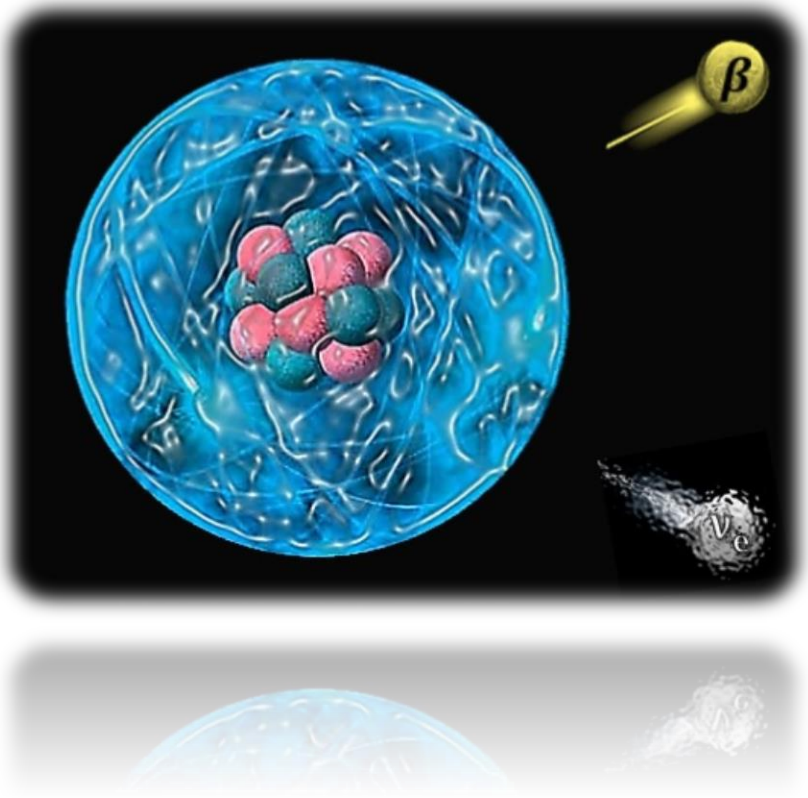


BetaShape and related studies

Nuclear Data Week 2020 – USNDP | Xavier Mougeot

list ceatech · LNHB Laboratoire National Henri Becquerel · Laboratoire National de Métrologie et d'Essais LNE · Institut Carnot CEA LIST · université Paris-Saclay

Outline

- Summary of the BetaShape formalism and status of the code
- Recent work about forbidden non-unique transitions with realistic nuclear structure
- Recent experimental work using silicon detectors
- Perspectives

BetaShape

Beta decay modelling

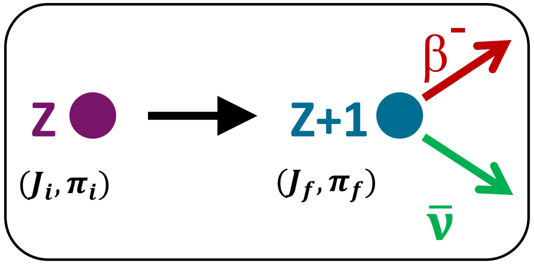
Beta spectrum

$$\frac{\mathrm{d}N}{\mathrm{d}W} \propto \underbrace{p\,W\,q^2}_{\text{Phase space}}\;\underbrace{F_0 L_0}_{\text{Coulomb part (Fermi function)}}\;\underbrace{C(W)}_{\text{Shape factor}}$$

Coulomb part (Fermi function)

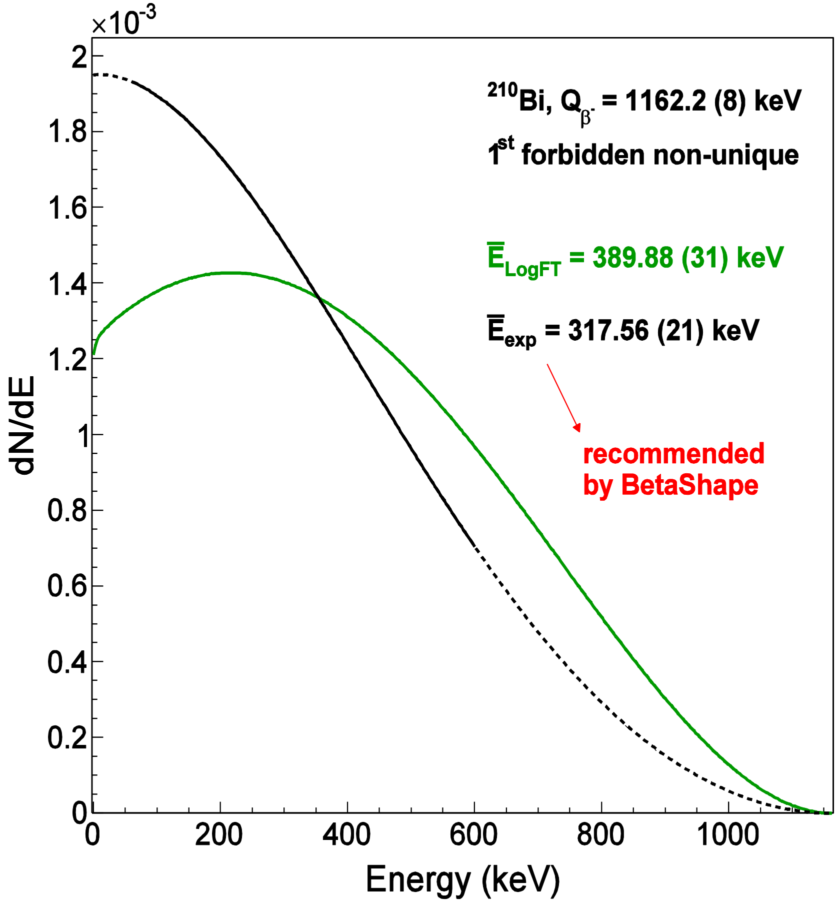
Phase space

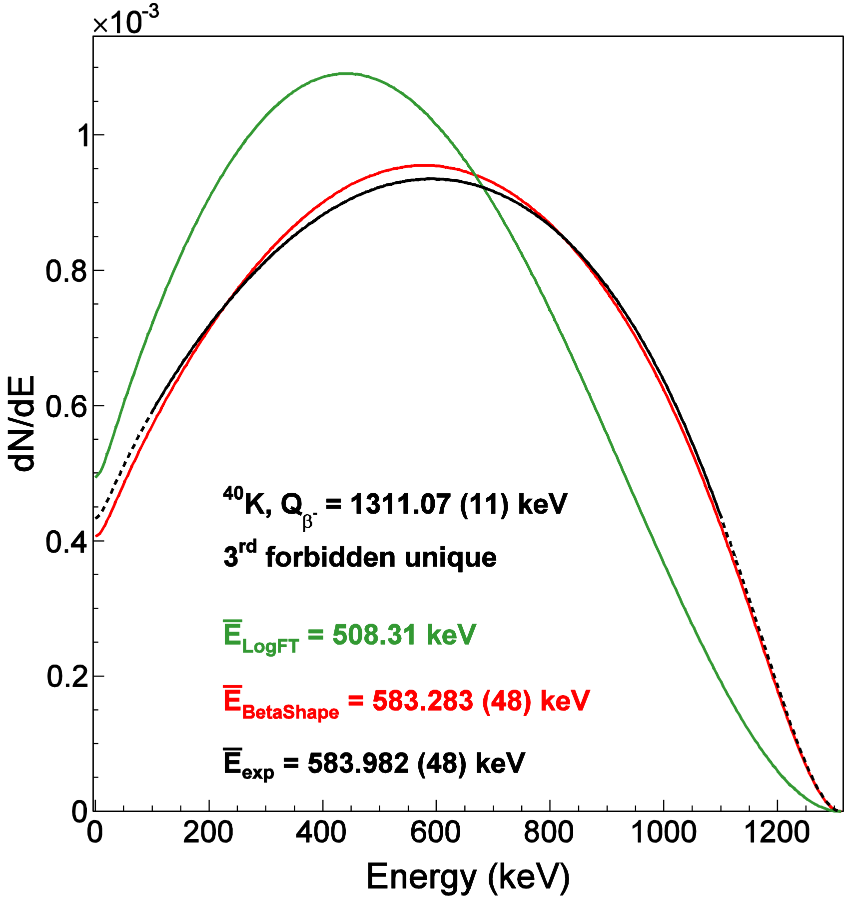
Shape factor



- Behrens and Bühring formalism
- Dirac equation solved numerically for beta particles (with spherical nucleus)
- Allowed and forbidden unique transitions
- Forbidden non-unique transitions with ξ approximation
- Bühring screening correction with Salvat's potentials
- Precise radiative corrections from Hardy's study of superallowed transitions
- Database of experimental shape factors (131 transitions)
- Propagation of uncertainties on Q-values and level energies
- Reads and writes to/from ENSDF files
- Provides beta and neutrino spectra for each transition, total spectrum for a radionuclide, mean energies, log *ft* values and report files

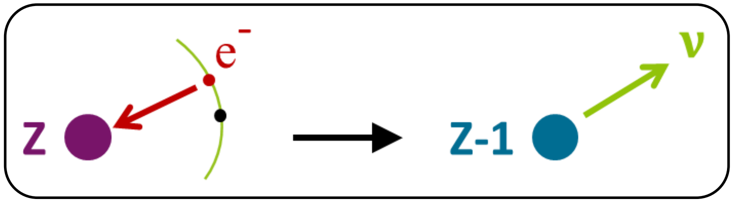
Examples of improved calculations





These two transitions are calculated as allowed by the LogFT program, which does not provide any beta spectrum.

Electron capture modelling



Total capture probability

$$\lambda_\varepsilon \propto \sum_{\kappa_x} n_{\kappa_x} C_{\kappa_x} q_{\kappa_x}^2 \beta_{\kappa_x}^2 B_{\kappa_x} \left(1 + \sum_{m,\kappa} P_{m\kappa}\right)$$

- κ_x: shell quantum number
- n_{κ_x}: relative occupation number
- C_{κ_x}: "shape" factor similar to $C(W)$ in β decay
- $q_{\kappa_x}^2$: ν momentum
- $\beta_{\kappa_x}^2$: amplitude of wave function
- B_{κ_x}: overlap and exchange corrections
- $\left(1 + \sum_{m,\kappa} P_{m\kappa}\right)$: shaking effects

+ hole effect (vacancy)
+ radiative corrections

Allowed and forbidden unique
→ no nuclear structure

If transition energy $\geq 2m_e$
→ **competition with a β^+ transition**

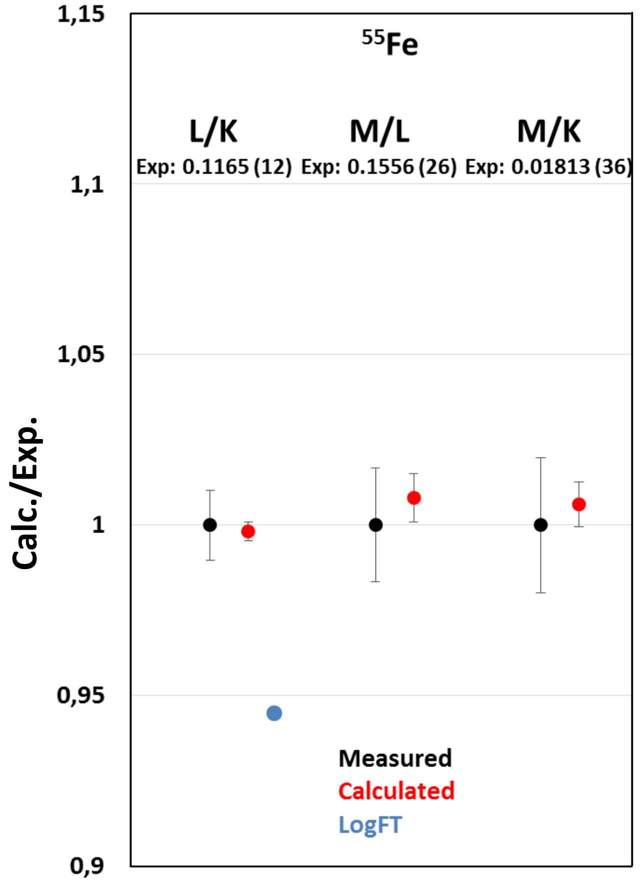
X. Mougeot, Appl. Radiat. Isot. 154, 108884 (2019)

Atomic wave functions

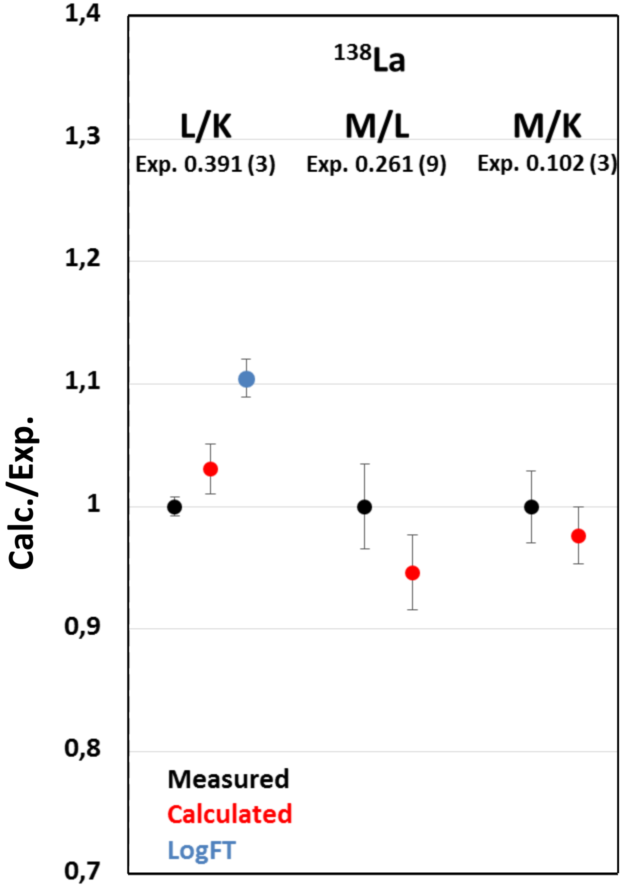
- Numerical solving of Dirac equation
- Forced convergence to relativistic DFT energies, with electron correlations

S. Kotochigova *et al.*, Phys. Rev. A 55, 191 (1997)

Comparison with measurements



Allowed



Second forbidden unique

J. Pengra *et al.*, Phys. Rev. C 5, 2007 (1972)
M. Loidl et al., Appl. Radiat. Isot. 134, 395 (2018)

F. Quarati *et al.*, Appl. Radiat. Isot. 109, 172 (2016)

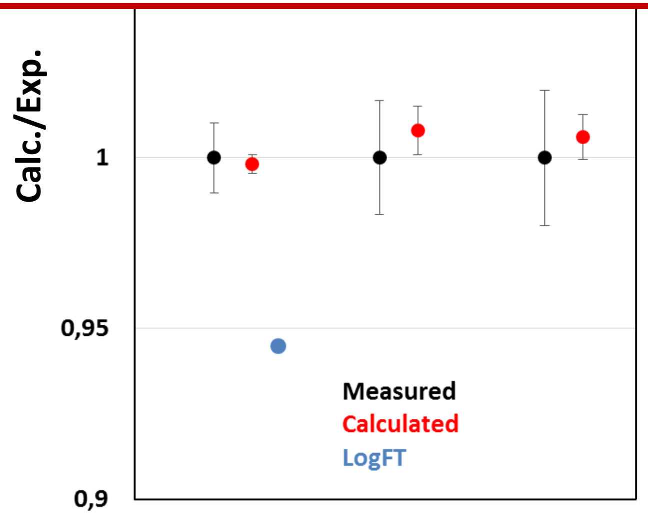
Comparison with measurements

New high-precision measurement of ^{138}La Q-values

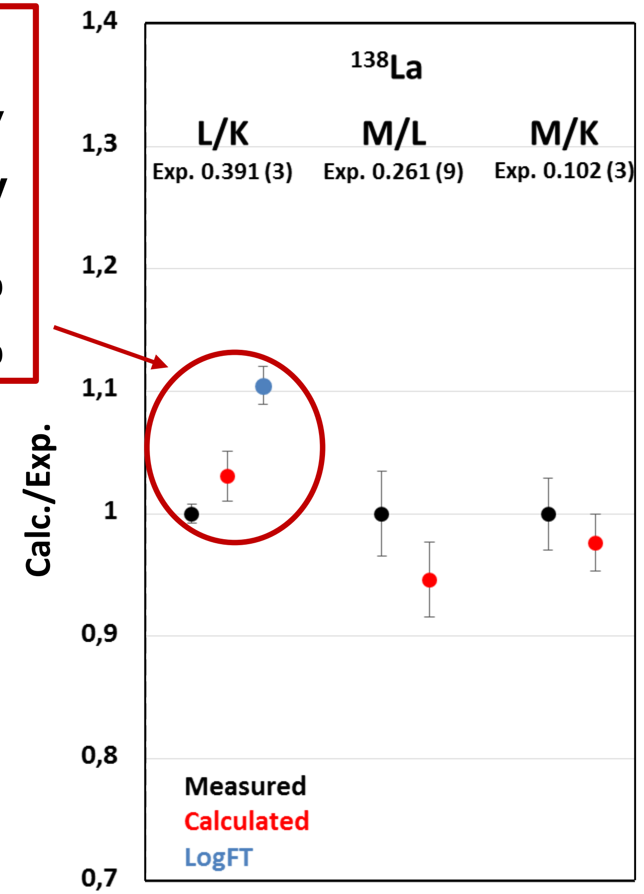
- AME2016: Q_ε = 1742(3) keV
- PRC 100 (2019) 014308: $\mathbf{Q_\varepsilon}$ **= 1748.41(34) keV**

→ **Exp. L/K = 0.391 (3)** ΔQ_ε<0.4%

→ **Calc. L/K = 0.3913 (26)** $\Delta(L/K)_{calc.}$~3%



Allowed



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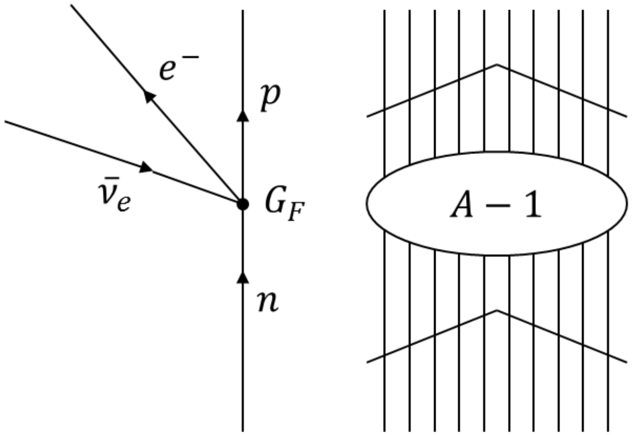
Recent work on theory

Theoretical shape factor

H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

$$C(W_e) = \sum_{K k_e k_\nu} \lambda_{k_e} \left[M_K^2(k_e,k_\nu) + m_K^2(k_e,k_\nu) - \frac{2\mu_{k_e}\gamma_{k_e}}{k_e W_e} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right]$$

Multipole expansion of hadron and lepton currents. Calculation of shape factors, half-lives, branching ratios, log *ft* values.



Fermi theory

- Vertex of the weak interaction is assumed to be pointlike. No propagation of $W^\pm$ boson.
- Effective coupling constant G_F.

Impulse approximation

- The nucleon is assumed to feel only the weak interaction.
- Other nucleons are spectators.

Forbidden non-unique transitions

Leading term for these transitions, simplifying the lepton current:

Relativistic matrix element

$$M_n(k_e,k_\nu) = K_n (pR)^{k_e-1}(qR)^{k_\nu-1}\left\{-\sqrt{\frac{2n+1}{n}}\,\underline{{}^VF^{(0)}_{n,n-1,1}} - \frac{\alpha Z}{2k_e+1}\,\underline{{}^VF^{(0)}_{n,n,0}(k_e,1,1,1)} - \left[\frac{WR}{2k_e+1}+\frac{qR}{2k_\nu+1}\right]\underline{{}^VF^{(0)}_{n,n,0}} - \frac{\alpha Z}{2k_e+1}\sqrt{\frac{n+1}{n}}\,\underline{{}^AF^{(0)}_{n,n,1}(k_e,1,1,1)} - \left[\frac{WR}{2k_e+1}-\frac{qR}{2k_\nu+1}\right]\sqrt{\frac{n+1}{n}}\,\underline{{}^AF^{(0)}_{n,n,1}}\right\}$$

Nuclear structure models are non-relativistic

→ Small component of nucleon wave function estimated from large (non-relativistic) component.

OR

→ **Conserved Vector Current (CVC) hypothesis**

- Comes from gauge invariance of the weak interaction.
- Relationships between non-relativistic and relativistic vector matrix elements.

Realistic nuclear structure

Nuclear state described as a **superposition of nucleon states**

$$\langle \xi_f J_f || T_\lambda || \xi_i J_i \rangle = \hat{\lambda}^{-1} \sum_{a,b} \langle a || T_\lambda || b \rangle \left\langle \xi_f J_f \left\| \left[c_a^\dagger \tilde{c}_b \right]_\lambda \right\| \xi_i J_i \right\rangle$$

transition matrix element

tensor rank

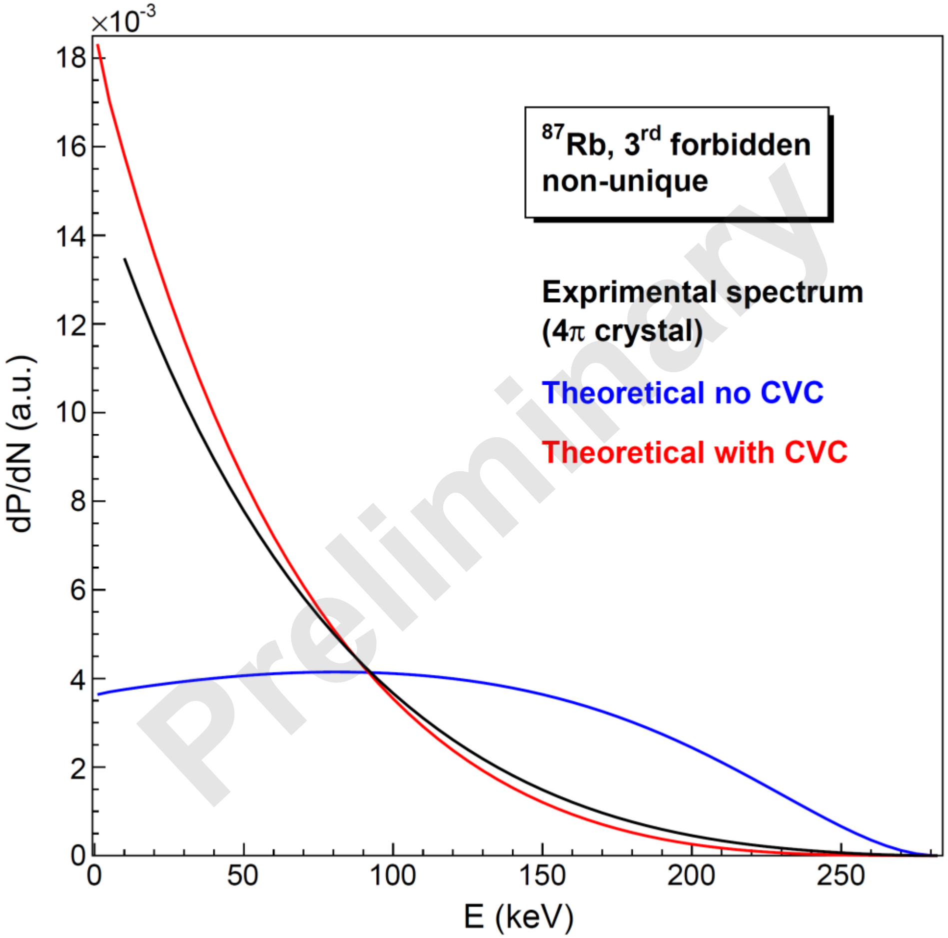
single particle matrix element

one-body transition density

One-body transition densities must be given by a nuclear structure model.

NushellX@MSU: spherical shell model, effective Hamiltonians fitted on nuclear data, widely used.

Preliminary study of ^{87}Rb



$^{87}\mathrm{Rb}\ (\mathrm{gs}, 3/2^-) \rightarrow {}^{87}\mathrm{Sr}\ (\mathrm{gs}, 9/2^+)$

- Third forbidden non-unique transition
- NushellX $^{56}\mathrm{Ni}$ doubly magic core, jj44 model space, jj44b interaction
- Preliminary measurement from the European MetroBeta project (4π $RbGd_2Br_7$ crystal)

→ CVC hypothesis mandatory for an accurate description of the spectrum

Preliminary study of ^{36}Cl

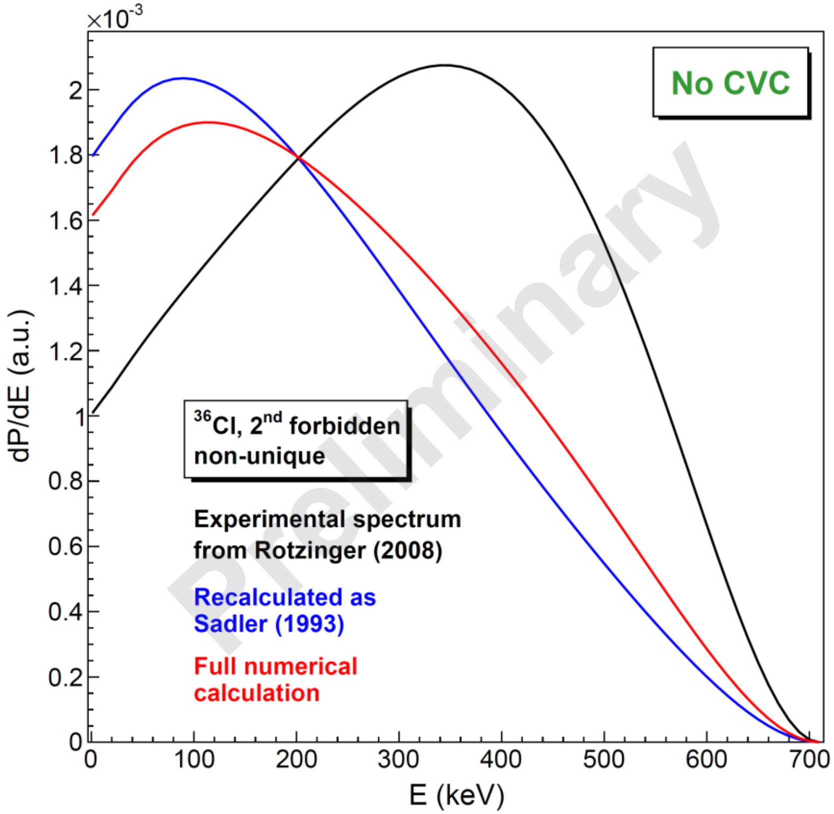
Precise measurement exists

Rotzinger et al., J. Low Temp. Phys. 151, 1087 (2008)

Detailed theoretical study (with approximations)

Sadler, Behrens, Z. Phys. A 346, 25 (1993)

→ Matrix elements are correctly recalculated



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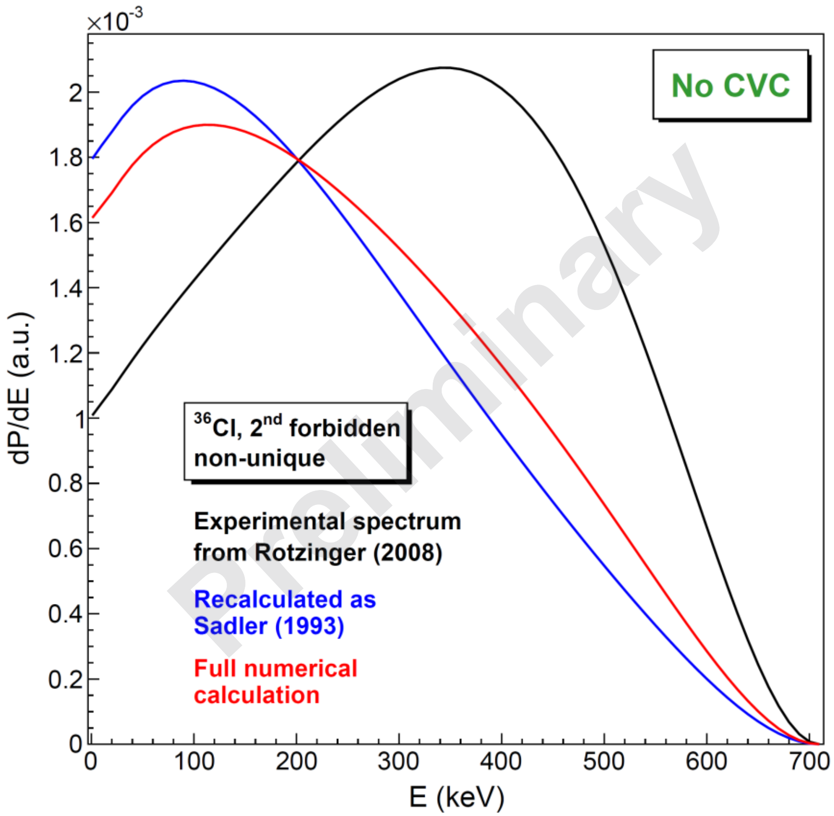

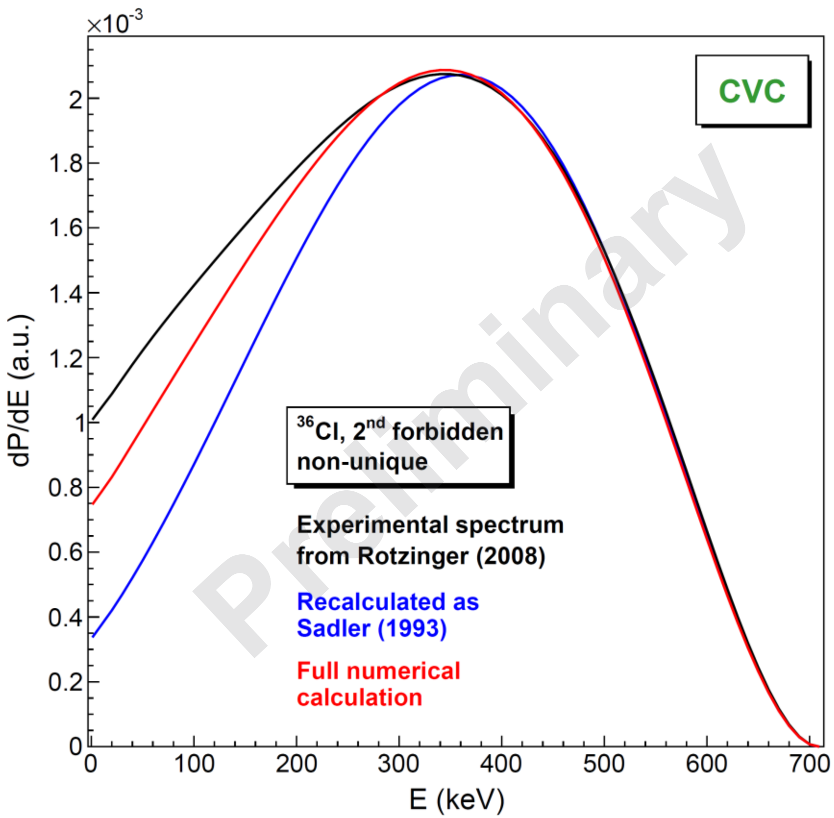

→ CVC hypothesis mandatory + Influence of lepton current treatment

Reactor antineutrino anomaly

PHYSICAL REVIEW C **100**, 054323 (2019)

First-forbidden transitions in the reactor anomaly

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[1]*Instituut voor Kern- en Stralingsfysica, KU Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium*
[2]*Department of Physics, University of Jyväskylä, P.O. Box 35, FI-40014 University of Jyväskylä, Finland*

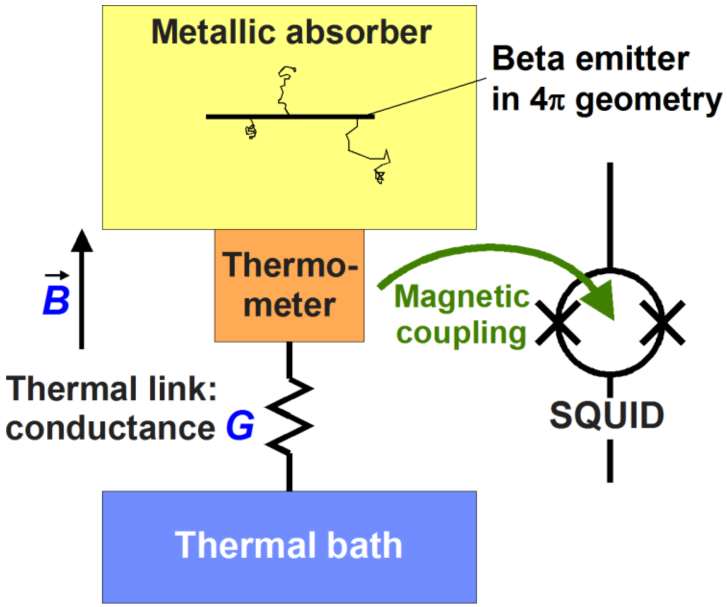
(Received 22 August 2019; published 20 November 2019)

We describe here microscopic calculations performed on the dominant forbidden transitions in reactor antineutrino spectra above 4 MeV using the nuclear shell model. By taking into account Coulomb corrections in the most complete way, we calculate the shape factor with the highest fidelity and show strong deviations from allowed approximations and previously published results. Despite small differences in the *ab initio* electron cumulative spectra, large differences on the order of several percent are found in the antineutrino spectra. Based on the behavior of the numerically calculated shape factors we propose a parametrization of forbidden spectra. Using Monte Carlo techniques we derive an estimated spectral correction and uncertainty due to forbidden transitions. We establish the dominance and importance of forbidden transitions in both the reactor anomaly and spectral shoulder analysis with their respective uncertainties. Based on these results, we conclude that a correct treatment of forbidden transitions is indispensable in both the normalization anomaly and spectral shoulder.

- Conclusion: first forbidden non-unique transitions could explain by themselves the reactor anomaly
- Drawbacks: **no account of CVC hypothesis**, simplified lepton current

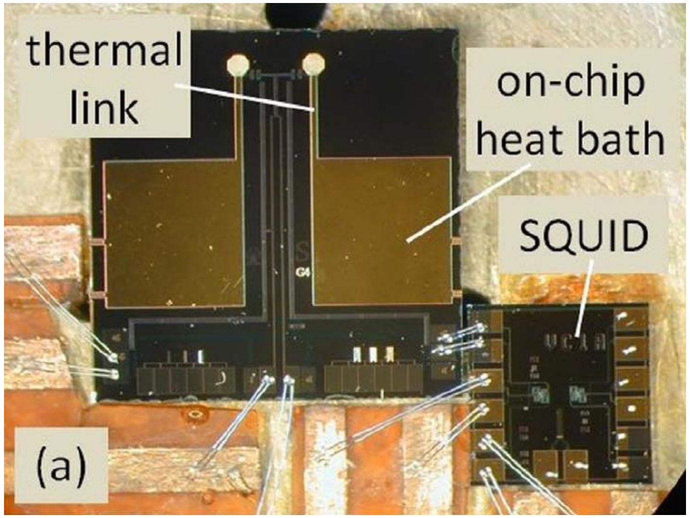
Recent work on experiments

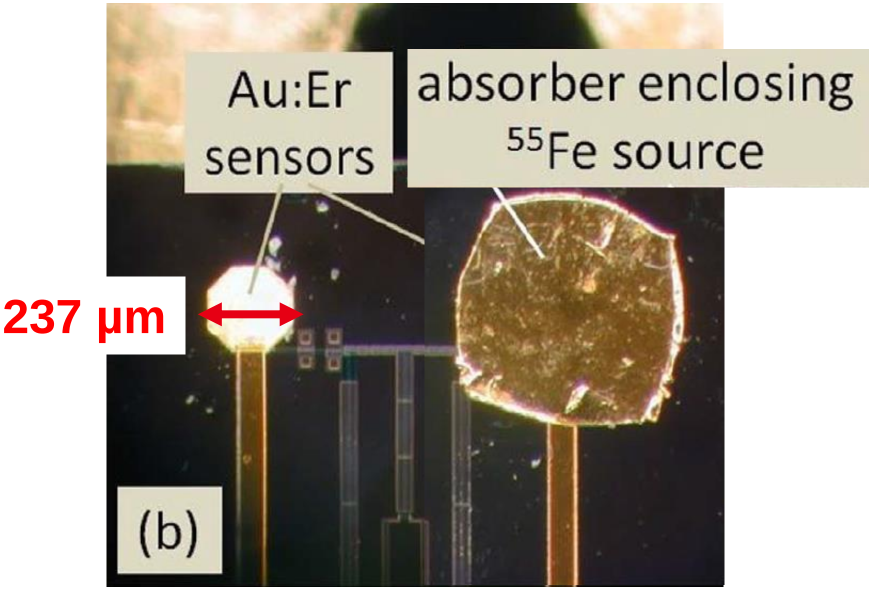
Metallic magnetic calorimetry



System cooled down to 10 mK

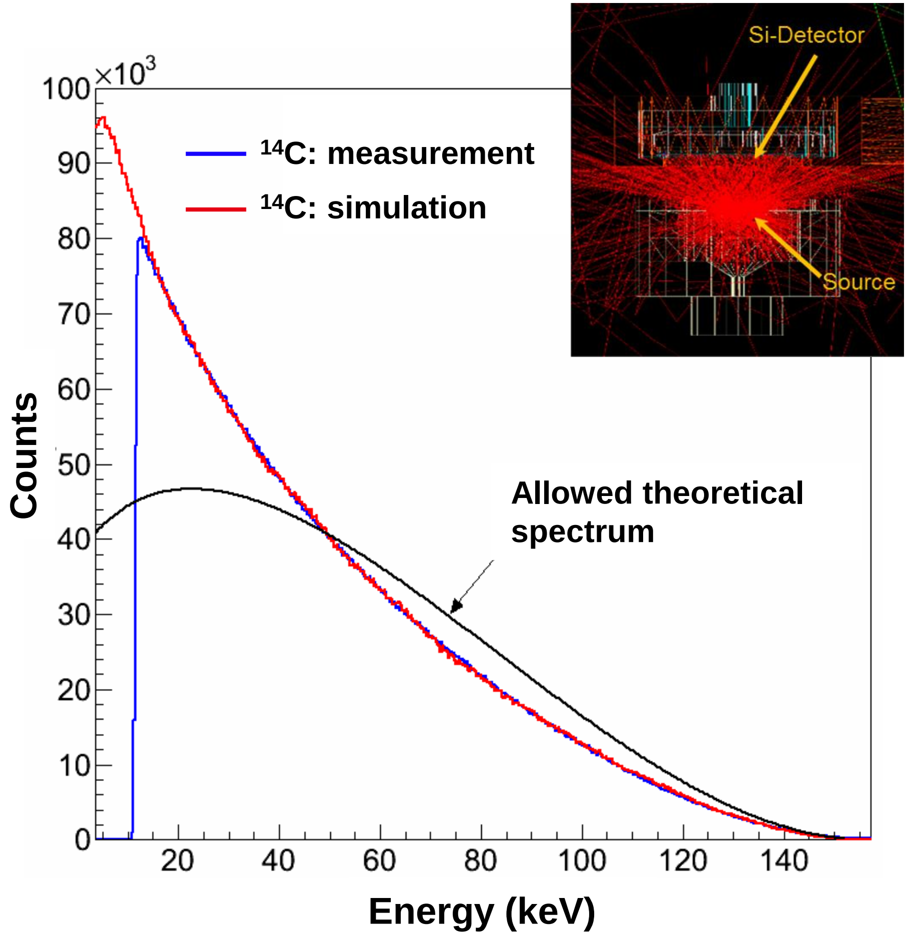
M. Loidl *et al.*, App. Radiat. Isot. 134, 395 (2018)

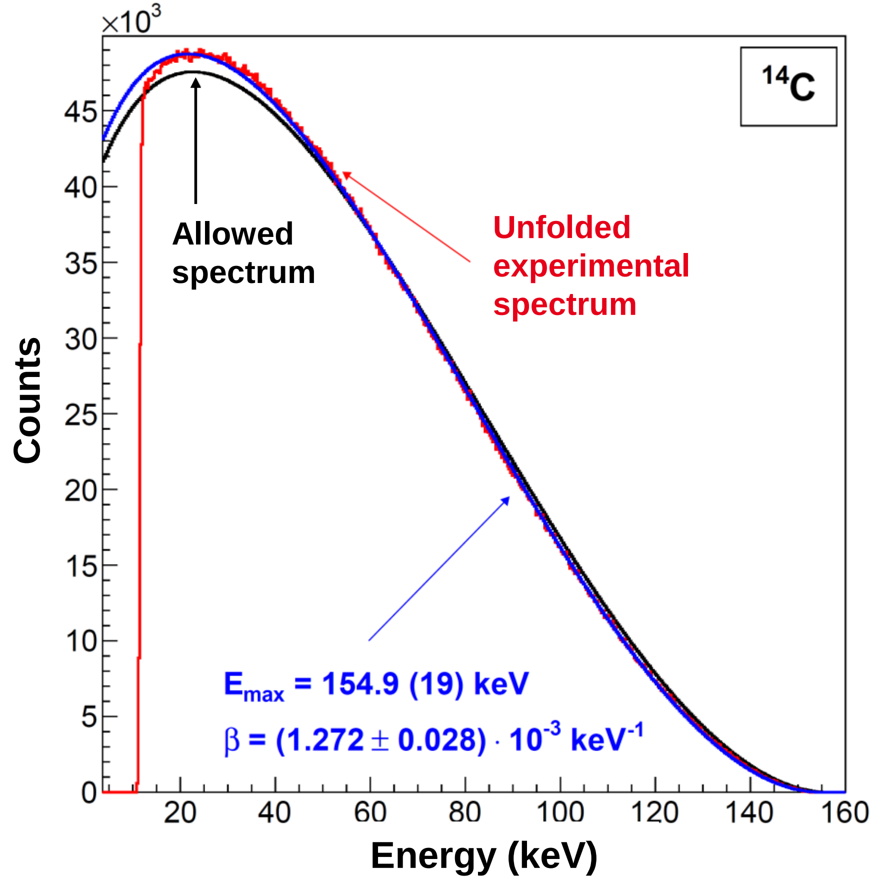




Single silicon detector

Charlène Bisch (PhD 2011-2014)





Experimental shape factor

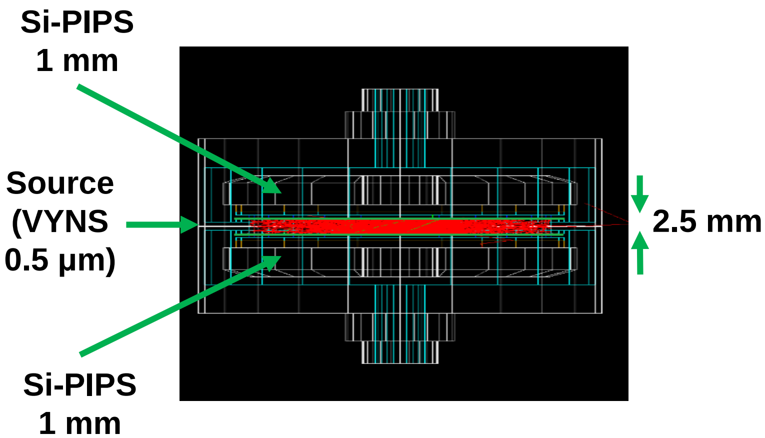
$$C(E) = 1 + \beta(E_{max} - E)$$

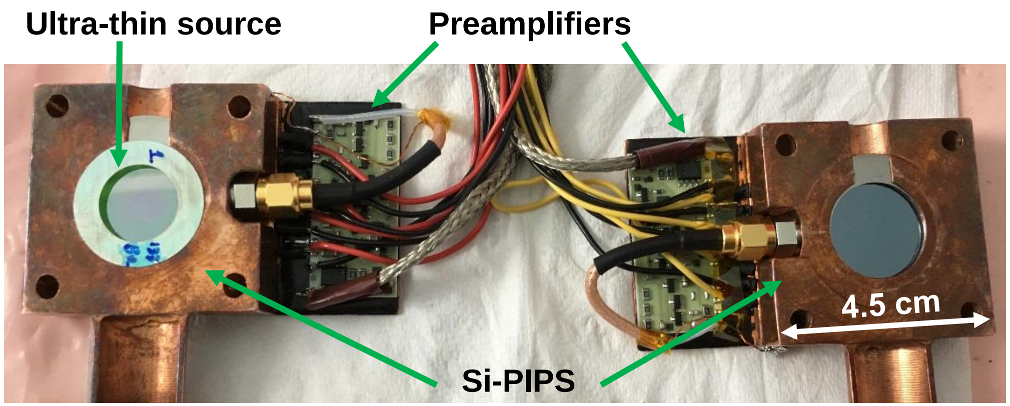
Excellent agreement with

Kuzminov, *Physics of Nuclei*, 63 (7) (2000)

Quasi 4π geometry

Abhilasha Singh (PhD 2017-2020)

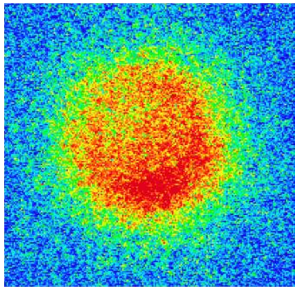




Configuration for measurement

Specific source preparation technique

Radioactive deposit

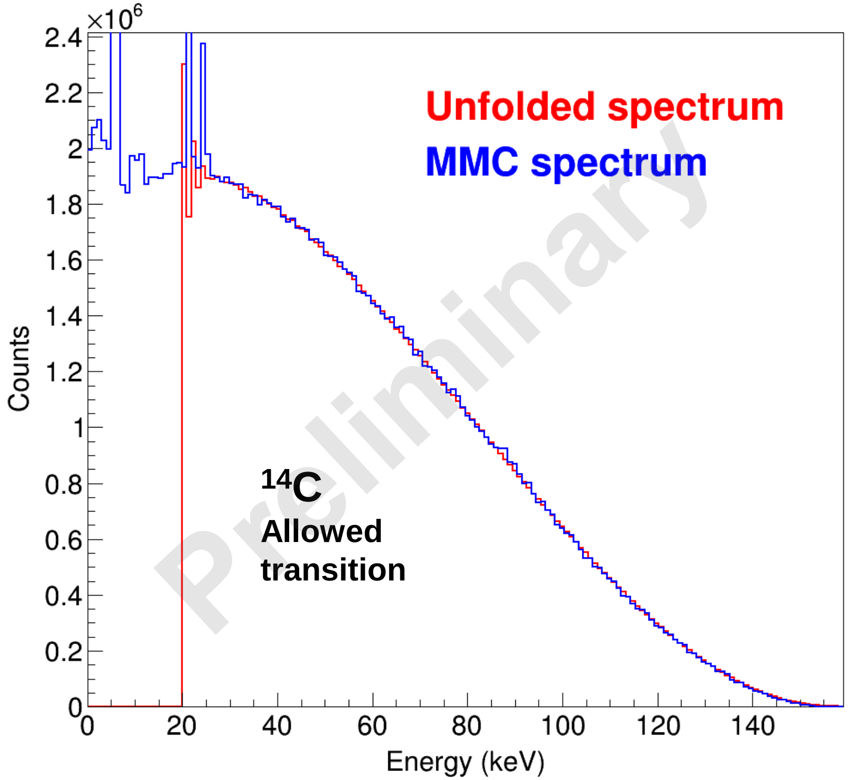


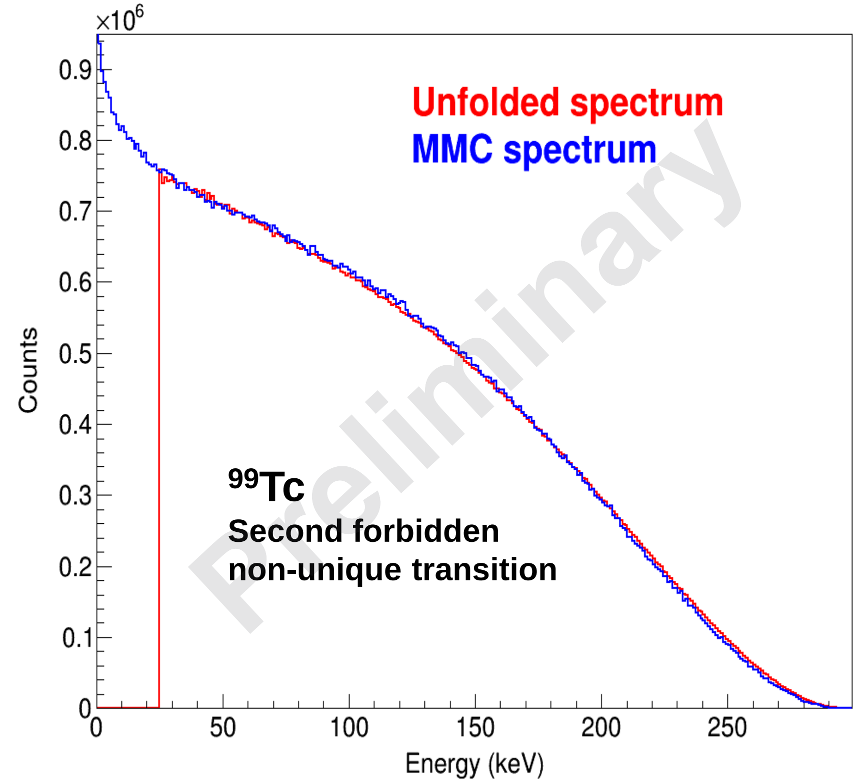
Autoradiography

Unsealed sources:	0.5 to 0.7 µm thick
Sealed sources:	1 to 1.5 µm thick
Typical activity:	~1 kBq

Measurements of ^{14}C and ^{99}Tc spectra

Unfolding process for silicon measurements: response matrix built from precise Monte Carlo simulations.





→ Spectrum much closer to allowed shape than in

Kuzminov, *Physics of Nuclei*, 63 (7) (2000)

→ Not consistent with existing measurement below 100 keV

Reich, Schüpferling, Z. Physik 271, 107 (1974)

Perspectives

Future work included in different projects

French metrology (2021-2024)

- Scientific Committee highlighted the importance of diffusing BetaShape. Funding of a Mac computer is ensured. Executables should be made available in 2021.
- Tabulation of atomic exchange effect for allowed and forbidden unique transitions.

European EMPIR project MetroMMC (2018-2021)

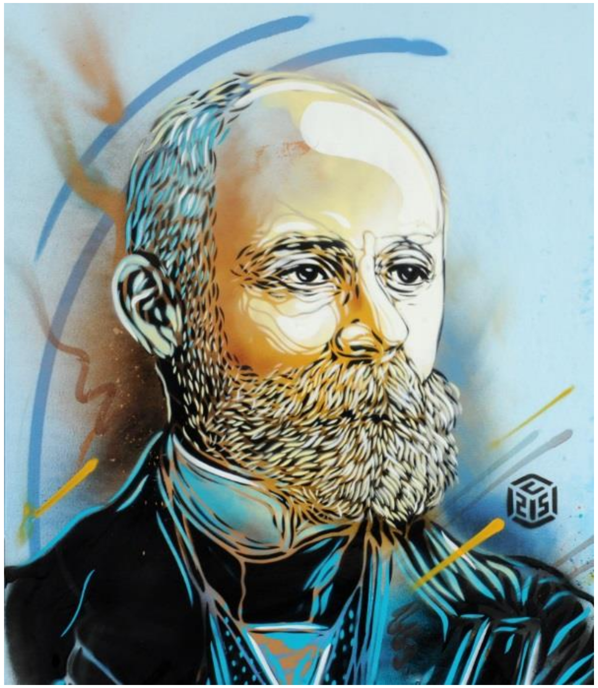
Development of a DFT code for precise atomic wave functions (CNRS). Coupling with a specific version of BetaShape done. Tests in progress.

European EMPIR project PrimA-LTD (2021-2024)

High-precision measurement and calculation of ^{55}Fe and ^{129}I decays. Nuclear structure from HFB + pnQRPA large-scale calculations.

French ANR (National Research Agency) project bSTILED (2021-2024)

PI: Oscar Naviliat-Cuncic (LPC Caen, France). High-precision measurement of ^{6}He spectrum for the search of tensor interactions in beta decay.



Thank you for your attention

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