Identifying critical behavior in heavy-ion collision observables: A mean-field hadronic transport approach

Agnieszka Wergieluk (UCLA/LBNL)

in collaboration with Volker Koch (LBNL)

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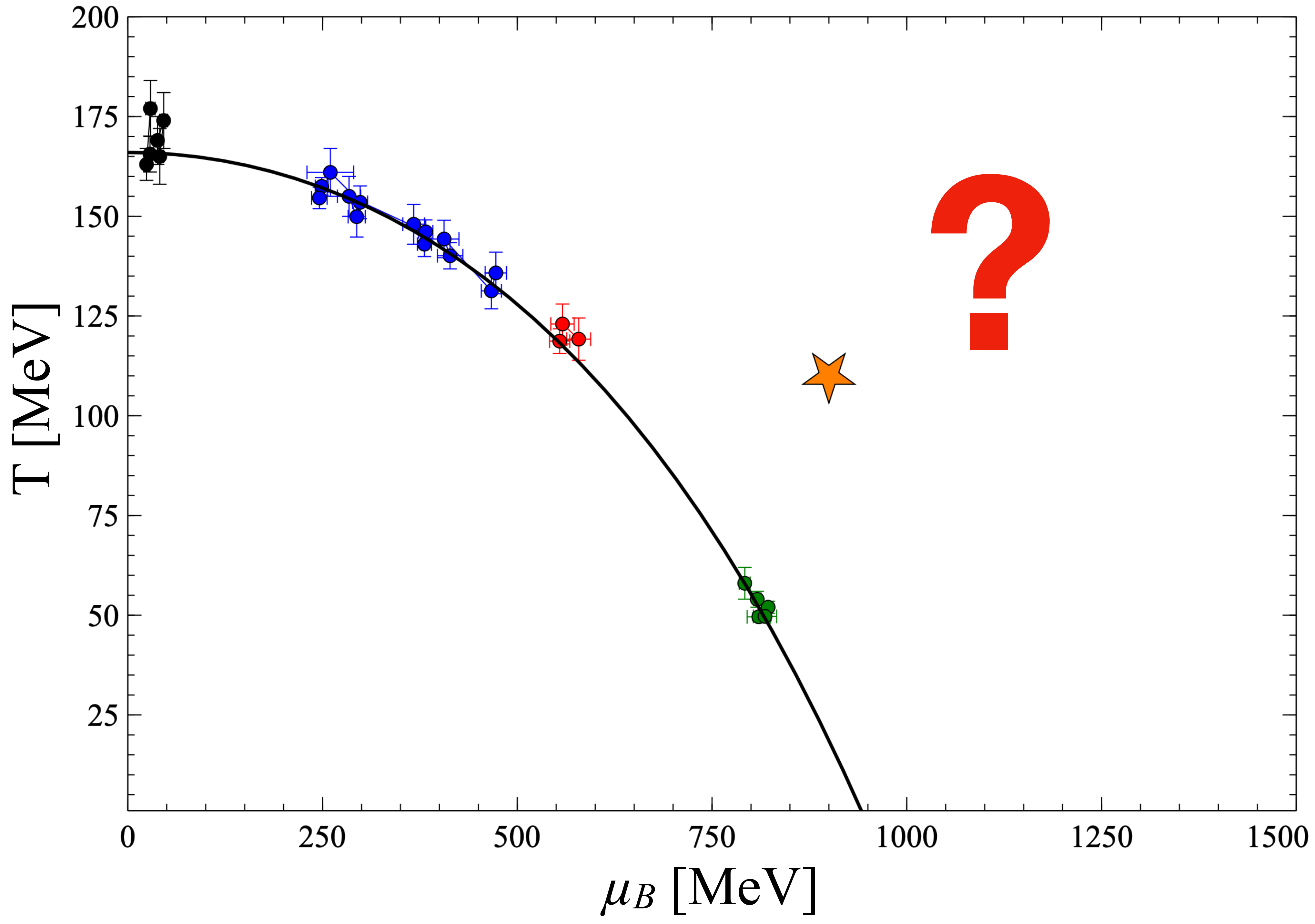
2020 Fall Meeting of the APS Division of Nuclear Physics

Motivation



Is there a critical point? Where is it? How will we know?
How precisely can we measure it?

We try to answer by comparing simulations with data

We need: thermodynamics (hydrodynamic evolution)
and particle dynamics (hadronic afterburner)

Hadronic phase is ever more important for low energies:
how is it affected by the QCD equation of state (EOS)?

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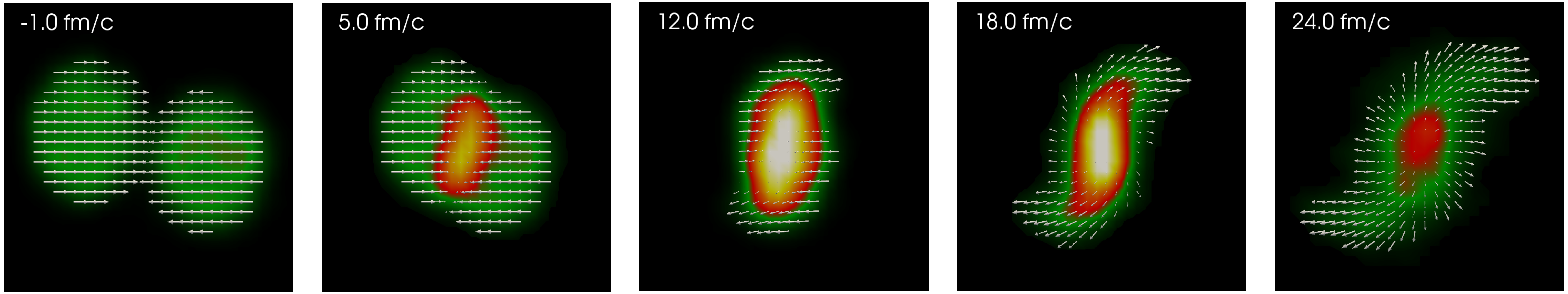

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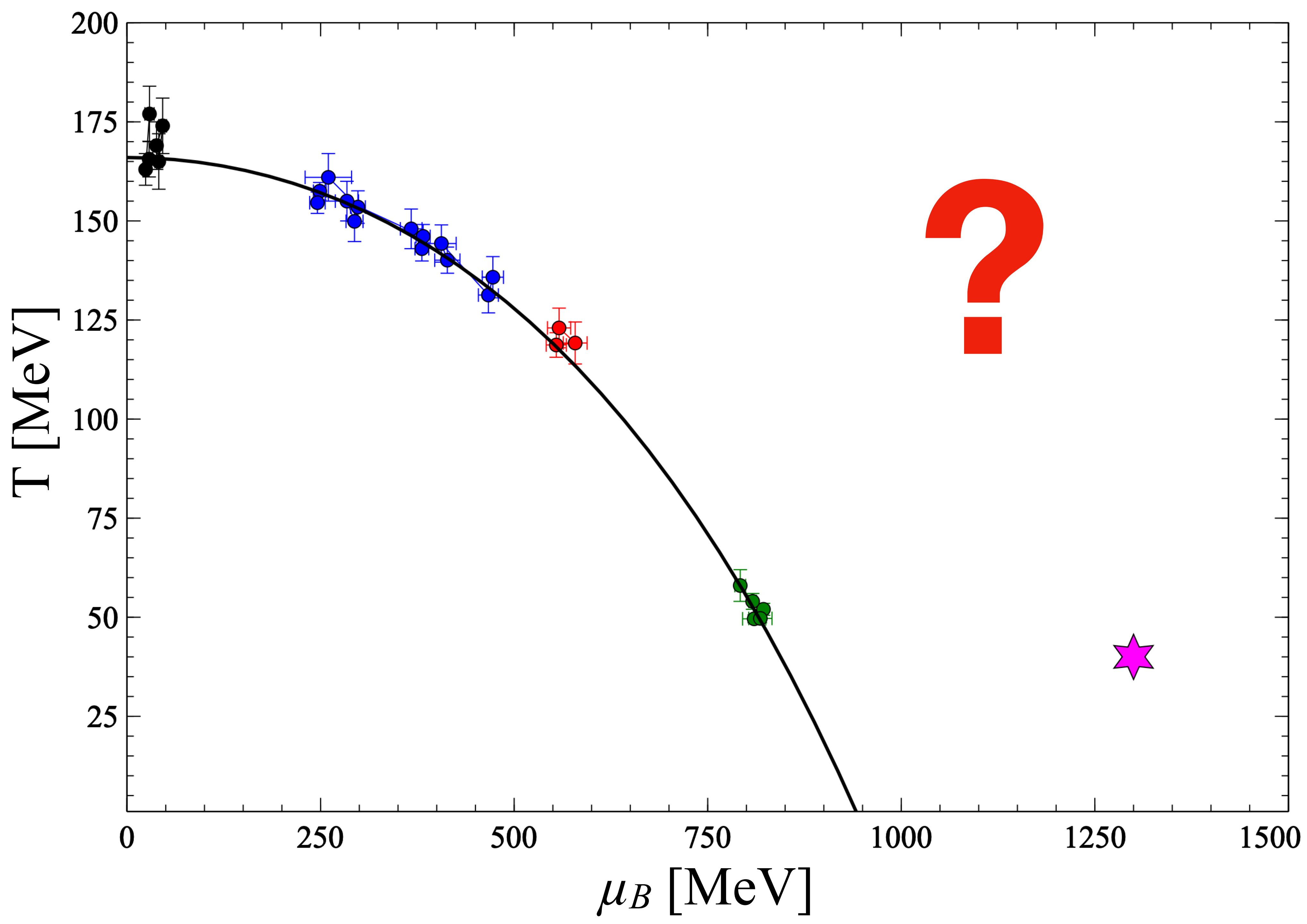

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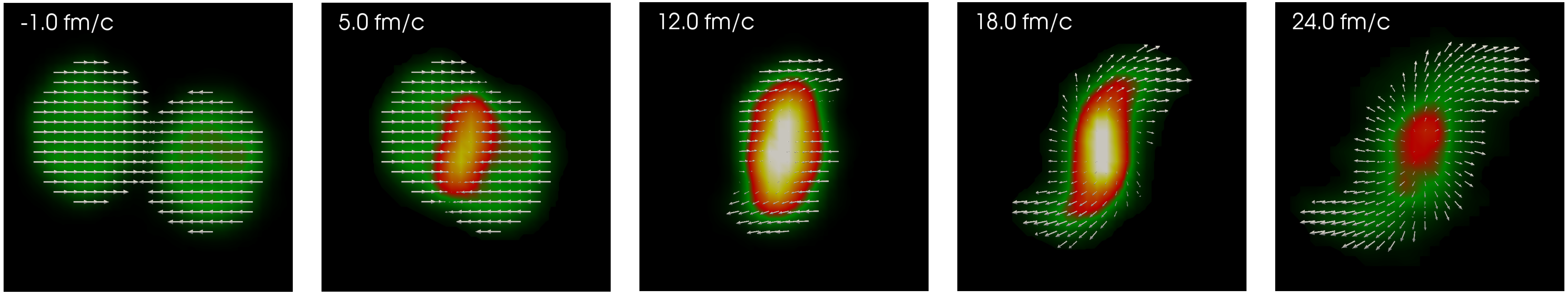

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- Flexible and systematic parameterization of the EOS of dense QCD matter
- Access to both thermodynamics and single-particle equations of motion for hadronic transport
- Relatively simple, baryon-number-density—dependent interactions: easy to use in hadronic transport (nucleons as the degrees of freedom, mean field level interactions only)

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We do not want to build a model that predicts the critical point.

We construct a consistent, flexible parameterization of critical behavior in hot and dense nuclear matter to use in simulations and compare the results with data.

Relativistic DF with vector-number-density—based interactions

G. Baym and S. A. Chin, "Landau Theory of Relativistic Fermi Liquids," Nucl. Phys. A **262**, 527 (1976)

1) Postulate the energy density of the system:

$$\mathcal{E} = \mathcal{E}[f_{\mathbf{p}}] = \int \frac{d^3p}{(2\pi)^3}\, \epsilon_{kin}\, f_{\mathbf{p}} \; + \; \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1} \left[j^0 j^0 - g^{00}\left(\frac{b_i - 1}{b_i}\right) j_\lambda j^\lambda \right]$$

← like Skyrme energy, but Lorentz covariant

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2) Calculate the quasiparticle energy and obtain equations of motion (EOMs):

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3) Use the Boltzmann equation + EOMs to get conservation laws, the energy-stress tensor:

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \left(\nabla_{\mathbf{p}} \varepsilon_{\mathbf{p}}\right) \cdot \left(\nabla_{\mathbf{r}} f_{\mathbf{p}}\right) - \left(\nabla_{\mathbf{r}} \varepsilon_{\mathbf{p}}\right) \cdot \left(\nabla_{\mathbf{p}} f_{\mathbf{p}}\right) = 0 \quad \longrightarrow \quad \partial_\nu T^{\mu\nu} = 0$$

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4) Obtain pressure from the energy-stress tensor: $P = \frac{1}{3}\sum_i T^{ii}\Big|_{\textbf{rest frame}}$

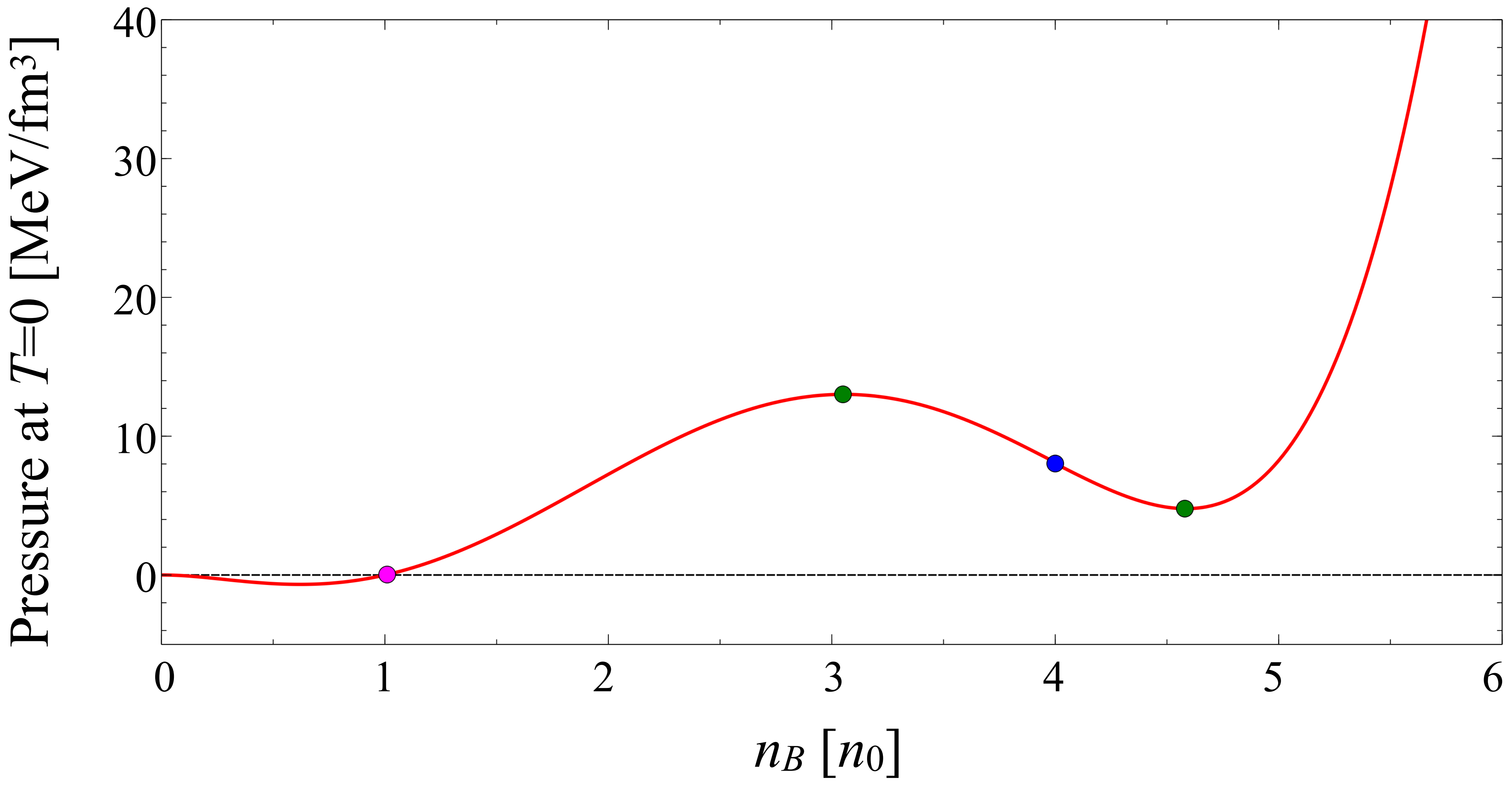
← used for parameterizing the EOS

Relativistic DF with 2 phase transitions

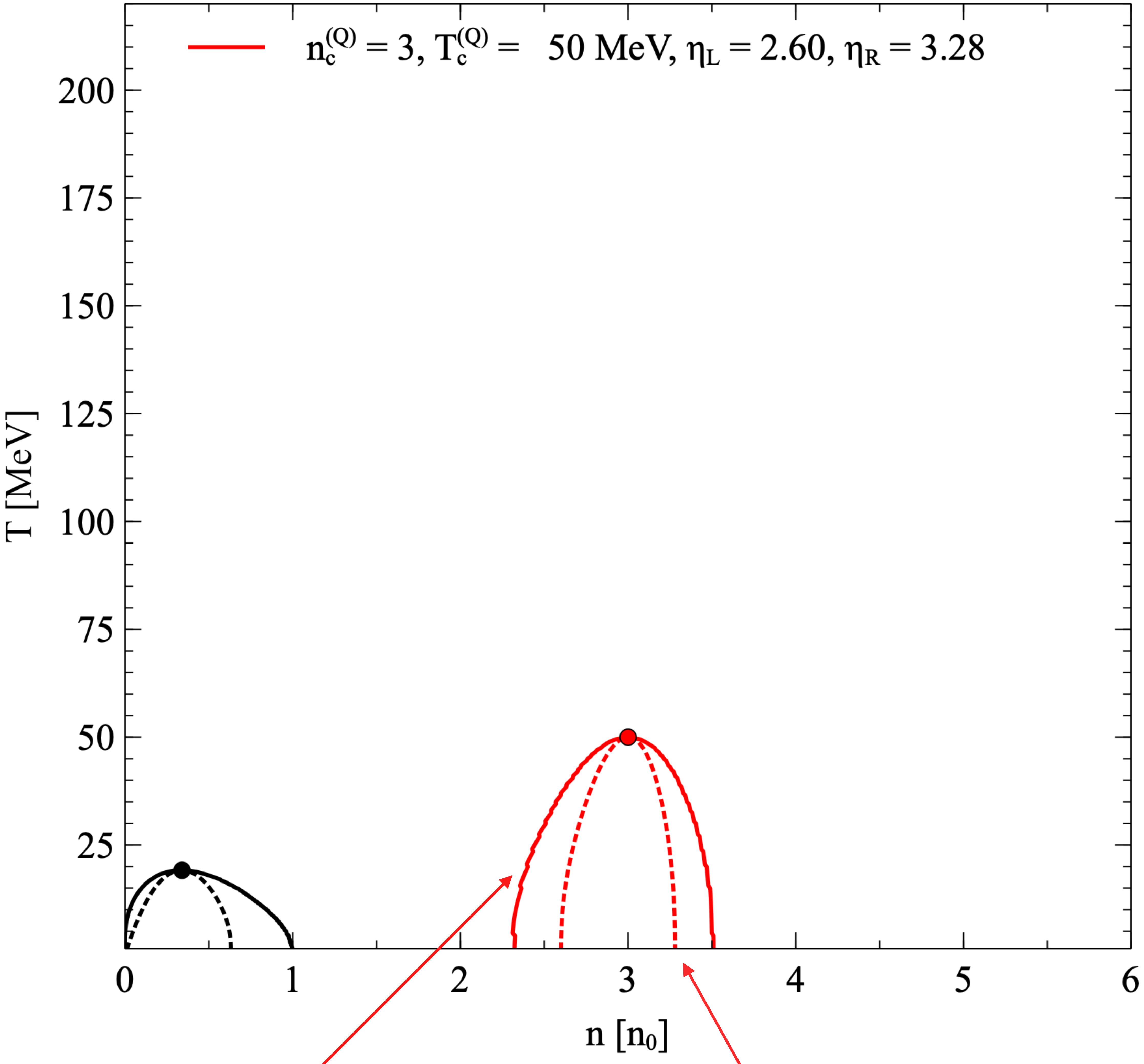
Systems with two 1st order phase transitions: nuclear and "quark/hadron"

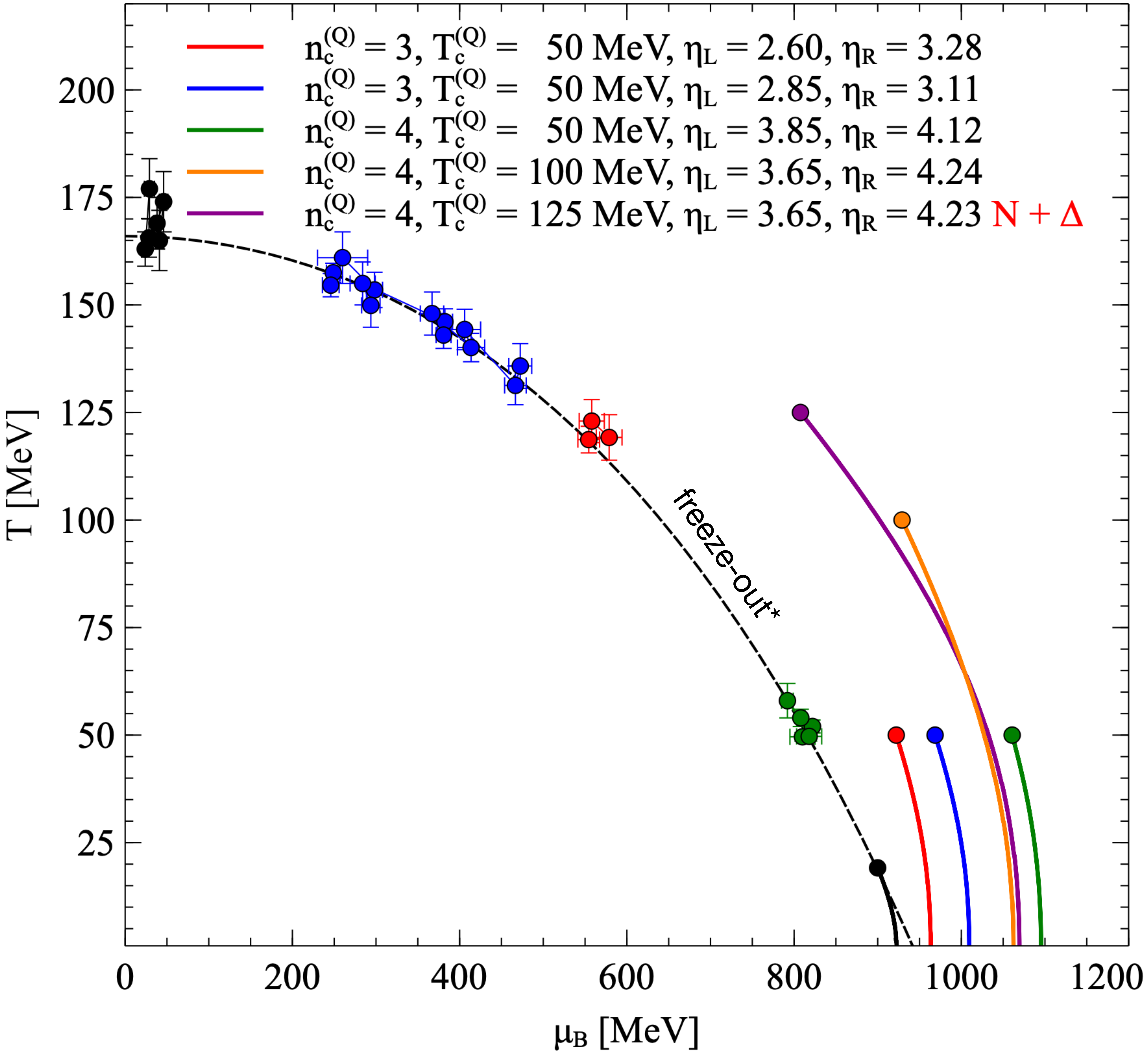
- Degrees of freedom: nucleons
- "Quark" matter coexists with dense nuclear matter, not vacuum
- 4 interactions terms = 8 parameters to fix: $\{C_1, C_2, C_3, C_4, b_1, b_2, b_3, b_4\}$

Pressure: $$P = g\int \frac{d^3p}{(2\pi)^3}\, T\, \ln\left[1 + e^{-\beta(\varepsilon_{\mathbf{p}}-\mu_B)}\right] + \sum_{i=1}^{N=4} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$



Phase diagram in the (T, n_B) and (T, μ_B) plane





solid lines = coexistence regions

dashed lines = spinodal regions

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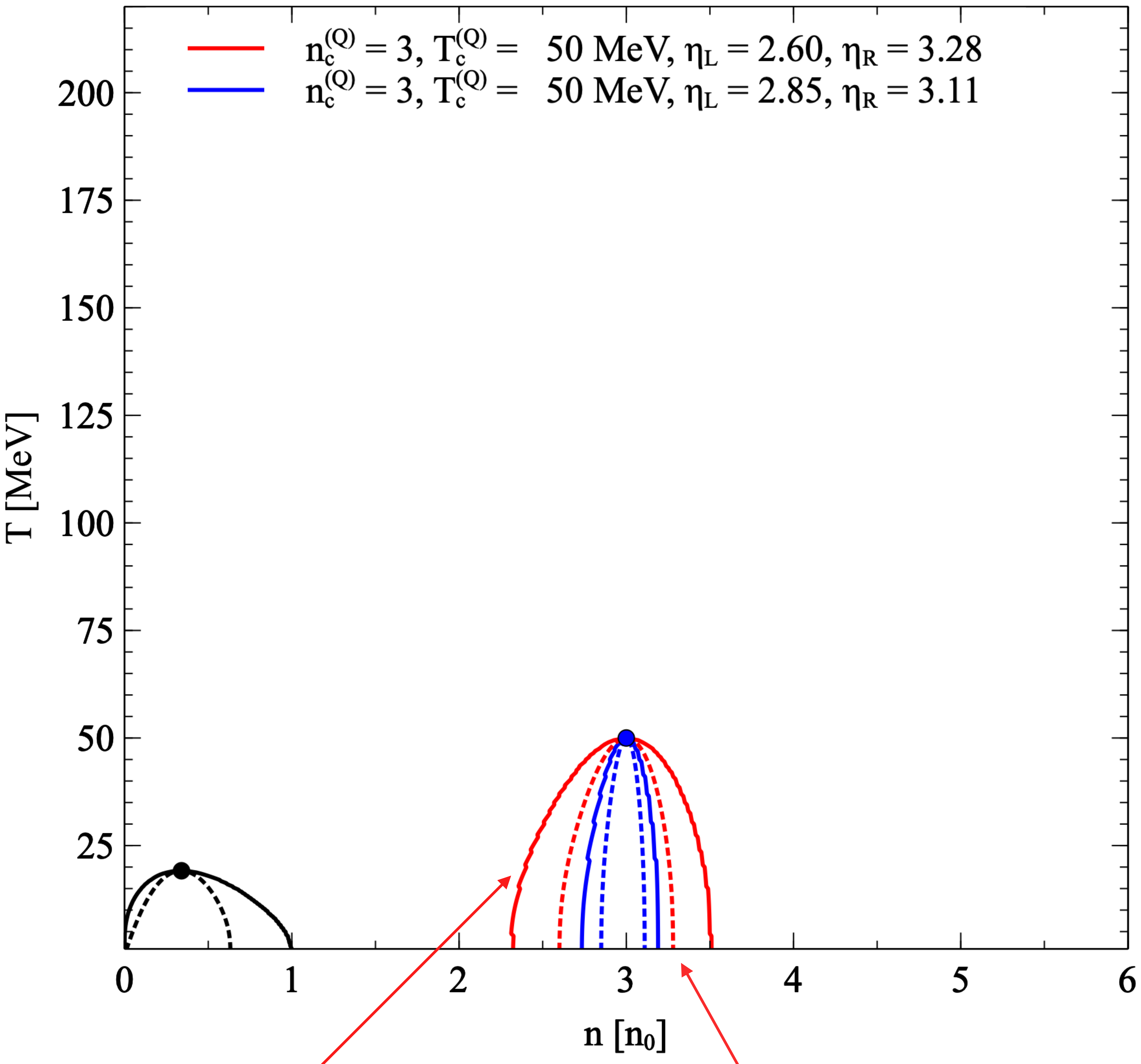

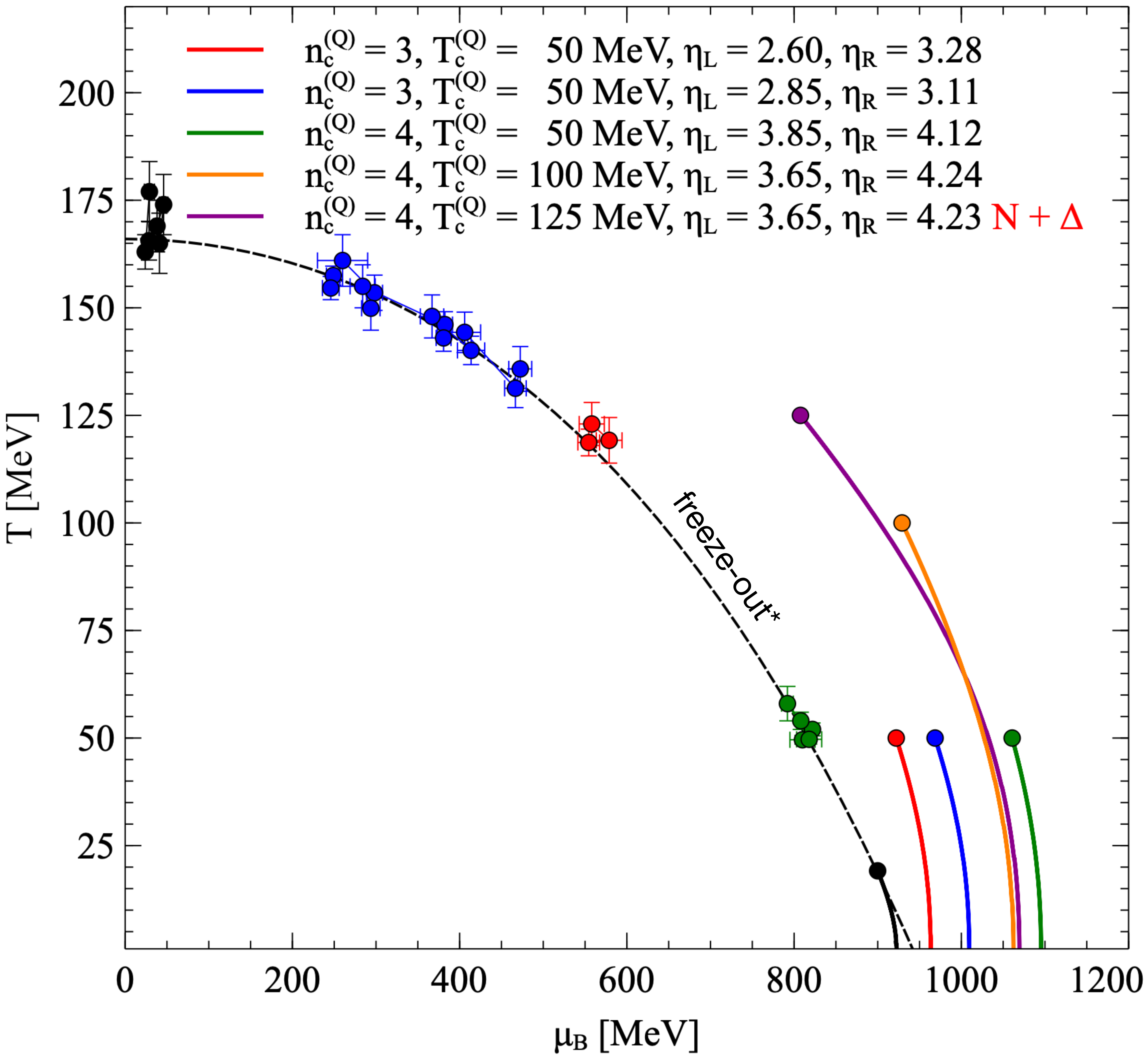

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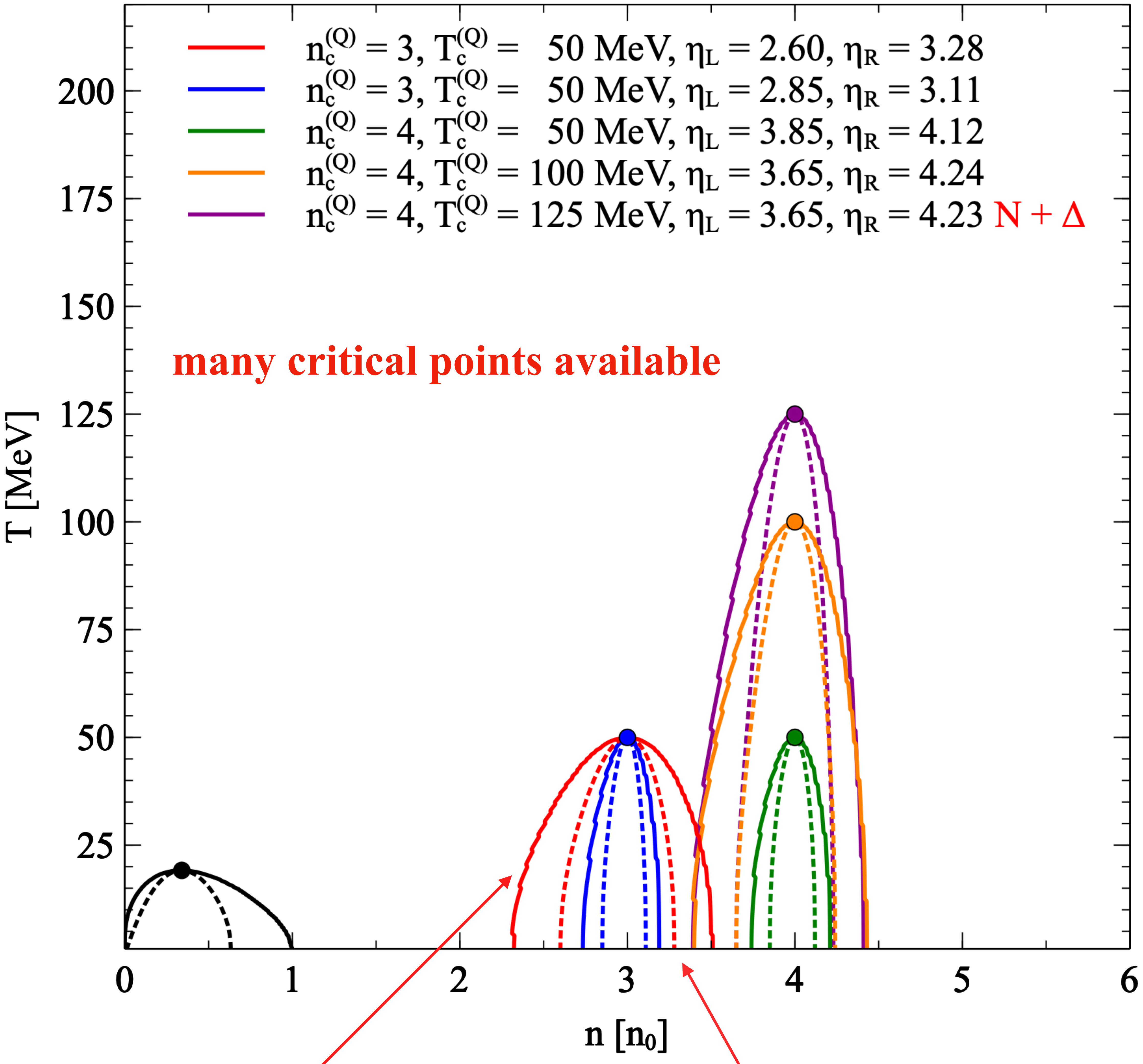

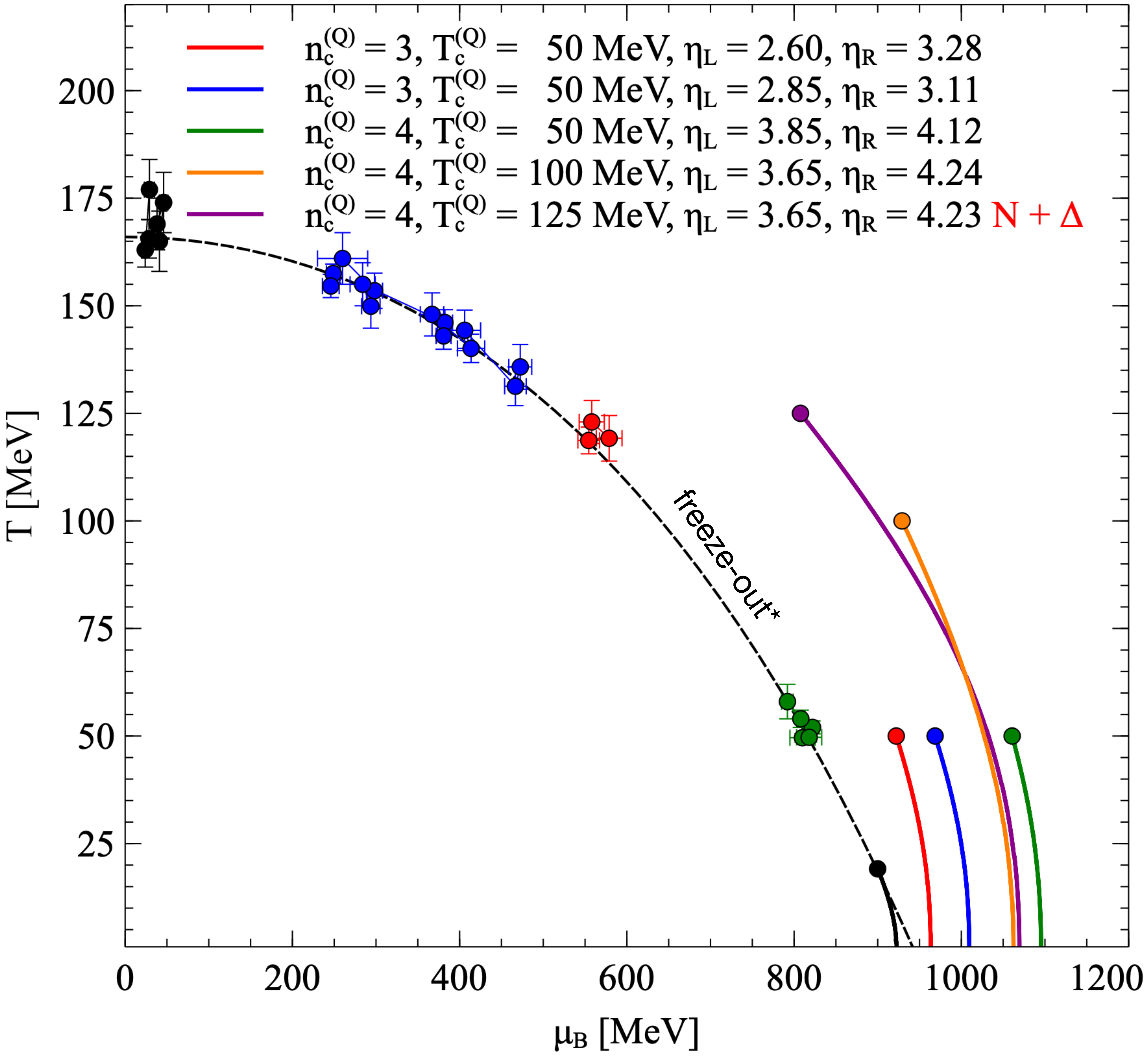

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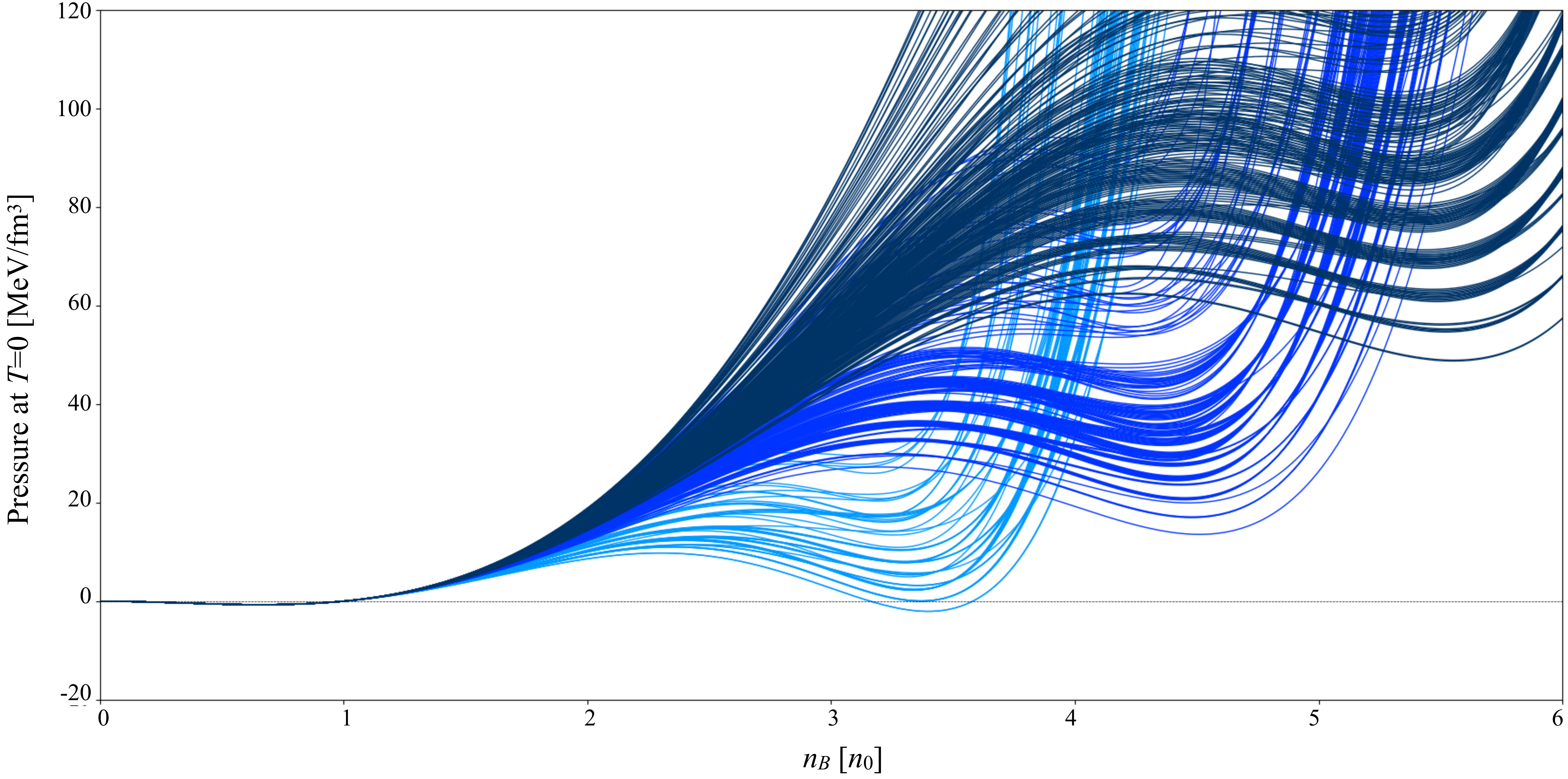
Many equations of state to try!



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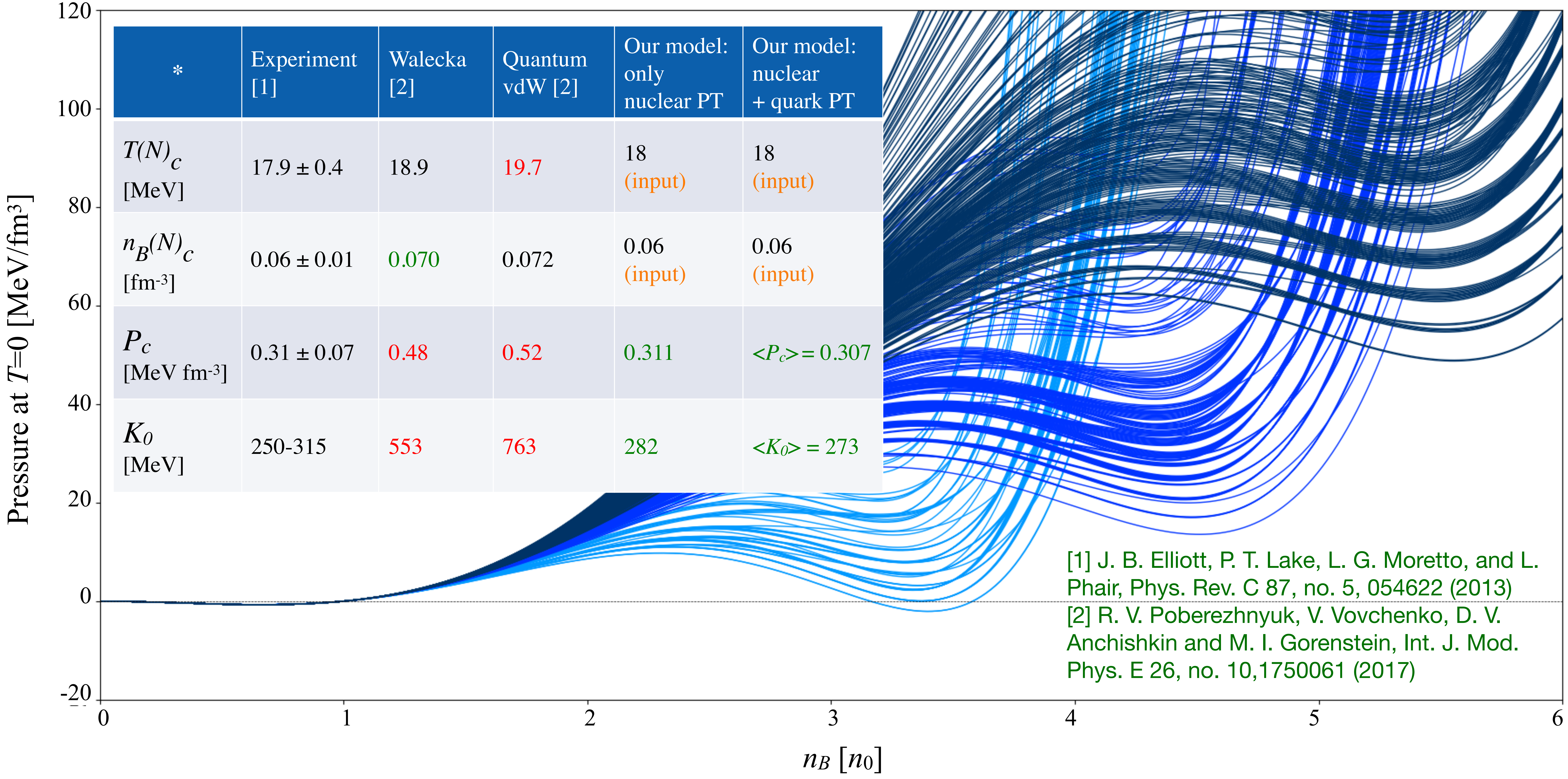
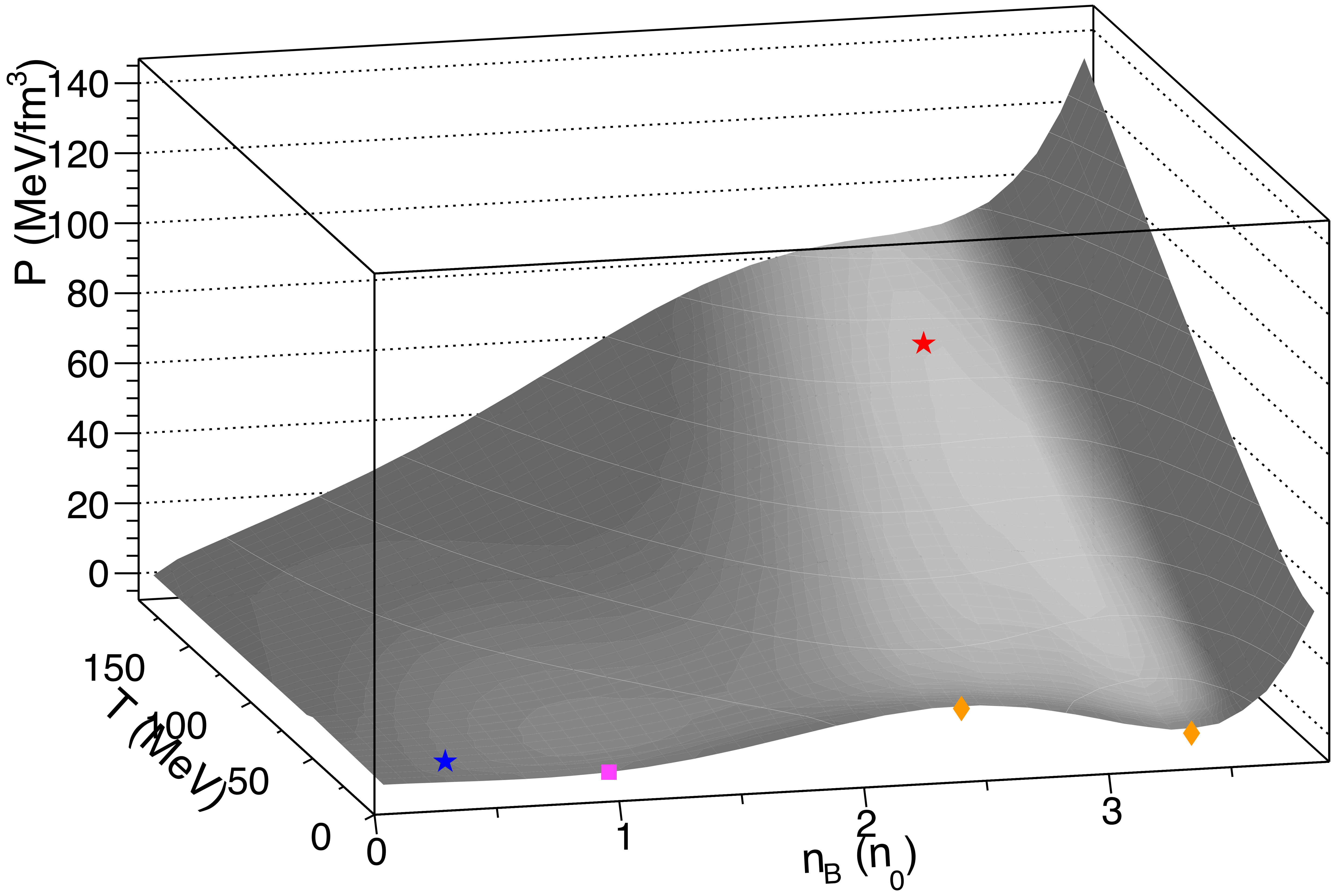


*	Experiment [1]	Walecka [2]	Quantum vdW [2]	Our model: only nuclear PT	Our model: nuclear + quark PT
$T(N)_c$ [MeV]	17.9 ± 0.4	18.9	19.7	18 (input)	18 (input)
$n_B(N)_c$ [fm^{-3}]	0.06 ± 0.01	0.070	0.072	0.06 (input)	0.06 (input)
P_c [MeV fm^{-3}]	0.31 ± 0.07	0.48	0.52	0.311	$\langle P_c \rangle$ = 0.307
K_0 [MeV]	250-315	553	763	282	$\langle K_0 \rangle$ = 273

[1] J. B. Elliott, P. T. Lake, L. G. Moretto, and L. Phair, Phys. Rev. C 87, no. 5, 054622 (2013)
[2] R. V. Poberezhnyuk, V. Vovchenko, D. V. Anchishkin and M. I. Gorenstein, Int. J. Mod. Phys. E 26, no. 10,1750061 (2017)

We choose one particular EOS to study



$n_0 = 0.160$ fm^{-3}, $E_B = -16.3$ MeV, $T_c^{(N)} = 18$ MeV, $n_c^{(N)} = 0.375 n_0$, $T_c^{(Q)} = 150$ MeV, $n_c^{(Q)} = 3 n_0$, $\eta_L = 2.4 n_0$, $\eta_R = 3.34 n_0$

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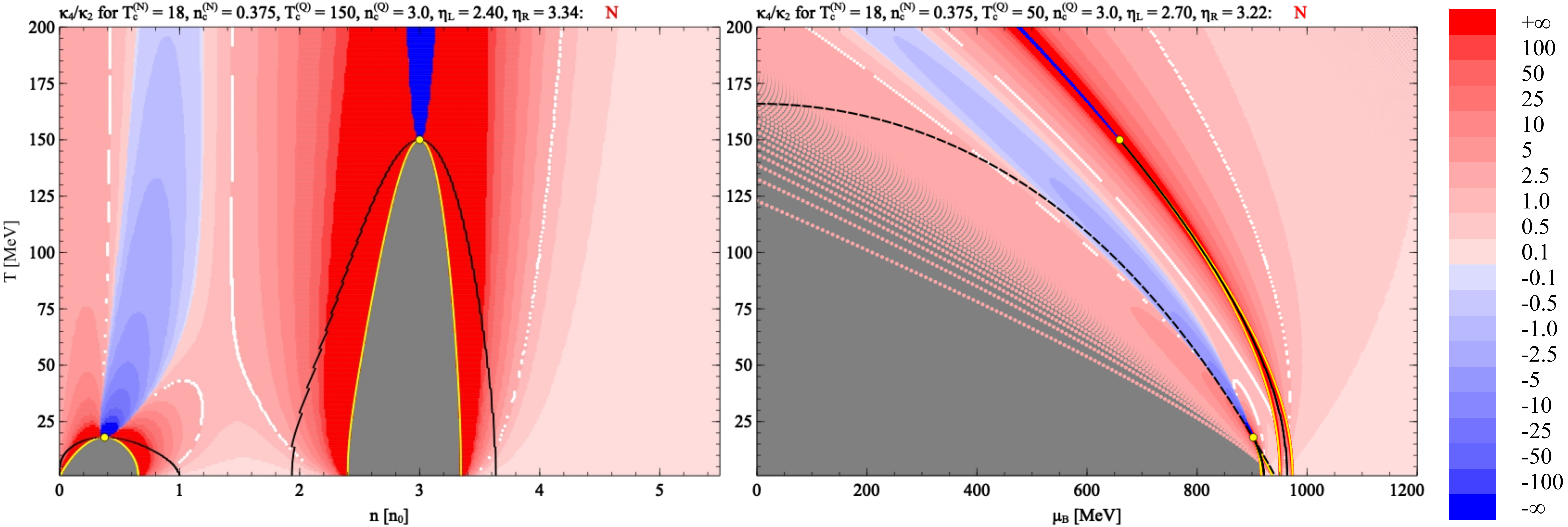

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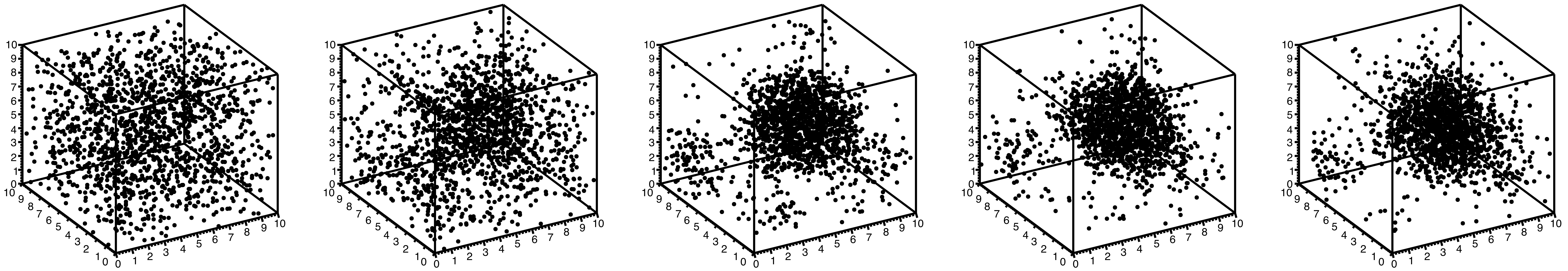
SMASH* results

We simulate dense nuclear matter in a periodic box.
First, we are going to initialize the system inside the spinodal region: super interesting and biggest effects expected!

0.0 fm/c | 50.0 fm/c | 100.0 fm/ | 150.0 fm/ | 200.0 fm/c



$L = 10$ fm, $n_B = 0.25 n_0$, $T = 1$ MeV, $N_{test} = 200$ ⟵ **Test particles are *necessary* for the density calculation**

No scattering or decays

* J. Weil et al., Phys.Rev. C94 (2016) no.5, 054905

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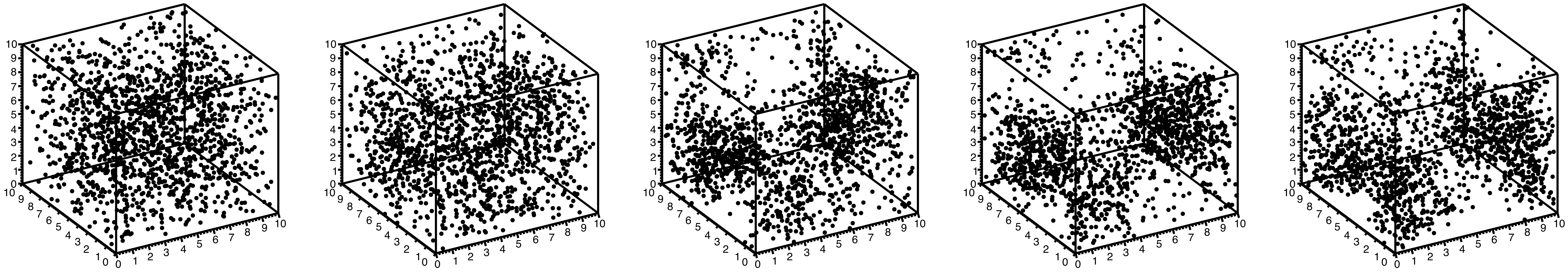

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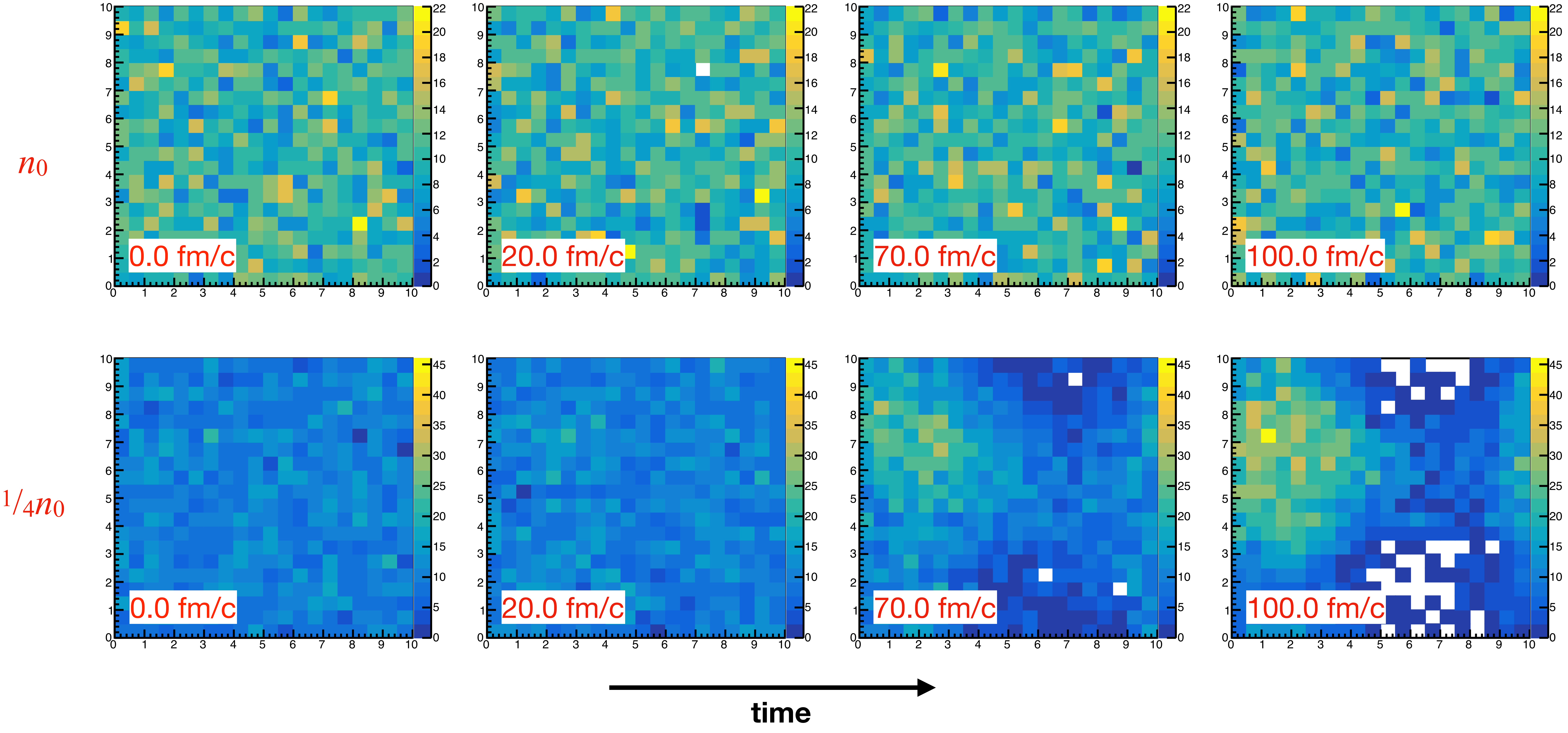

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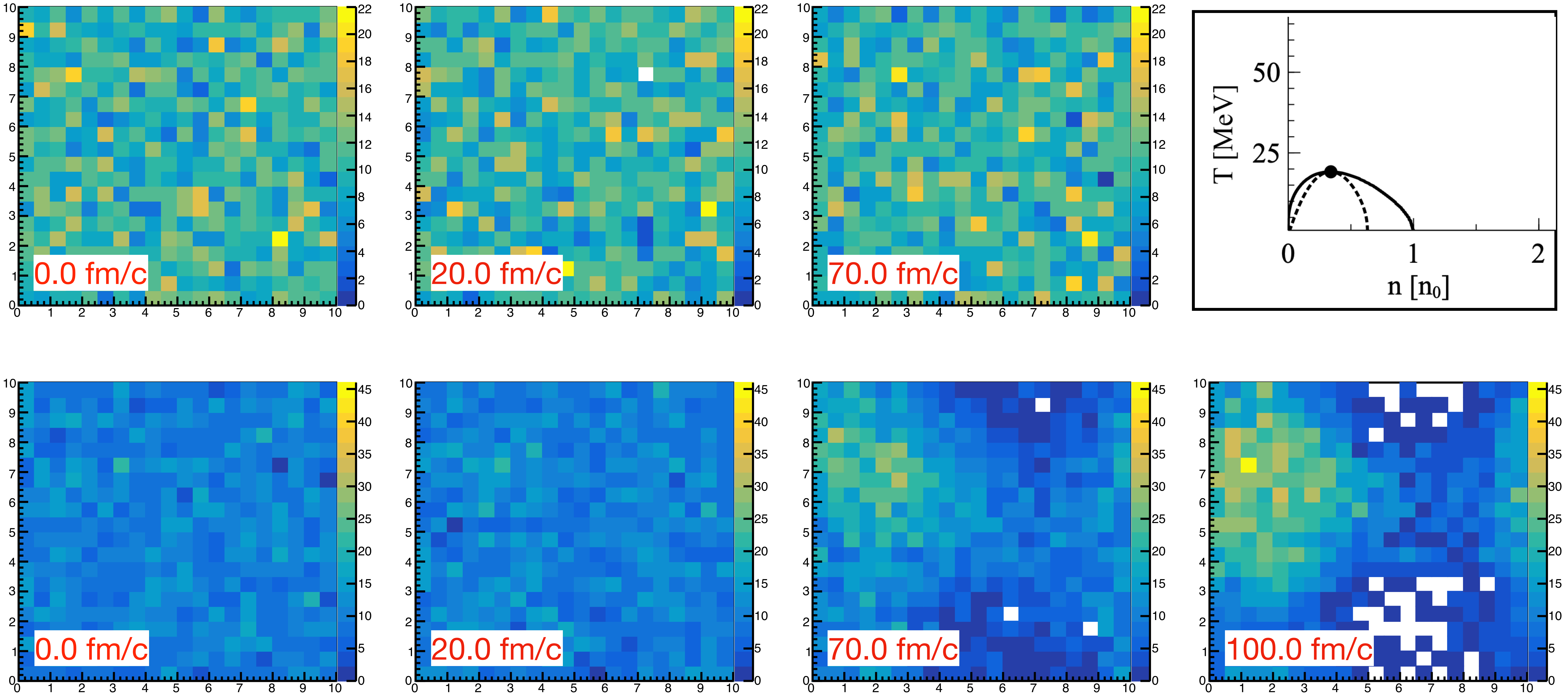

time

SMASH results: periodic box

L = 10 fm T = 1 MeV pair separation distribution (scaled for $t > 0$)

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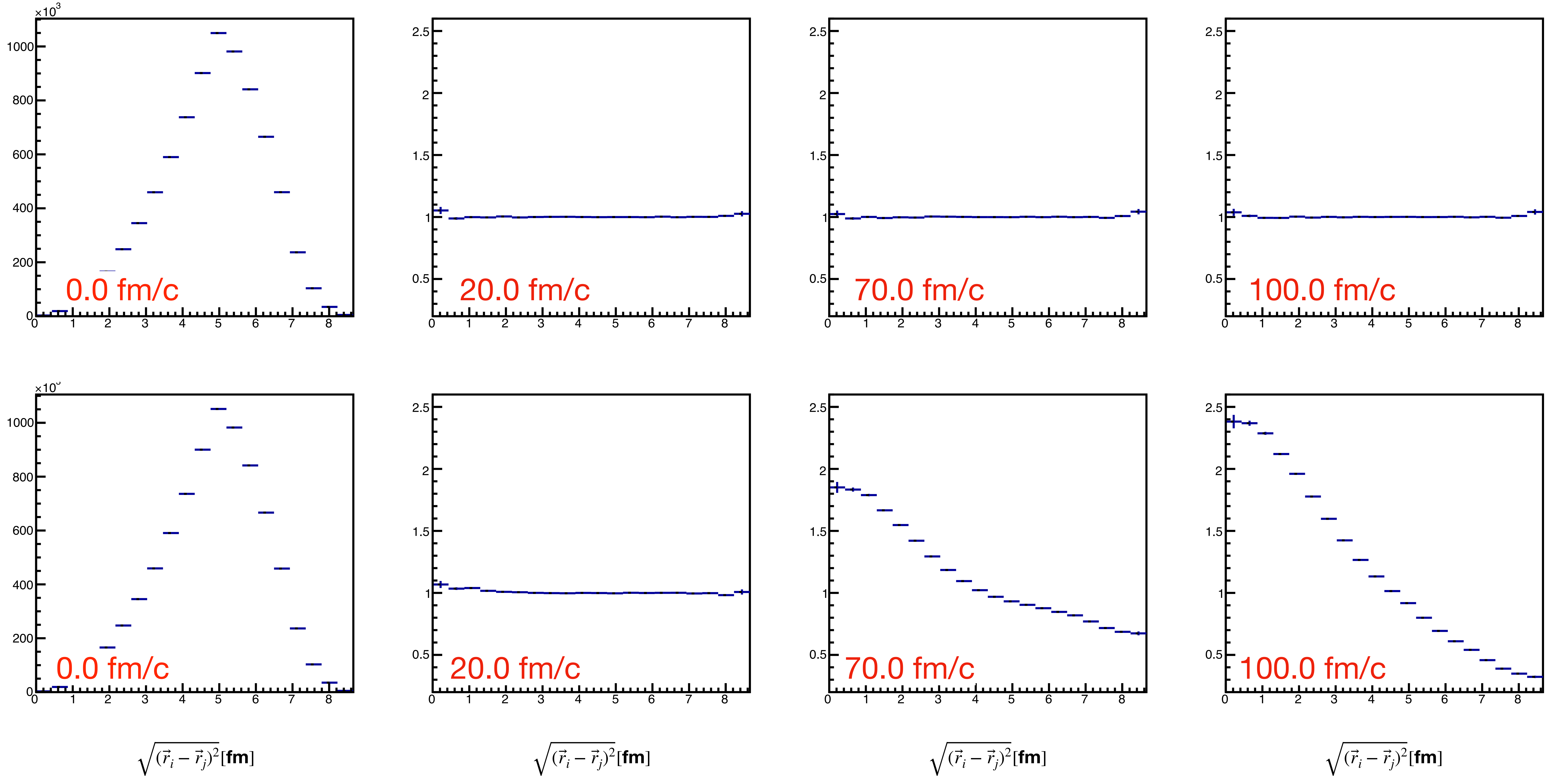

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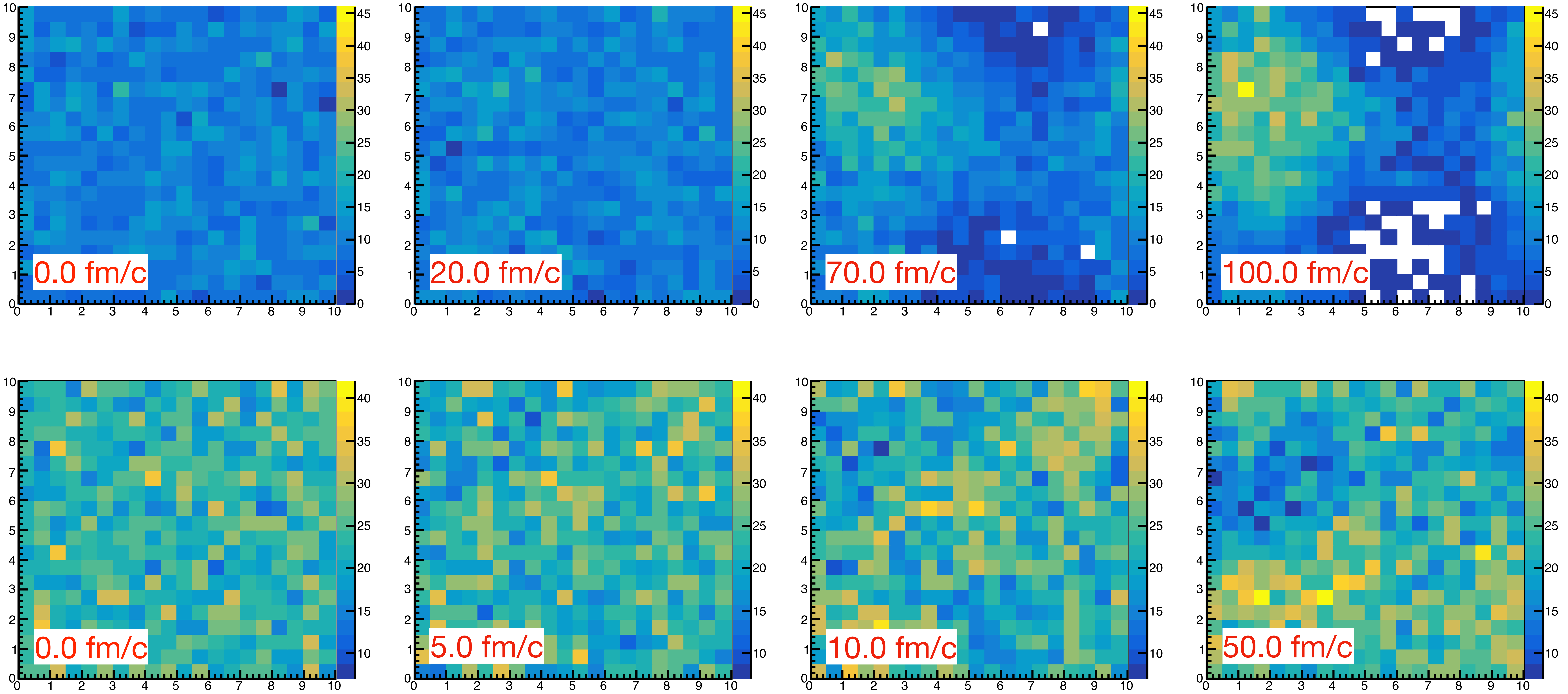

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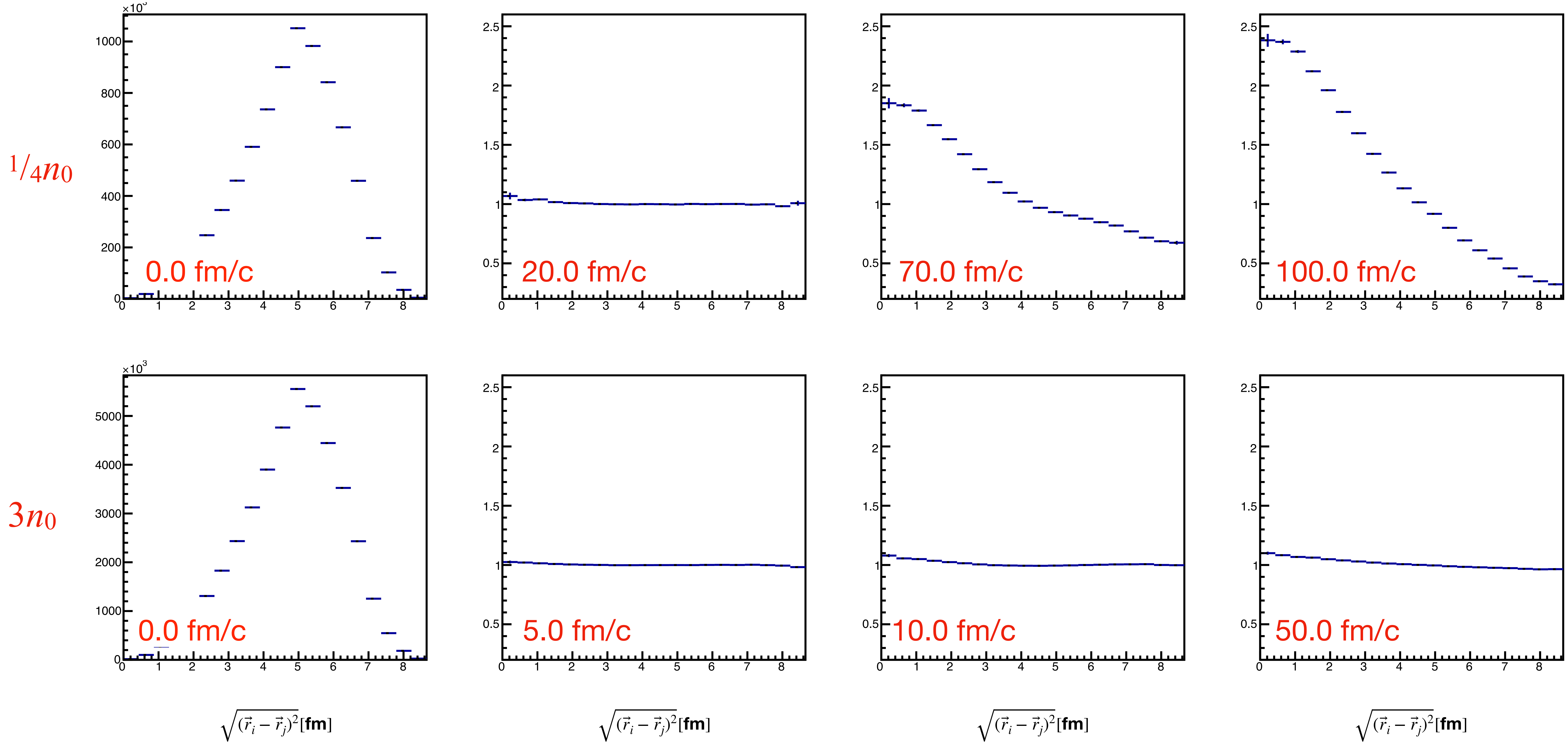
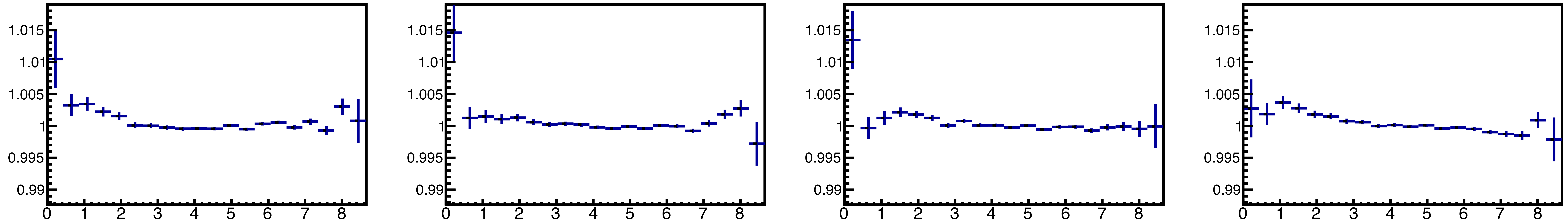


SMASH results: periodic box averaged over events

L = 10 fm T = 150 MeV pair separation distribution (scaled for $t > 0$)

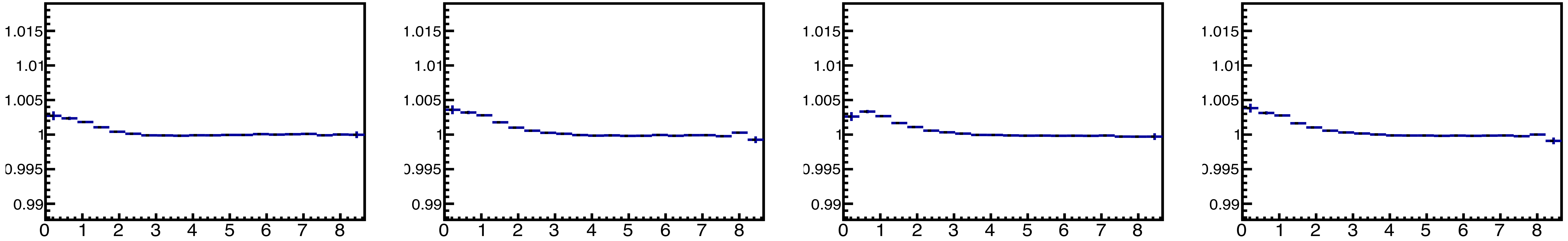
1 event:

$3n_0$



50 events:

$3n_0$



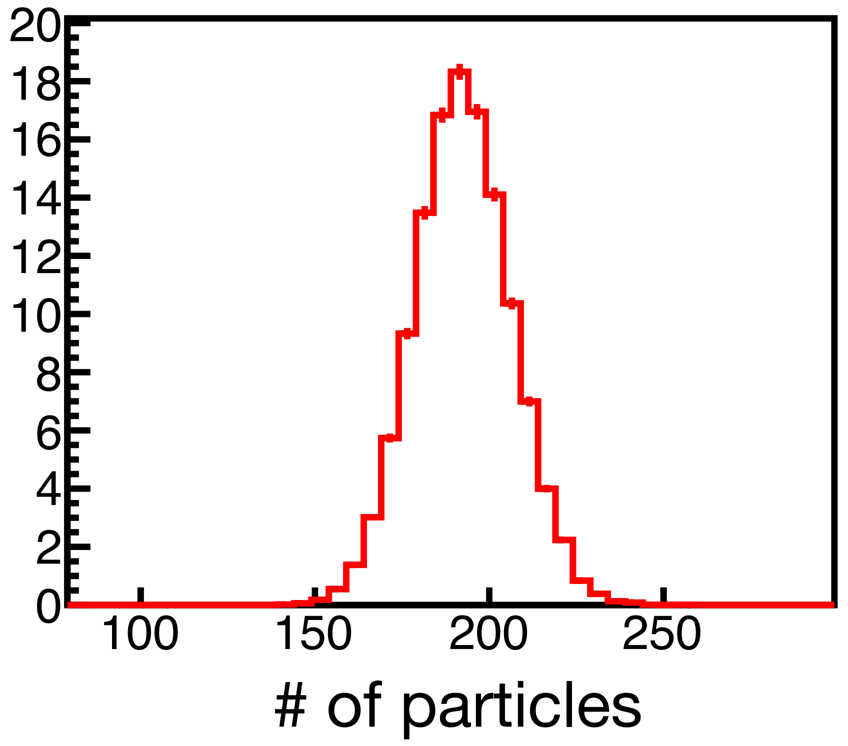
Averaging over many events needed especially for correlations at $T=T_c$

How can we *see* the phase separation for many events?
We can divide our box into cells (bins) and histogram the particle number distribution

SMASH results: periodic box, averaged over 250 events

L = 10 fm   T = 1 MeV $n_B = 3n_0$ **Cell (bin) length L = 2 fm**

The distribution becomes bimodal as the system separates!



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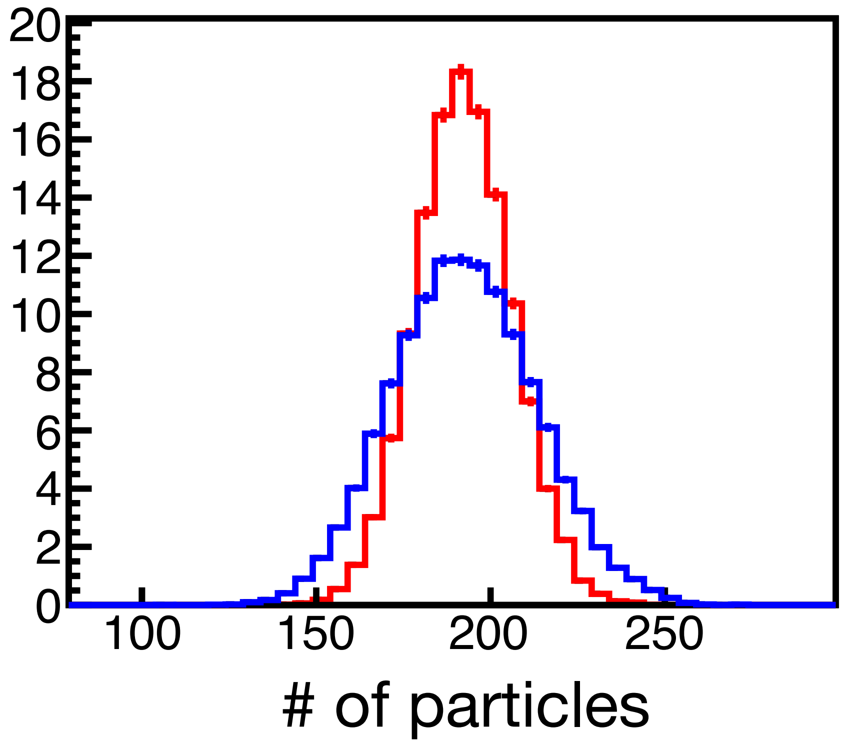
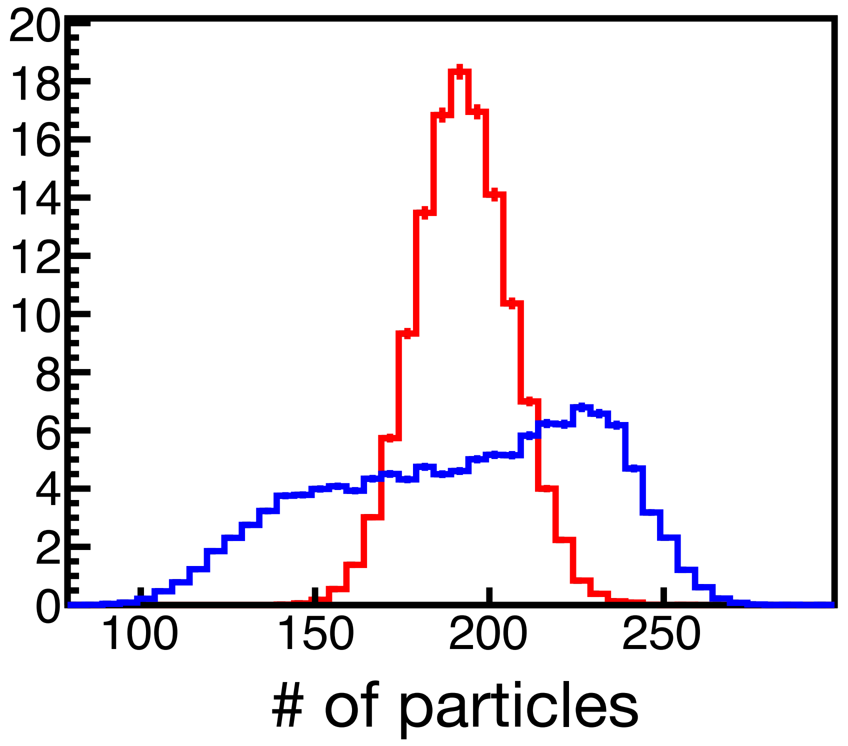
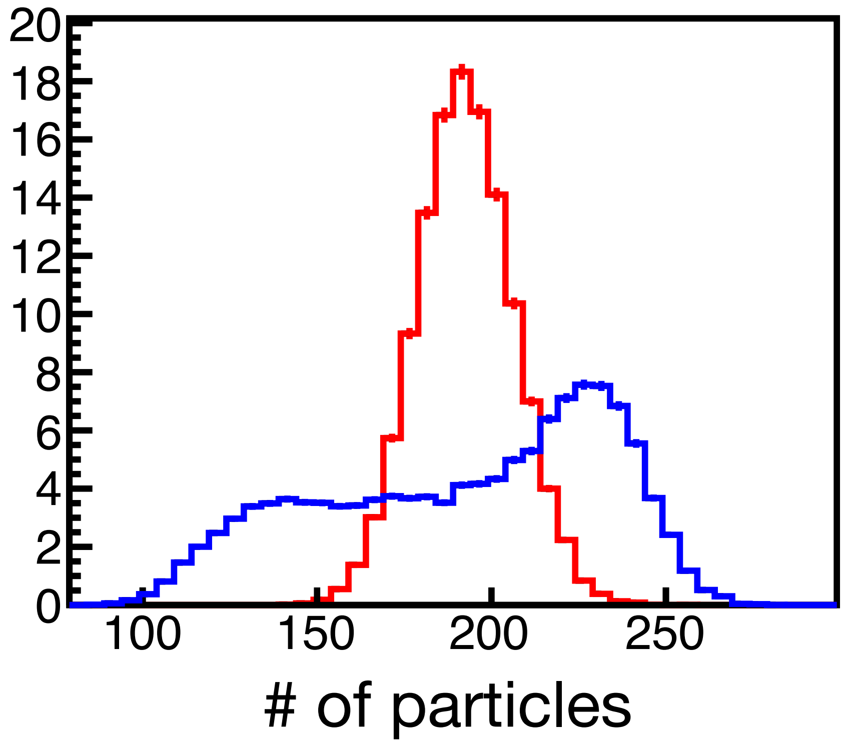


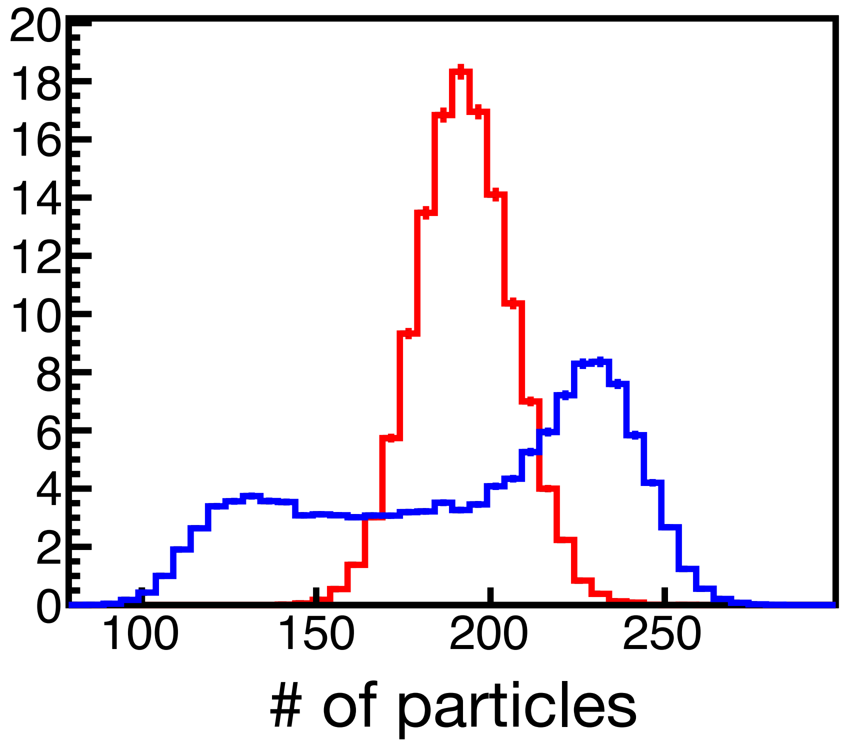
5.0 fm/c



15.0 fm/c



20.0 fm/c



30.0 fm/c

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L = 10 fm   T = 1 MeV $n_B = 3n_0$ **Cell (bin) length L = 2 fm**

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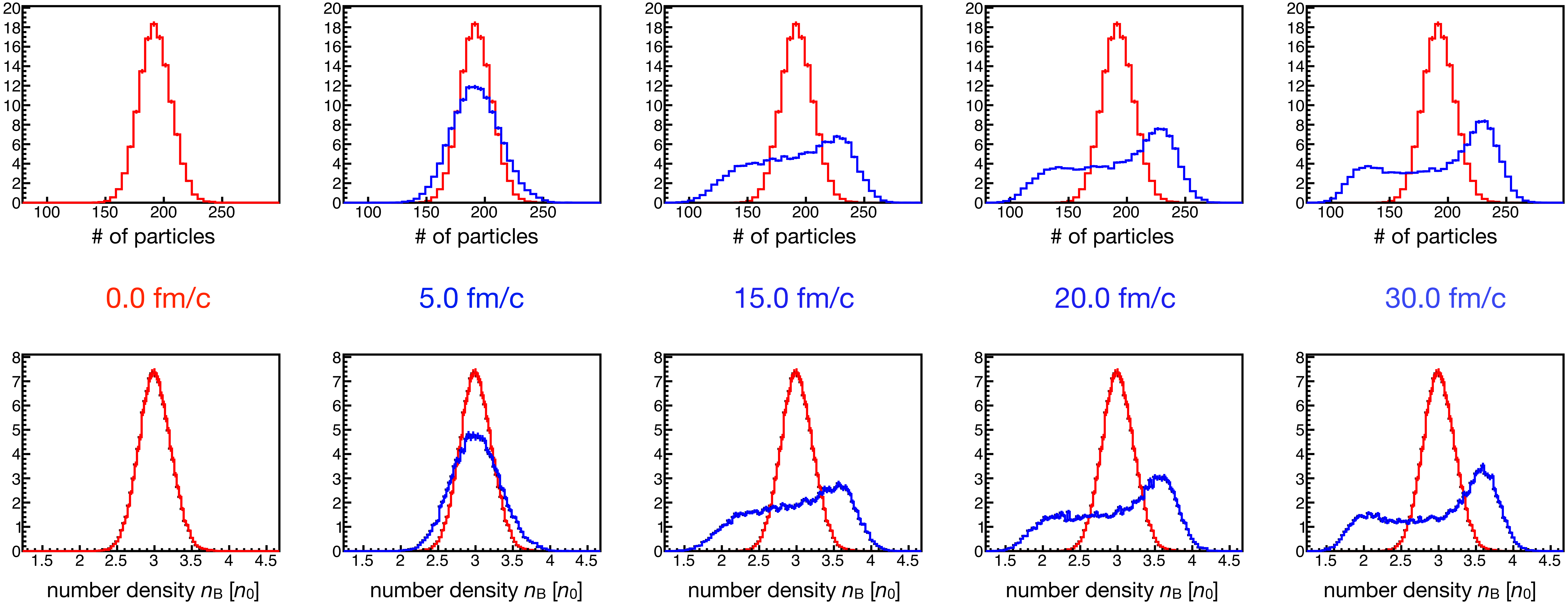

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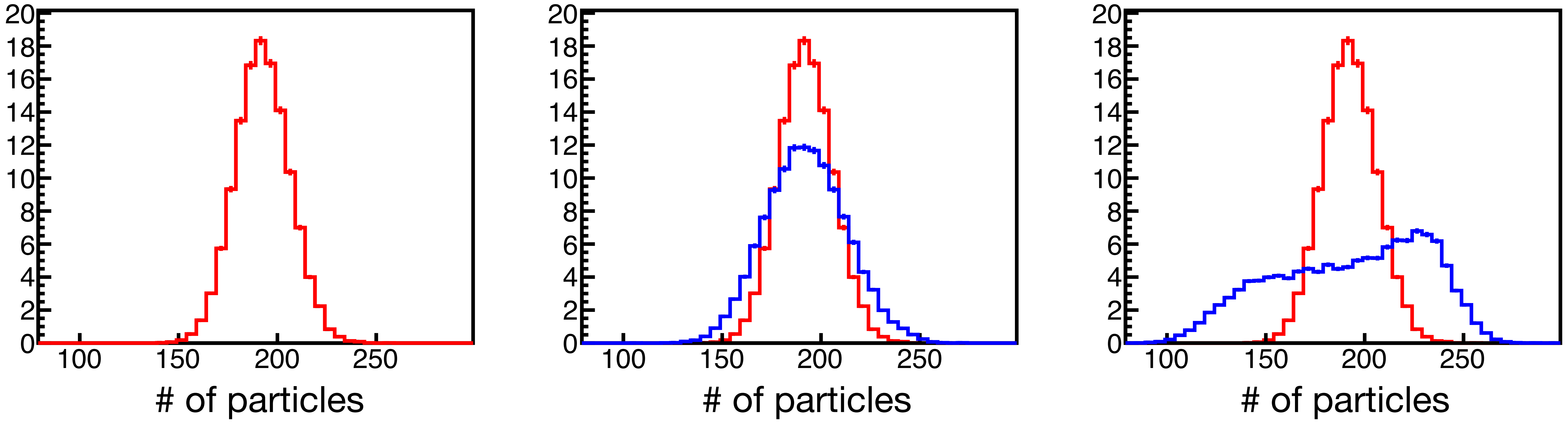

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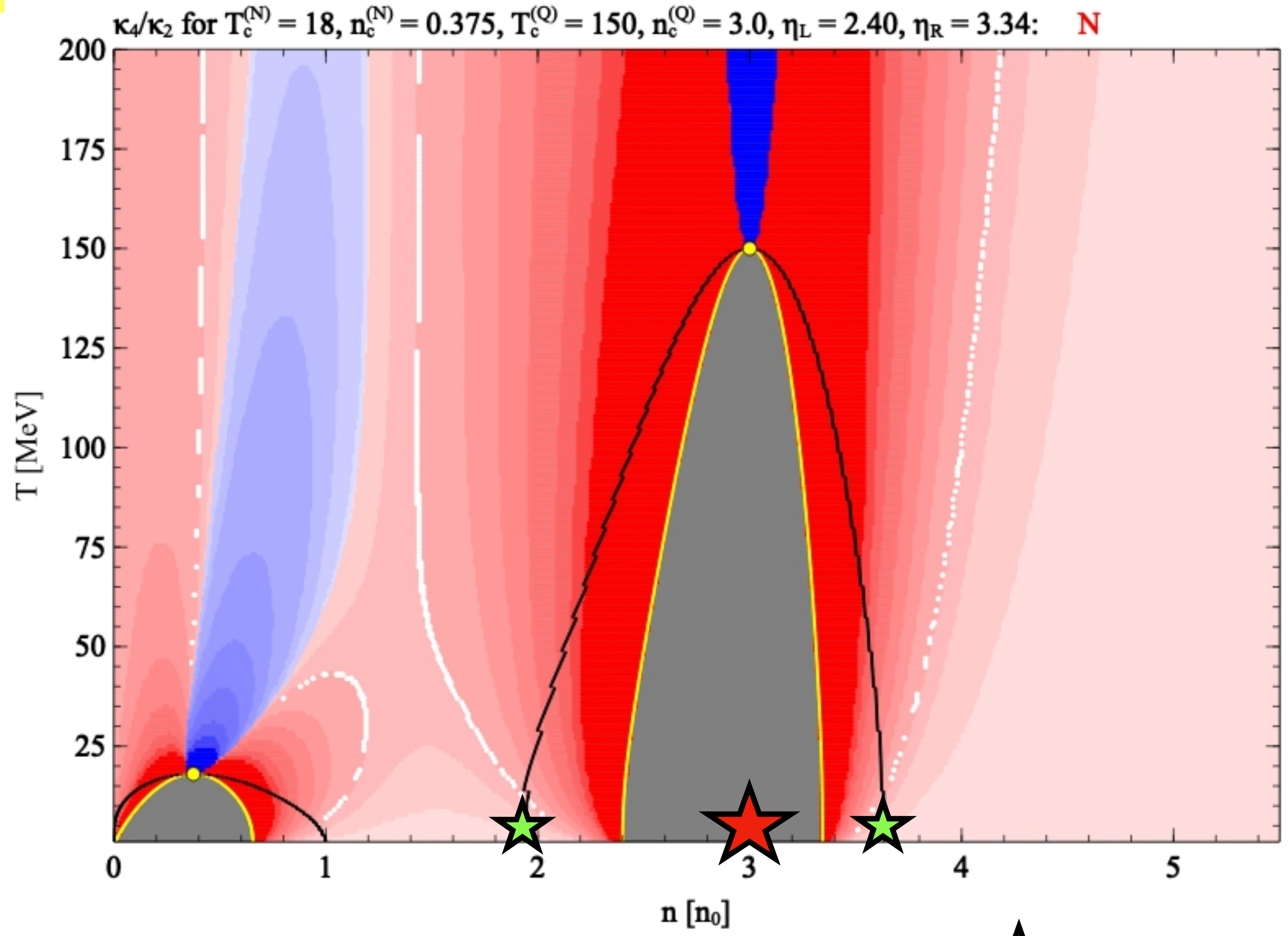

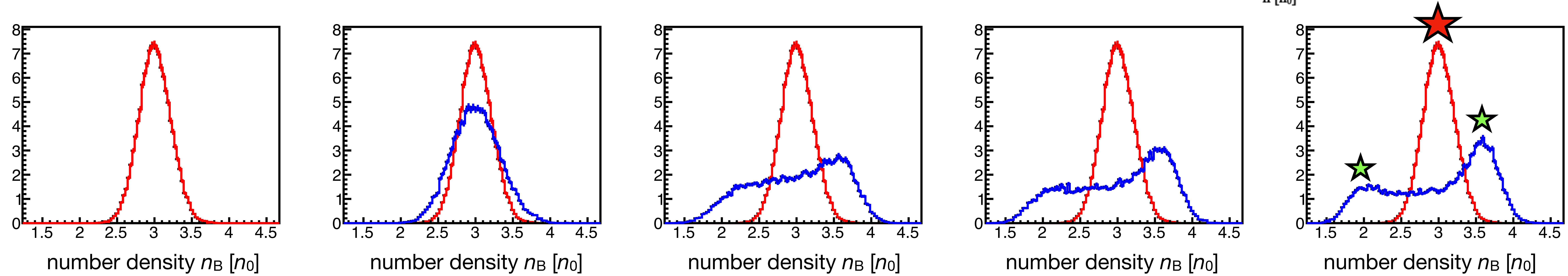

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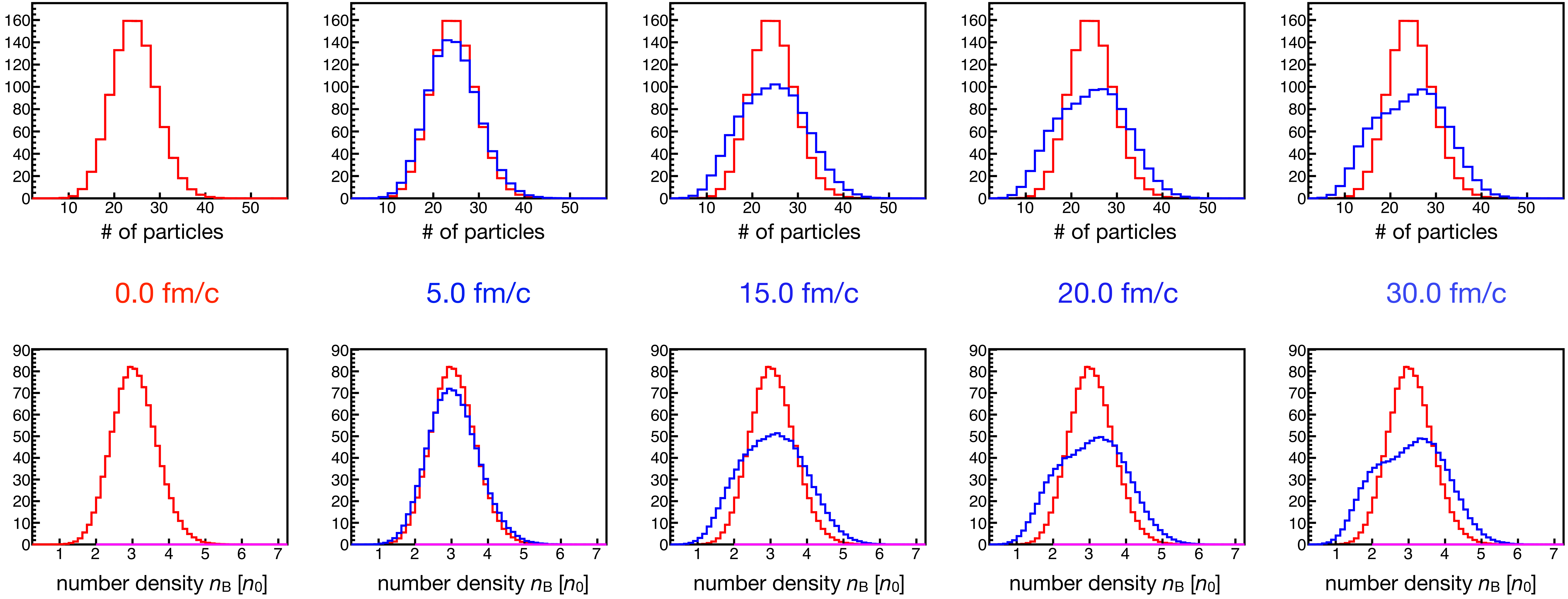
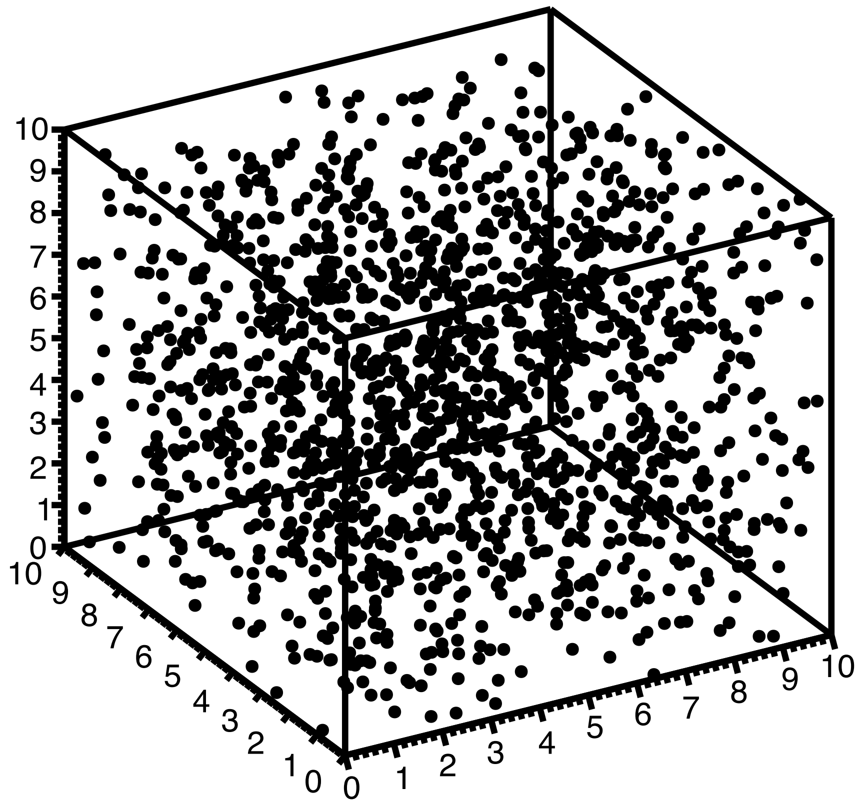
The degree of bimodality is sensitive to binning

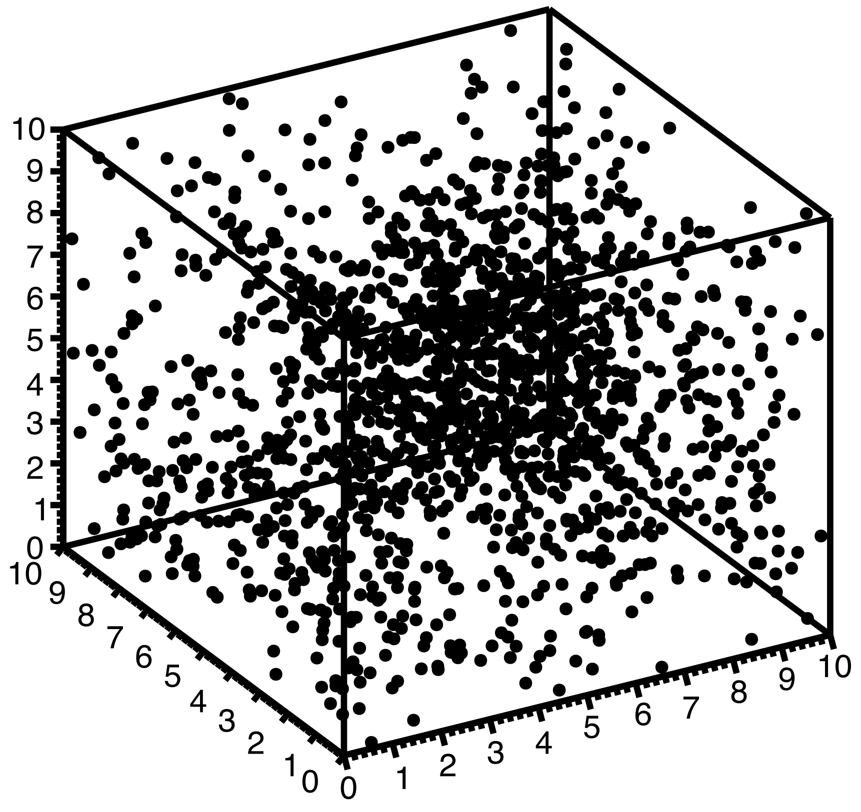


SMASH results: periodic box with “parallel ensembles”

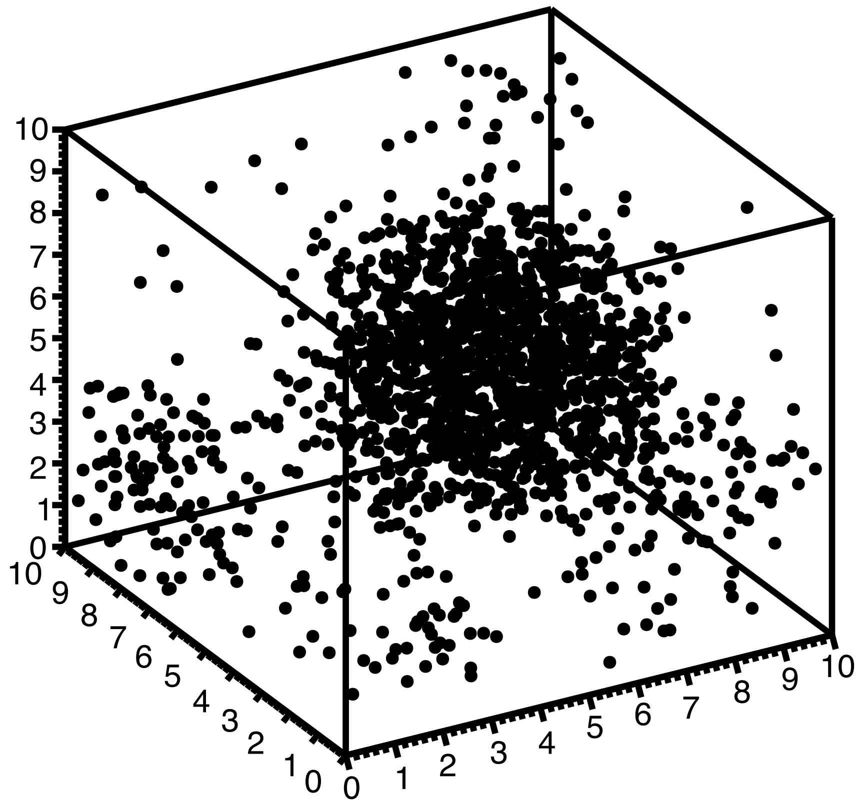
0.0 fm/c



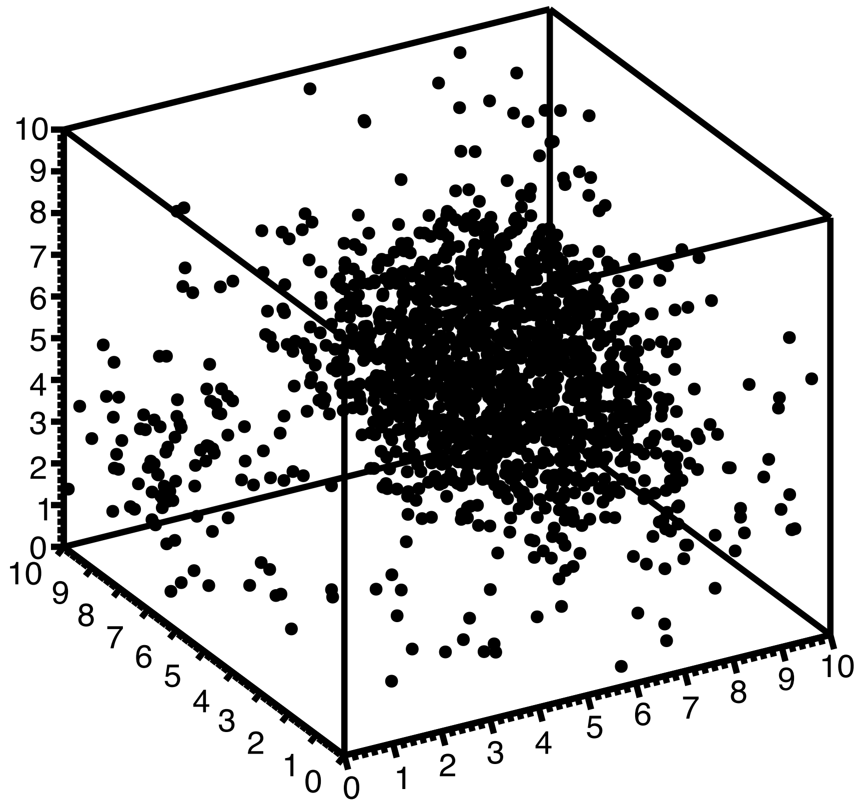
50.0 fm/c



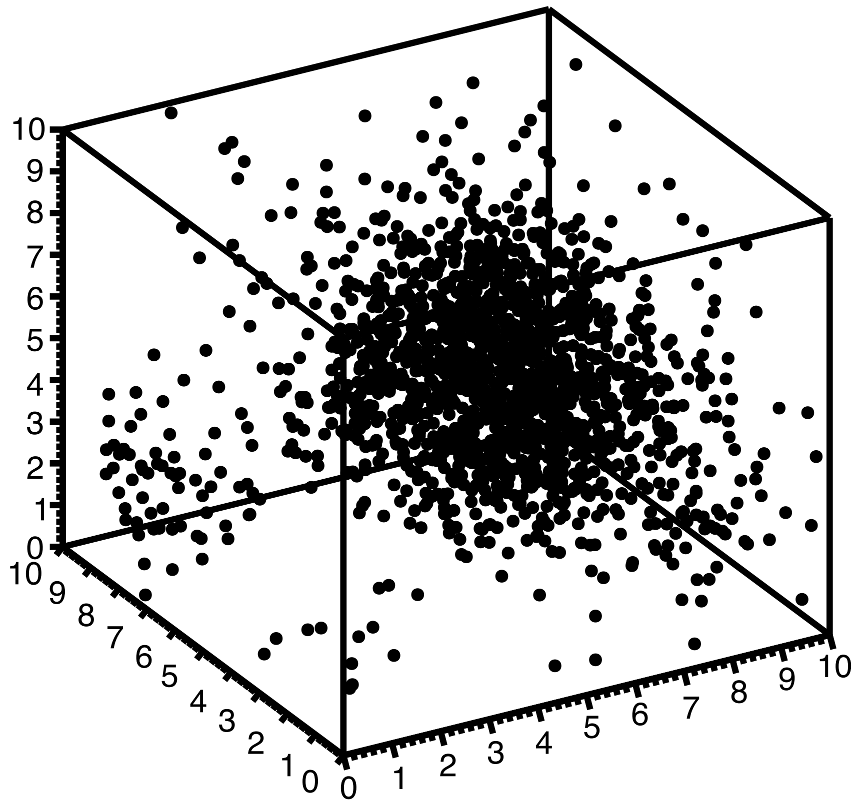
100.0 fm/



150.0 fm/

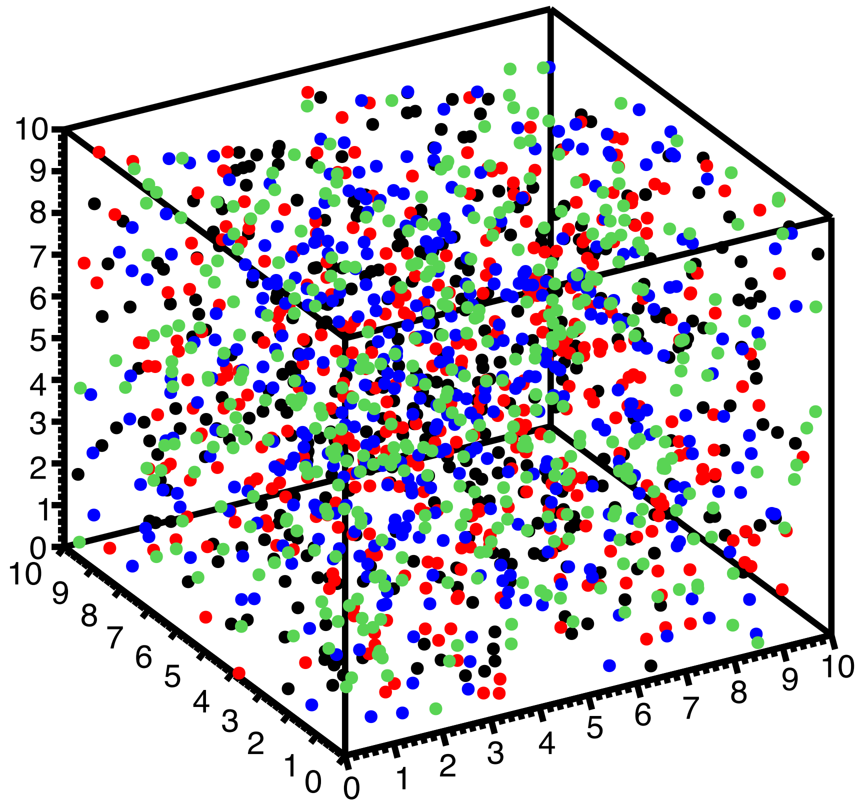


200.0 fm/c

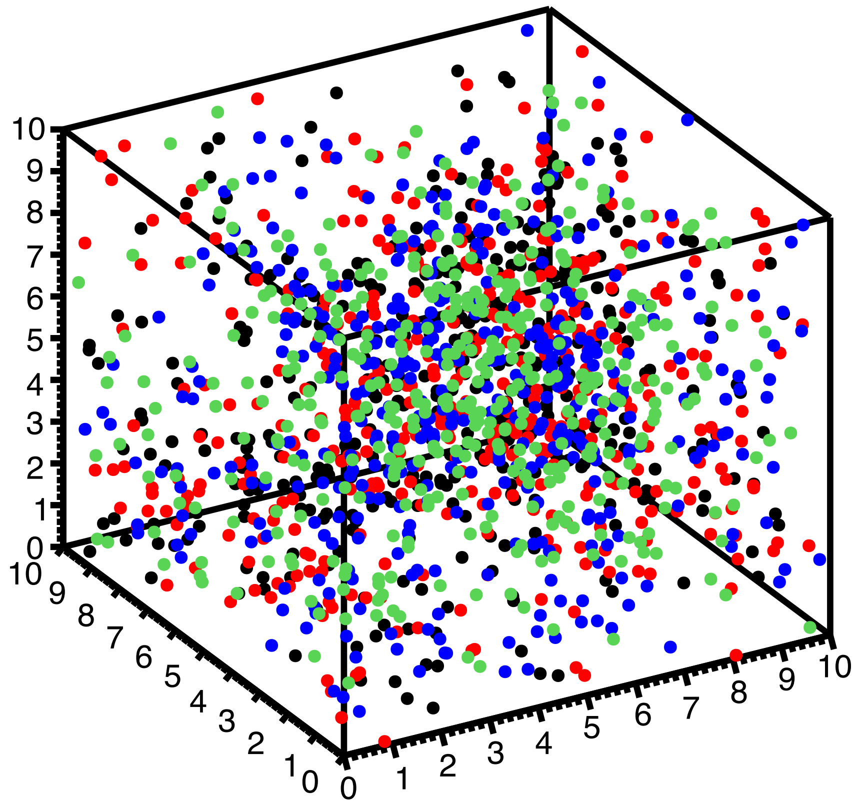


SMASH results: periodic box with “parallel ensembles”

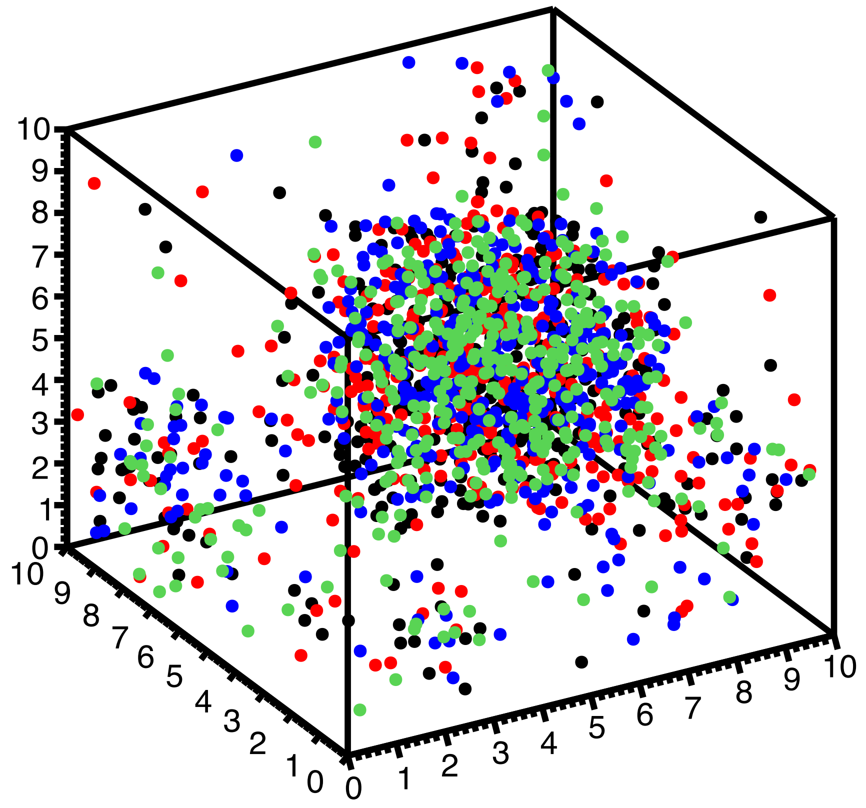
0.0 fm/c



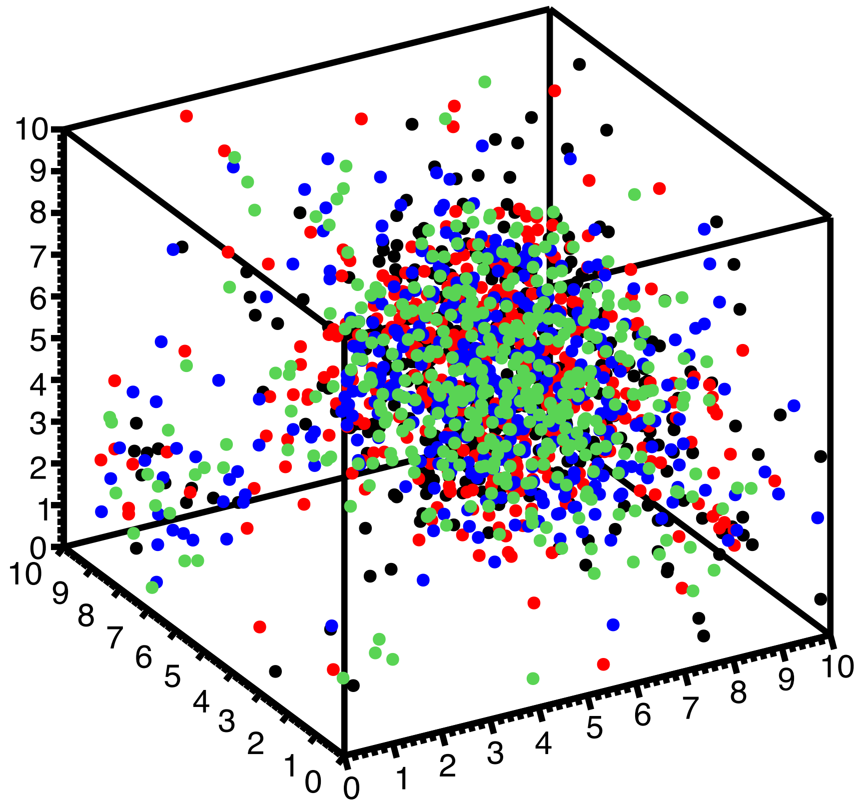
50.0 fm/c



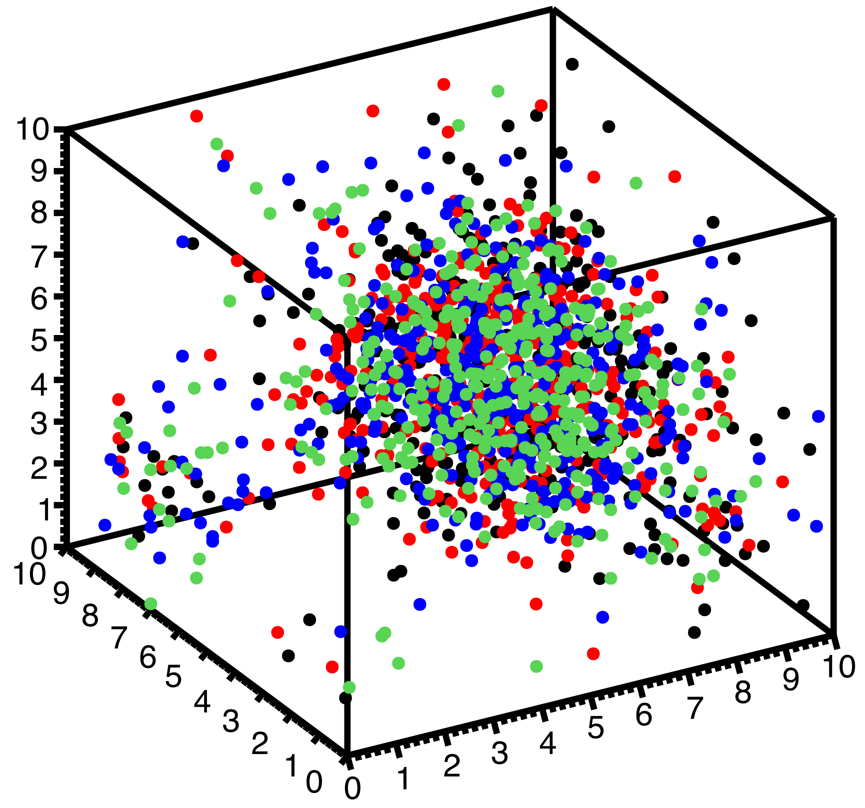
100.0 fm/



150.0 fm/



200.0 fm/c



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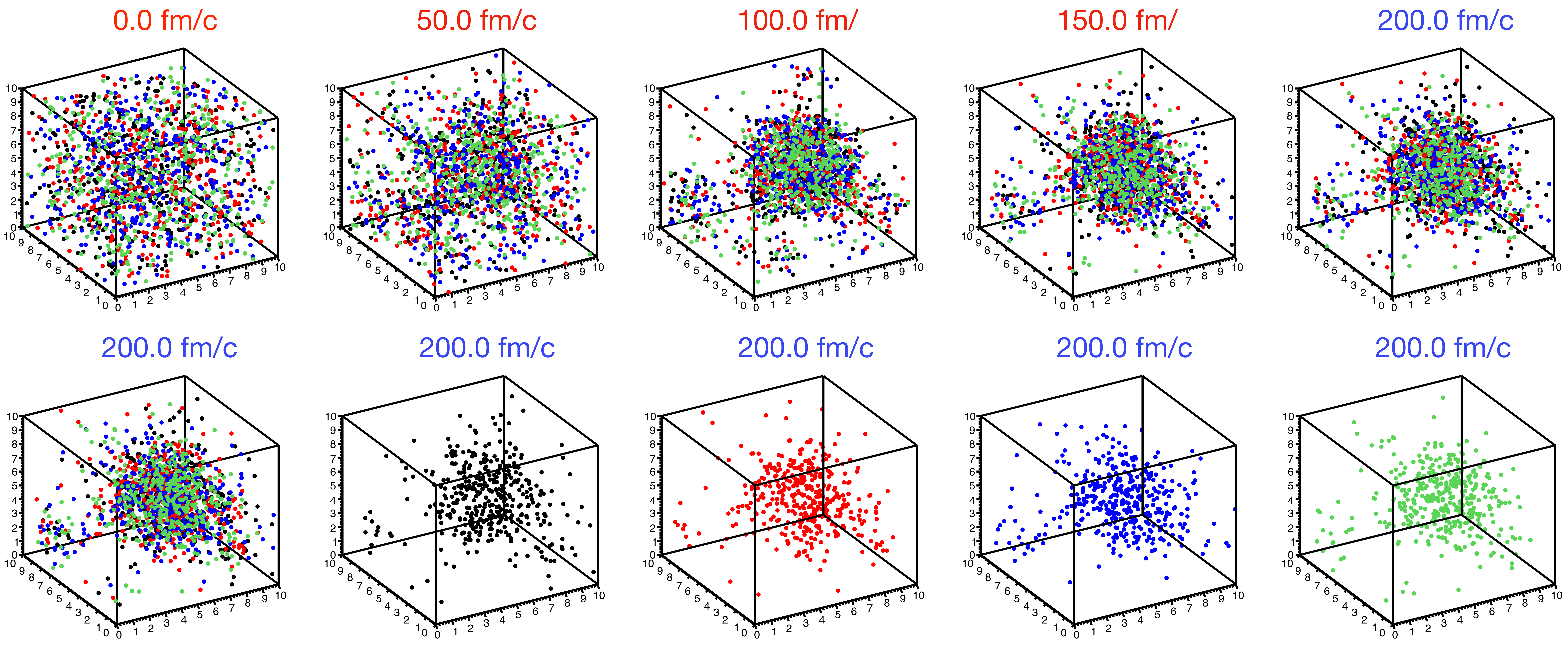
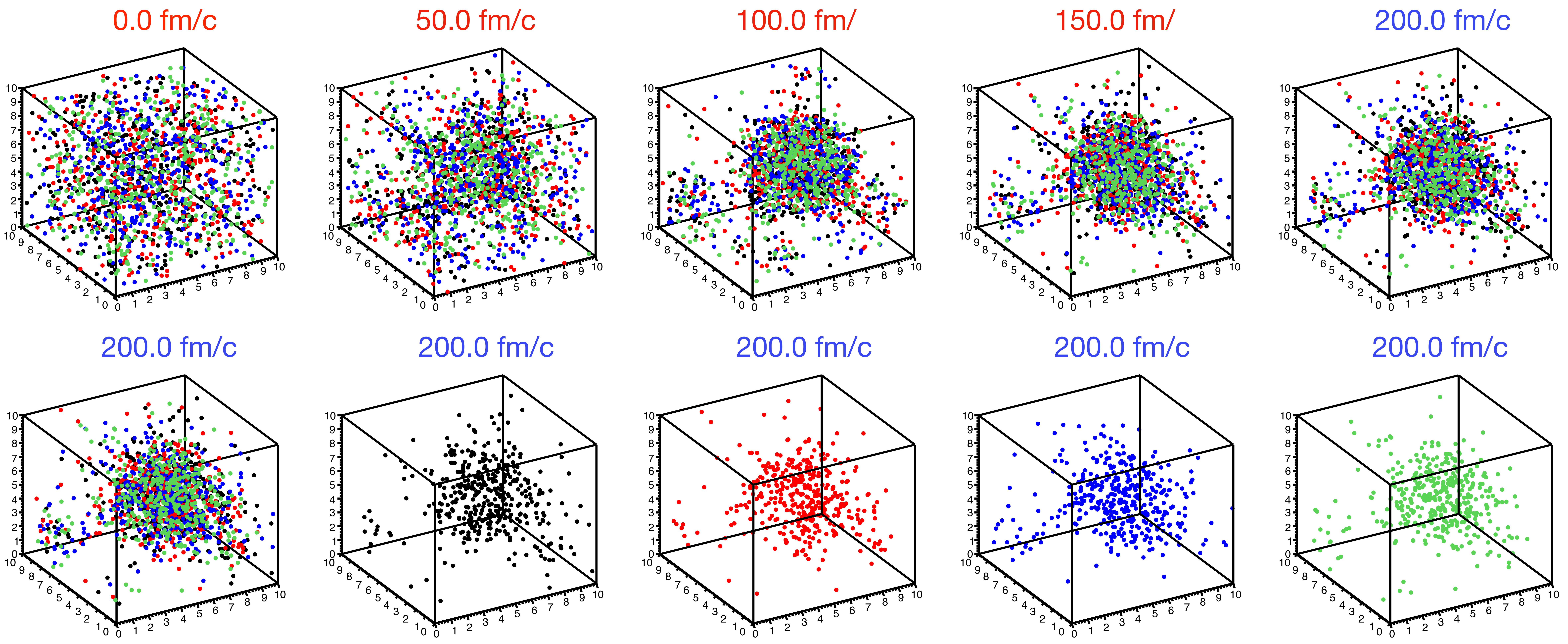


SMASH results: periodic box with “parallel ensembles”



We introduce “parallel ensembles” *post factum*:
We take # test particles N_T = # “parallel ensembles”⟶
and create “parallel boxes” at the analysis stage

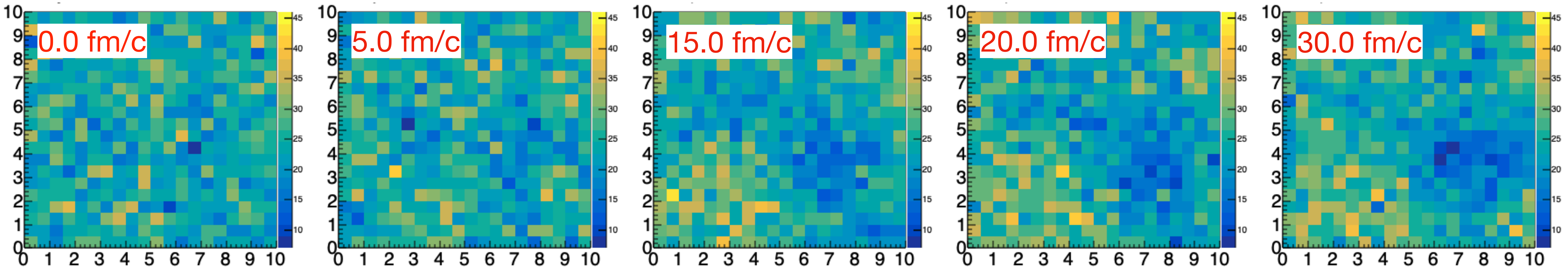
At $n_B = 3n_0$, this means 480 baryons instead of 24,000 test particles (N_T =50). This is more like experiment!

SMASH results: periodic box with “parallel ensembles”

L = 10 fm   T = 1 MeV   particle number projection onto the *xy*-plane   **NT = 20**

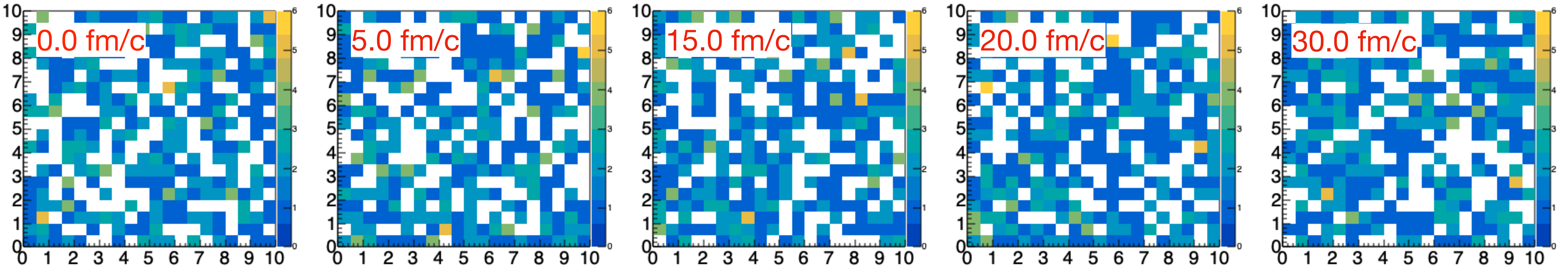
Full box:

$3n_0$



“Parallel ensembles”:

$3n_0$



SMASH results: periodic box with "parallel ensembles"

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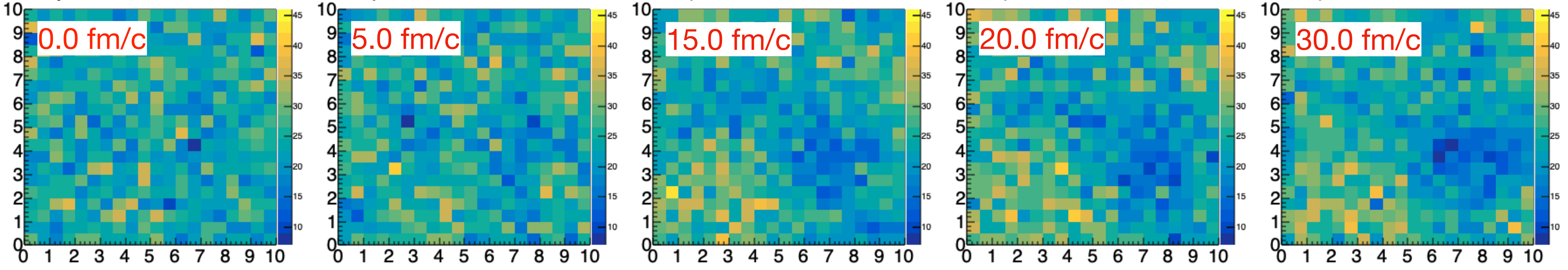
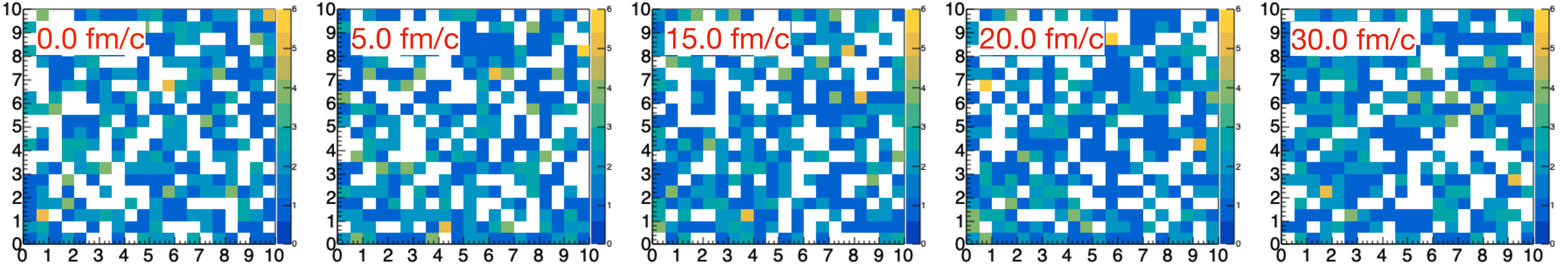


"Parallel ensembles":

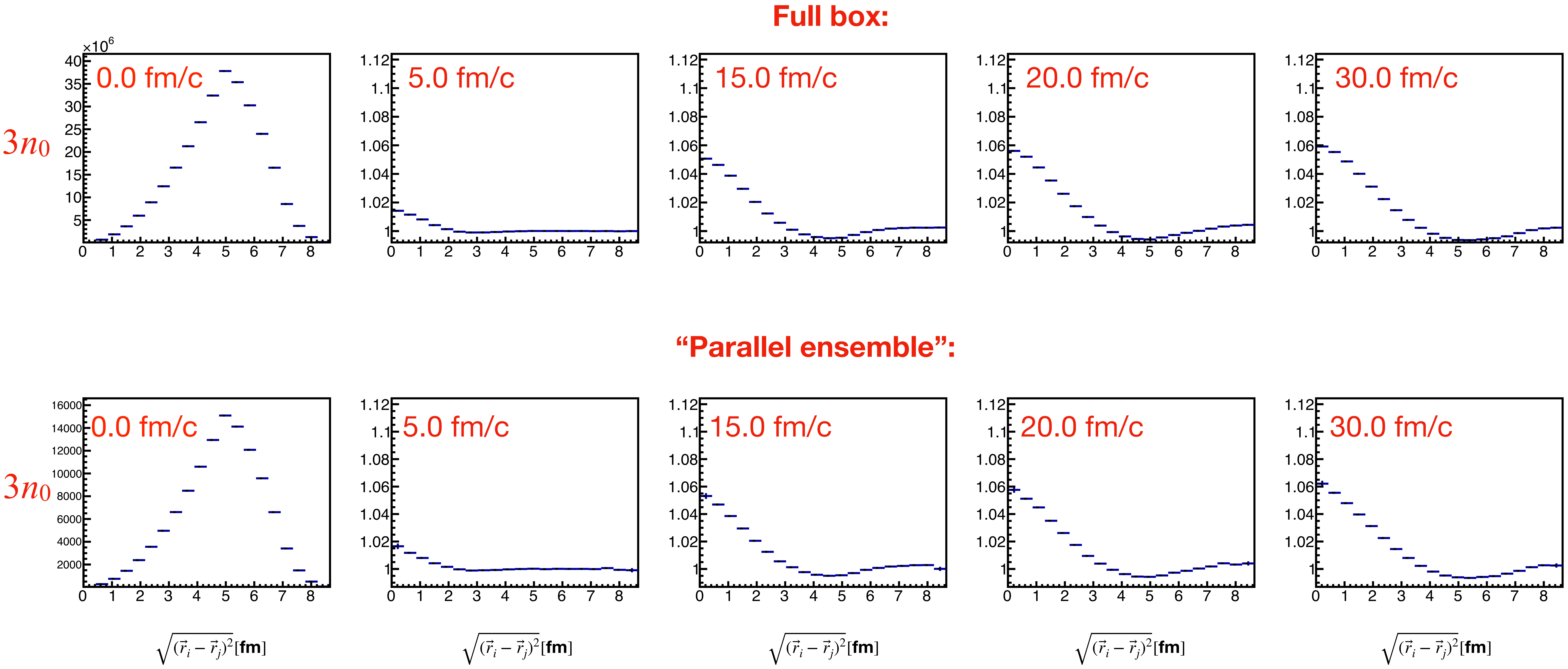
$3n_0$



Is information about phase separation preserved in "parallel ensembles"?

SMASH results: periodic box with “parallel ensembles”

L = 10 fm   T = 1 MeV   pair separation distribution (scaled for $t > 0$) $N_T = N_{\text{"parallel"}} = 50$



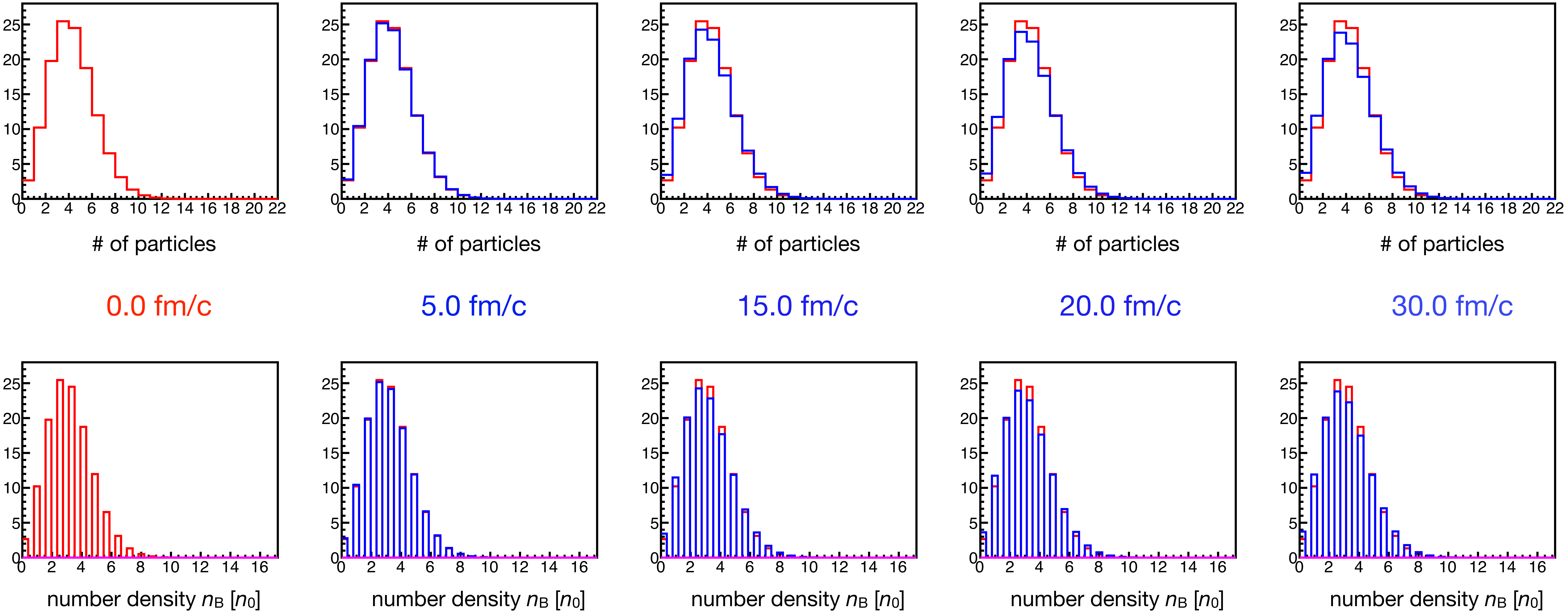
Is information about phase separation preserved in “parallel ensembles”? Yes!

SMASH results: periodic box with “parallel ensembles”

L = 10 fm   T = 1 MeV $n_B = 3n_0$ **Cell (bin) length L = 2 fm** $N_T = N_{\text{"parallel"}} =$ **50**

But bimodality is gone!

Same physics, same phase transition, but small particle numbers affect what the distribution looks like

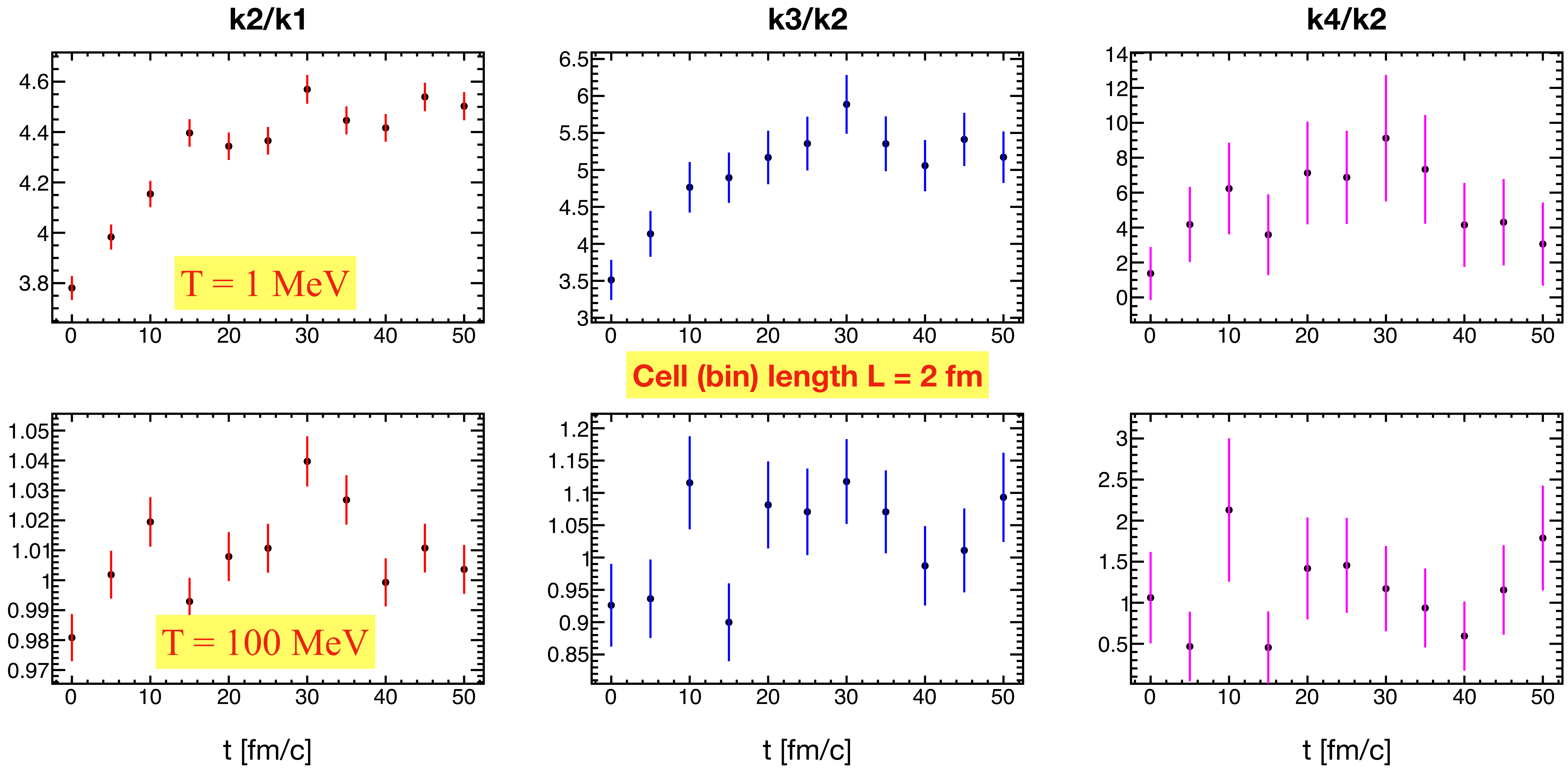


SMASH results: periodic box with "parallel ensembles"

L = 10 fm $n_B = 3n_0$

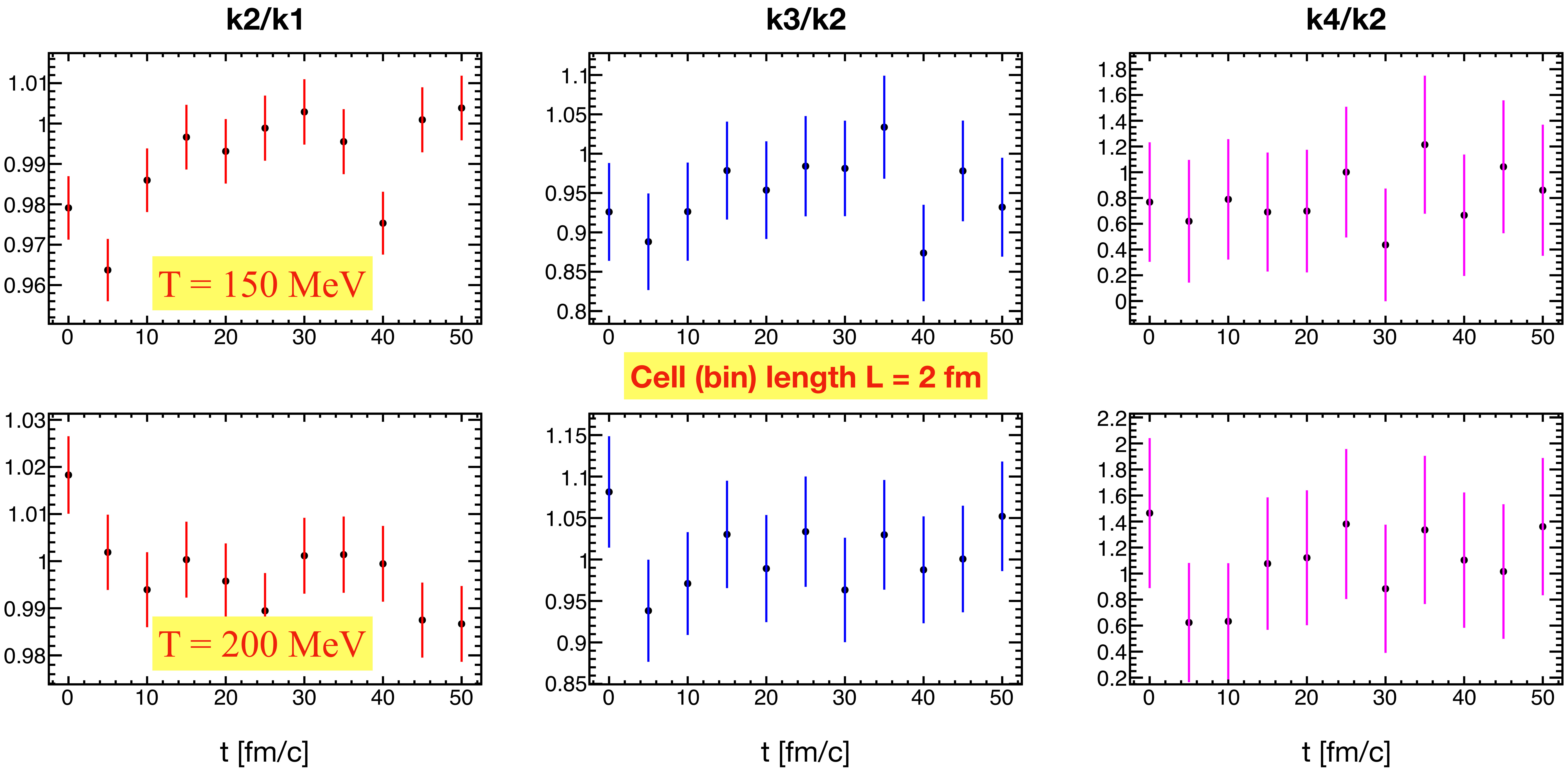
Cumulants of the baryon number

$N_T = N_{\text{"parallel"}} = 50$



SMASH results: periodic box with “parallel ensembles”

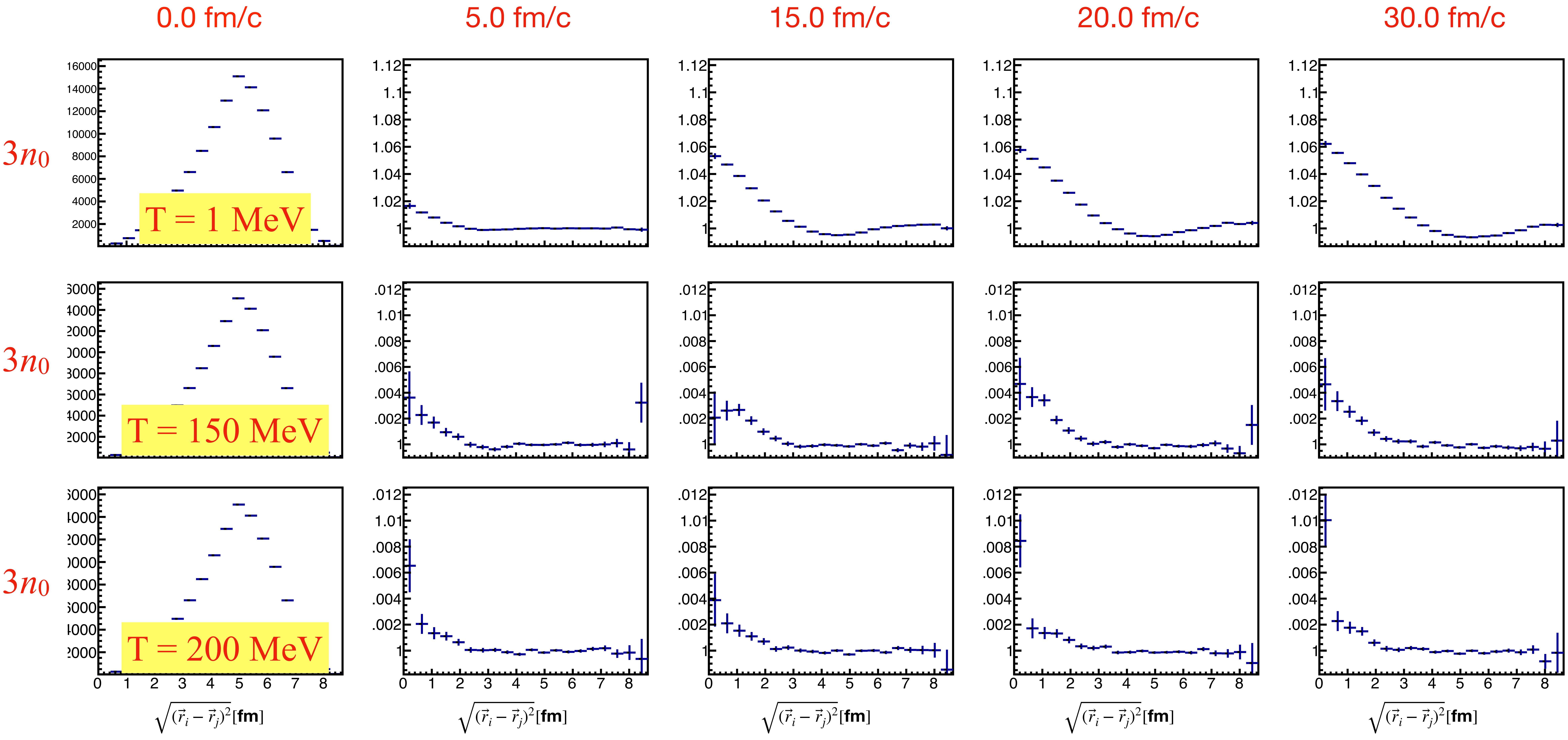
L = 10 fm $n_B = 3n_0$ **Cumulants of the baryon number** $N_T = N_{\text{"parallel"}} =$ **50**



SMASH results: periodic box with “parallel ensembles”

L = 10 fm

pair separation distribution (scaled for $t > 0$)



Summary

- Flexible parameterization of the dense QCD EOS including the known properties of nuclear matter
- Comprehensive approach to studying thermodynamics and dynamical evolution of heavy-ion collisions
- Promising initial results from implementation in SMASH

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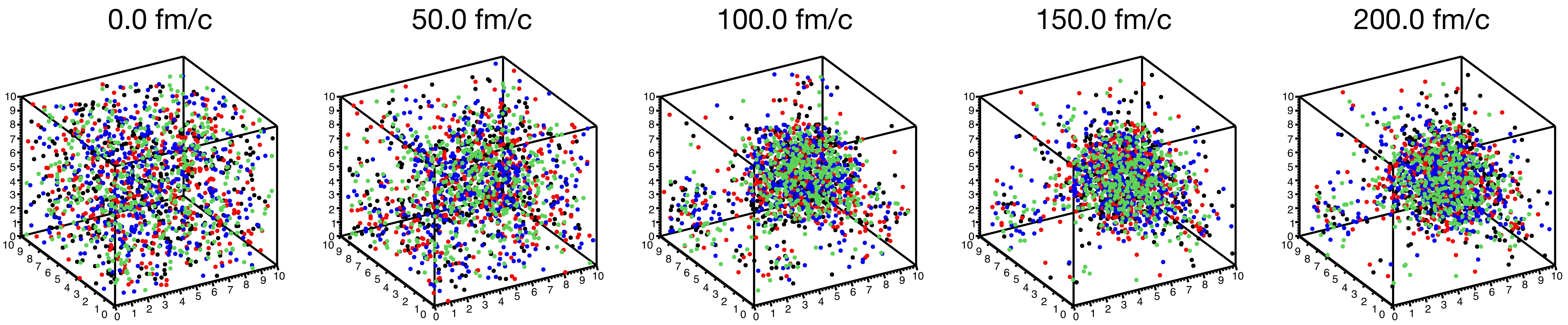
Thank you for your attention



$n_B = 0.25 n_0$

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