Recent Developments: BESHYDRO and BESHYDRO+

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BESHYDRO: Hydrodynamics at non-zero net baryon density

Baryon evolution: recent developments

The conservation laws for energy, momentum and the baryon charge are

$$\begin{aligned} d_\mu T^{\mu\nu} &= 0\,, \quad \text{with} \quad T^{\mu\nu} = e u^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}\,, \\ d_\mu N^\mu &= 0\,, \quad \text{with} \quad N^\mu = n u^\mu + n^\mu\,. \end{aligned}$$

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\end{aligned}
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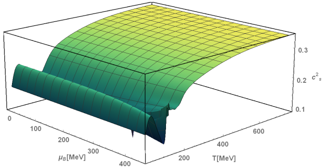
Required ingredients:

- Equation of State
  - At non-zero charge density: [A. Monnai et al, 1902.05095; J. Noronha-Hostler et al, 1902.06723]
  - With a critical point: [P. Parotto et al, 1805.05249]



Speed of sound from a Lattice-QCD-based Equation of State with a critical point [P. Parotto et al, 1805.05249]

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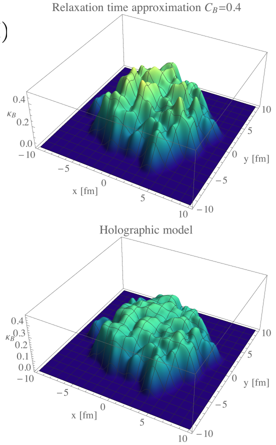
- Transport coefficients
  - Shear and bulk viscosity $(\eta/s)(\mu, T)$, $(\zeta/s)(\mu, T)$: e.g. [J. Noronha-Hostler et al, 0811.1571; G. Denicol, 1512.01538]
  - Baryon diffusion coefficient $\kappa_n(\mu, T)$: e.g. [G. Denicol et al, 1804.10557; R. Rougemont, 1507.06972; O. Soloveva et al, 1911.08547; Fotakis et al, 1912.09103]



Baryon diffusion coefficients from kinetic theory and holographic theory [L. Du et al, 1807.04721]

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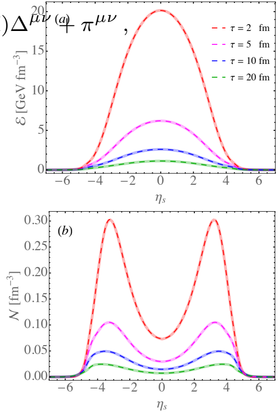
- (3+1)-d hydrodynamic simulation
  - MUSIC [G. Denicol et al, 1804.10557]
  - BESHYDRO [L. Du and U. Heinz, 1906.11181]



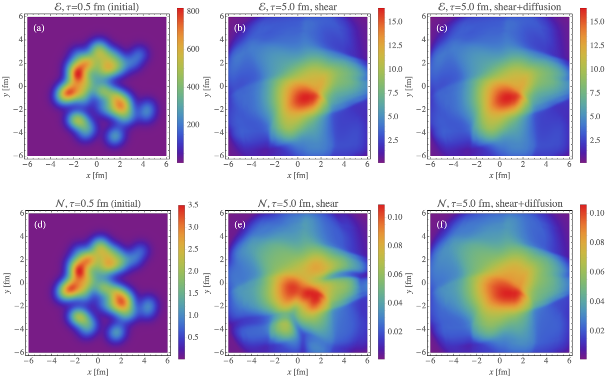
(1+1)D comparison ($r = 0$) between MUSIC and BESHYDRO [L. Du and U. Heinz, 1906.11181], both using DNMR theory [G. Denicol et al, 1004.5013; G. Denicol et al, 1202.4551]

Baryon diffusion: smoothen baryon gradients

- The relaxation equation for baryon diffusion:

$$u^\nu \partial_\nu n^\mu = -\frac{1}{\tau_n}\left[n^\mu - \kappa_n \nabla^\mu\left(\frac{\mu}{T}\right)\right] + \dots,$$

where the baryon diffusion coefficient κ_n controls the response of diffusion current to the driving force.



Energy and baryon evolution in transverse plane [L. Du and U. Heinz, 1906.11181]

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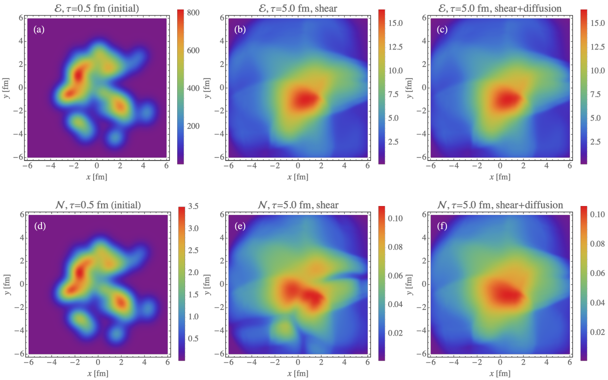
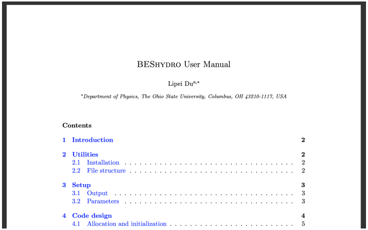


Energy and baryon evolution in transverse plane [L. Du and U. Heinz, 1906.11181]

- Baryon diffusion leaves no pronounced signatures in the evolution of the energy density but smoothes out gradients in baryon density [L. Du and U. Heinz, 1906.11181].
- An active topic: see also e.g. [A. Monnai, 1204.4713; C. Shen et al, 1704.04109; G. Moritz et al, 1711.08680; G. Denicol et al, 1804.10557; M. Li and C. Shen, 1809.04034].

BESHYDRO: internal workings



BESHYDRO User Manual

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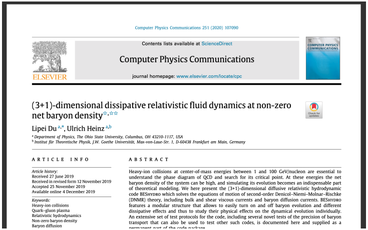

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(3+1)-dimensional dissipative relativistic fluid dynamics at non-zero net baryon density[☆,☆☆]

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ABSTRACT

Heavy-ion collisions at center-of-mass energies between 1 and 100 GeV/nucleon are essential to understand the phase diagram of QCD and search for its critical point. At these energies the net baryon density of the system can be high, and simulating its evolution becomes an indispensable part of theoretical modeling. We here present the (3+1)-dimensional diffusive relativistic hydrodynamic code BESHYDRO which solves the equations of motion of second-order Denicol–Niemi–Molnar–Rischke (DNMR) theory, including bulk and shear viscous currents and baryon diffusion currents. BESHYDRO features a modular structure that allows to easily turn on and off baryon evolution and different dissipative effects and thus to study their physical effects on the dynamical evolution individually. An extensive set of test protocols for the code, including several novel tests of the precision of baryon transport that can also be used to test other such codes, is documented here and supplied as a permanent part of the code package.

- Physics, internal workings and validation schemes are well documented:
  - BESHYDRO paper [L. Du and U. Heinz, 1906.11181]
  - User manual ▸ on GitHub
- With documents and validation schemes, BESHYDRO can be a good tool for knowing internal workings of hydrodynamic code, and it is especially suitable for doing exploratory studies.

BESHYDRO: validation schemes

Code validation is essential

- avoid issues in numerical schemes
- assure the results are from the physics, not numerical issues

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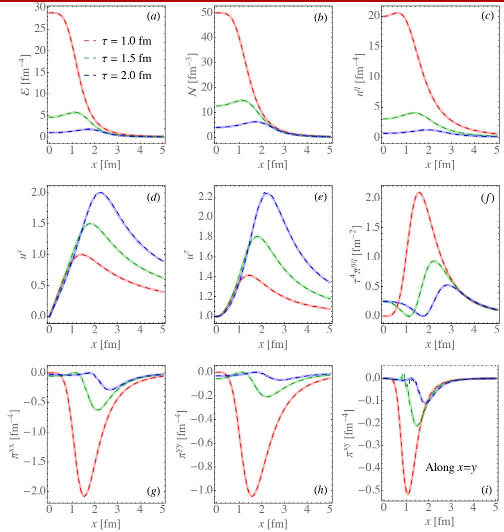

Test of transerverse evolution with Gubser flow [L. Du and U. Heinz, 1906.11181]

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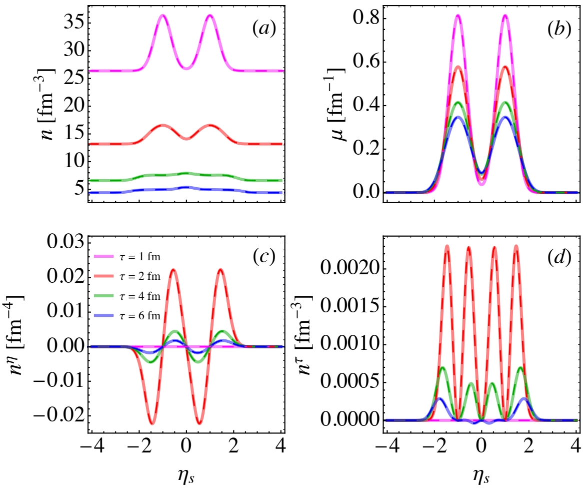

Test of longitudinal evolution [L. Du et al, in preparation]

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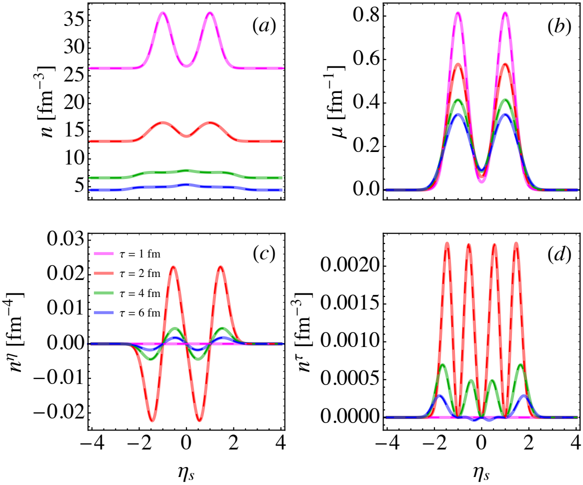
- stay tuned for more validation schemes, which can be used for other codes ▸ on GitHub



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BESHYDRO+: Fluctuation dynamics near the QCD critical point

Critical fluctuations: necessity of off-equilibrium dynamics

- Off-equilibrium effects of the critical fluctuations is essential: 1. critical slowing-down [Hohenberg and Halperin, *Rev. Mod. Phys.*, 1977]; 2. the QCD matter created in heavy-ion collisions evolves very rapidly;

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- Hydro+ framework: conventional hydrodynamics coupled to slowly evolving critical modes. The slowest mode in the system created in heavy-ion collisions is, $\delta(s/n)_p$ [M. Stephanov and Y. Yin, 1712.10305].

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- Introduce a phase-space density for the slow degrees of freedom, $\phi_Q(\boldsymbol{x})$, via the Wigner transform of the two-point correlation of $\delta(s/n)_p$ [M. Stephanov and Y. Yin, 1712.10305; see also X. An et al, 1902.09517 and 1912.13456]:

$$\phi_Q(\boldsymbol{x}) \sim \int_{\Delta x} \left\langle \delta\frac{s}{n}\left(\boldsymbol{x}+\frac{\Delta \boldsymbol{x}}{2}\right) \delta\frac{s}{n}\left(\boldsymbol{x}-\frac{\Delta \boldsymbol{x}}{2}\right)\right\rangle e^{iQ\cdot\Delta x}\,.$$

  dependence of two point correlation on $\boldsymbol{x}$ of scale ℓ (homogeneity scale of fluid), on $\Delta\boldsymbol{x}$ of scale ξ (correlation length);

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  dependence of two point correlation on $\boldsymbol{x}$ of scale ℓ (homogeneity scale of fluid), on $\Delta\boldsymbol{x}$ of scale ξ (correlation length);
- With the scale separation $\ell \gg \xi$, in the hydrodynamic limit $Q \to 0$, i.e. $Q \ll \xi^{-1}$ $(Q = |\boldsymbol{Q}|)$, the equilibrium value $\bar{\phi}_0$ is [M. Stephanov and Y. Yin, 1712.10305; Y. Akamatsu et al, 1811.05081]

$$\bar{\phi}_0 = V\left\langle\left(\delta\frac{s}{n}\right)^2\right\rangle = \frac{c_p}{n^2}\,,$$

  where $c_p = nT(\partial(s/n)/\partial T)_p$ is the heat capacity.

Critical fluctuations: off-equilibrium dynamics

- The equations of motion of slow modes are of relaxation form [M. Stephanov and Y. Yin, 1712.10305; Y. Akamatsu et al, 1811.05081; X. An et al, 1912.13456]

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where u^μ the flow velocity and $u^\mu \partial_\mu$ the time-derivative in local rest frame.

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$$\bar{\phi}_Q = \bar{\phi}_0 f_2(Q\xi) = \left[\left(\frac{c_p}{n^2}\right)\left(\frac{\xi}{\xi_0}\right)^2\right]\frac{1}{1+(Q\xi)^2},$$

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- Near the critical point, the Q-dependent relaxation rate Γ_Q is [Y. Akamatsu et al, 1811.05081; X. An et al, 1912.13456]

$$\Gamma_Q = \Gamma_\xi f_\Gamma(Q\xi) = \left[2\left(\frac{\lambda_T}{c_p \xi^2}\right)\left(\frac{\xi_0}{\xi}\right)^2\right]\left[(Q\xi)^2(1+(Q\xi)^2)\right],$$

with λ_T being the heat conductivity (note: “model B” in use, $\Gamma_\xi \propto \xi^{-4}$, *cf.* [K. Rajagopal et al, 1908.08539]).

Critical fluctuations: back-reaction to the fluid

- Dynamics ϕ_Q results from competition between expansion of the system and the relaxation of ϕ_Q. Introduce the "critical Knudsen number" to characterize this competition quantitatively:

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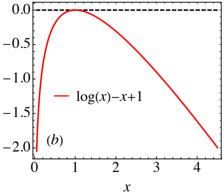
- The slow modes are additional, non-thermal degrees of freedom, which contribute to the entropy density, $s_{(+)}(e,n,\phi) \equiv s_{\mathrm{eq}}(e,n) + \Delta s$, with [M. Stephanov and Y. Yin, 1712.10305] (denoting $x \equiv \phi_Q/\bar{\phi}_Q$)

$$\Delta s(e,n,\phi) = \int dQ \frac{Q^2}{(2\pi)^2}\left[\log\frac{\phi_Q}{\bar{\phi}_Q} - \frac{\phi_Q}{\bar{\phi}_Q} + 1\right].$$

Note: Q^2 in the phase space factor.



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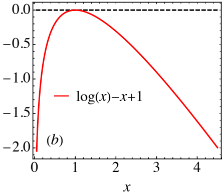

- The largest contribution to $|\Delta s|$ arises from modes with intermediate $Q \sim Q_{\rm max}$.
- Corrections to μ, T and p calculable given Δs.

Setup: parameterization and background fluid

- Parameterization: $c_p = s^2/((\mu/T)n)$ [Y. Akamatsu et al, 1811.05081], $\lambda_T \propto T^2$ and $\xi(T)$ [K. Rajagopal et al, 1908.08539];

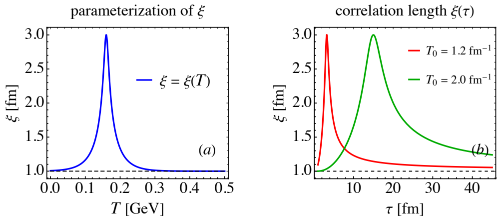

Illustration of the parameterization $\xi(T)$, and its evolution in a Bjorken expanding fluid

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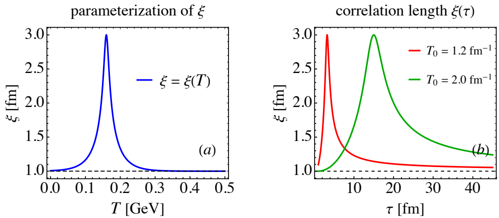
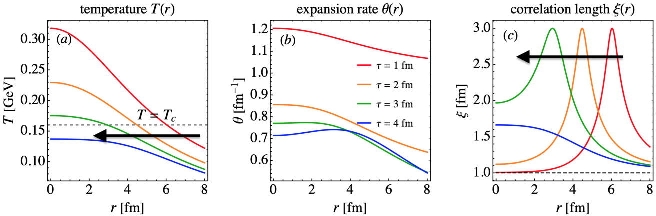
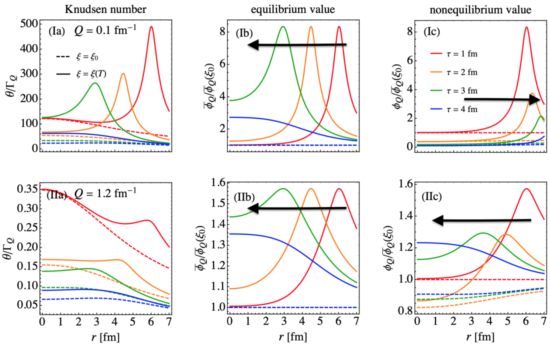


Illustration of the parameterization $\xi(T)$, and its evolution in a Bjorken expanding fluid

- Ideal Gubser flow [S. Gubser, 1006.0006] with no back-reaction; temperature profile given as $T(\tau, r) = C/(\tau \cosh^{2/3} \rho(\tau, r))$ (other quantities from conformal EoS, e.g., $e \propto T^4$ and $n \propto T^3$).





- Equations of motion
$$(u^\tau \partial_\tau + u^r \partial_r)\phi_Q = -\Gamma_Q\,(\phi_Q - \bar\phi_Q)\,,$$
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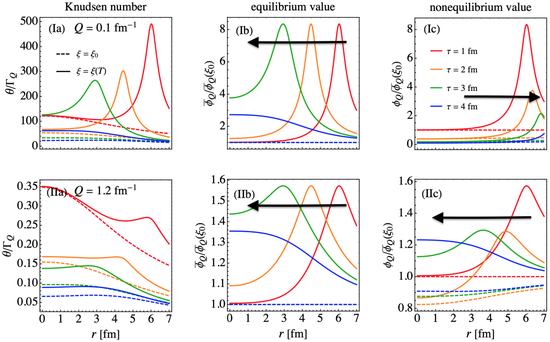
Dashed lines: $\xi = \xi_0$; solid lines: $\xi = \xi(T)$ [L. Du, U. Heinz, K. Rajagopal, and Y. Yin, 2004.02719].

Off-equilibrium dynamics: dependence on Q and ξ



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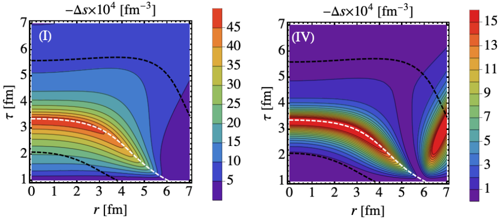
- Two effects at play: (1) the initial peak in the fluctuations carried outward by advection (outward through $u^r\partial_r$-term); (2) out-of-equilibrium fluctuations relax but more slowly due to critical slowing down (inward through Γ_Q-term);
- Small Q modes ($Q < \xi_{max}^{-1}$): relaxation is invisible because of large Kn(Q). Large Q modes ($Q \gtrsim \xi_0^{-1}$): relax more quickly to equilibrium and the initial peak dissipates more rapidly; advection is invisible.

Off-equilibrium effects: on entropy density and eccentricities

- Test different aspects of the dynamics:

$$\bar{\phi}_Q = \left[\left(\frac{c_p}{n^2}\right)\left(\frac{\xi}{\xi_0}\right)^2\right] f_2(Q\xi)\,,$$

$$\Gamma_Q = \left[2\left(\frac{\lambda_T}{c_p\xi^2}\right)\left(\frac{\xi_0}{\xi}\right)^2\right] f_\Gamma(Q\xi)\,.$$



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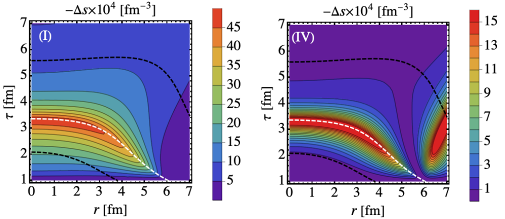

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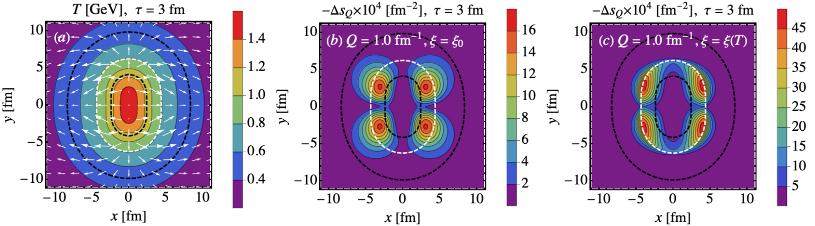

Non-equilibrium slow-mode entropy correction for elliptically deformed Gubser flow
(a) temperature and flow profile, (b) with $\xi = \xi_0$, (c) with $\xi = \xi(T)$ [L. Du, U. Heinz, K. Rajagopal, and Y. Yin, 2004.02719].

- The ellipticity is slightly increased by the correction from slow modes (of relative order $\lesssim 10^{-4}$).

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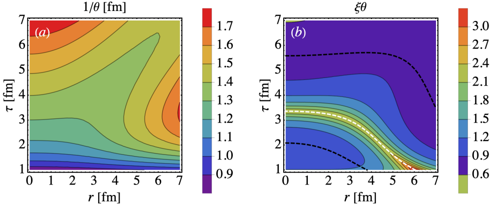

(a) estimation of hydrodynamic homogeneity length ℓ (b) estimation of $\xi/\ell \sim \xi\theta$
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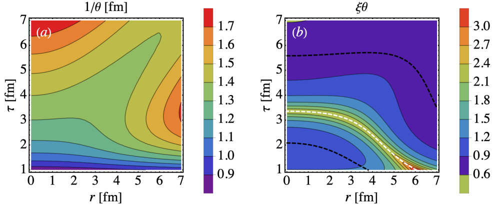

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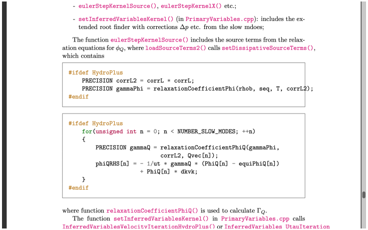
- One sees that the framework gets challenged mostly in an arrow region around T_c but elsewhere it works well, even at very early times where the homogeneity length ℓ is short.



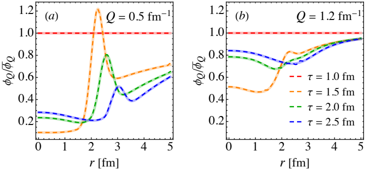
- eulerStepKernelSource(), eulerStepKernelX() etc.;
- setInferredVariablesKernel() (in PrimaryVariables.cpp): includes the extended root finder with corrections Δp etc. from the slow mdoes;

The function eulerStepKernelSource() includes the source terms from the relaxation equations for ϕ_Q, where loadSourceTerms2() calls setDissipativeSourceTerms(), which contains

```
#ifdef HydroPlus
    PRECISION corrL2 = corrL * corrL;
    PRECISION gammaPhi = relaxationCoefficientPhi(rhob, seq, T, corrL2);
#endif
```

```
#ifdef HydroPlus
    for(unsigned int n = 0; n < NUMBER_SLOW_MODES; ++n)
    {
        PRECISION gammaQ = relaxationCoefficientPhiQ(gammaPhi,
                           corrL2, Qvec[n]);
        phiQRHS[n] = - 1/ut * gammaQ * (PhiQ[n] - equiPhiQ[n])
                     + PhiQ[n] * dkvk;
    }
#endif
```

where function relaxationCoefficientPhiQ() is used to calculate Γ_Q.
The function setInferredVariablesKernel() in PrimaryVariables.cpp calls InferredVariablesVelocityIterationHydroPlus() or InferredVariables UtauIteration



Document of BESHYDRO+ and validation of BESHYDRO+ [L. Du, U. Heinz, K. Rajagopal, and Y. Yin, 2004.02719]

Physics, internal workings and validation schemes are well documented:

- BESHYDRO+ paper [L. Du, U. Heinz, K. Rajagopal, and Y. Yin, 2004.02719]
- User manual ▸ on GitHub

Conclusions

Conclusions

- BESHYDRO: Dissipative hydrodynamics at non-zero net baryon density
  - Baryon diffusion can have effects on observables and should be included in hydrodynamic simulations for Beam Energy Scan studies;
  - BESHYDRO is designed for this purpose and well documented.
- BESHYDRO+: Fluctuation dynamics near the QCD critical point
  - Different aspects of the off-equilibrium dynamics controlling the evolution of critical fluctuations are analyzed in details;
  - It provides useful guidance for more realistic studies on observables affected by critical fluctuations;
  - Back-reaction off-equilibrium effects of critical fluctuations are small on the bulk properties of the fluid;

Thank you very much!