Freeze-out prescription for critical fluctuations

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Work in progress with K. Rajagopal, M. Stephanov, R. Weller, Y. Yin

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Hydro+ fluctuations on the QGP side

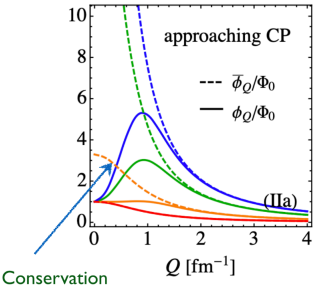
- Fluctuations of the order parameter : Signatures of critical point
- Critical slowing down : Fluctuations lag behind their equilibrium values as the fluid expands and cools
- Hydro+ identifies the slowest set of modes Stephanov,Yin, 2017 , framework for simultaneous evolution of average conseved densities and slow mode

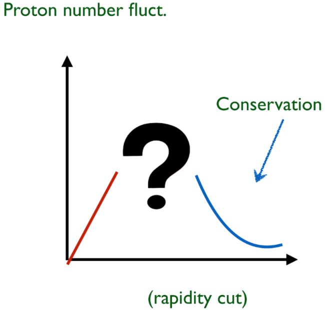
$$\phi_Q(x) = \int e^{iQ\Delta x} \left\langle \delta \frac{s}{n}(x_+) \delta \frac{s}{n}(x_-) \right\rangle , \; x_\pm = x \pm \frac{\Delta x}{2}$$

- Attempts to simulate Hydro+ in simplified settings Rajagopal et al., 2019 (RRWY), Du et al., 2020
- Next step: To freeze-out these fluctuations

Overview of the talk

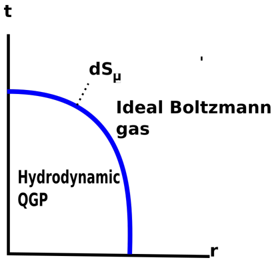
- Motivate the prescription we use to freeze-out hydro+ fluctuations
- Develop explicit example of how we do this for RRWY set up
- Qualitative predictions for experiment
- Summarize this work in progress

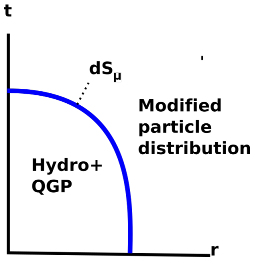




Borrowed from Yi Yin's talk at the INT workshop , 2020

Cooper-Frye freeze out





$$\langle f_A \rangle (x,p) = e^{-\frac{E_A(x,p)-\mu}{T}}$$

$$f_A(x,p) = \langle f_A \rangle (x,p) + \underbrace{\delta f_A(x,p)}_{\text{critical fluctuations}}$$

$$N_A = \int dS_\mu \int Dp\, p^\mu\, f_A(x,p)$$

Critical fluctuations in particle number

- We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with the critical sigma field

$$\delta m_A \approx g_A \sigma$$

- Modified particle distribution function:

$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

- σ field correlations in equilibrium:

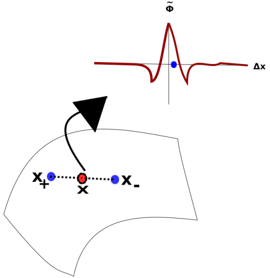
$$\langle \sigma \rangle \quad = \quad 0\,, \quad \langle \sigma(x_+)\sigma(x_-) \rangle = \frac{T e^{-\frac{|\Delta x|}{\xi}}}{4\pi |\Delta x|}$$

Prescription for freeze-out of critical fluctuations

$\tilde{\phi}(x, \Delta x) = \int_Q e^{iQ\Delta x}\phi_Q(x)$

$\langle \sigma(x_+)\sigma(x_-)\rangle = Z\,\tilde{\phi}(x, \Delta x)$

Z fixed so that $\langle \sigma\sigma\rangle$ reduces to the equilibrium expression when ϕ_Q given by OZ eqn.



$$\langle \delta N_A^2\rangle_\sigma = g_A^2 Z \int dS_\mu J_A^\mu(x_+) \int dS'_\nu J_A^\nu(x_-)\tilde{\phi}(x, \Delta\tilde{x})$$

$$J_A^\mu = d_A \int Dp\, p^\mu \frac{\partial \langle f_A\rangle}{\partial m_A}$$

Project Overview: Freeze-out of two systems near the critical point

$$u \cdot \partial\phi_Q = -\Gamma(Q)(\phi - \bar{\phi}_Q)$$

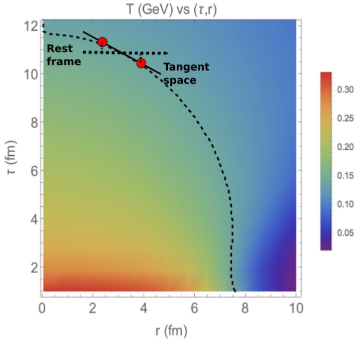
CP along $\mu = 0$	CP at non-zero μ
Order parameter non-conserved	Order parameter conserved
Model A	Model H
$\Gamma(Q) \propto 1 + (Q\xi)^2$	$\Gamma(Q) \propto (Q\xi)^2$
Rajagopal, Ridgway, Weller, Yin, 2019	In progress

Hydro+ in a simplified setting

Rajagopal, Ridgway, Weller, Yin, 2019

- Flow: Boost invariant azimuthally symmetric
- Freeze-out condition: $T(x) = 0.14\text{GeV}$ $(T_c \sim 0.16\,\text{GeV})$

$$\begin{aligned}\langle \delta N_A^2 \rangle &= g_A^2\, Z \int d^3x_+ \int d^3x_-\, I_A(x,\Delta x)\, \tilde{\phi}(x,\Delta\tilde{x}) \\ &\approx g_A^2\, Z \int d^3x \int d^3\Delta x\, I_A(x,\Delta x)\, \tilde{\phi}(x,\Delta\tilde{x})\end{aligned}$$



$$\mathbf{I_A(x,\Delta x) = n(x_+)\cdot J_A(x_+)\, n(x_-)\cdot J_A(x_-)}$$

$$\mathbf{n\cdot J_A = d_A \int Dp\, \frac{\partial \langle f_A \rangle}{\partial m_A}\, n\cdot p}$$

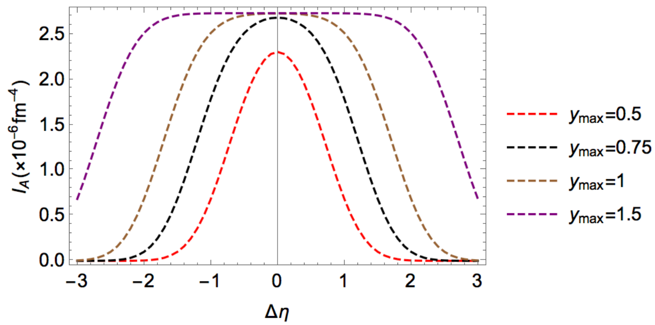
$\Delta\tilde{x}$ is the projection of Δx onto the local rest frame at x.

Δx dependence of I_A $\boxed{\int d^3\Delta x\, I_A(x,\Delta x)\,\tilde{\phi}(x,\Delta\tilde{x})}$

- No spatial dependence when integrated over full phase space

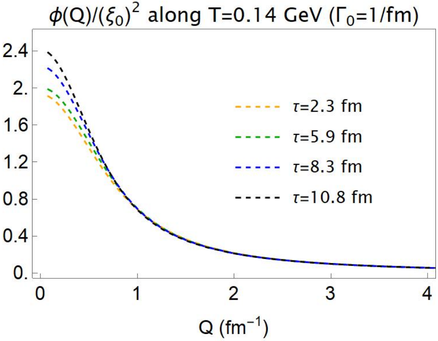
$$n\cdot J_A = \frac{d_A m_A}{T_f}\int Dp\, e^{-\frac{u\cdot p}{T}}\,\frac{n\cdot p}{u\cdot p}$$

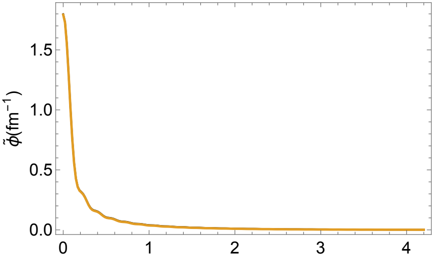
$$\langle \delta N_A^2\rangle = g_A^2\, Z\, I_A \int d^3x\phi_0(x)$$



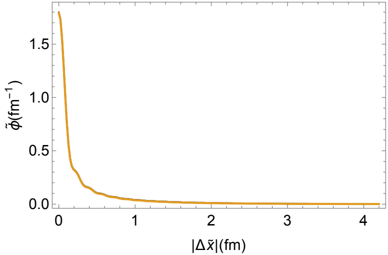
Rapidity cuts in the lab frame ⇔ Cuts in spatial rapidity on the FHS

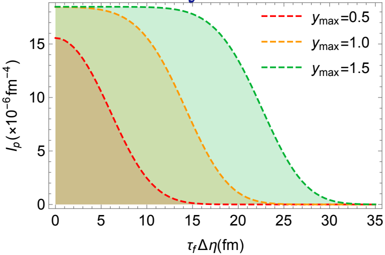
Output from Hydro+ simulation

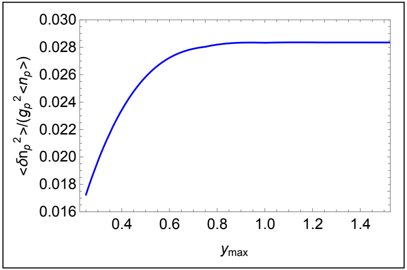




Proton number fluctuations due to ϕ_Q in Model A







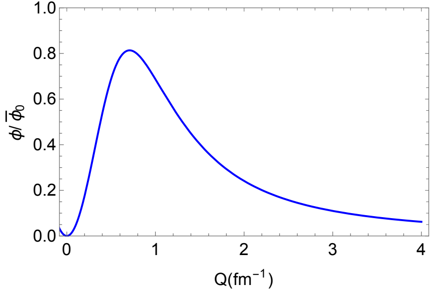
$\frac{\langle \delta n_p^2 \rangle}{g_p^2 \langle n_p \rangle}$ at $\tau = 8.35$ on the freeze-out hypersurface

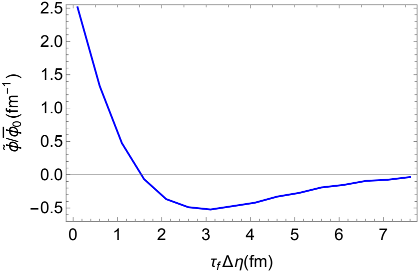
$$\langle \delta N_p^2 \rangle = \int \langle \delta n_p^2 \rangle \, d^3x$$

$$\frac{\langle \delta n_p^2 \rangle}{g_p^2 \langle n_p \rangle} = \frac{\int I_p(x, \Delta x) \tilde{\phi}(x, \Delta\tilde{x}) \, d^3\Delta x}{\int f_p(x, q) \, Dq}$$

Model H calculation for the same set up is in progress

Model H : ϕ_Q from an analytically solvable Bjorken model



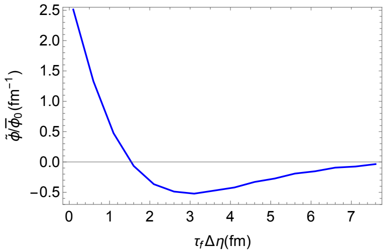


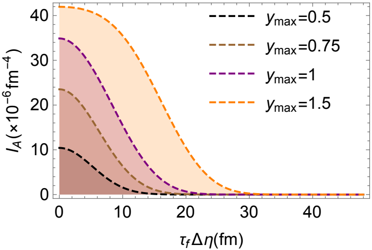
$$\frac{d\phi_Q}{d\tau} = -\Gamma[Q\xi](\phi_Q - \bar{\phi}_Q)$$

$$\Gamma[x] = \Gamma_0 \left(\frac{\xi_0}{\xi}\right)^3 K(x)\,, \quad \Gamma_0 = 1\,\text{fm}^{-1}\,, \quad \xi_0 = 1\text{fm}$$

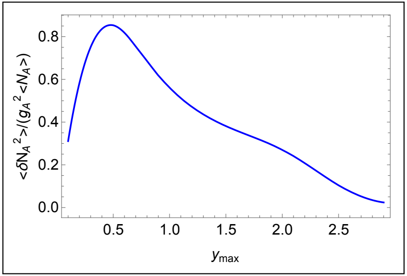
$$\bar{\phi}_Q = \frac{\bar{\phi}_0}{1 + (Q\xi)^2}$$

Model H: Effects of conservation-1



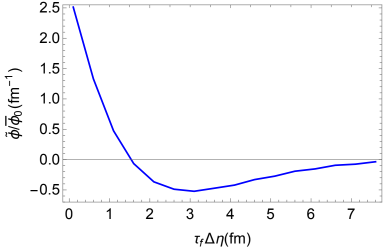


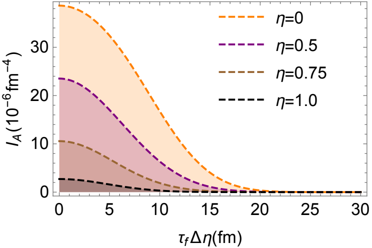
$$\frac{\left\langle \delta N_A^2 \right\rangle}{g_A^2 \left\langle N_A \right\rangle} = \frac{\tau_f \int d\eta \int d\Delta\eta\, I_A(\eta, \Delta\eta)\, \tilde{\phi}(\eta, \Delta\eta)}{\int d\eta \int Dp\, f(\eta, p)}$$

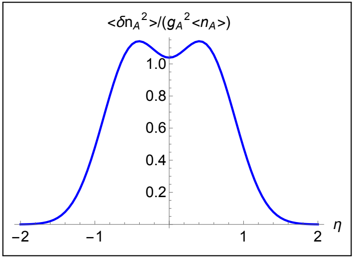


- At $y_{\max} << 1$, $\dfrac{\left\langle \delta N_A^2 \right\rangle}{\left\langle N_A \right\rangle} \propto y_{\max}$
- As $y_{\max} \to \infty$, the effect of charge conservation takes over $\left\langle \delta N_A^2 \right\rangle \approx 0$

Model H: Effects of conservation-2







- Ratio of fluctuation density to mean density at different points on the freeze-out hypersurface

- Patch size for Oliinychenko and Koch, 2019?

Summary

- Demonstrated the freeze-out of Hydro+ fluctuations in a simplified setting RRWY, 19.
- Need to finish the integration over the full freeze-out hypersurface to calculate $\left\langle \delta N_A^2 \right\rangle$
- Qualitative prediction for a non-monotonic behavior of $\left\langle \delta N_p^2 \right\rangle / (g_p^2 \left\langle N_p \right\rangle)$ with acceptance
- Numerical simulation for Hydro+ with Model H dynamics in progress
- The procedure could be extended to more general scenarios with no symmetries
- A comparative study of $\left\langle \delta N_A^2 \right\rangle$ as a function of m_A needs to be done
- The procedure should be extended for higher cumulants