BEST COLLABORATION Annual Meeting

Cooper-Frye particlization with spectral functions

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Introduction

- There are unstable resonance hadrons, with finite lifetime
  - there is non-zero variation in the rest energy (mass).

- This can be quantified as the probability distribution, or density of states within specific mass bin
  - the spectral function (SF)

- Why do we care about SF and finite width in hybrid approach for heavy ion collisions?
  - It can alter hadronic chemistry as the variation in resonance mass can change the multiplicity.
  - It is also crucial in electromagnetic probes. (e.g. dileptons and photons)

Framework

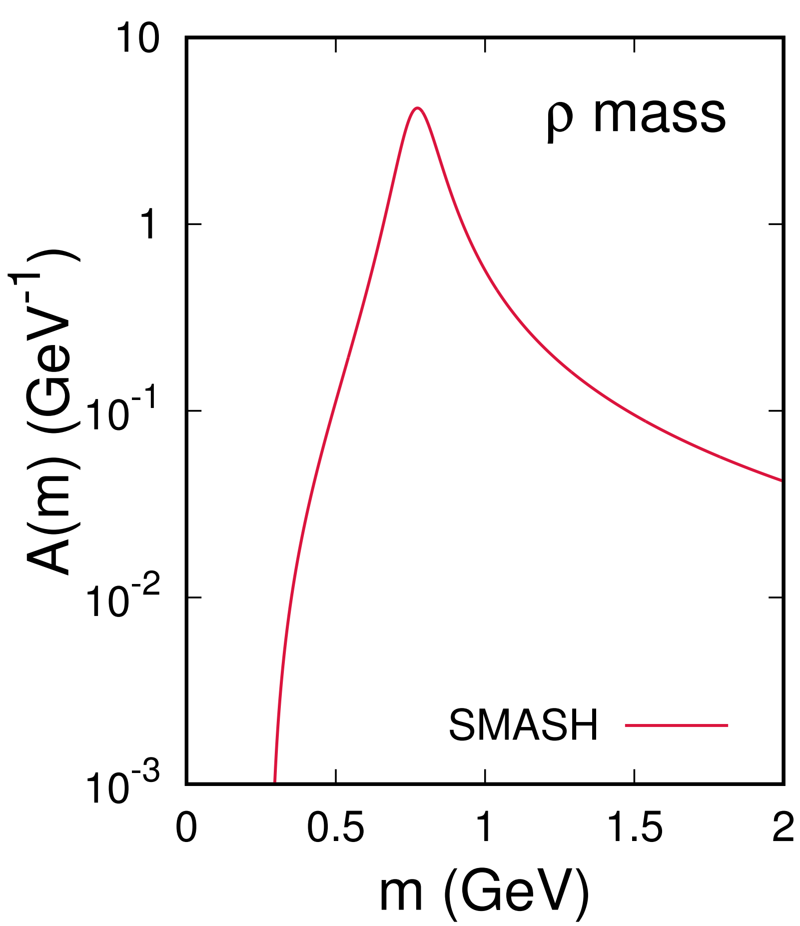
Spectral functions in SMASH

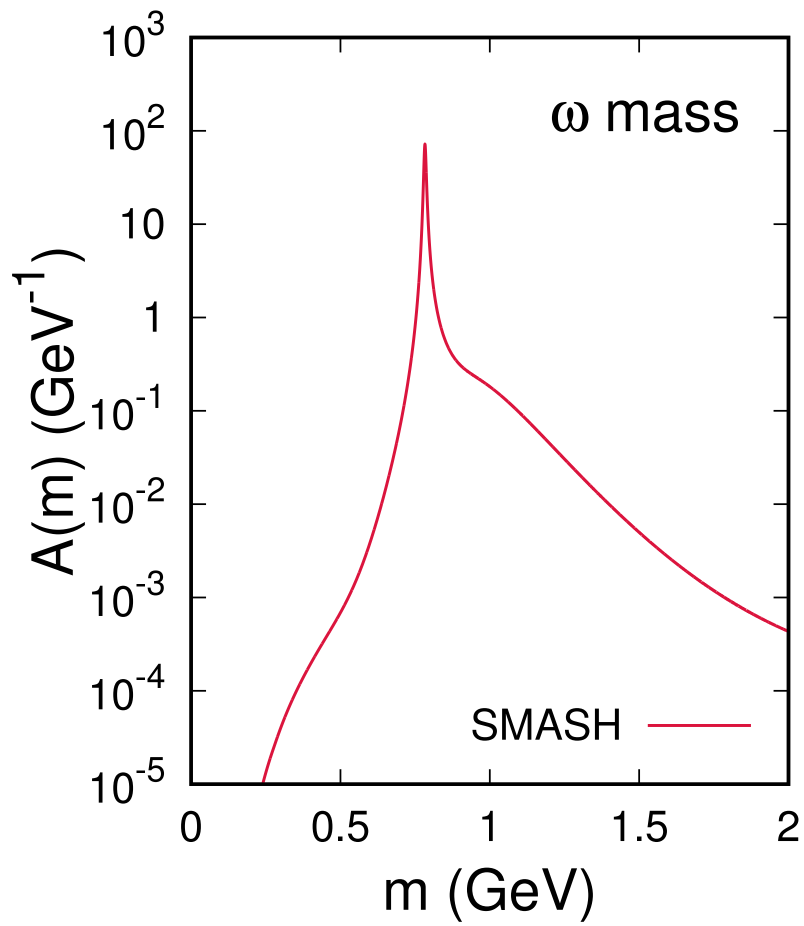
In SMASH, which is a microscopic transport of hadronic system,

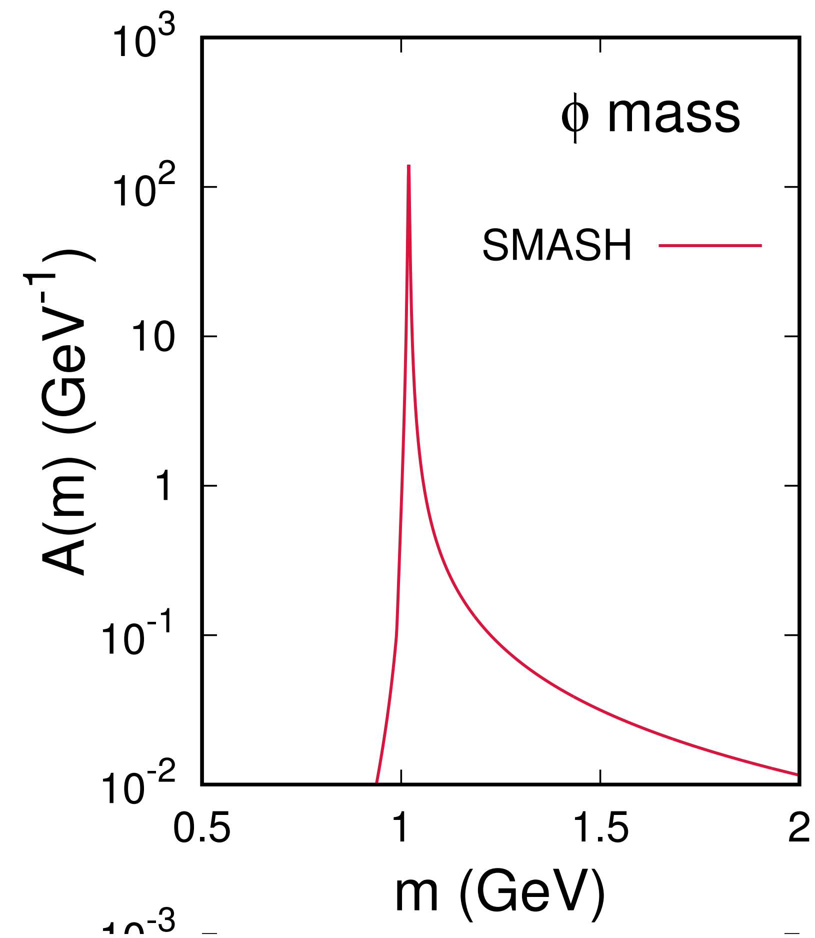
J. Weil *et al*. (2016)

Breit-Wigner SF $\mathcal{A}(m) = \frac{2}{\pi} \frac{m^2\Gamma(m)}{(m^2 - m_{\text{pole}}^2)^2 + m^2\Gamma^2(m)}$

with mass-dependent width from D. M. Manley and E. M. Saleski (1992)







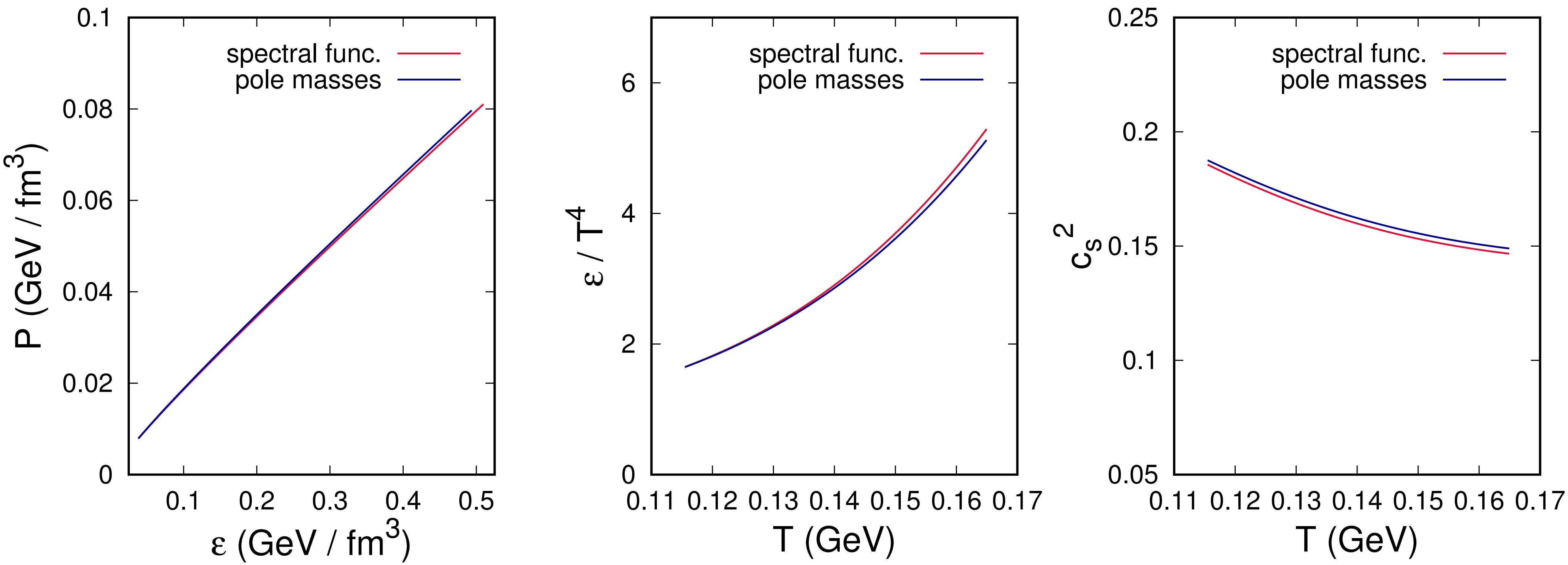
Framework

Spectral functions in SMASH

Thermodynamics with spectral functions

$$\epsilon = \sum_i g_i \int dm\, \mathcal{A}_i(m) \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, f_{0,i}(\mathbf{q}, m)\, E_{\mathbf{q}}$$

$$P = \sum_i g_i \int dm\, \mathcal{A}_i(m) \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, f_{0,i}(\mathbf{q}, m)\, \frac{|\mathbf{q}|^2}{3\, E_{\mathbf{q}}}$$



Framework

Spectral functions in SMASH

Thermodynamics with spectral functions

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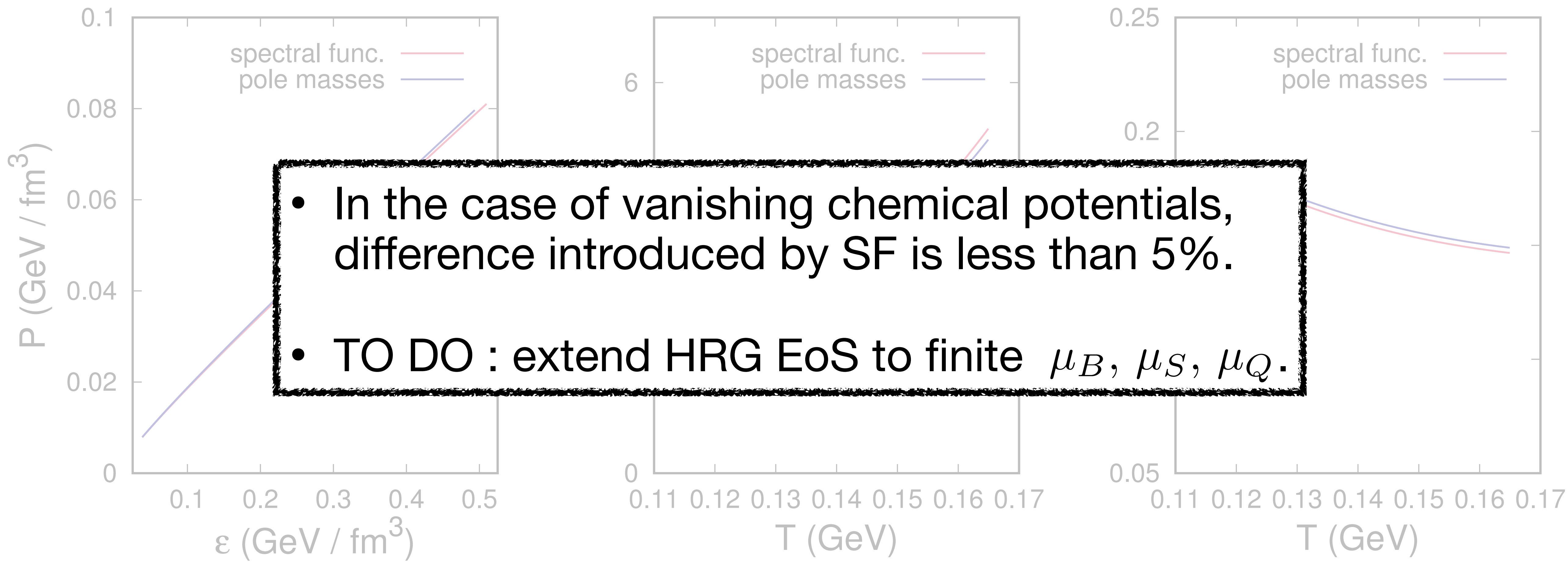


- In the case of vanishing chemical potentials, difference introduced by SF is less than 5%.
- TO DO : extend HRG EoS to finite $\mu_B,\, \mu_S,\, \mu_Q$.

Framework

Extension of the Cooper-Frye formalism

F. Cooper and G. Frye (1972)

Beginning with the energy-stress tensor

$$T^{\mu\nu} = \sum_i g_i \int dm\, \mathcal{A}_i(m) \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E_\mathbf{p}} f(x,\mathbf{p},m)$$

spectral function

Cooper-Frye with spectral function

$$\frac{dN_i}{dm\, d^3\mathbf{p}} = \frac{g_i}{(2\pi)^3} \int_\Sigma d^3\Sigma_\mu\, \mathcal{A}_i(m) \frac{p^\mu}{E_\mathbf{p}} f(x,\mathbf{p},m)$$

Energy/momentum conservation

$$\begin{aligned}
\mathcal{P}^\mu_{\mathrm{tot}} &= \sum_i \int dm \int d^3\mathbf{p}\, p^\mu \frac{dN_i}{dm\, d^3\mathbf{p}} \\
&= \int_\Sigma d^3\Sigma_\nu \sum_i g_i \int dm\, \mathcal{A}_i(m) \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E_\mathbf{p}} f(x,\mathbf{p},m) \\
&= \int_\Sigma d^3\Sigma_\nu T^{\mu\nu}
\end{aligned}$$

Framework

Extension of the Cooper-Frye formalism F. Cooper and G. Frye (1972)

Beginning with the energy-stress tensor

$$T^{\mu\nu} = \sum_i g_i \int dm\, \mathcal{A}_i(m) \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E_\mathbf{p}} f(x, \mathbf{p}, m)$$

spectral function

Cooper-Frye with spectral function

$$\frac{dN_i}{dm\, d^3\mathbf{p}} = \frac{g_i}{(2\pi)^3} \int_\Sigma d^3\Sigma_\mu\, \mathcal{A}_i(m) \frac{p^\mu}{E_\mathbf{p}} f(x, \mathbf{p}, m)$$

Mass distribution

$$\begin{aligned}
\frac{dN_i}{dm} &= \int d^3\mathbf{p}\, \frac{dN_i}{dm\, d^3\mathbf{p}} \\
&= g_i \int_\Sigma d^3\Sigma_\mu\, \mathcal{A}_i(m) \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu}{E_\mathbf{p}} f(x, \mathbf{p}, m) \\
&= \int_\Sigma d^3\Sigma_\mu\, j_i^\mu(x, m)\, \mathcal{A}_i(m) \quad \rightarrow \quad \int_\Sigma u^\mu d^3\Sigma_\mu\, n_i(x, m)\, \mathcal{A}_i(m)
\end{aligned}$$

Framework

Extension of the Cooper-Frye formalism F. Cooper and G. Frye (1972)

Viscous corrections to the distribution function

There are a few formalisms in the market.

- Grad's expansion with 14-moments approximation
  - Expansion in the power of momentum
    D. Teaney (2003)

- Chapman-Enskog expansion with relaxation time approximation
  - Expansion in the power of Knudsen number $(\mathrm{Kn} \sim l_{\mathrm{mfp}} \nabla_\mu u^\mu)$
    P. Bozek (2010)

- Resummed viscous correction
  - Exponentiate the distribution function
    M. McNelis, D. Everett and U. Heinz 1912.08271
    S. Pratt and G. Torrieri (2010)

Framework

Extension of the Cooper-Frye formalism F. Cooper and G. Frye (1972)

Bulk viscous correction to the distribution function

In the case of the (leading-order) Chapman-Enskog expansion

$$\delta f_{\rm bulk} = -\frac{\Pi}{\hat{\zeta}}\, f_0(1\pm f_0)\,\frac{p^\mu p^\nu}{(p\cdot u)}\left(\frac{1}{3}\Delta_{\mu\nu} + c_s^2\, u_\mu u_\nu\right)$$

Normalization :
$$\begin{aligned} -3\,\Pi &= g_{\mu\nu}\,\delta T^{\mu\nu}_{\rm bulk} \\ &= \sum_i g_i \int dm\,\mathcal{A}_i(m)\,m^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}}\,\delta f_{{\rm bulk},i}(\mathbf{p}) \end{aligned}$$

$$\hat{\zeta} = \frac{1}{3}\sum_i g_i \int dm\,\mathcal{A}_i(m)\,m^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_{\mathbf{k}}}\, f_{0,i}(1\pm f_{0,i})\left[\frac{m^2}{3E_{\mathbf{k}}} + \left(\frac{1}{3} - c_s^2\right)E_{\mathbf{k}}\right]$$

Viscous correction to the number density :

$$\begin{aligned} \delta n_{{\rm bulk},i} &= \frac{g_i}{2\pi^2}\,\frac{\Pi}{\hat{\zeta}}\sum_{l=1}^{\infty}(\pm 1)^{l+1} e^{l\mu/T}\times \\ &\int dm\,\mathcal{A}_i(m)\,m^2 T^2\left[\left(\frac{1-3\,c_s^2}{l}\right)K_2\left(\frac{lm}{T}\right) - c_s^2\frac{m}{T}K_1\left(\frac{lm}{T}\right)\right] \end{aligned}$$

Framework

Extension of the Cooper-Frye formalism F. Cooper and G. Frye (1972)

Shear viscous correction to the distribution function

Grad's expansion with 14-moment approximation is used for the $\delta f_{\mathrm{shear}}$ in the current MUSIC+SMASH hybrid simulations.

$$\delta f_{\mathrm{shear}} = f_0(1 \pm f_0)\frac{\pi_{\mu\nu}p^{\mu}p^{\nu}}{2\,(\epsilon + P)\,T^2}$$

NOTE : All of the shear, bulk and diffusive corrections can be obtained within each framework mentioned before.

TO DO : determine $\delta f_{\mathrm{shear}}$, δf_{bulk} and $\delta f_{\mathrm{diffusion}}$ from thermodynamic integrals within a single and consistent framework.

Framework

Sampling Procedure

In each (discretized) hypersurface element

1. Determine the mean multiplicity of each hadronic specie

$$\langle N \rangle = u^\mu \Delta^3 \Sigma_\mu \int dm\, \mathcal{A}_i(m)\ [n_{0,i}(x,m) + \delta n_{\text{bulk},i}(x,m)]$$

2. In the case of resonance, sample the mass from

$$\text{PDF}(m) \propto \mathcal{A}_i(m)\ [n_{0,i}(x,m) + \delta n_{\text{bulk},i}(x,m)]$$

3. Then sample momentum from

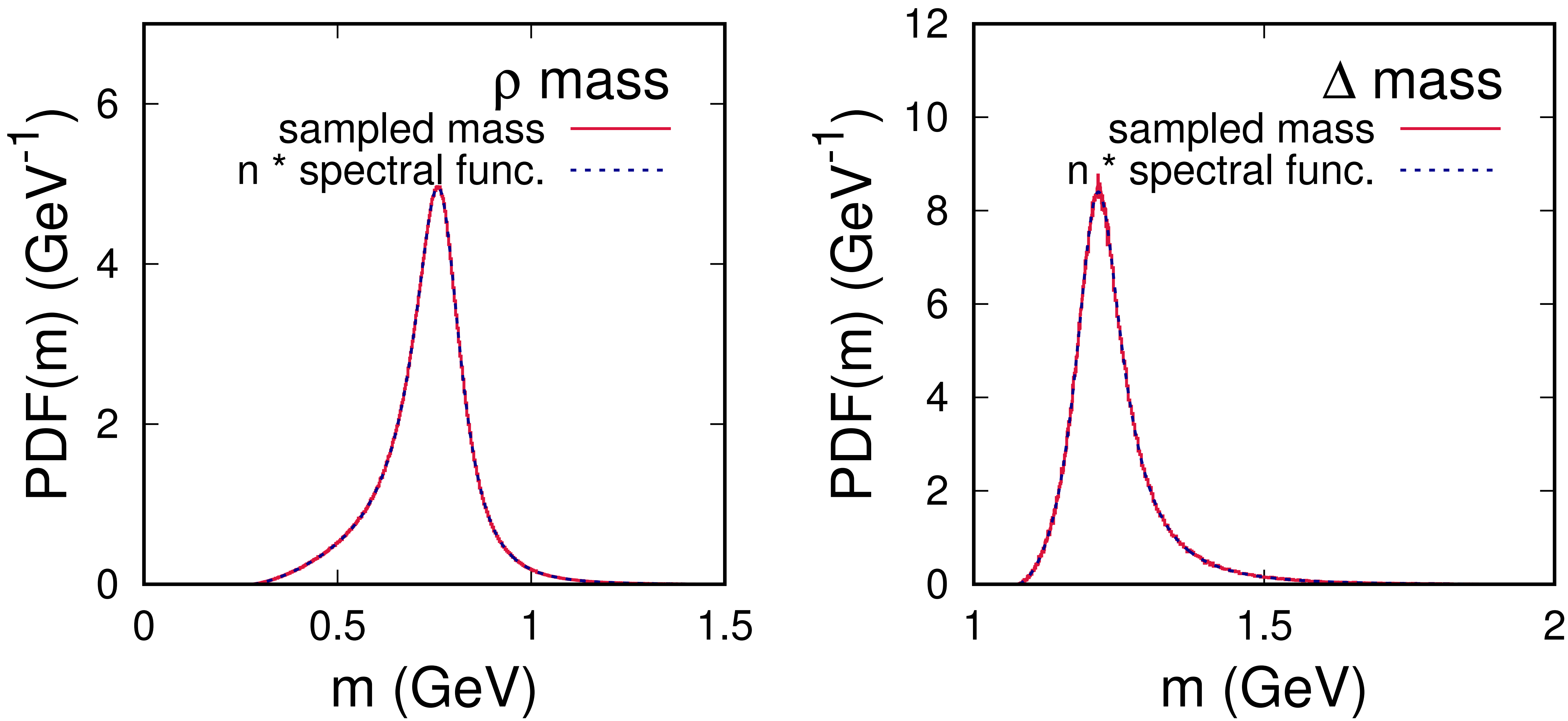
$$\frac{dN_i}{d^3\mathbf{p}} \propto \frac{p^\mu \Delta^3 \Sigma_\mu}{E_\mathbf{p}}\ [f_{0,i}(x,\mathbf{p},m) + \delta f_{\text{shear},i}(x,\mathbf{p},m) + \delta f_{\text{bulk},i}(x,\mathbf{p},m)]$$

Go over all hypersurface elements.

Results

Verification of mass sampling in a thermal box

Mass distribution is different from the spectral function itself, due to the mass-dependent number density.



Results

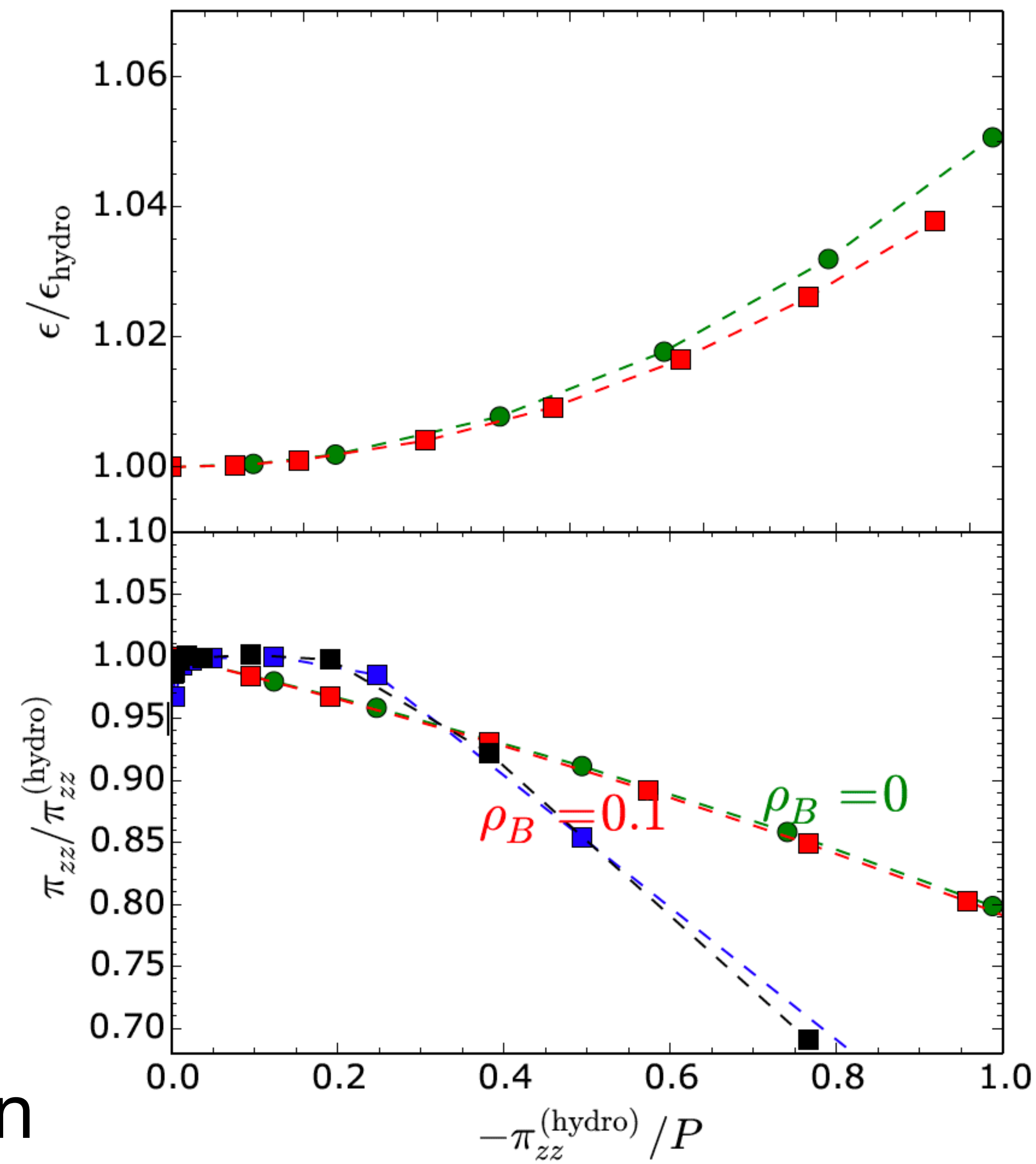
Energy/momentum conservation check in a box

For an arbitrary shear stress tensor satisfying $\pi^{xx} = \pi^{yy} = -\pi^{zz}/2$,

- Chapman-Enskog $\delta f_{\rm shear}$ works better to yield the desired value of π^{zz}, for small $\pi^{\mu\nu}$.

- Resummed $\delta f_{\rm shear}$ results in a slight deviation in π^{zz}, but it is not an issue for typical size of $\pi^{\mu\nu}$ in ultra-relativistic heavy ion collisions. $\left(\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}} \sim 0.02\,(\epsilon + P)\right)$

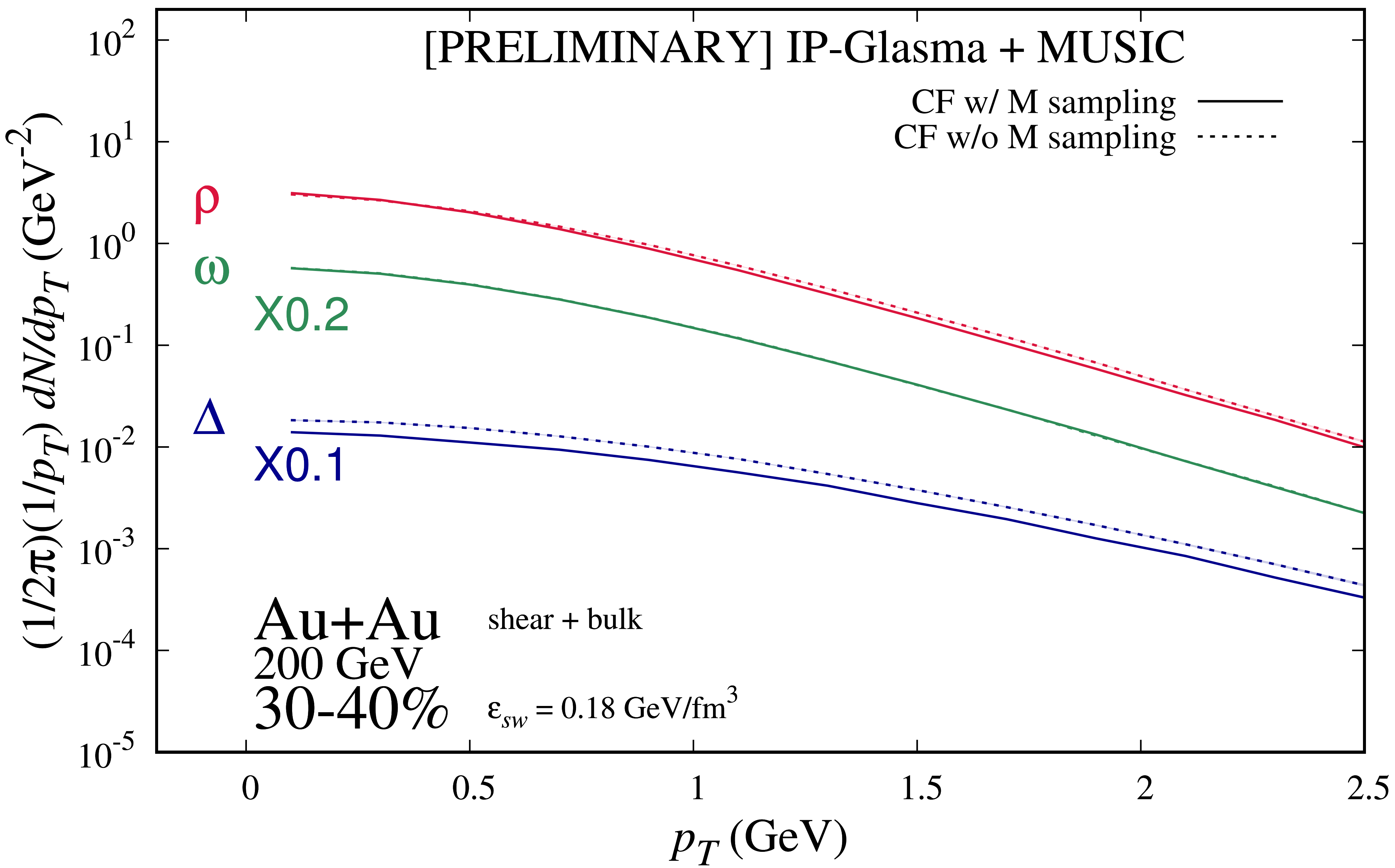
TO DO : conservation check with bulk viscous correction



Calculation by Scott Pratt

Results

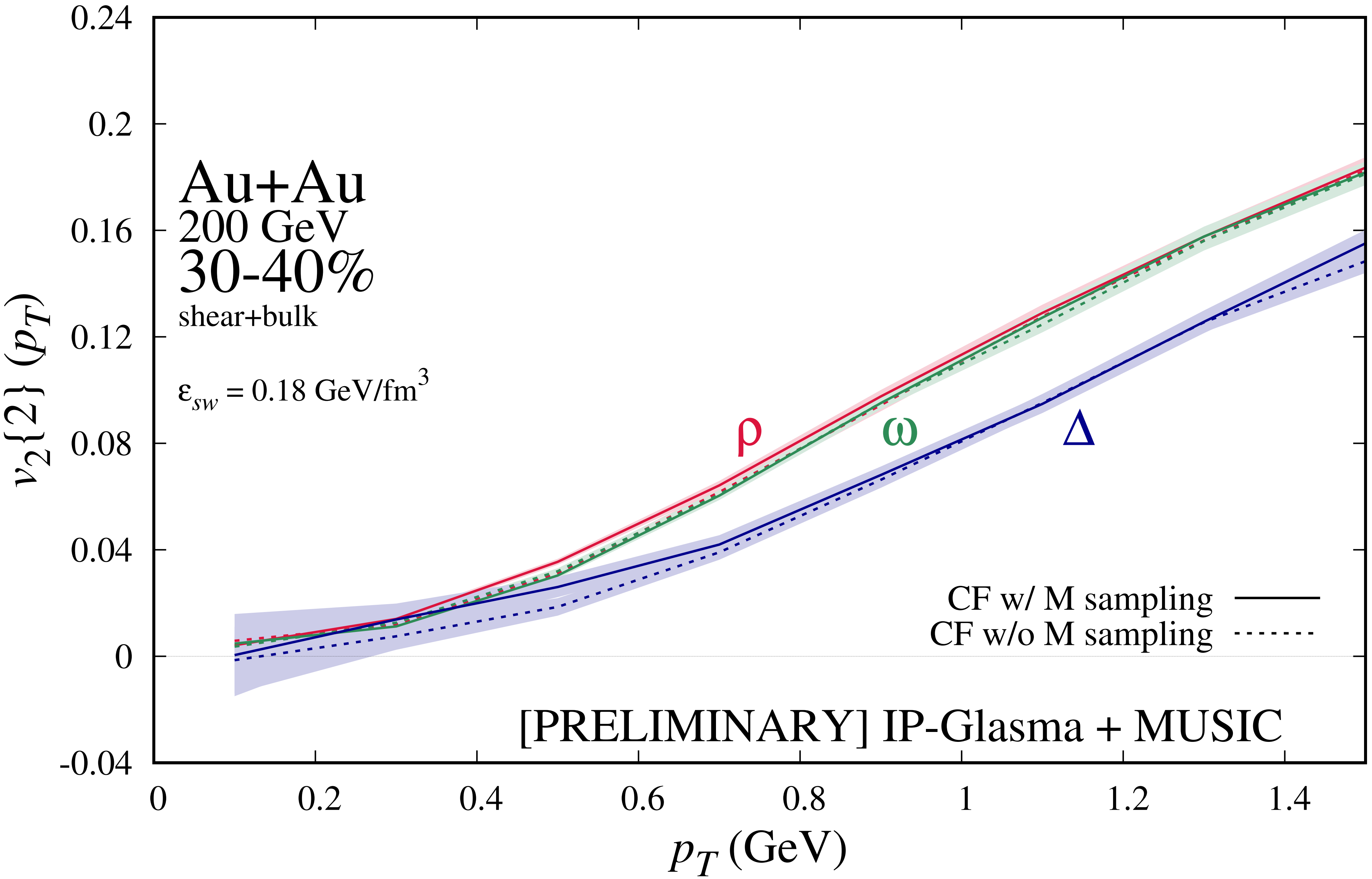
Single-particle distributions of resonances at particlization



- There is up to ~30% change in the multiplicity of Δ .
- Modification of the momentum space distribution is negligible.

Results

Flow anisotropy of resonances at particlization



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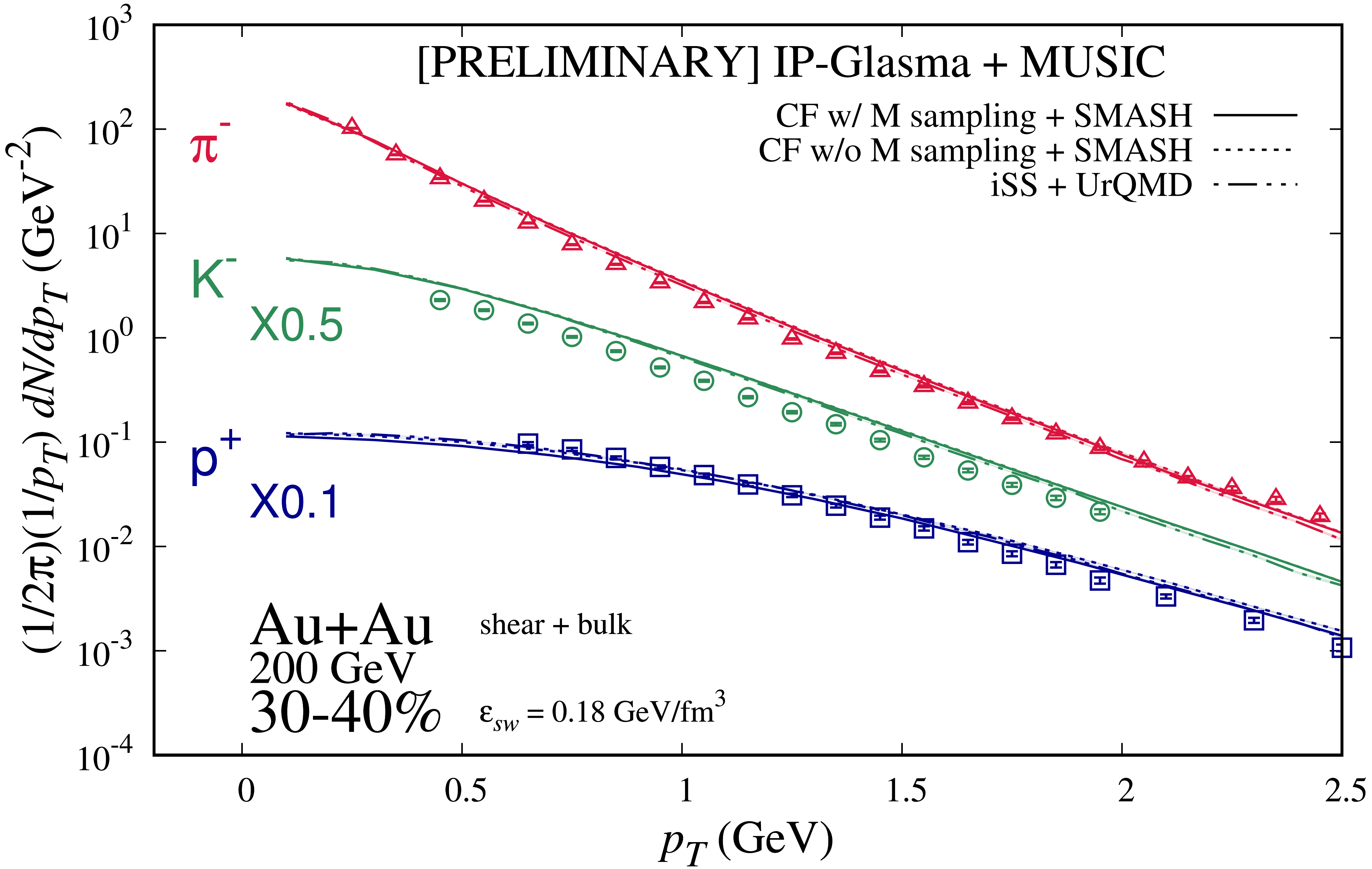
Results

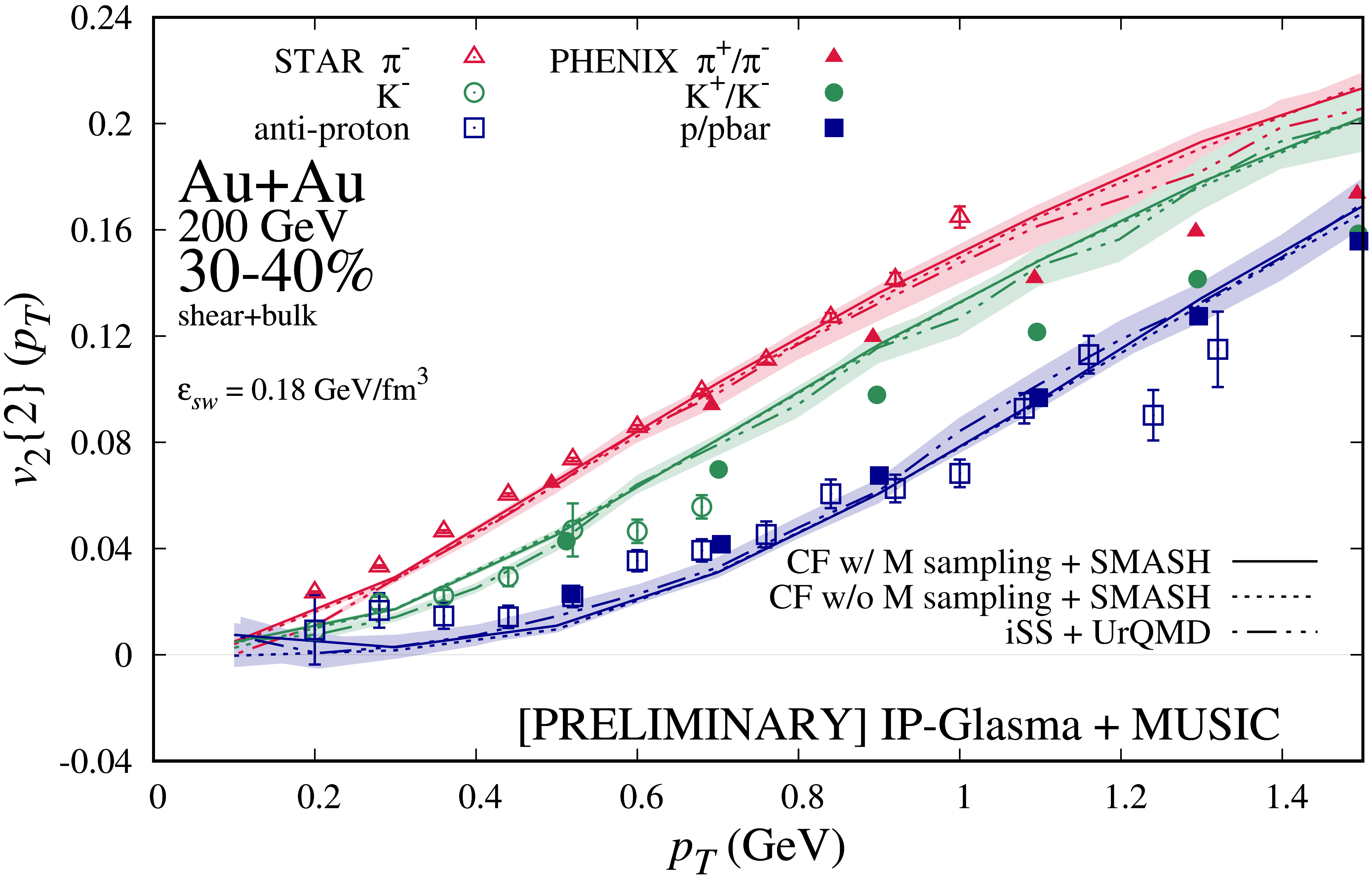
Single-particle distributions of final-state hadrons



- Spectral functions have small effect (~ 10% for proton), in the case of ultra-relativistic heavy ion collisions.

Results

Flow anisotropy distributions of final-state hadrons



- Spectral functions have quite small effect (< 5%), in the case of ultra-relativistic heavy ion collisions.

Conclusions

- Finite widths of resonances are taken into account by having spectral functions at the Cooper-Frye particlization.

- At the $\mu_B = 0$ regime, spectral functions do not change the thermodynamics much. (less than 5% difference)

- In the case of ultra-relativistic heavy ion collisions, spectral functions also have small effects on the final-state flow anisotropy of hadrons.

TO DO list

- Extend toward finite μ_B, μ_S, μ_C regime as it should be addressed in low-energy heavy ion collisions.

- Consistently determine shear, bulk and diffusive corrections to distribution function and corresponding transport coefficients.