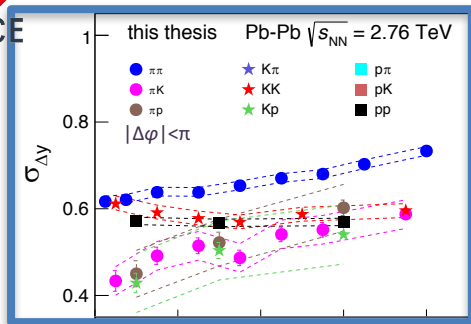
ALICE

BF RMS Widths and Integrals

$$0.2 \leq p_T^{\pi,K} \leq 2.0 \text{ GeV}/c$$

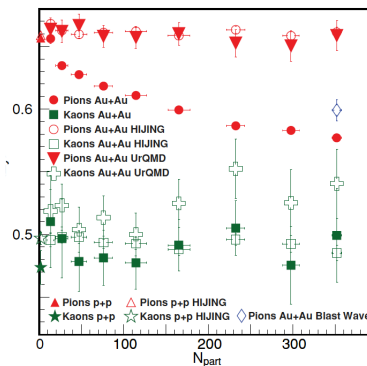
$$0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c$$

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 JP et al. (ALICE), PRL + PRC In Preparation



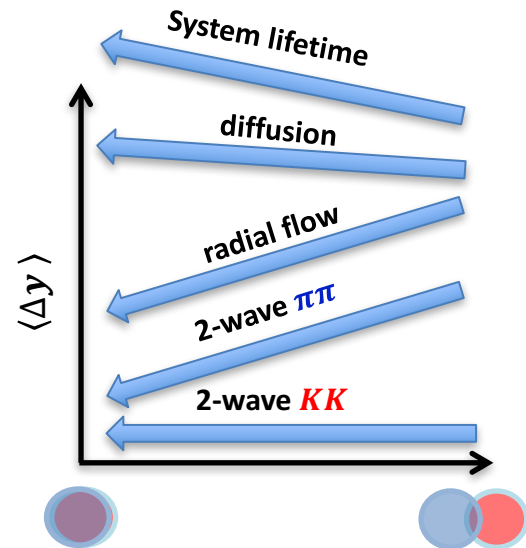
B(Δy) RMS Widths:

- KK & pp no centrality dependence; $\pi\pi$ & cross-species pairs narrow towards central collisions
- Similar values for all cross-species pairs.
- Qualitatively consistent with radial flow and two-wave quark production
 -> detailed modeling required to distinguish them.



STAR PRC 82, 024905 (2010)

Au-Au @ 200 GeV
 $0.2 < p_T < 0.6 \text{ GeV}/c$





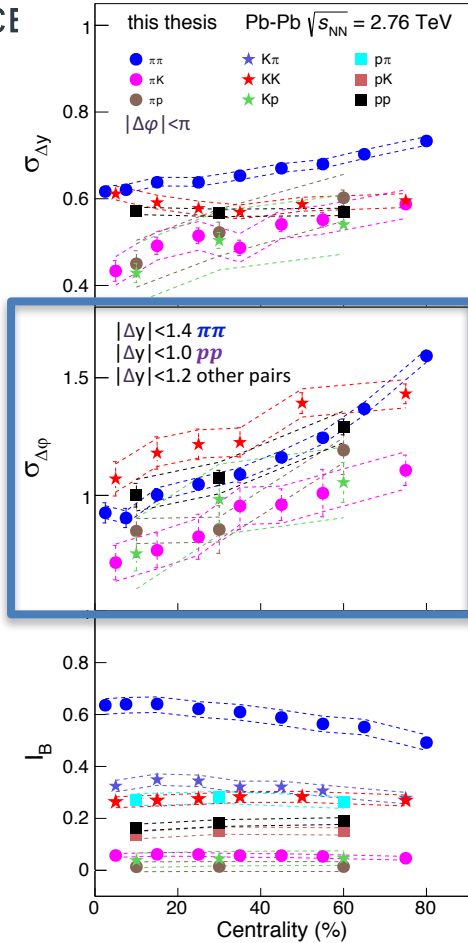
ALICE

BF RMS Widths and Integrals

$$0.2 \leq p_T^{\pi,K} \leq 2.0 \text{ GeV}/c$$

$$0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c$$

JP PhD dissertation, arXiv:1911.02234
 JP (for ALICE), NPA 982 (2019) 315–318
 JP et al. (ALICE), PRL + PRC In Preparation



$B(\Delta\phi)$ RMS Widths:

- Different values for different species pairs
 -> radial flow affects pairs of different mass differently.
- Widths for pp is same with $\pi\pi$ due to different Δy range. Other effects?
- All species pairs narrow towards central collision -> qualitatively radial flow > diffusion.
- More detailed information on radial flow profile in context of hadron species pairs.



ALICE

BF RMS Widths and Integrals

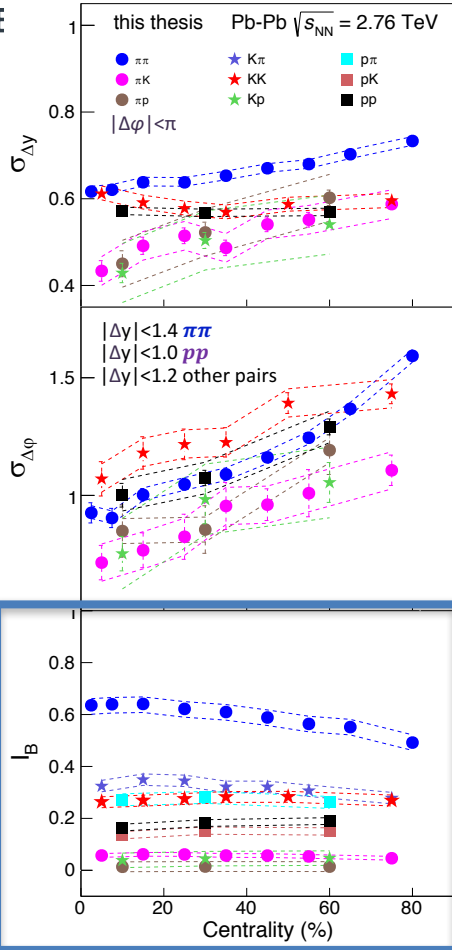
$$0.2 \leq p_T^{\pi,K} \leq 2.0 \text{ GeV}/c$$

$$0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c$$

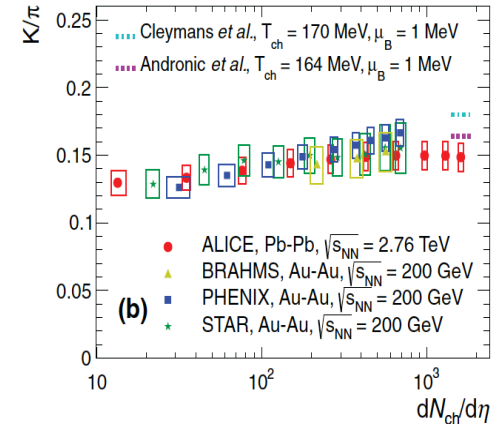
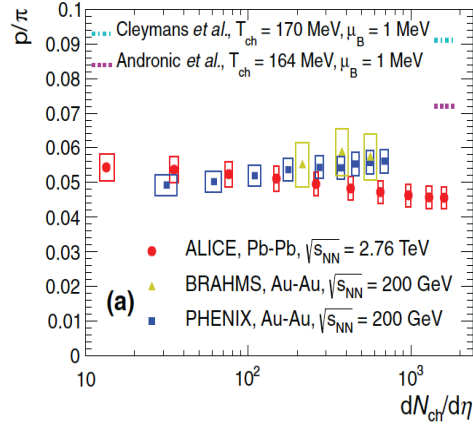
JP PhD dissertation, arXiv:1911.02234

JP (for ALICE), NPA 982 (2019) 315–318

JP et al. (ALICE), PRL + PRC In Preparation



ALICE, PRC 88, 044910 (2013)



Balance Function Integrals

- 1st measurement of hadron species pairing probability (within acceptance).
- Sum of integrals of π triggered, K triggered, p reference BFs ~ 0.65 .
- Minimal centrality dependence for most pairs, but $\pi\pi$ increasing towards central collisions \rightarrow $B(\pi\pi)$ losses beyond acceptance more for peripheral than central collisions.
- Hadron species pairing probabilities very different from single hadron ratios. e.g. $K\pi$ not larger than KK by π/K ratio; pp larger than pK . \rightarrow better constraint for models.



Balance Function experimental outlook



ALICE

π^+	π^-
$u \bar{d}$	$\bar{u} d$
K^+	K^-
$u \bar{s}$	$\bar{u} s$
p	\bar{p}
$u u$	$\bar{u} \bar{u}$
d	\bar{d}
Λ^0	$\bar{\Lambda}^0$
u	\bar{u}
$s d$	$\bar{s} \bar{d}$
Ξ^-	$\bar{\Xi}^+$
d	\bar{d}
$s s$	$\bar{s} \bar{s}$

\pm — General conserved charges:

- **e**: electric charge
- **S**: strangeness
- **B**: baryon number

✓: previous works

✓: JP PhD dissertation 2019 (ALICE)

★: JP et al. (STAR) BES work in progress

$B^{\alpha\beta}(\Delta y, \Delta\varphi)$		h^\pm	π^\pm	K^\pm	p/\bar{p}	$\Lambda^0/\bar{\Lambda}^0$	$\Xi^-/\bar{\Xi}^+$
e		✓					
e			✓✓★	✓★	✓★		
e	S		✓★	✓✓★	✓★	★	★
e	B		✓★	✓★	✓★	★	★
S	B			★	★	★	★
e	S B			★	★	★	★



- differential $B^{\alpha\beta}(p_T^\alpha, p_T^\beta)$
- $B^{\alpha\beta}$ w.r.t event plane
- $B^{\alpha\beta}$ in jets

Summary & Conclusions

➤ Generalized BF (between different particle species) key observable

- > understand balancing between quarks
- > have access to the timing
- > equivalent to net-baryon C_2 for critical point search

➤ GBF (Pb-Pb @ 2.76 TeV):

- **Three 1st**
 - 1st GBF measurement of full species matrix of $\pi^\pm, K^\pm, p/\bar{p}$.
 - 1st 2D differential measurement of PID BF.
 - 1st measurement of hadron species pairing probability.
- **$B(\Delta y)$ Widths:**
 - qualitatively consistent with radial flow and two-wave quark production
-> a detailed model required to distinguish them.
- **$B(\Delta\varphi)$ Widths:**
 - qualitatively radial flow > diffusion.
 - more info on radial flow profile in context of hadron pairs.
- **BF Integrals:**
 - minimal centrality dependence.
 - hadron pairing probabilities different from single hadron ratios.
-> better constraint for models.

➤ From Model Comparisons

- Models need to properly account for balancing charge production & transport mechanisms.

My contributions in BF research:

JP PhD dissertation, arXiv:1911.02234

JP (for ALICE), Nuclear Physics A 982 (2019) 315–318

JP et al. (ALICE), PRL + PRC In Preparation “BF of π, K, p ”

JP (for ALICE), J. Phys.: Conf. Ser. 832 (2017) 012044

Basu, Gonzalez, JP, et al. arXiv:2001.07167, Submitted to PRC

Gonzalez, Marin, Guevara, JP, et al. PRC 99, 034907 (2019)

JP et al. (STAR), PRL In Preparation “BF of π, K, p, Λ and Ξ ”

**More exciting results in
STAR BES coming soon!**



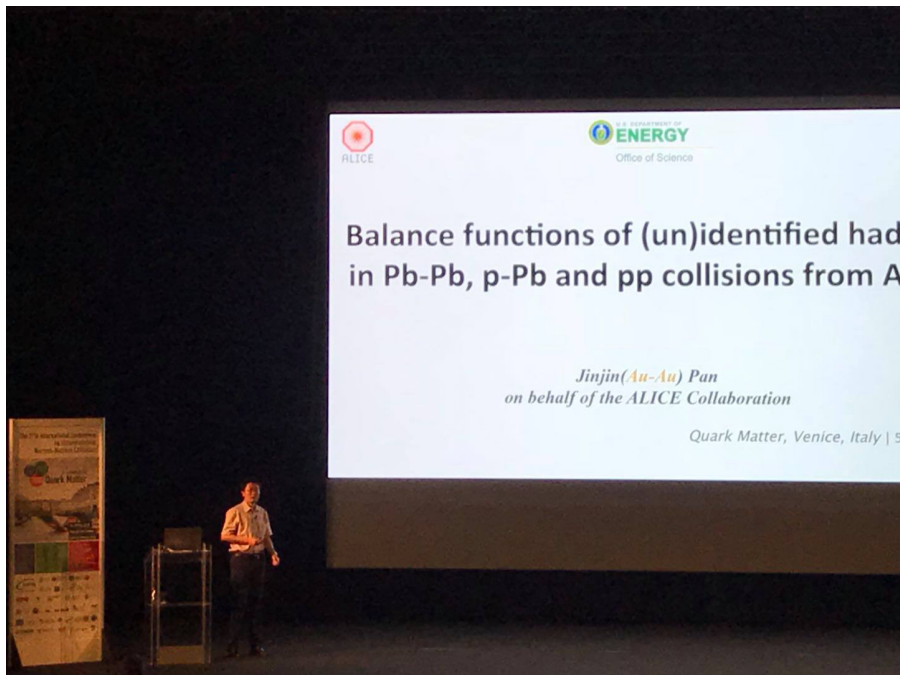


Thank you!



Quark Matter 2018 Talk

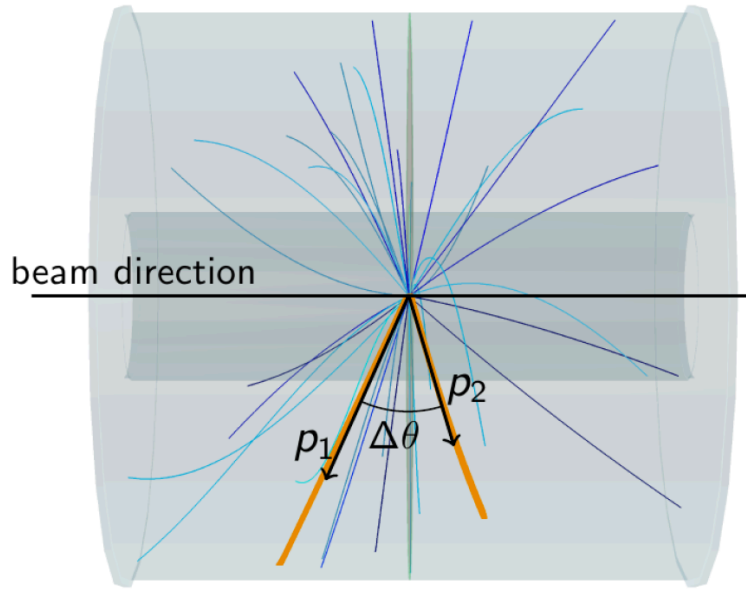
ALICE Shift Leader (Honor) – Pb-Pb runs 2018



Look forward to learning at BNL

Back-up

Momentum Space Variables



Transverse plane

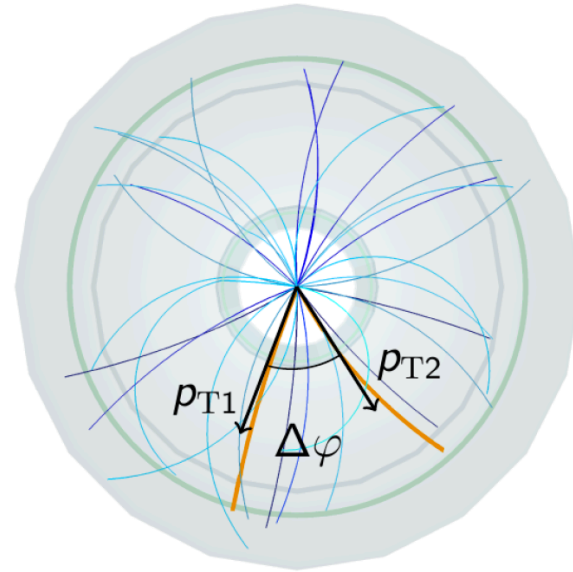


Fig. A. Zaborowska

p — particle momentum

p_T — transverse momentum

φ — azimuthal angle

θ — polar angle

η — pseudorapidity

y — rapidity

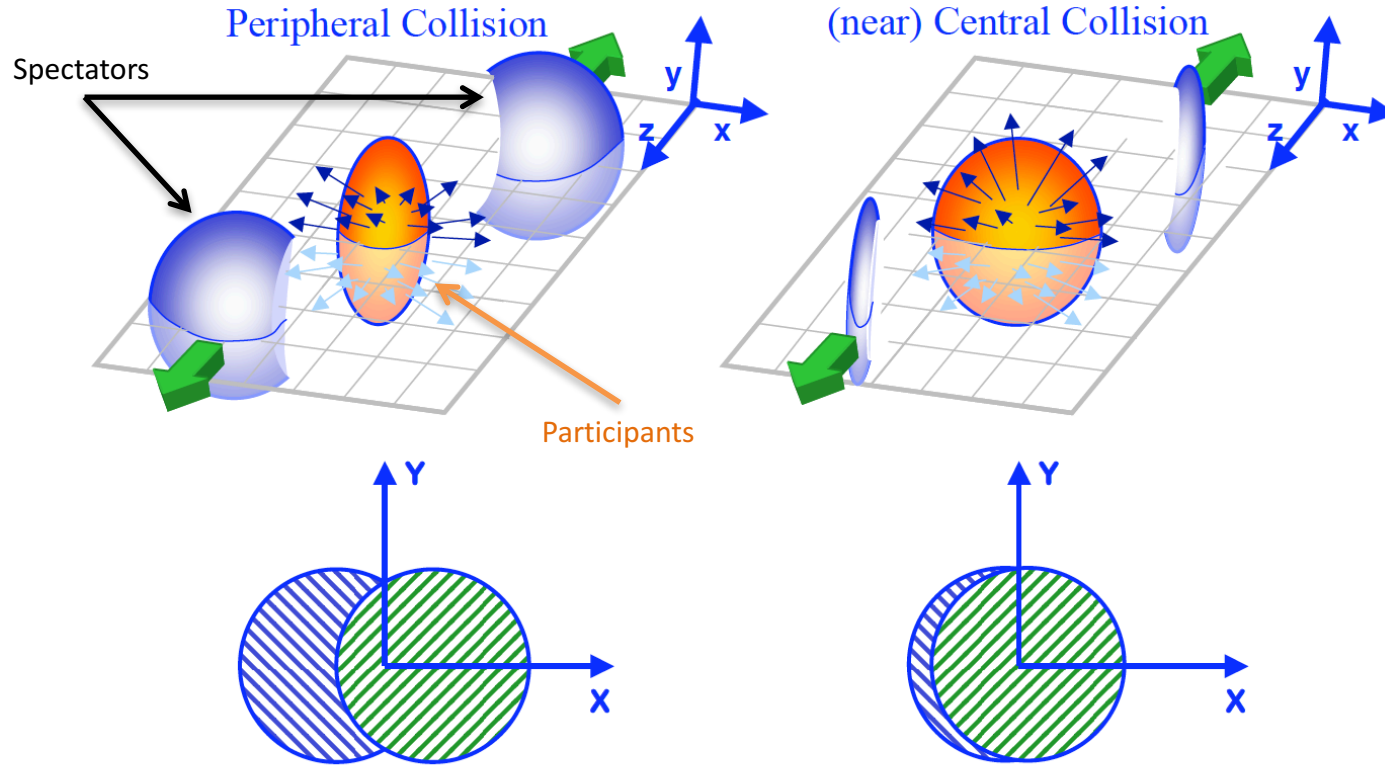
$$p_T^2 = p_x^2 + p_y^2$$

$$\eta = -\ln \left[\tan\left(\frac{\theta}{2}\right) \right] = \frac{1}{2} \ln \frac{|p| + p_z}{|p| - p_z}$$

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

Lorentz invariant

Centrality of Relativistic Heavy Ion Collisions

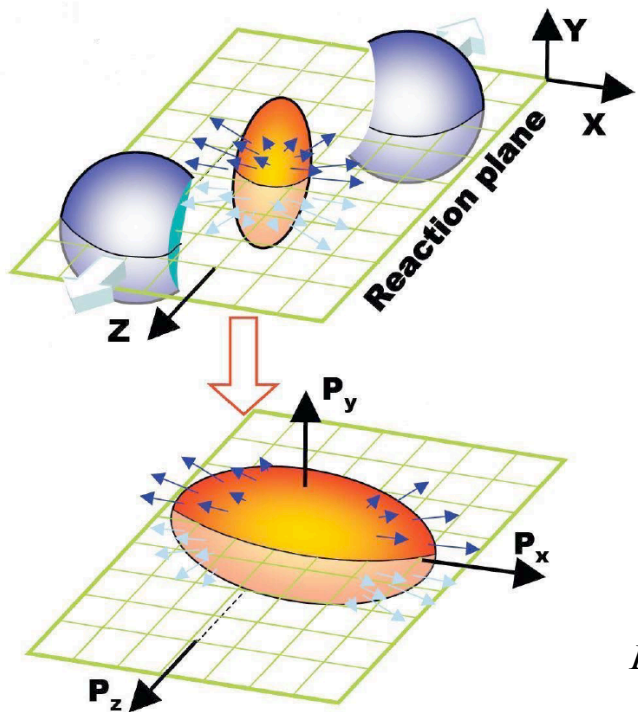


Centrality measured by the multiplicity of charged particles

Definition of Anisotropic Flow

- Flow refers to a collective expansion of matter.
- The system follows an anisotropic expansion.
- Anisotropy in the azimuthal particle distribution are studied in terms of the Fourier decomposition.

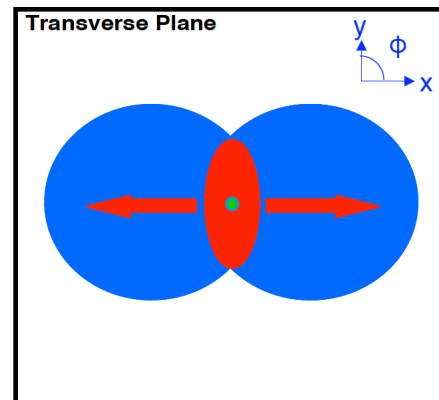
Voloshin and Zhang. Z.Phys.,C70:665-672,1996.



Spatial Anisotropy

density gradient \rightarrow pressure
for anisotropic expansion

Momentum Anisotropy



azimuthal angle around the beam axis

$$E \frac{dN^3}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} (1 + 2v_1 \cos(\phi - \Psi_{RP}) + 2v_2 \cos(2(\phi - \Psi_{RP})) + \dots)$$

$$v_n = \langle \cos[n(\phi - \Psi_{RP})] \rangle$$

directed flow

elliptic flow

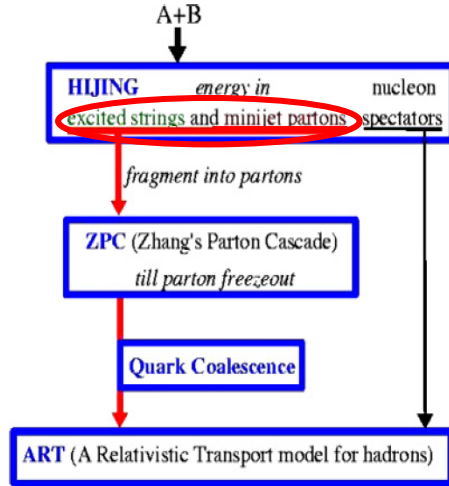
reaction plane

Hiroshi Masui (2008)

Models Used In This Work

AMPT

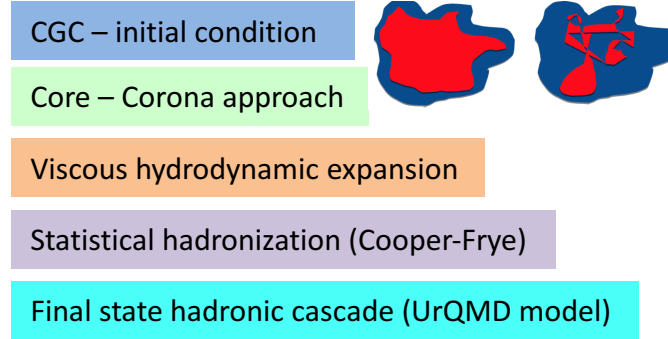
Structure of AMPT model with string melting



HIJING

- QCD Lund jet fragmentation
- Hard parton scatterings dominate
- Emphasis on mini-jets in pp, pA & AA

EPOS 3.0



Event Generation:
A. Knospe, C. Markert

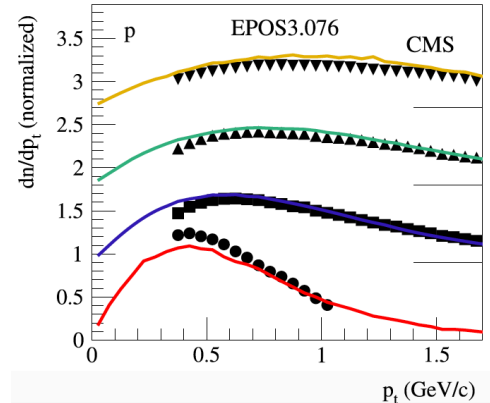
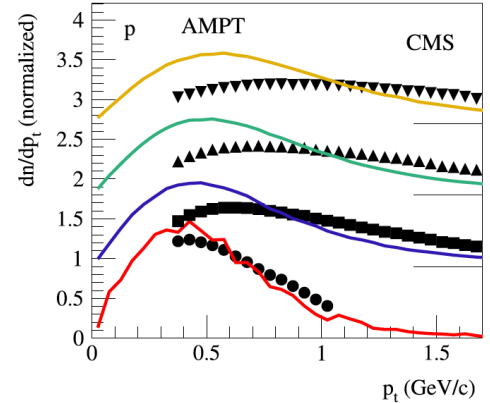


UrQMD

Hadronic relativistic dynamics
Event Generation: W.J. Llope



Werner et al. NPA 931(2014)83–91

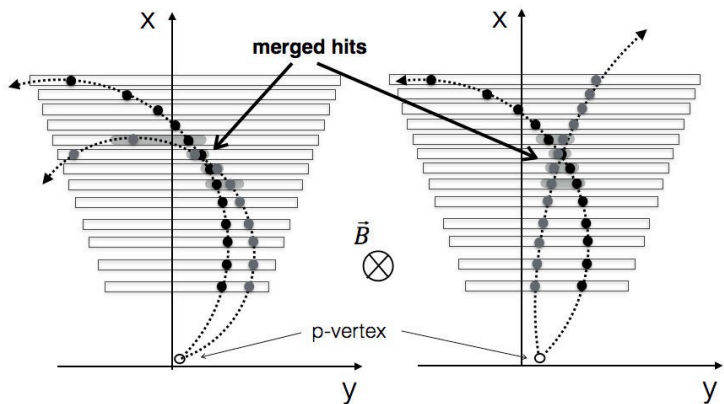


Track Crossing Correction

The solution / correction

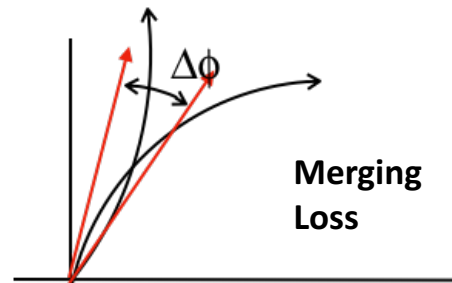
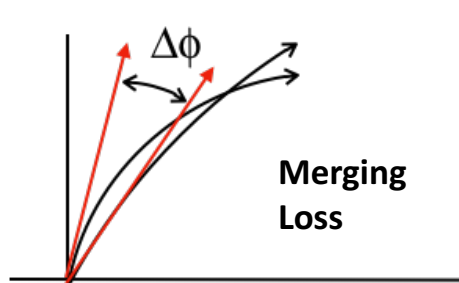
The cause of track crossing

tracks with $\Delta y \sim 0$

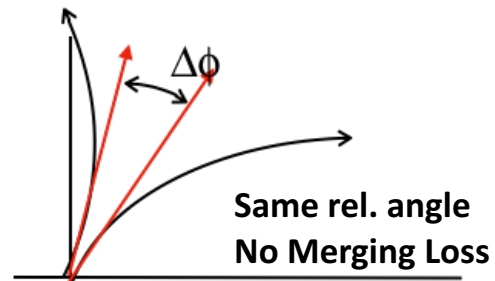
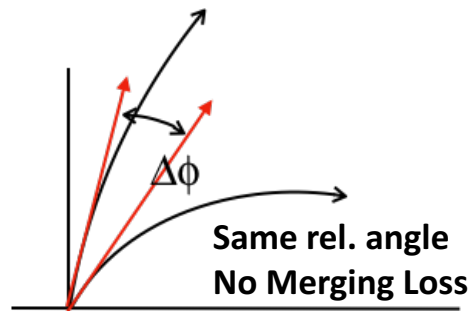


Like-sign: p_T -ordered analysis

Unlike-sign: charge-ordered analysis



At given $\Delta\phi$, count un-merged pairs and use count at $-\Delta\phi$



Correcting The Collision Centrality Bin Width Effects

Gonzalez, Marin, Guevara, JP, Basu, Pruneau, PRC 99, 034907 (2019)

2-Particle Normalized Cumulant (for multiplicity m)

$$R_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m) = \frac{\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)}{\rho_1(\eta_1, \varphi_1 | m)\rho_1(\eta_2, \varphi_2 | m)} - 1$$

Weighted Average

$$\rho_1^{(Bin,k)}(\eta_1, \varphi_1) = \bar{\rho}_1^{(k)}(\eta_1, \varphi_1) = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m)\rho_1(\eta_1, \varphi_1 | m)$$

$$\rho_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \bar{\rho}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m)\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)$$

Uncorrected

$$R_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \frac{\rho_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2)}{\rho_1^{(Bin,k)}(\eta_1, \varphi_1)\rho_1^{(Bin,k)}(\eta_2, \varphi_2)} - 1$$

$$= \frac{\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m)\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)}{\left[\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m)\rho_1(\eta_1, \varphi_1 | m) \right] \left[\frac{1}{Q_k} \sum_{m'=m_{min,k}}^{m_{max,k}} q(m')\rho_1(\eta_2, \varphi_2 | m') \right]} - 1$$

Weighted Average

$$\bar{R}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m)R_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)$$

$$= \left[\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \frac{\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)}{\rho_1(\eta_1, \varphi_1 | m)\rho_1(\eta_2, \varphi_2 | m)} \right] - 1$$

$$\rho_1(\eta, \varphi | m) = \langle n \rangle_m P_1(\eta, \varphi | m)$$

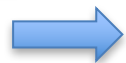
$$\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m) = \langle n(n-1) \rangle_m P_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)$$

P_1, P_2 — Probability Densities



$$R_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \alpha \frac{\bar{P}_2(\eta_1, \varphi_1, \eta_2, \varphi_2)}{\bar{P}_1(\eta_1, \varphi_1)\bar{P}_1(\eta_2, \varphi_2)} - 1$$

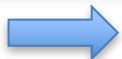
$$\alpha = \frac{\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m)\langle n(n-1) \rangle_m}{\left(\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m)\langle n \rangle_m \right)^2}$$



$$\bar{R}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \beta \frac{\bar{P}_2(\eta_1, \varphi_1, \eta_2, \varphi_2)}{\bar{P}_1(\eta_1, \varphi_1)\bar{P}_1(\eta_2, \varphi_2)} - 1$$

$$\beta = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \frac{\langle n(n-1) \rangle_m}{\langle n \rangle_m^2}$$

Correction

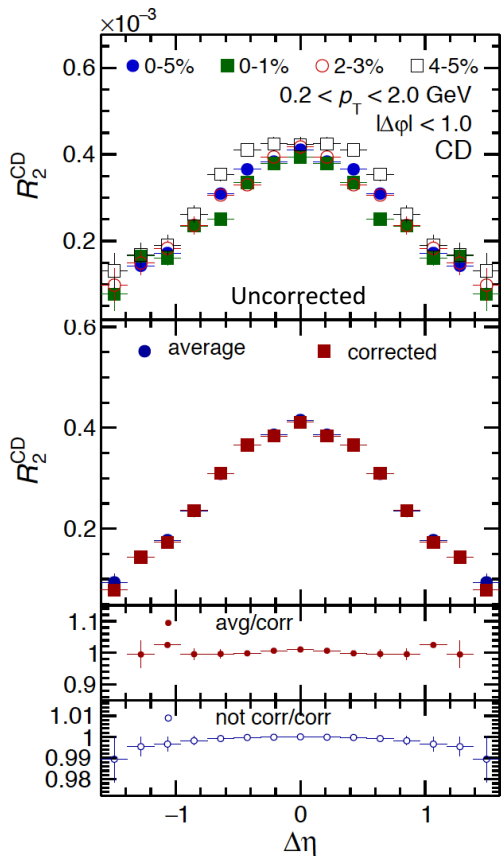
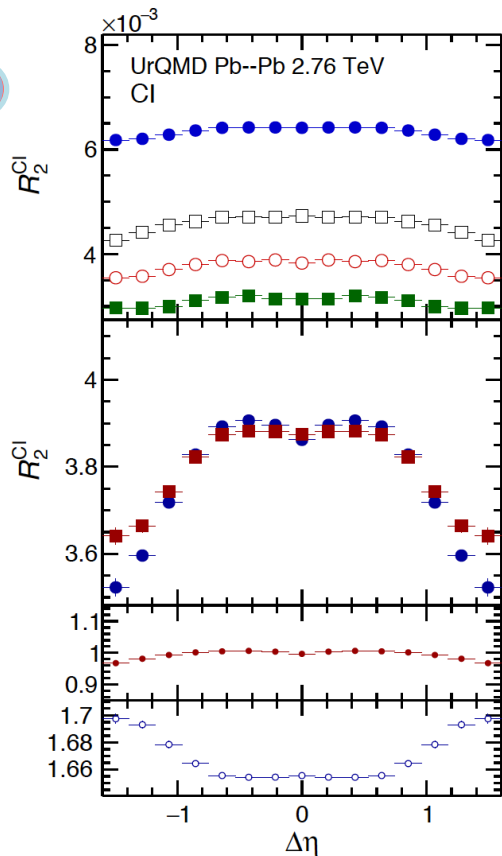


$$\bar{R}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \beta\alpha^{-1}(R_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) + 1) - 1$$

Correcting The Collision Centrality Bin Width Effects

Gonzalez, Marin, Guevara, JP, Basu, Pruneau, PRC 99, 034907 (2019)

UrQMD (100k events) — Unidentified Hadrons



$$R_2^{CI} = \frac{1}{4} [R_2^{+-} + R_2^{++} + R_2^{-+} + R_2^{--}]$$

$$R_2^{CD} = \frac{1}{4} [R_2^{+-} - R_2^{++} + R_2^{-+} - R_2^{--}]$$

- Results corrected agree with those obtained with the weighted mean within 1% for both R_2^{CI} and R_2^{CD} .
- The correction enables reasonably accurate corrections of the R_2 correlators in the context of HIJING and UrQMD models.
- Given these models provide relatively realistic representations of single and pair particle spectra, the correction method should provide reasonably reliable bin-width corrections of R_2 correlation functions measured at any heavy ion collider.