Progress in Lattice QCD

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The BEST plan for lattice QCD

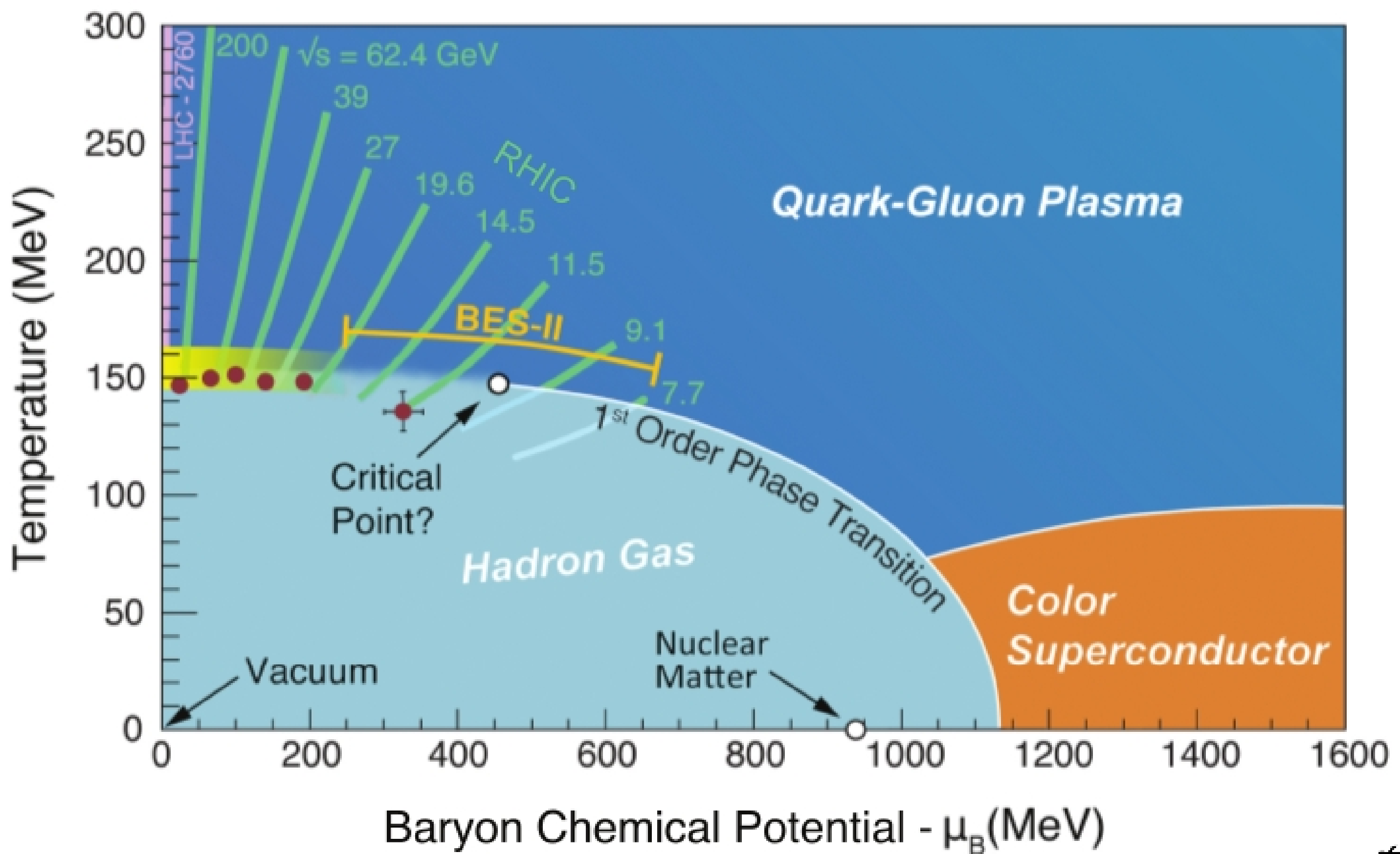
- characterize bulk thermodynamics and fluctuations of conserved charges on the freeze-out line
- provide the equation of state for BES
- (constrain) the location of the critical point

BEST COLLABORATION

CRC-TR 211 Strong-interaction matter under extreme conditions

DFG Deutsche Forschungsgemeinschaft

Exploring the phase diagram of strong-interaction matter

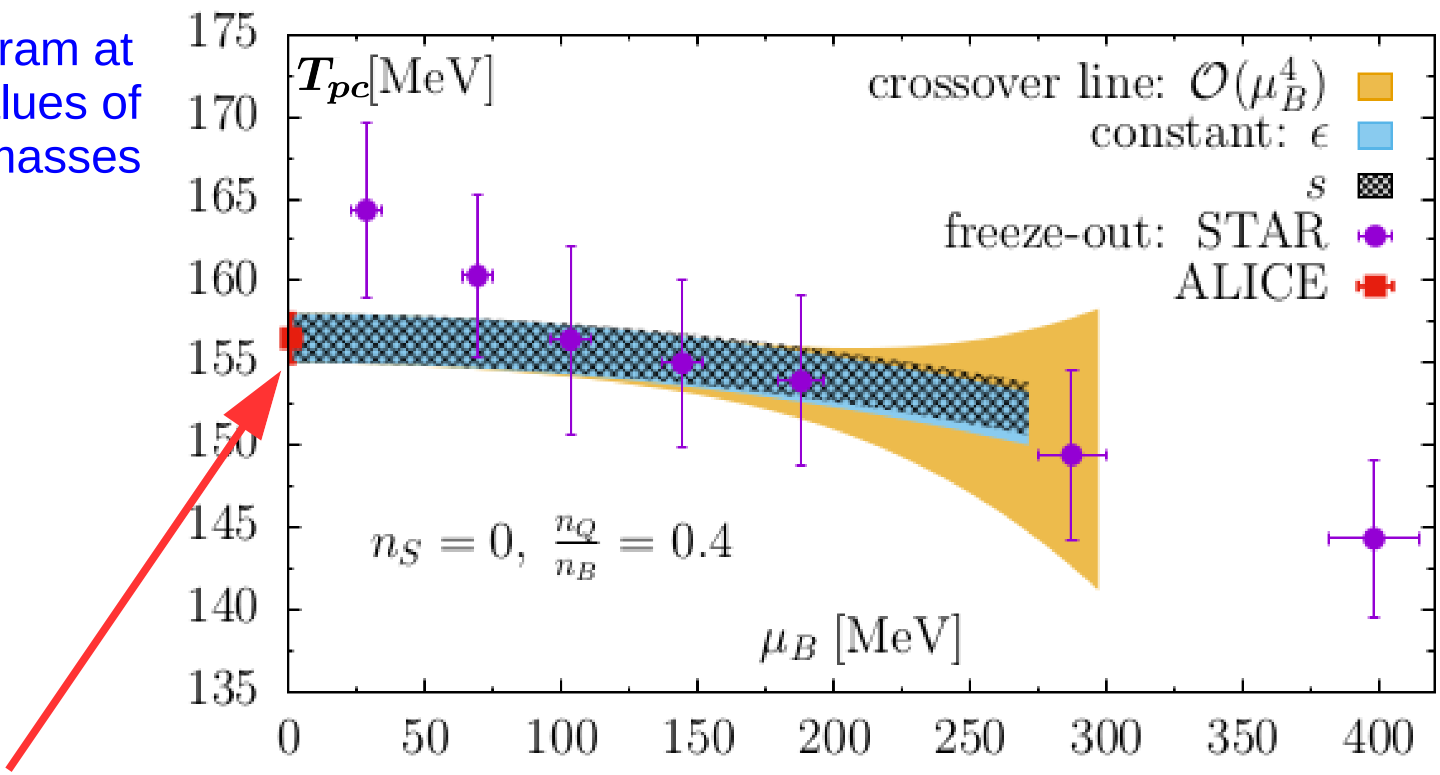


Where is the
critical point?

Phases of strong-interaction matter

$$T_{pc}(\mu_B) = T_{pc}\left(1 + \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 + \kappa_4\left(\frac{\mu_B}{T_c}\right)^4 + \ldots\right)$$

phase diagram at physical values of the quark masses



A. Andronic et al., Nature 561 (2018) 321

$T_{pc} = (156.5 \pm 1.5)$ MeV

$\kappa_2 = 0.012(4)$
$\kappa_4 = 0.000(4)$

A. Bazavov et al. [HotQCD], Phys. Lett. B795, 15 (2019), arXiv:1812.08235

$T_{pc} = (158.0 \pm 0.6)$ MeV

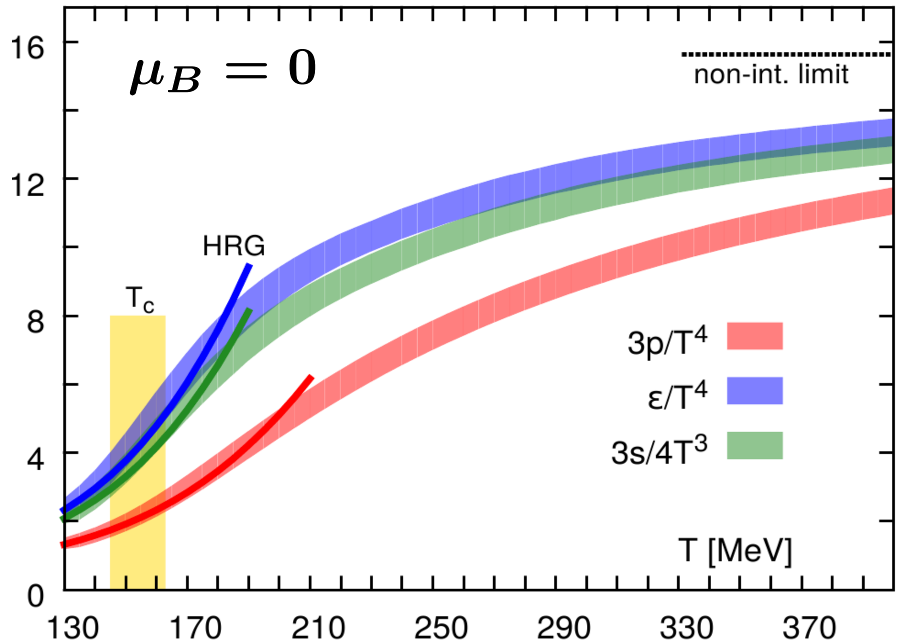
$\kappa_2 = 0.0153(18)$
$\kappa_4 = 0.00032(67)$

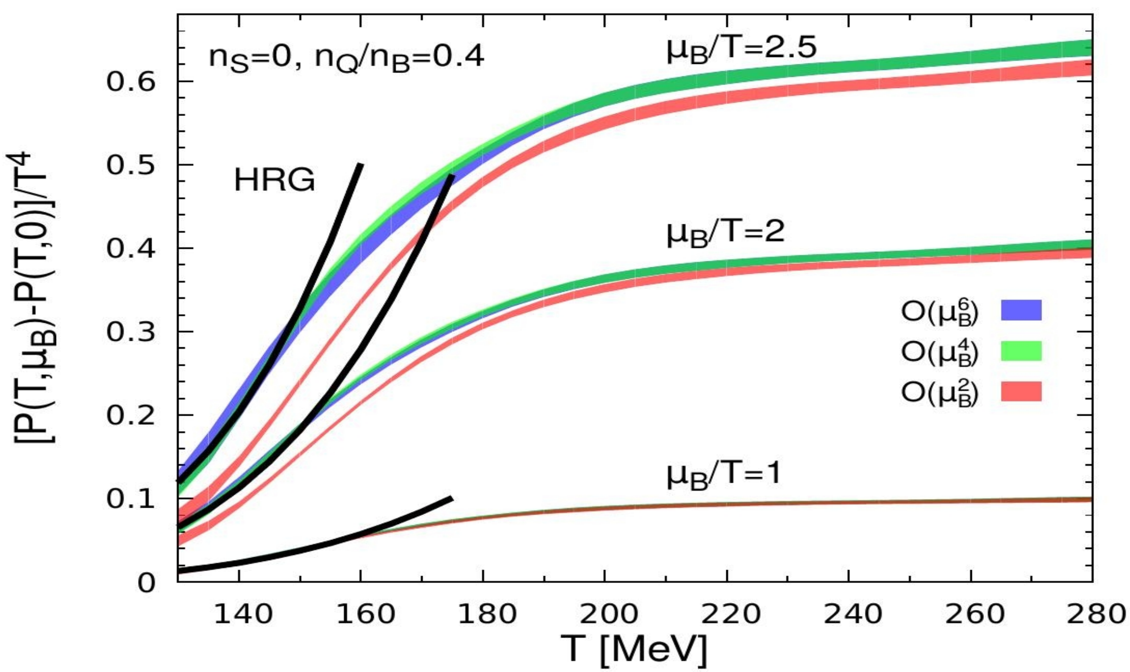
S. Borsanyi, et al, arXiv:2002.02821

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B}{24}\left(\frac{\mu_B}{T}\right)^4 + \frac{\chi_6^B}{720}\left(\frac{\mu_B}{T}\right)^6 + \dots$$

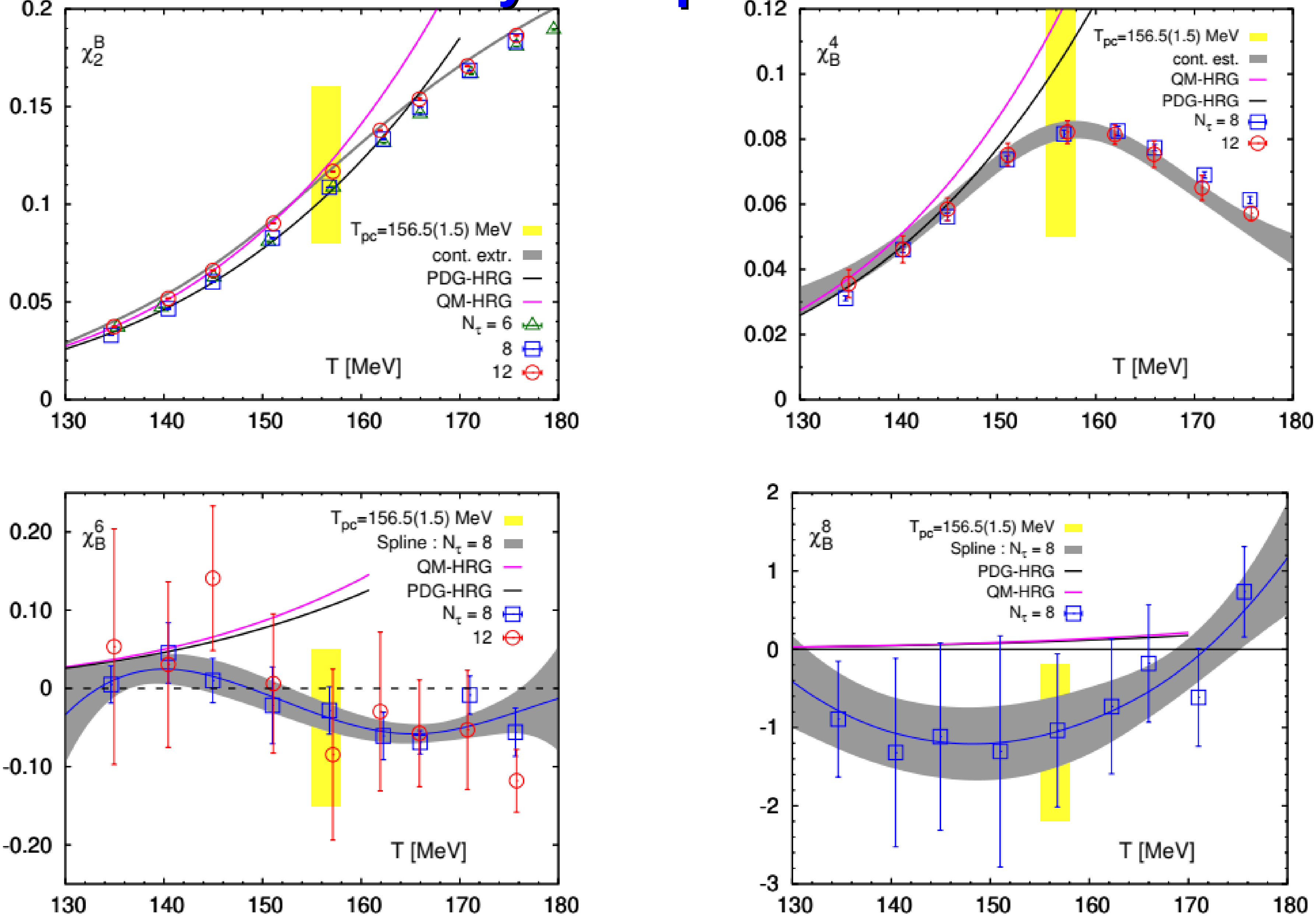
(10-30)% contribution to total pressure at $\mu_B/T = 2$





The EoS is well controlled for $\mu_B/T \le 2$
or equivalently $\sqrt{s_{NN}} \ge 19.6$ GeV

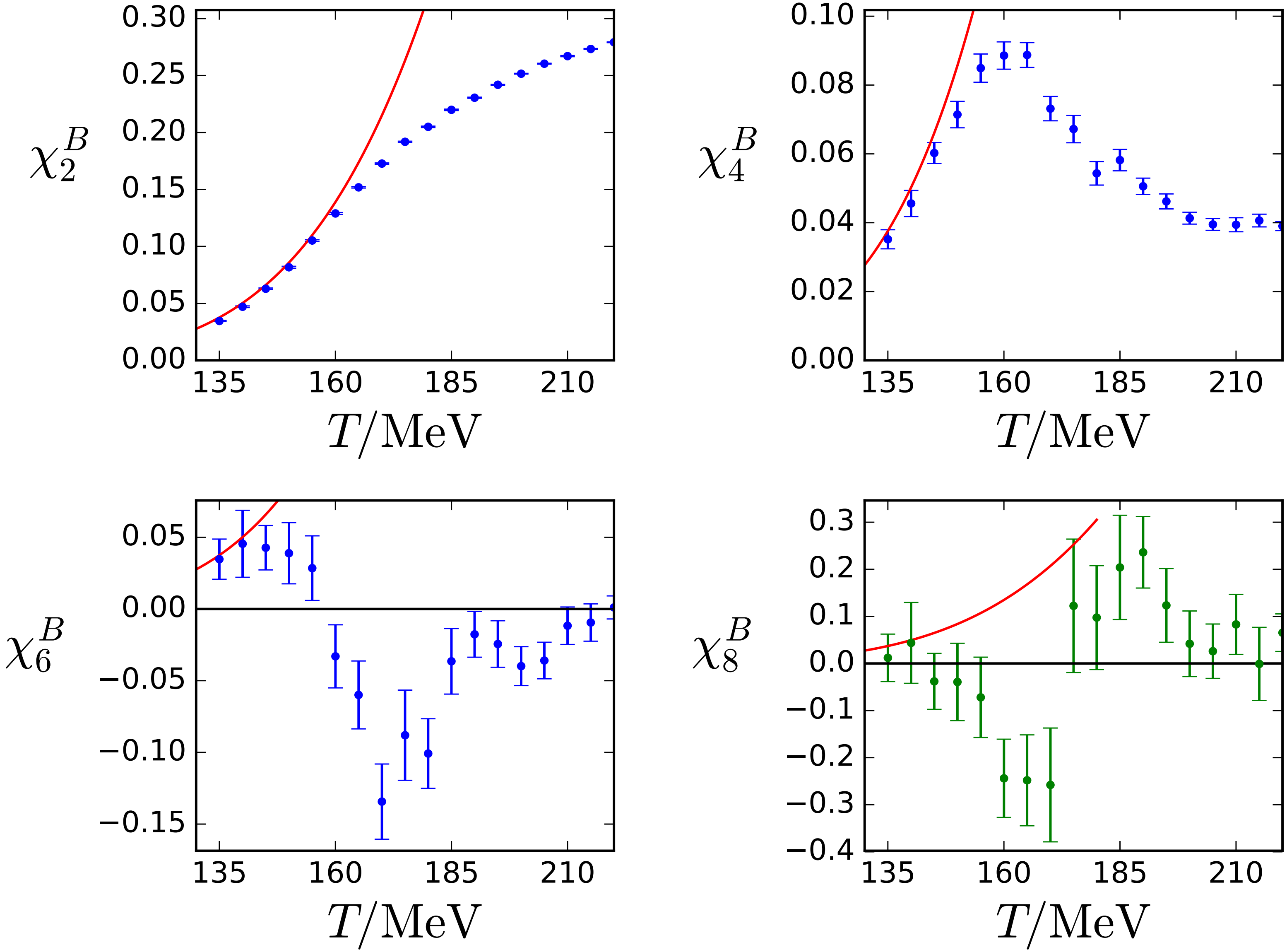
Up to 8th order cumulants are used frequently
– Taylor expansion –



A. Bazavov et al. (HotQCD), Phys. Rev. D 101 (2020) 074502, arXiv:2001.08530

Up to 8th order cumulants are used frequently
– imag. chem. pot extrapolations –

$48^3 \times 12$



S. Borsanyi et al. , JHEP 10 (2018) 205, arXiv:1805.04445

HRG vs. QCD
net baryon-number fluctuations

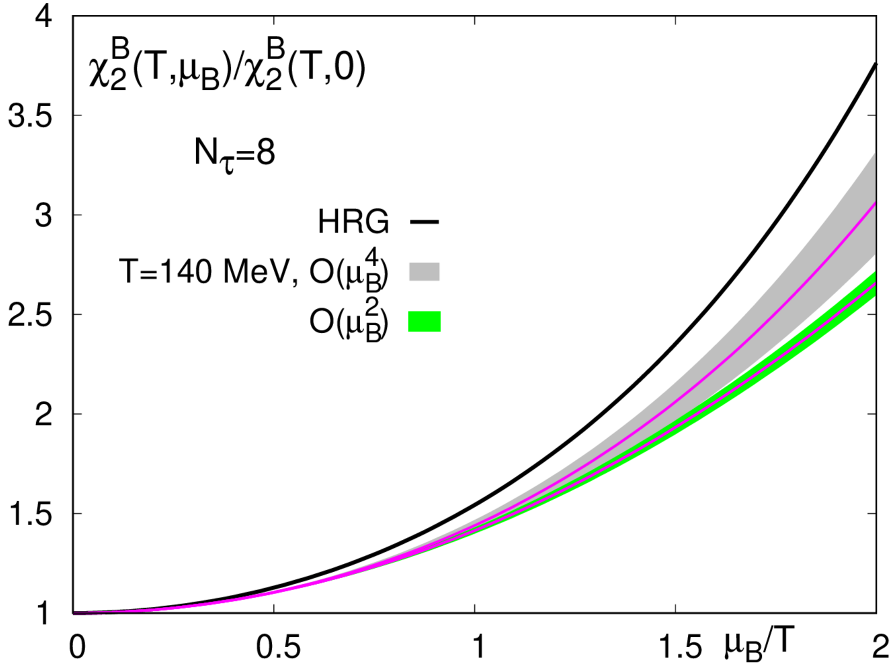
$$\mu_B/T > 0$$

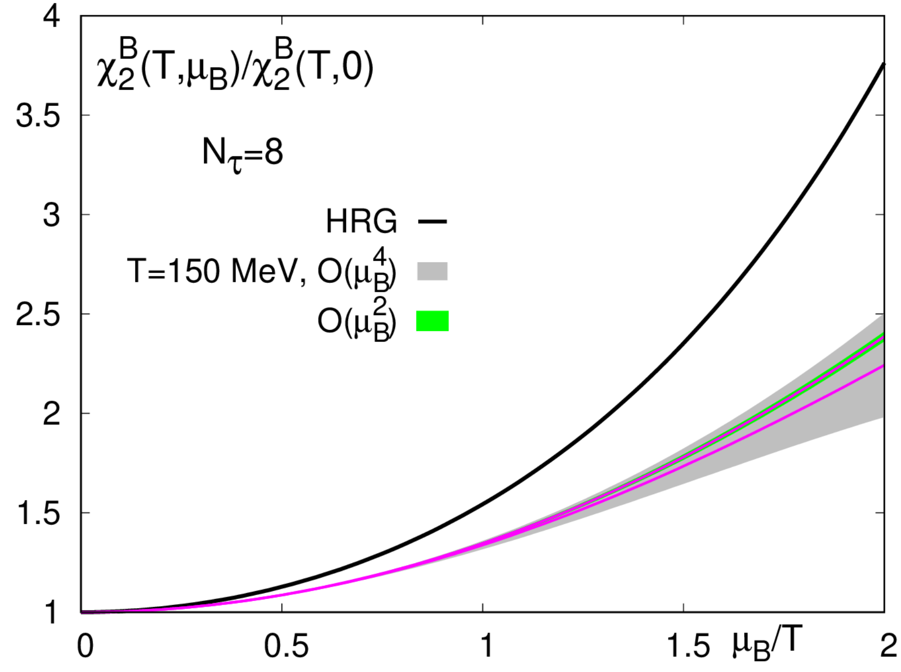
for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2}\chi_4^B\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}\chi_6^B\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^6)$$

– **agreement between HRG and QCD will start to deteriorate for T>140 MeV**

– **net baryon-number fluctuations in QCD always smaller than in HRG for T>150 MeV**





HRG vs. QCD
net baryon-number fluctuations

$$\mu_B/T > 0$$

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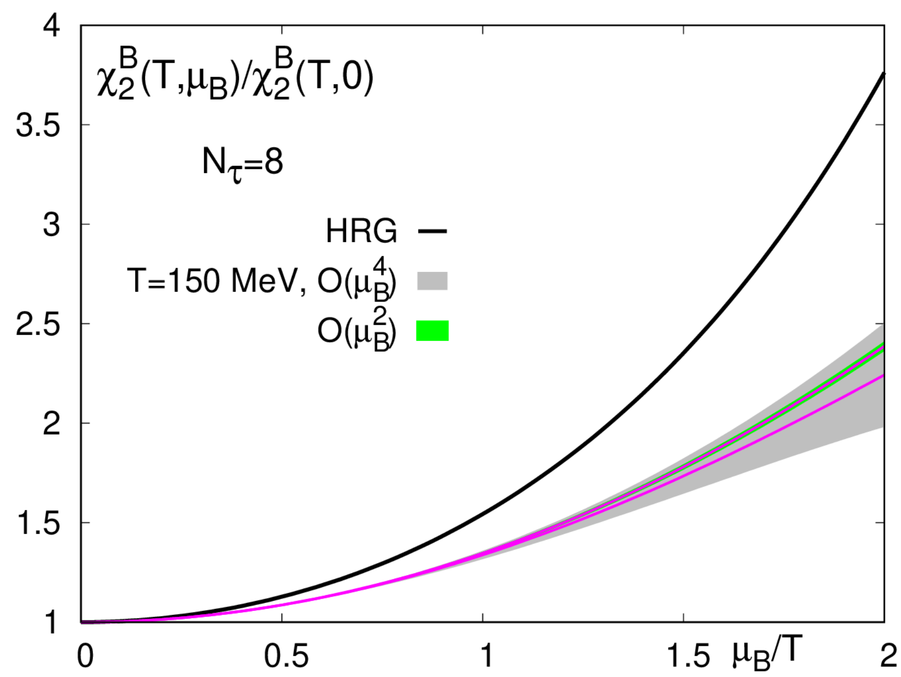
$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2}\chi_4^B\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}\chi_6^B\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^6)$$

- **agreement between HRG and QCD will start to deteriorate for T>140 MeV**
- **net baryon-number fluctuations in QCD always smaller than in HRG for T>140 MeV**

no evidence for enhanced net baryon-number fluctuations for

$$T \geq \ 135\text{MeV}\ ,\ \mu_B \leq 2T$$

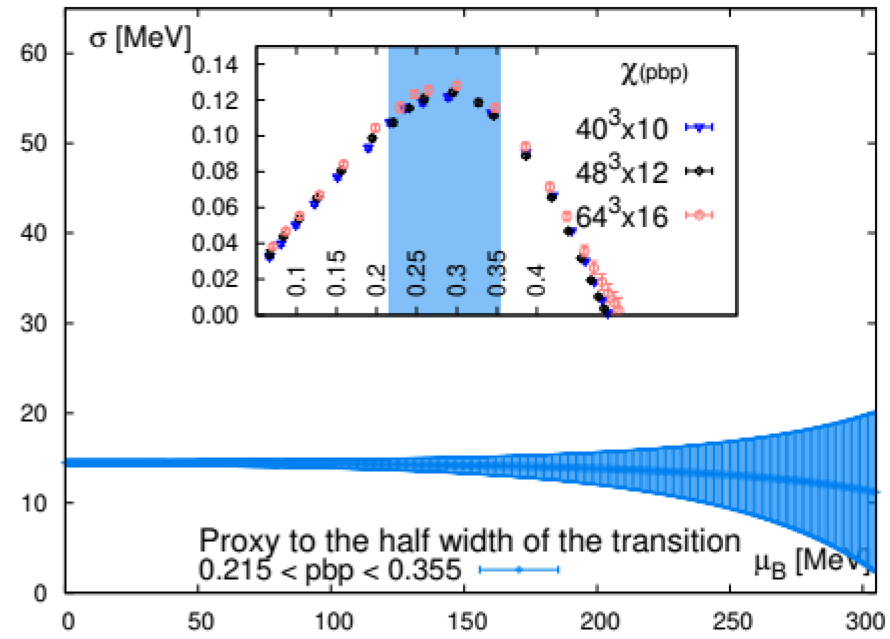
no evidence for getting closer to a "critical region"



No evidence for enhanced fluctuations for $\mu_B/T < 2$

– fluctuations of conserved charges stay smaller than HRG values

– width of chiral susceptibility does not become narrower with increasing chemical potential



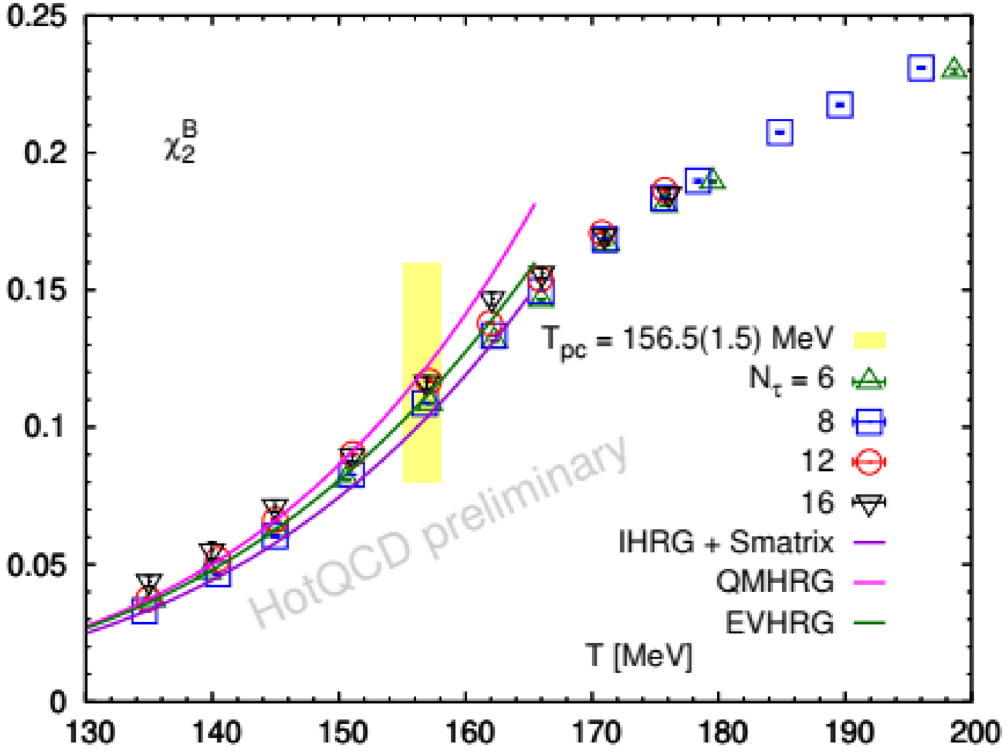
S. Borsanyi, et al, arXiv:2002.02821

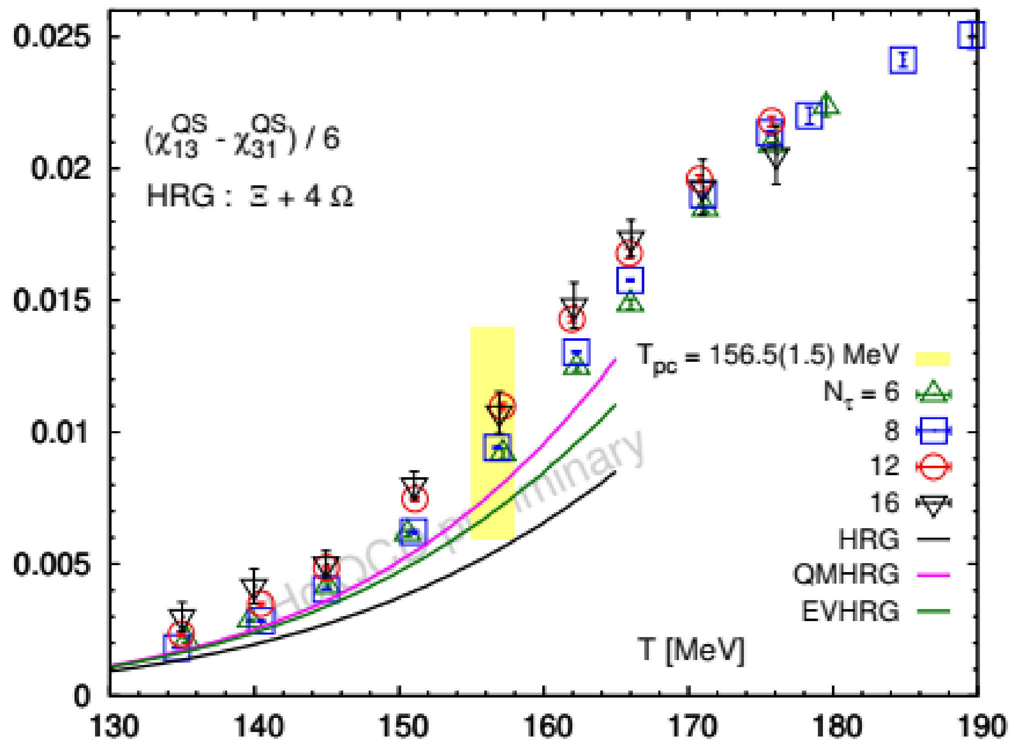
Deviations from non-interacting HRG

– including additional strange resonances a'la QMHRG seems to be mandatory

– deviations from HRG apparent already in lower order cumulants at $T \simeq 140\ \mathrm{MeV}$

– interacting-HRG with repulsive force only (EVHRG or S-matrix) is insufficient





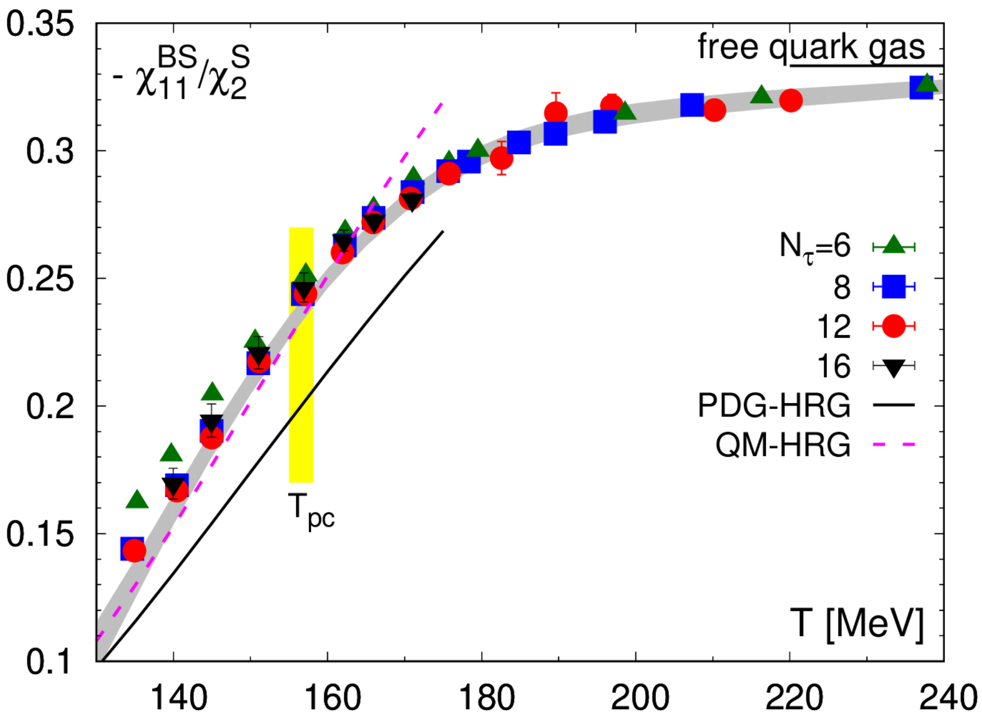
Ratio of baryon number – strangeness correlation and net strangeness fluctuation

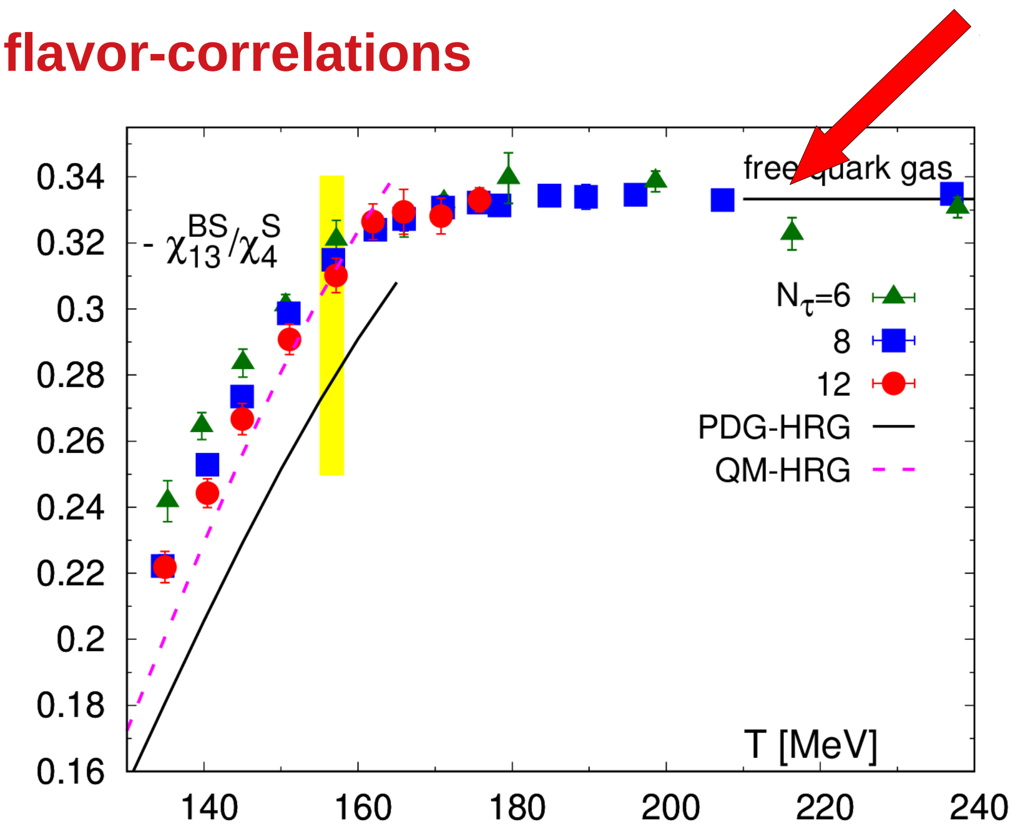
2[nd] and 4[th] order cumulants

evidence for experimentally not yet observed strange baryons?

rapid approach to free quark gas limit

BS ratios probe flavor-correlations





conserved charge ⟺ quark number fluctuations:

$$\chi_{13}^{BS} = -\frac{1}{3}\chi_{13}^{us} - \frac{1}{3}\chi_{13}^{ds} - \frac{1}{3}\chi_4^{s}$$

$$-\frac{\chi_{13}^{BS}}{\chi_4^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{13}^{us}}{\chi_4^s}$$

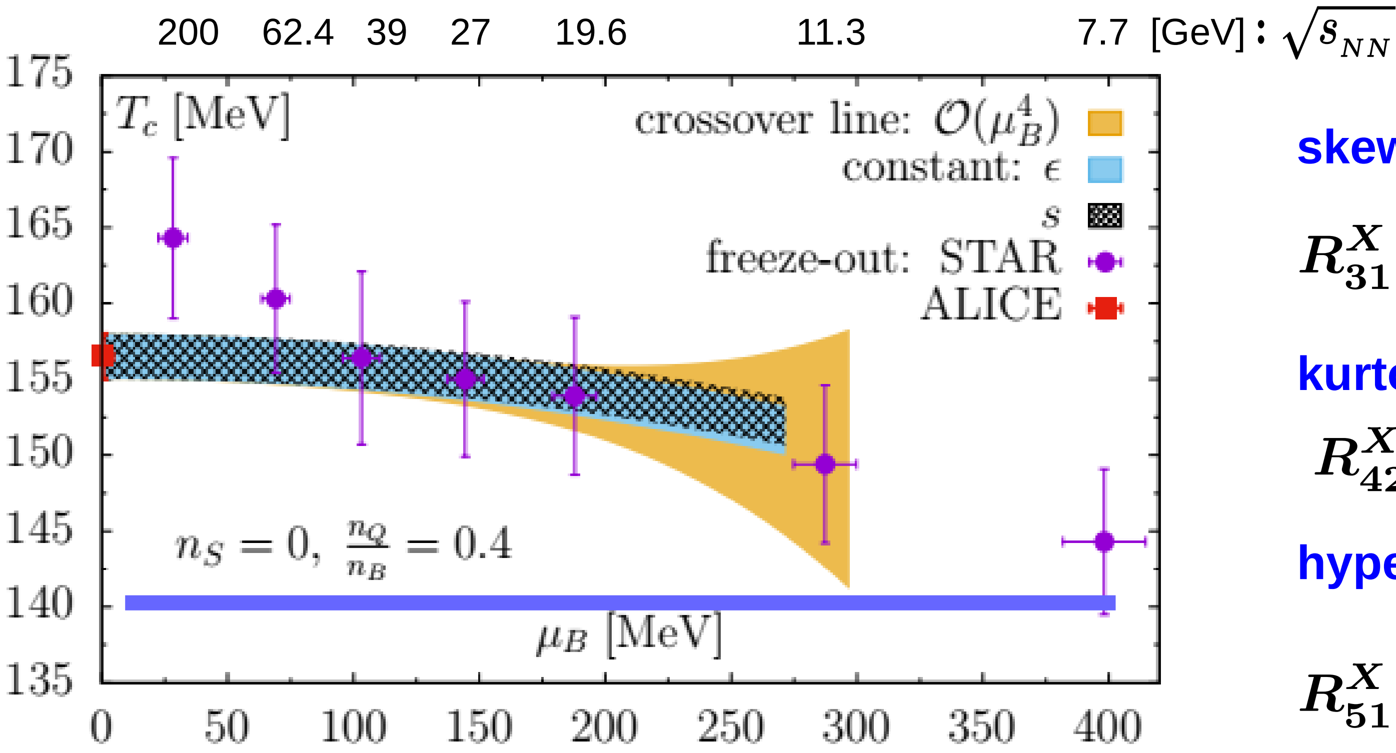
$$\mathcal{O}(g^6 \ln g^2)$$

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Lett. B 523 (2001) 143

Cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101, 074502 (2020)
arXiv:2001.08530

update from new, high statistics lattice QCD results (and a new STAR beam energy)



skewness

$$R_{31}^X = \left(\frac{S\sigma^3}{M}\right)_X = \frac{\chi_3^X}{\chi_1^X}$$

kurtosis

$$R_{42}^X = (\kappa\sigma^2)_X = \frac{\chi_4^X}{\chi_2^X}$$

hyper-skewness

$$R_{51}^X = \left(\frac{S^H\sigma^5}{M}\right)_X = \frac{\chi_5^X}{\chi_1^X}$$

hyper-kurtosis

$$R_{62}^X = (\kappa^H\sigma^4)_X = \frac{\chi_6^X}{\chi_2^X}$$

X=proton (experiment, equilibrium??)

X=baryon (lattice QCD, equilibrium thermo !!)

Cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101, 074502 (2020)
arXiv:2001.08530

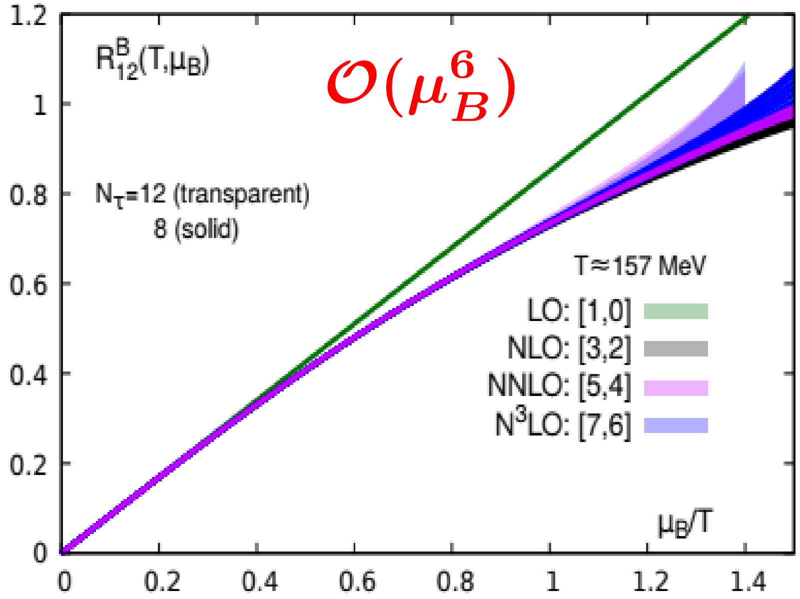
update from new, high statistics lattice QCD results (and a new STAR beam energy)

$$n_S = 0,\ n_Q/n_B = 0.4 :$$

$$R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=1}^{l_{max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

skewness

$$R^X_{12} = (M/\sigma^2)_X = \frac{\chi^X_1}{\chi^X_2}$$



$\mathcal{O}(\mu_B^4)$

skewness

$$R^X_{31} = \left(\frac{S\sigma^3}{M}\right)_X = \frac{\chi^X_3}{\chi^X_1}$$

kurtosis

$$R^X_{42} = (\kappa\sigma^2)_X = \frac{\chi^X_4}{\chi^X_2}$$

$\mathcal{O}(\mu_B^2)$

hyper-skewness

$$R^X_{51} = \left(\frac{S^H\sigma^5}{M}\right)_X = \frac{\chi^X_5}{\chi^X_1}$$

hyper-kurtosis

$$R^X_{62} = (\kappa^H\sigma^4)_X = \frac{\chi^X_6}{\chi^X_2}$$

Skewness, kurtosis, hyper-skewness and hyper-kurtosis ratios on the pseudo-critical line

new STAR data:

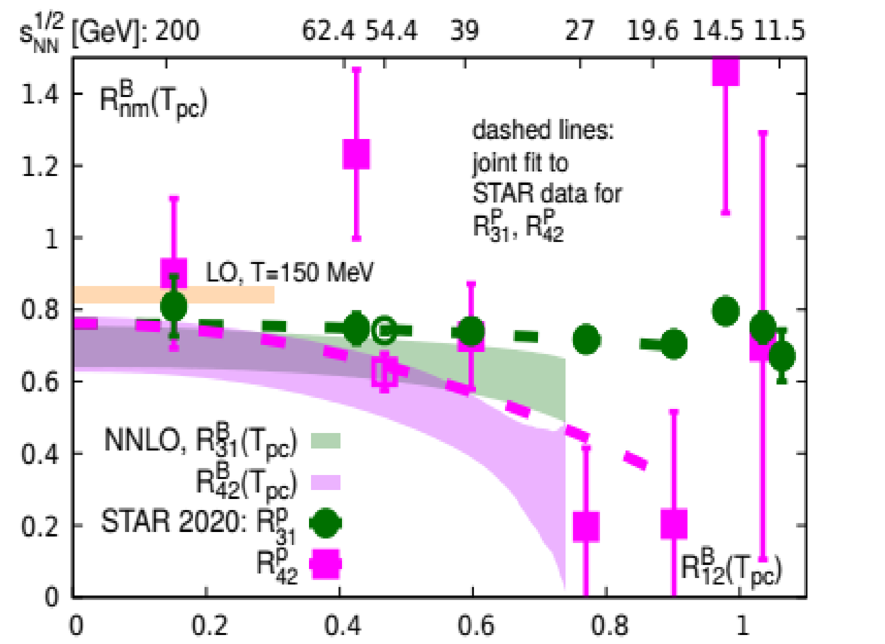
$\sqrt{s_{NN}} = 54.4$ GeV

$$R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=1}^{l_{max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

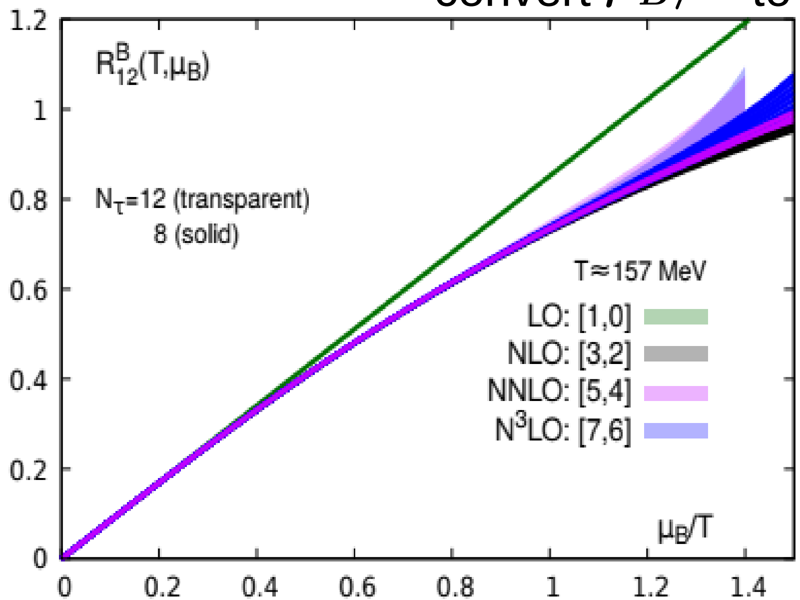
$$T_{pc}(\mu_B) = T_{pc}\left(1 + \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 + \ldots\right)$$

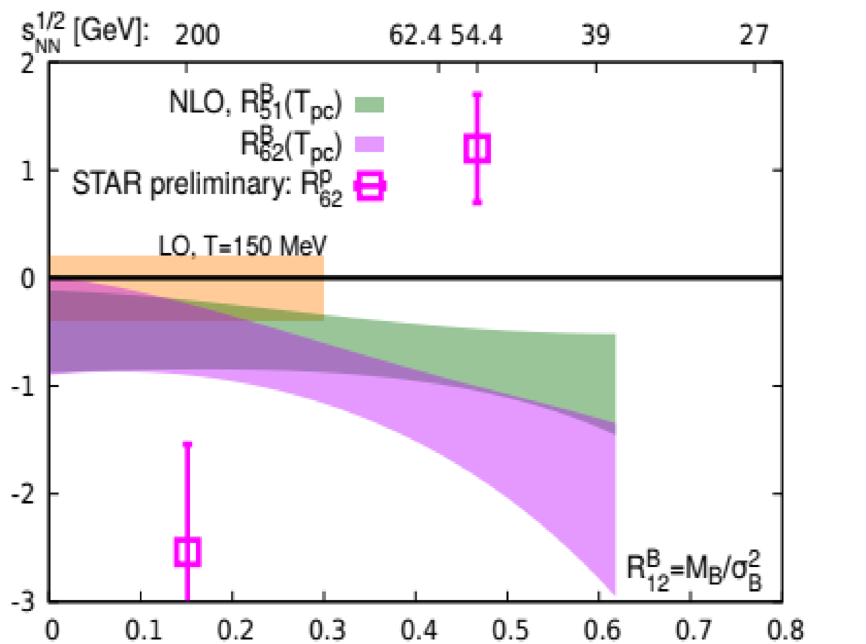
$$T_{pc} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

$$\kappa_2 = 0.012(4)$$



convert μ_B/T to M_B/σ_B^2

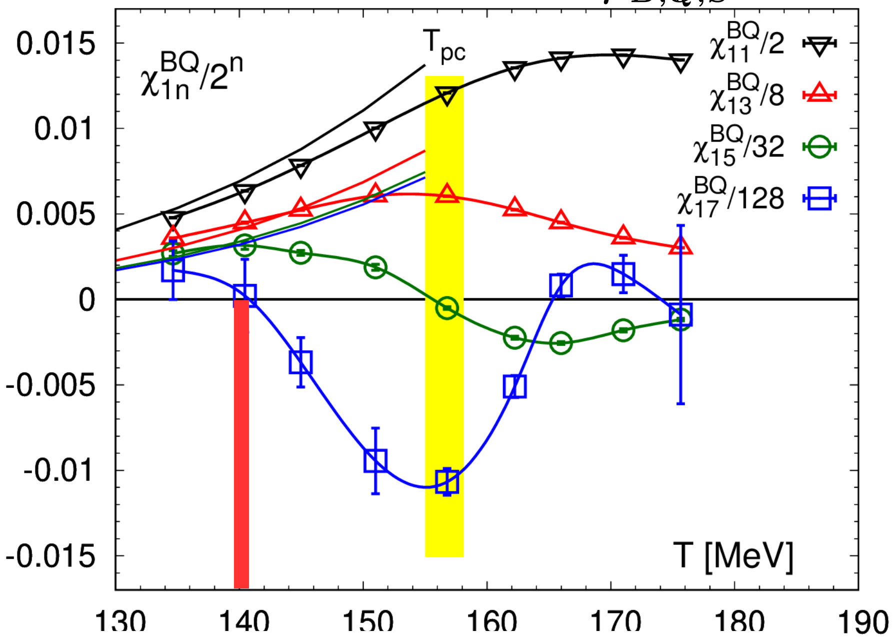


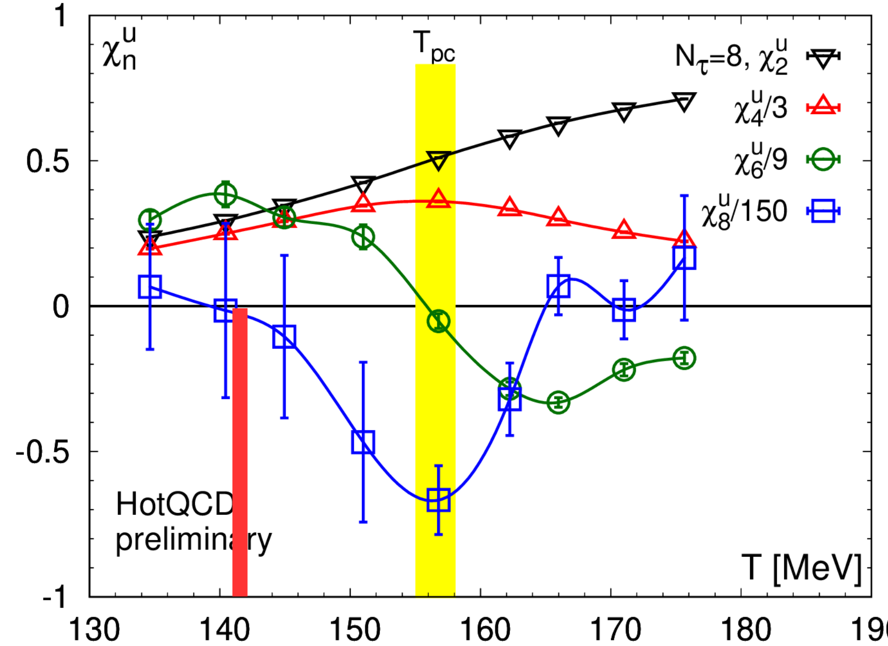


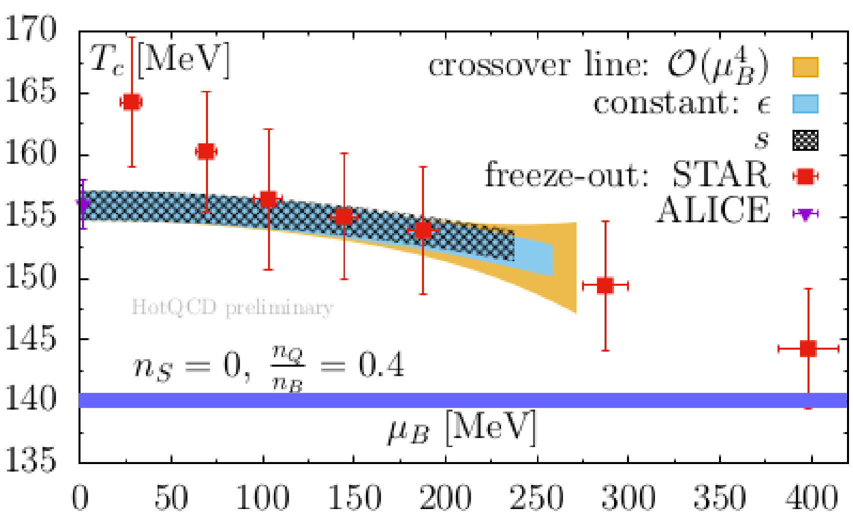
Critical behavior and higher order cumulants

$$\chi_{1n}^{BQ} = \left.\frac{\partial^{n+1} P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_Q^n}\right|_{\mu_{B,Q,S}=0}$$

$$\chi_n^u = \left.\frac{\partial^n P/T^4}{\partial\hat{\mu}_u^n}\right|_{\mu_{u,d,s}=0}$$







many 8th order cumulants turn negative for

$$T^- \gtrsim (140 - 145)\ \mathrm{MeV}$$

plausible scenario:

$$T_{CEP} < 140\mathrm{MeV}\ ,\ \mu_B^{CEP} > 400\mathrm{MeV}$$

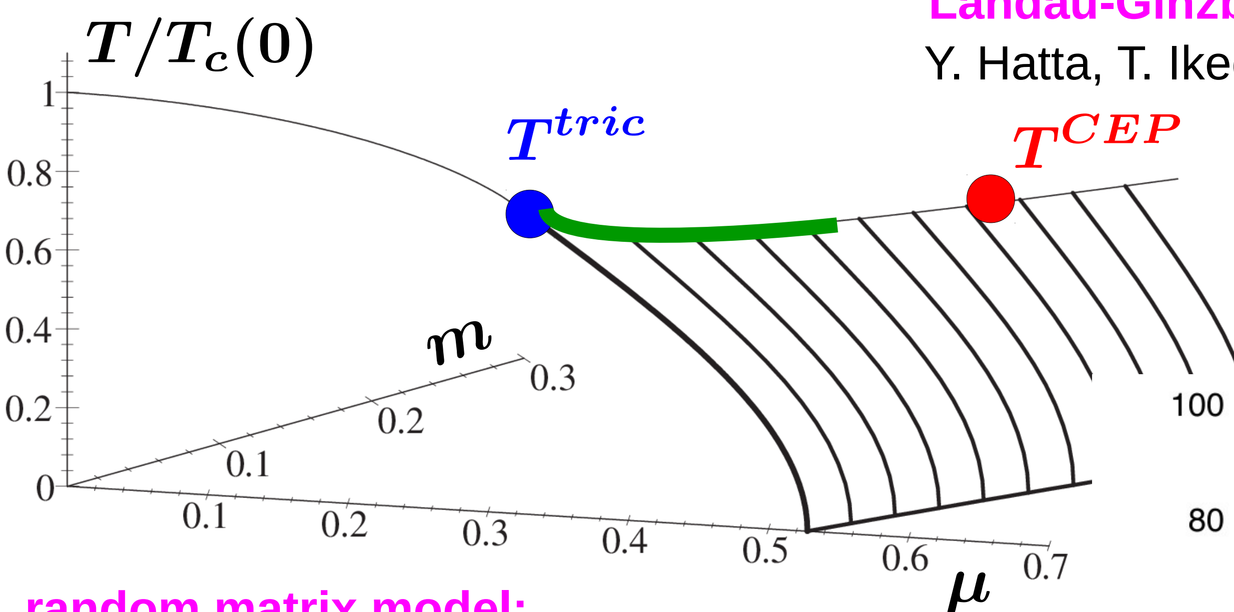
Phase diagram of QCD with two light flavors of mass m as calculated from RMT, Landau-Ginzburg (O(4)) & NJL models

$$T_c(m) = T^{tric} - c\ m^{2/5}$$

non − universal, but c > 0

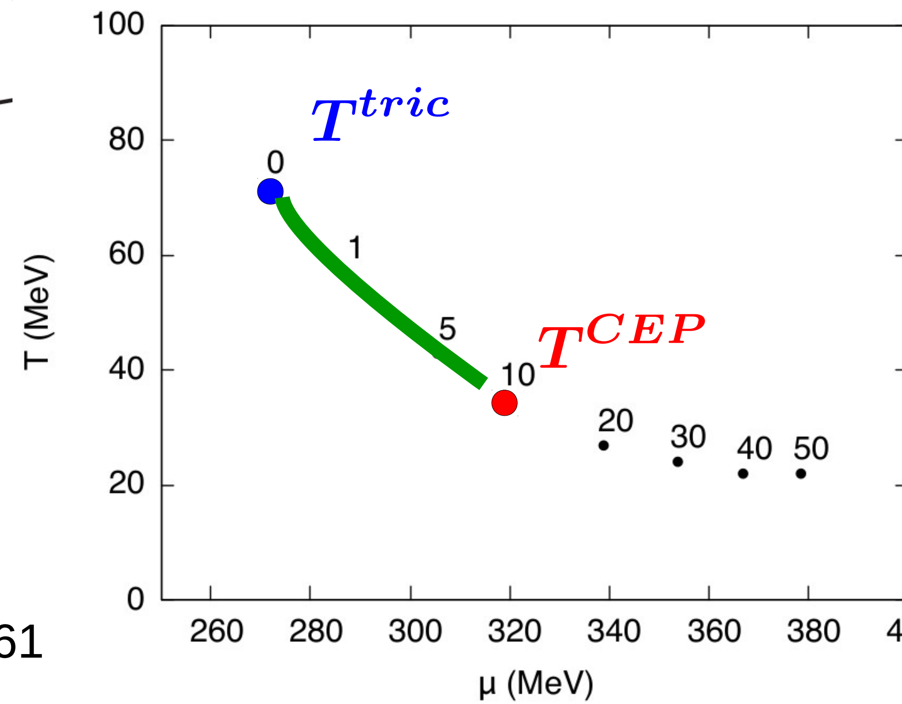
Landau-Ginzburg potential:
Y. Hatta, T. Ikeda, Phys. Rev. D67 (2003) 014028

QCD&RMT:
M.A. Stephanov, K. Rajagopal, E.V. Shuryak, PRL 81, 4816 (1998)

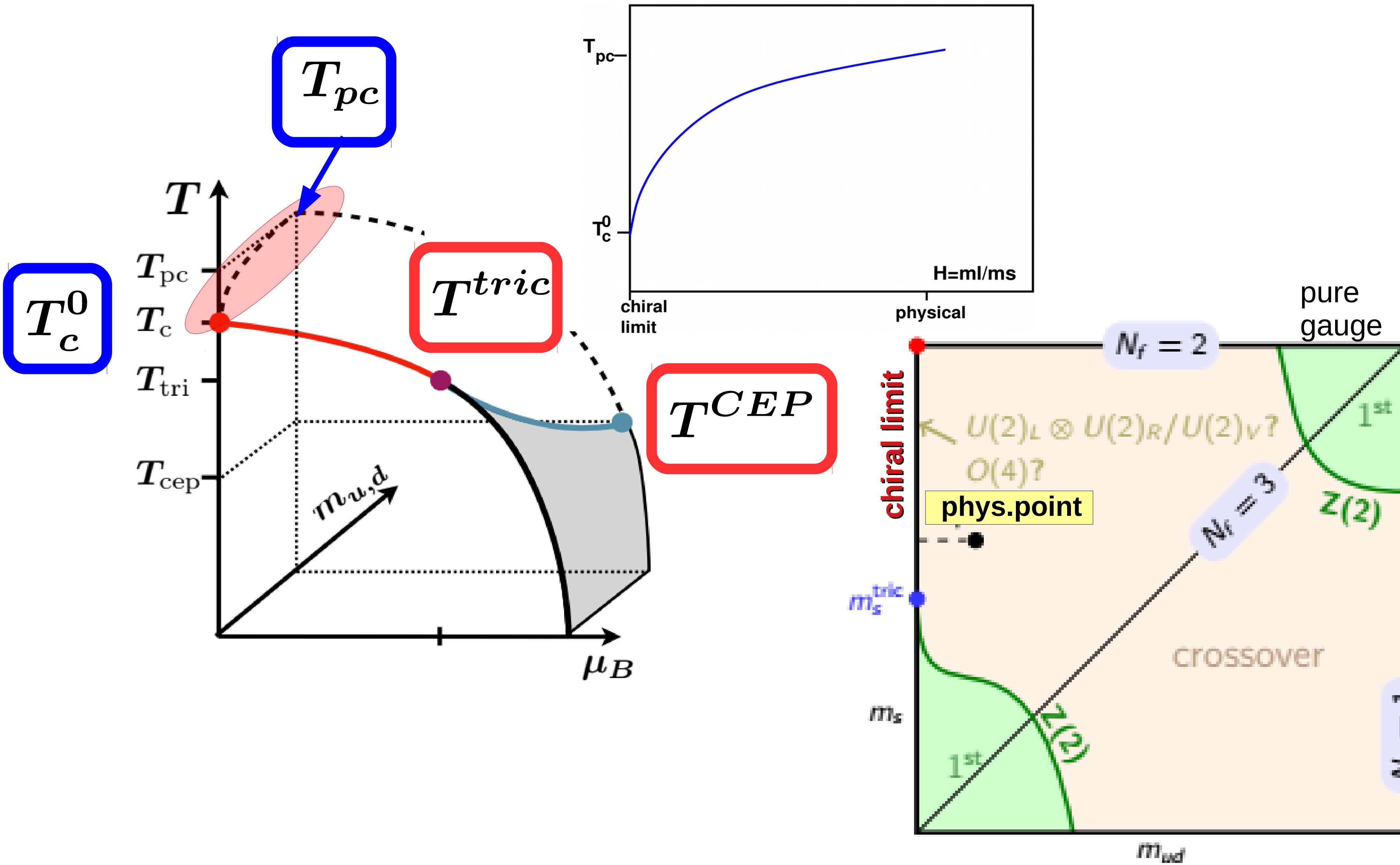


random matrix model:
A.M. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.M. Verbaarschot, Phys. Rev. D 58 (1998) 096007

NJL model:
M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361



Phases of strong-interaction matter



determination of T_c^0 puts an upper limit on T^{CEP}, if c>0

Critical behavior in QCD

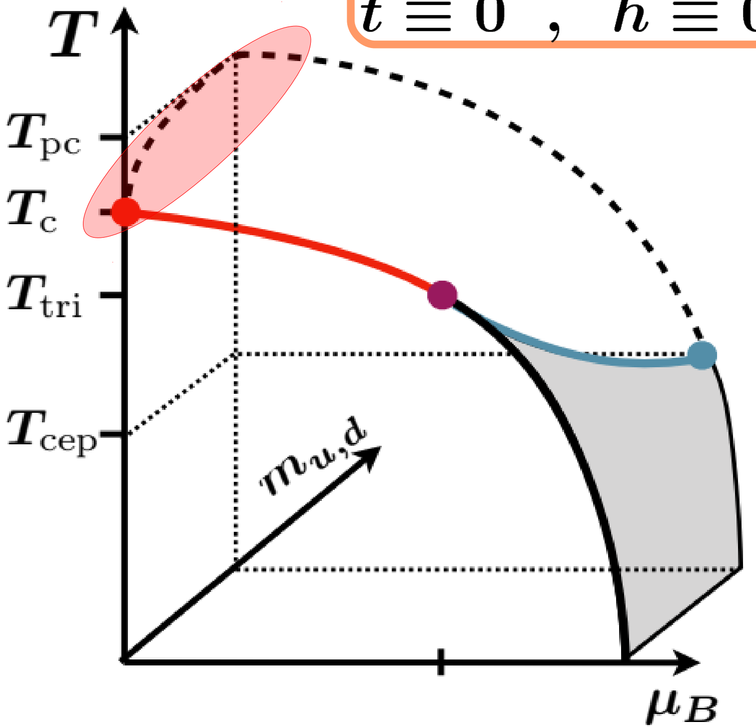
- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular — regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

critical line: $t \equiv 0 \quad , \quad h \equiv 0$

$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2 \quad , \quad h \sim \frac{m_q}{T_c}$$



question: **Where is the chiral PHASE TRANSITION for $m_u = m_d = 0$ located?**

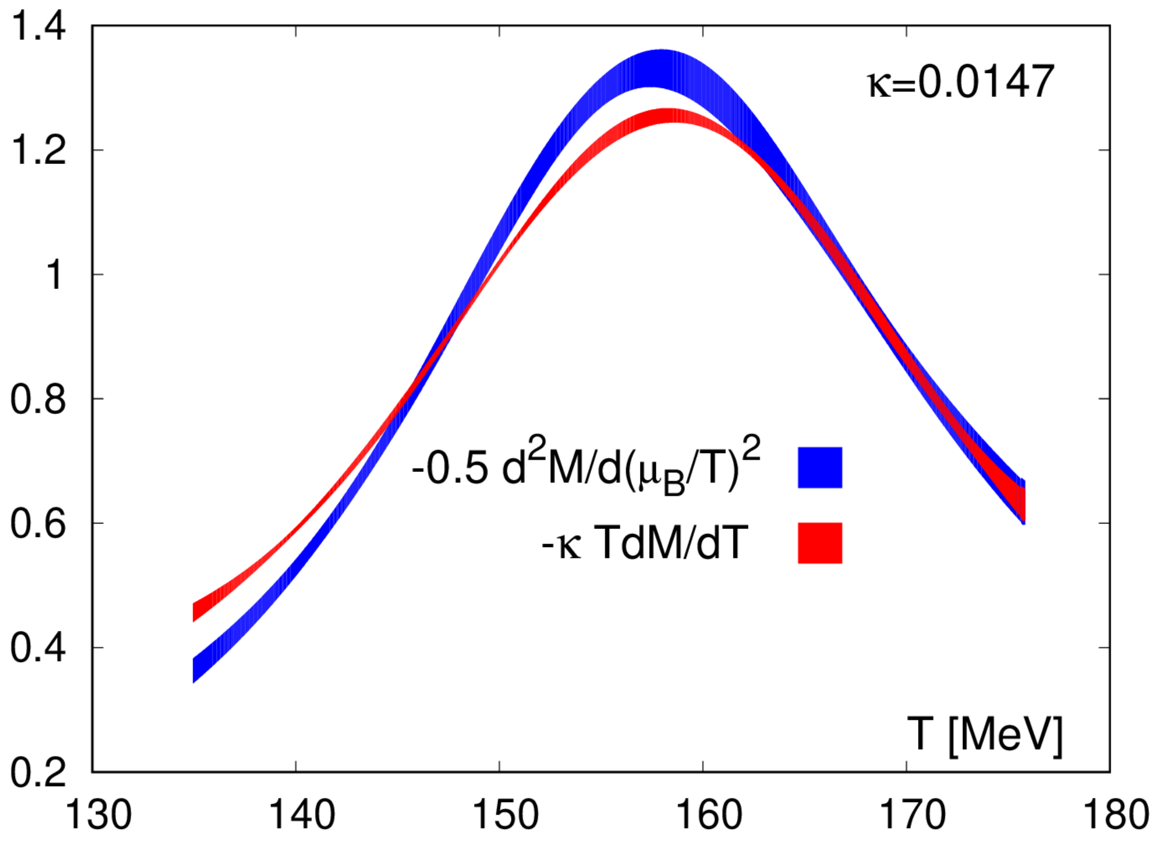
here only: $\mu_q \equiv 0$

Critical behavior and higher order cumulants

critical behavior in chiral observables: the T-derivative of the chiral condensate

a mixed susceptibility

$$\Delta_{ls}(T,\mu_B) = \Delta_{ls}(T,0) + \frac{1}{2}\frac{\partial^2 \Delta_{ls}}{\partial(\mu_B/T)^2}\bigg|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$



$$\frac{\partial^2}{\partial(\mu_B/T)^2} \simeq \frac{\partial}{\partial T}$$

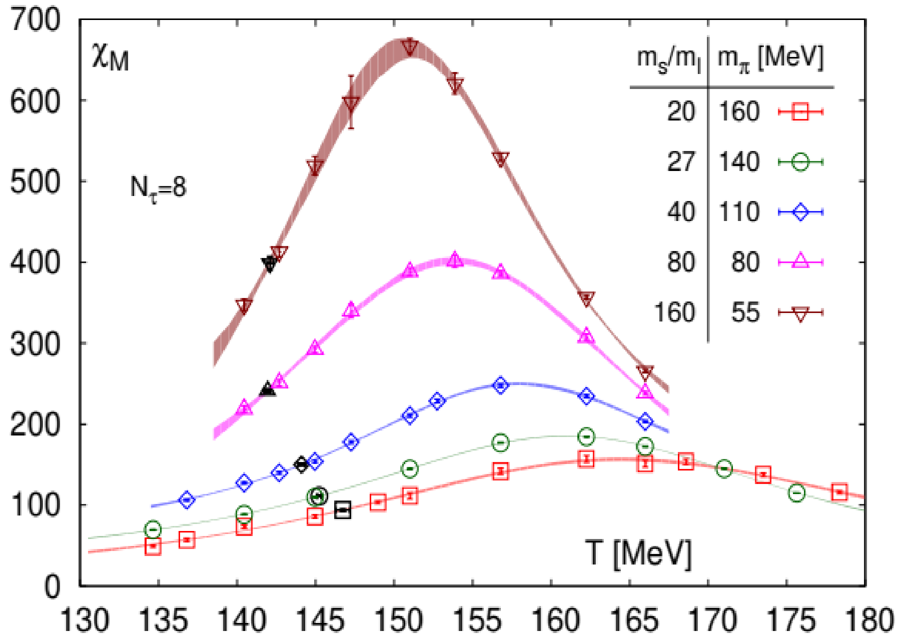
$$t \sim \frac{T-T_c}{T_c} + \kappa_2\left(\frac{\mu_B}{T}\right)^2$$

curvature of crossover line

Chiral PHASE TRANSITION in (2+1)-flavor QCD

A. Lahiri et al, QM 2018, arXiv:1807.05727
H.T. Ding et al (HotQCD), arXiv:1903.04801

– determination of the chiral PHASE Transition temperature



– physical strange quark mass
– vary light quark mass

$$55\ \mathrm{MeV} \le m_\pi \le 160\ \mathrm{MeV}$$

– use new estimators for pseudo-critical temperatures

$$T_\delta,\ T_{60}$$

– control finite volume effects

$$2 \le m_\pi L \le 5$$

– extrapolate to infinite volume limit and chiral limit

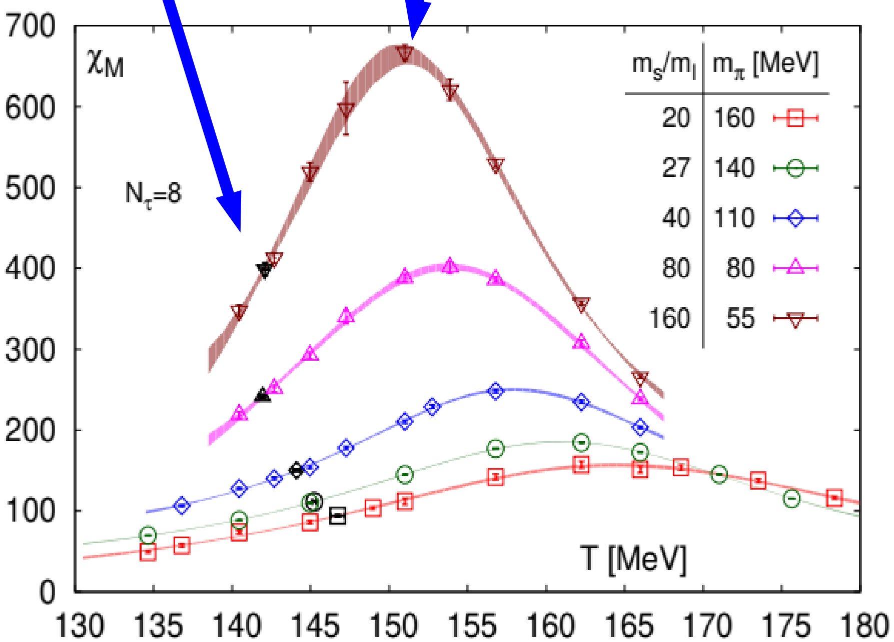
$$1/aT = 6,\ 8,\ 12$$

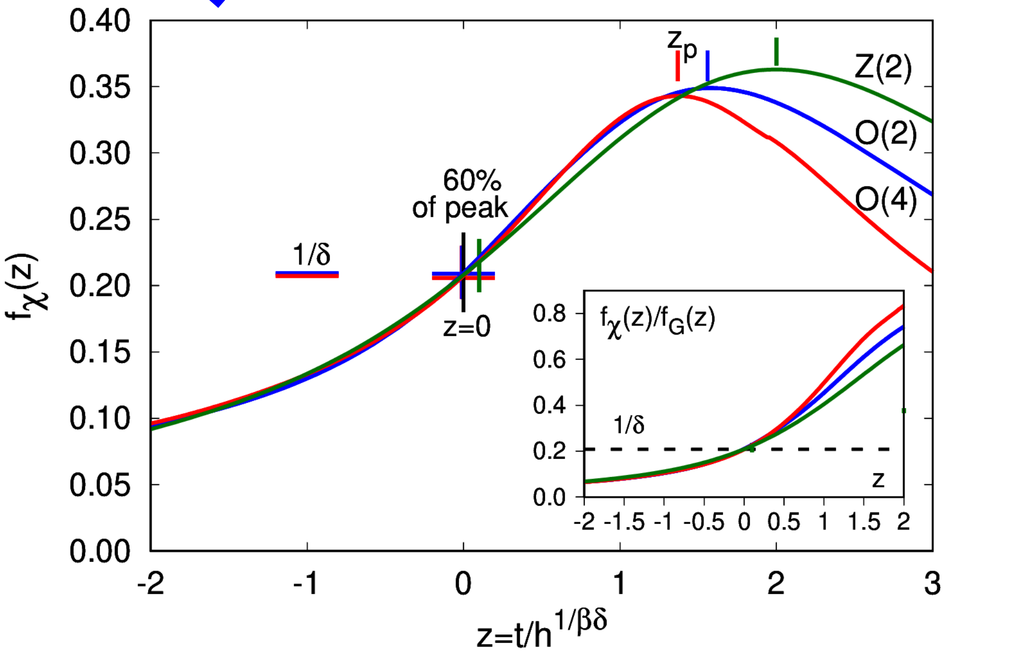
Chiral **PHASE TRANSITION** temperature

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

$z \equiv z_{60}$ $z \equiv z_p$

choose observable that "minimizes" the influence of H>0 correction





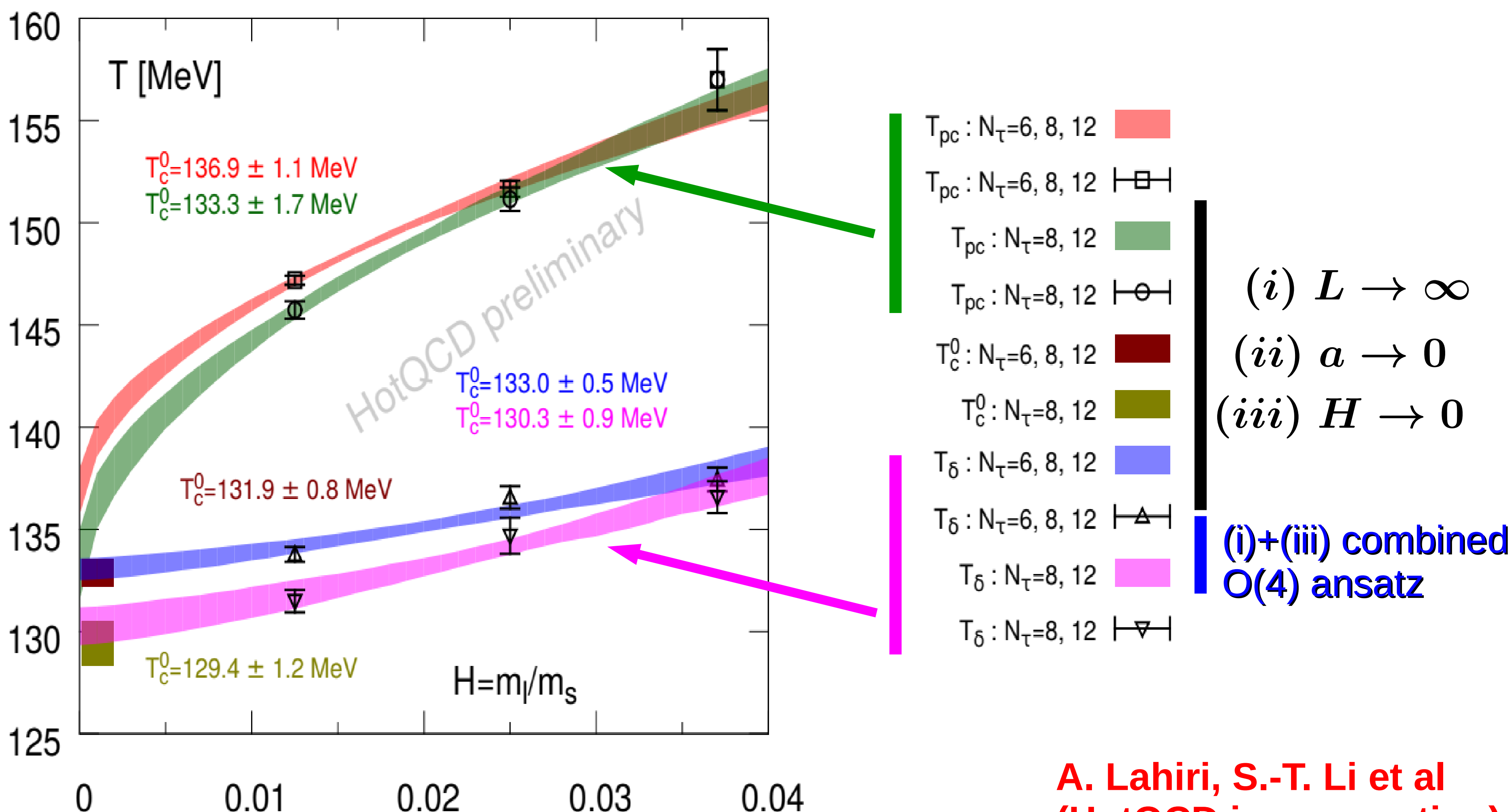
T_{60} temperature at which χ_M reaches 60% of its maximal value

estimators for $\sim T_c^0$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z)}{f_G(z)} + \text{regular} = \begin{cases} 1/\delta & , z = 0 \\ \sim 0.5 & , z = z_p \end{cases} \Rightarrow T_\delta$$

Finite size & and quark mass scaling

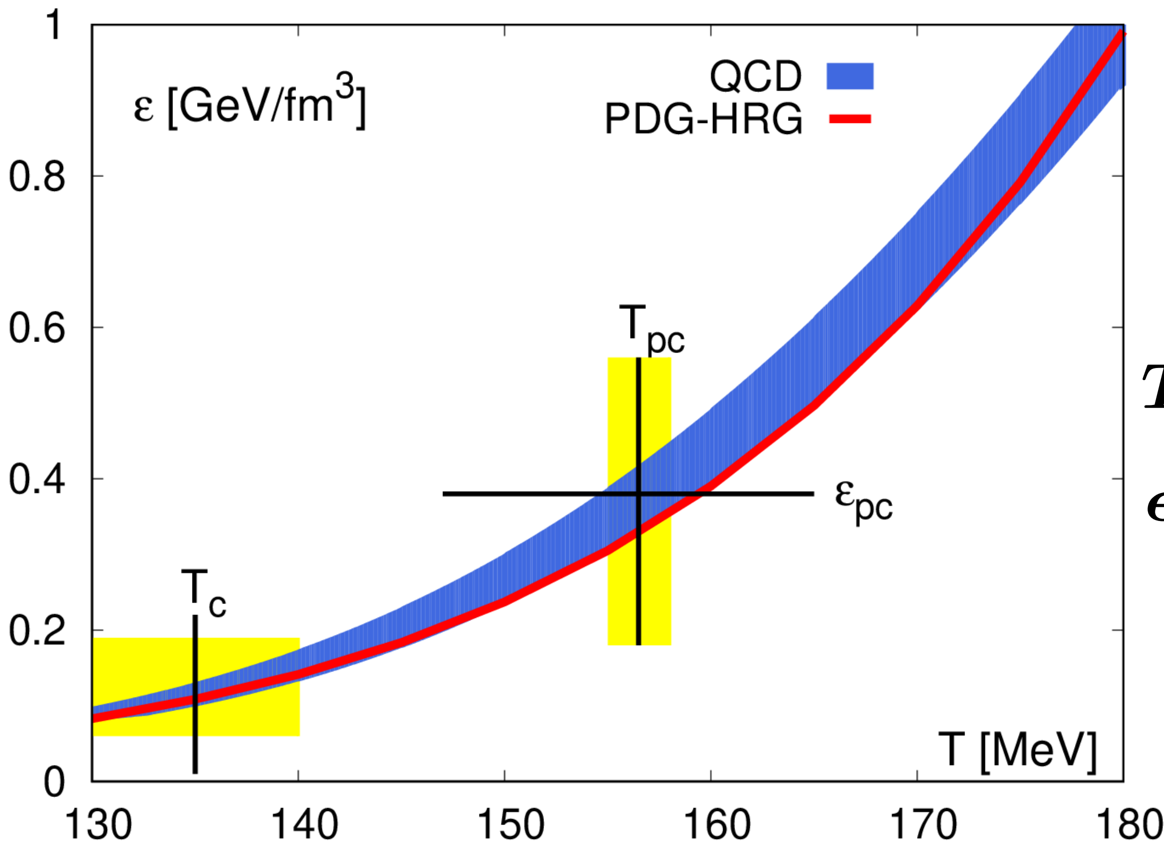
$$T_{\delta,pc}(H,L) = T_c^0 \left(1 + \frac{z_{\delta,pc}(z_L)}{z_0} H^{1/\beta\delta}\right) + \text{ sub leading}$$



A. Lahiri, S.-T. Li et al (HotQCD in preparation)

Crossover transition parameters
– and chiral limit –

PDG: Particle Data Group hadron spectrum



$\mu_B/T = 0$

physical quark masses

$T_{pc} = (156.5 \pm 1.5)$ MeV

$\epsilon_{pc} = (0.42 \pm 0.06)\mathrm{GeV/fm}^3$

chiral limit

$T_c = 132^{+3}_{-6}$ MeV

$\epsilon_c \simeq 0.15(5)$ GeV/fm^3

compare with:

$\epsilon^{\text{nucl. mat.}} \simeq 150$ MeV/fm^3

$\epsilon^{\text{nucleon}} \simeq 450$ MeV/fm^3

A. Bazavov et al. (HotQCD) ,
Phys. Rev. D90 (2014) 094503
and arXiv:1812.08235

Transition parameters in the chiral limit

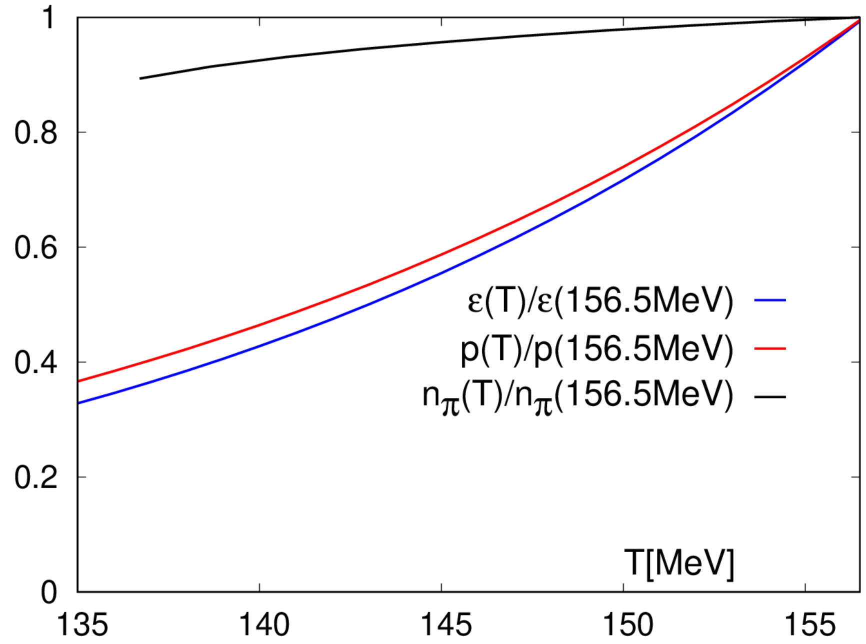
What drives the chiral transition?

– hadron resonance gas in the interval (132-156.5) MeV
– pion mass varies from 0 to its physical values

in the range $T \simeq (130 - 156.5)$ MeV :

contributions to total energy density and pressure change by a factor 3
but, pion density stays roughly constant

$$n_\pi \simeq 0.12/\mathrm{fm}^3$$



Conclusions

– no evidence for a 1st order transition in QCD for pion masses $m_\pi \geq 55$ MeV

– the chiral phase transition in QCD is **likely to be 2nd order**

– the chiral phase transition is (20-25) MeV smaller than the pseudo-critical temperature for physical values of the quark masses

$$T = 132^{+3}_{-6}\mathrm{MeV}$$

– the chiral phase transition occurs at a pion density

$$n_\pi \simeq 0.12/\mathrm{fm}^3$$

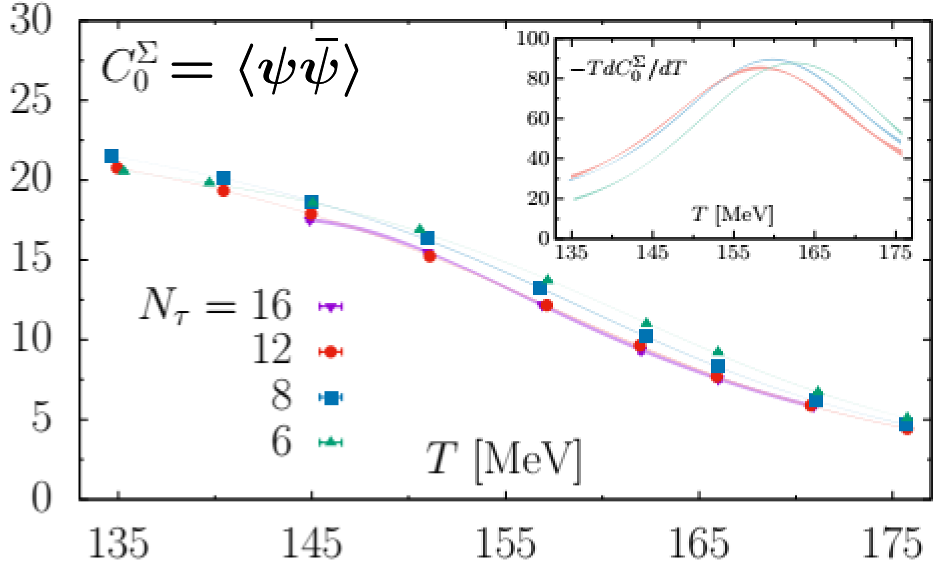
– basic structure of skewness and kurtosis ratios measured by STAR follows pattern found in lattice QCD calculations; hyper-kurtosis needs more statistics?

the new high statistics data at $\sqrt{s_{NN}} = 54.4$ GeV are exciting

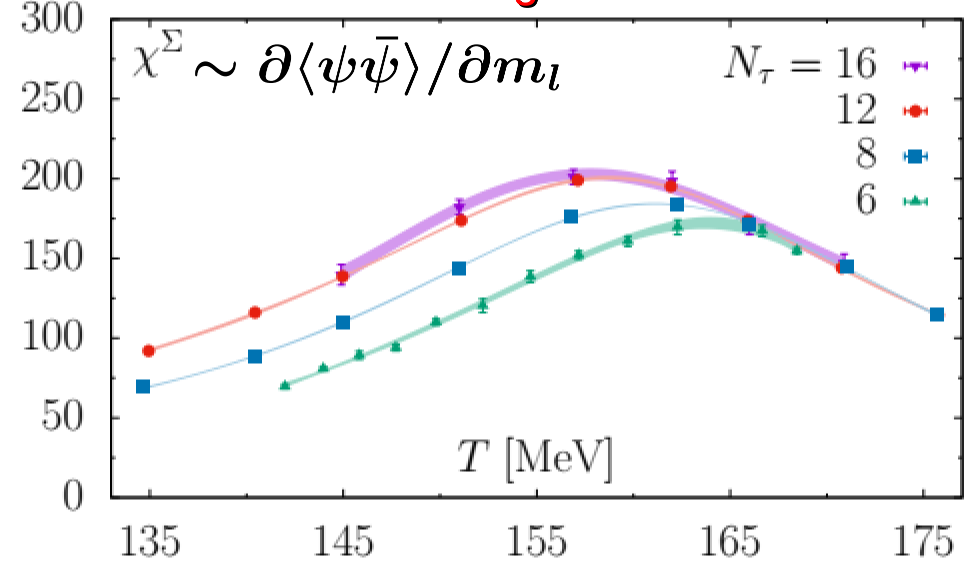
– **a critical endpoint with $T^{CEP} < 140$ MeV makes it difficult to be observed in experimental searches at RHIC (in collider mode)**

Pseudo-critical temperatures from chiral observables

mixed

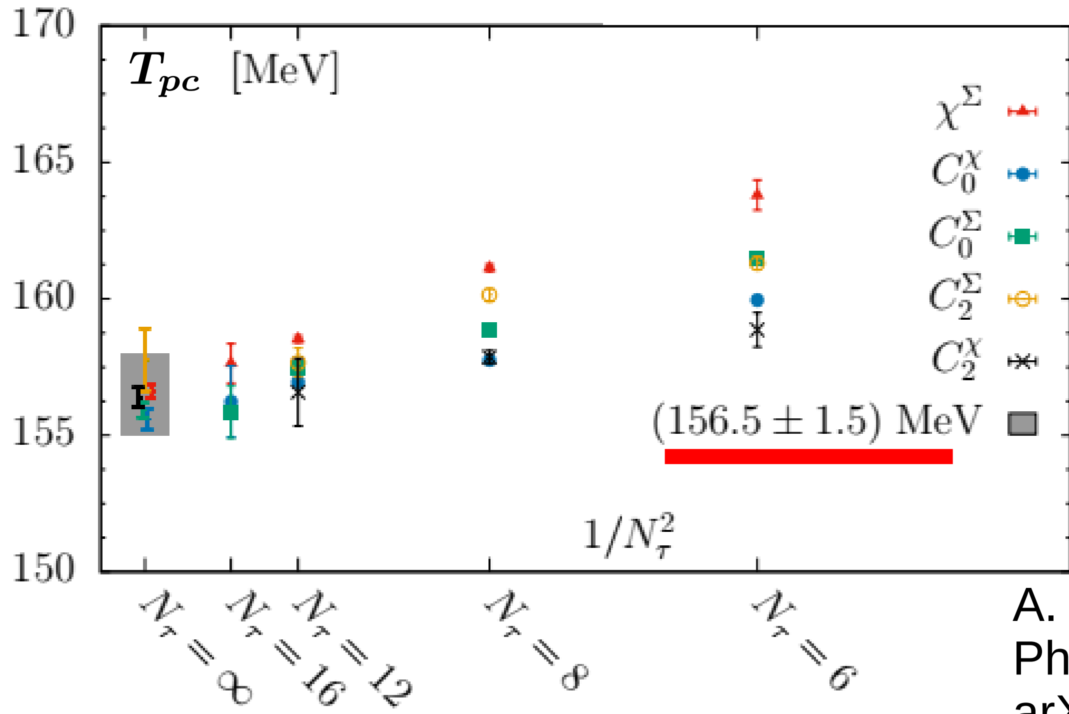


magnetic



physical
light & strange
quark masses;

continuum
extrapolated



$M \equiv \langle \psi\bar{\psi} \rangle^{n_f=2}$

$\chi^\Sigma \Leftrightarrow \chi_M$

$C_0^\Sigma \Leftrightarrow T(\partial M/\partial T)$

$C_2^\Sigma \Leftrightarrow \partial^2 M/\partial(\mu_B/T)^2$

A. Bazavov et al [HotQCD], Phys. Lett. B795, 15 (2019), arXiv:1812.08235