Microcanonical particlization of relativistic hydrodynamics

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with Volker Koch and Shuzhe Shi
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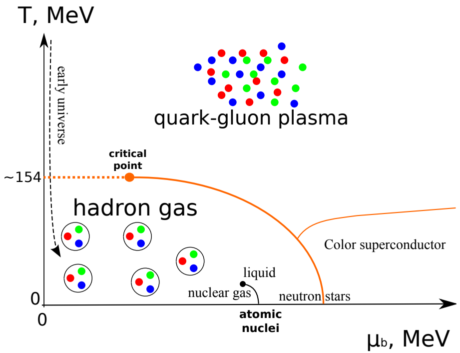
arXiv:2001.08176v1

BEST COLLABORATION

May 16, 2020

Where is phase transition/critical point?



Generic critical point features:

- Correlation length increases: critical opalescence
- Fluctuations increase

In heavy ion collisions: do they survive until measurement?
Need dynamical modelling.

Dynamical approaches with phase transition

- Transport: which degrees of freedom?
  - PHSD or AMPT with potentials generating phase transition?
  - RSP code with NJL potentials Marty, PRC 92, 015201 (2015), arXiv:1412.5375
    only mesons, code abandoned(?)
  - Danielewicz transport Fundam.Theor.Phys. 95 (1999) 69-84, arXiv:nucl-th/9808013
    selected hadrons with $m(S)$, $m = 0$ matches QGP entropy
    Agnieszka is working in a related direction, see her talk
- Hydrodynamics with
  - EoS with phase transition (Maxwell construction)
  - Surface tension terms
    Steinheimer, Randrup Eur. Phys. J. A52 (2016) no.8, 239
    Pratt PRC 96 (2017) no.4, 044903
  - Stochastic terms
    Kapusta et al, PRC 85, 054906 (2012)
    Kumar et al., Nucl. Phys. A **925**, 199 (2014)
    Nahrgang et al., arXiv:1804.05728
  - Fields with stochastic terms
    Nahrgang et al., PRC **84**, 024912 (2011)
    Herold et al., J. Phys. G **41**, no. 11, 115106 (2014)

Important: not to lose fluctuations at particlization

Standard particlization in hydro + transport hybrids

On average by events:

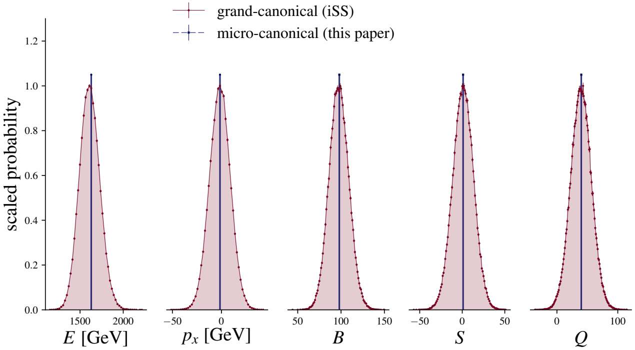
How many particles cross a moving surface $\equiv$
are produced from a hypersurface element with a normal $d\sigma_\mu$?
Cooper-Frye formula:

$$dN = \frac{g}{(2\pi\hbar c)^3}\frac{p^\mu}{p^0} f(p^\alpha u_\alpha, T, \mu) d^3p \, d\sigma_\mu = j^\mu d\sigma_\mu$$

- Cooper-Frye formula does not specify multiplicity distribution
- Standard choice $P(N) = Poisson(\bar{N})$
  motivated by grand-canonical ensemble + classical statistics
- Particles in different cells sampled independently

Grand-canonical versus microcanonical sampling

Usual grand-canonical particlization assumes independent particles
But particles should be correlated due to conservation laws



AuAu, 19.6 GeV, 30-40% central collisions

E-by-e conservation laws are necessary to study fluctuations

State of the art before our work

"In principle, these sampling fluctuations are constrained by energy-momentum, baryon number and charge conservation. However, the exact implementation of these constraints is non-trivial and will have to be left for future studies."

[Shen:2014vra, "The iEBE-VISHNU code package for relativistic heavy-ion collisions"]

"an exact implementation of the realistic thermal fluctuations in iEBE-VISHNU is non-trivial. Here, we take [Poisson distribution] as a basic assumption, and then focus on investigating how the effects of volume fluctuations, hadronic evolution, resonance decays, etc., influence the multiplicity fluctuations of final produced hadrons"

[Li:2017via, "Noncritical fluctuations of (net) charges and (net) protons from the iEBE-VISHNU hybrid model"]

"One might proceed further to take into account conserved charges, as discussed in the previous section. Unfortunately, the latter is highly nontrivial, owing to precisely the same difficulties to explicitly incorporate global charge conservation at hadronization in most hydrodynamical models."

[Ma:2019hmc, "Hydrodynamic results on multiplicity fluctuations in heavy-ion collisions"]

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Systematically taking conservation laws into account

Quantities to conserve:

$$\begin{pmatrix} P^{\mu}_{tot} \\ B_{tot} \\ S_{tot} \\ Q_{tot} \end{pmatrix} = \sum_{\substack{\text{cells} \\ i}} \int \begin{pmatrix} p^{\mu}_{i} \\ B_{i} \\ S_{i} \\ Q_{i} \end{pmatrix} \frac{p^{\nu} d\sigma_{\nu}}{p^{0}} f_i(p^{\alpha}u_{\alpha}, T, \mu_i) \frac{g_i d^3 p}{(2\pi\hbar)^3}$$

Conservation laws applied independently to parts of the hypersurface:

patches

Plan of the talk:

- Microcanonical particlization in a single patch
- Splitting into patches

Systematically taking conservation laws into account

Distribution to sample:

$$P(N, \{N_s\}^{\text{species}}, \{x_i\}_{i=1}^N, \{p_i\}_{i=1}^N) = \mathcal{N}$$

$$\left(\prod_s \frac{1}{N_s!}\right) \prod_{i=1}^N \frac{g_i}{(2\pi\hbar)^3} \frac{d^3 p_i}{p_i^0} p_i^\mu d\sigma_\mu \, f_i(p_i^\nu u_\nu, T, \mu_i) \times$$

$$\delta^{(4)}(\sum_i p^\mu - P^\mu_{tot}) \, \delta^{B_{tot}}_{\sum_i B_i} \, \delta^{S_{tot}}_{\sum_i S_i} \, \delta^{Q_{tot}}_{\sum_i Q_i}$$

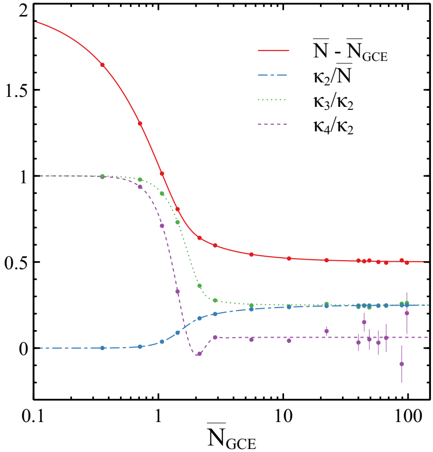
- Total quantities conserved
- Variations of T, μ, u within patch taken into account
- Turns into standard microcanonical sampling in case of one cell
- Sampled with Metropolis algorithm

Testing the sampling I: one cell, simple box

Sampling is already non-trivial, several works devoted to this case

Werner:1995mx, Becattini:2004rq, Begun:2005qd

Massless particles: analytically known



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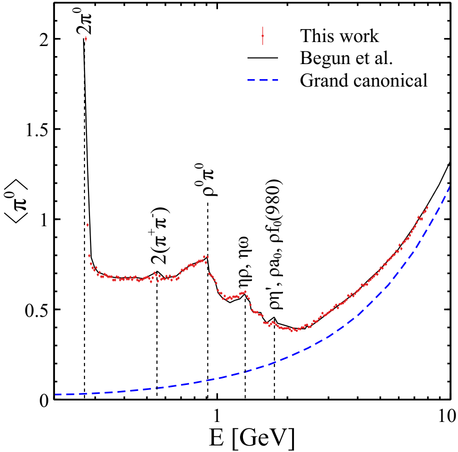
Full hadron gas



Fast open-source microcanonical sampler!

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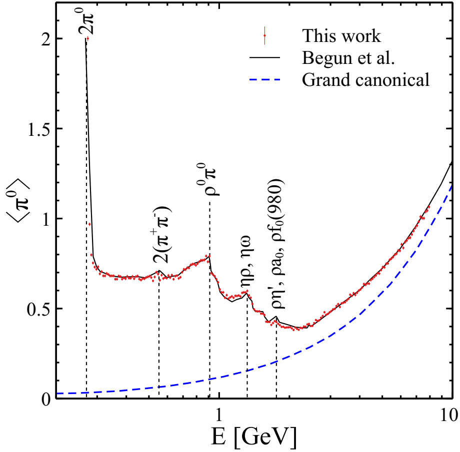
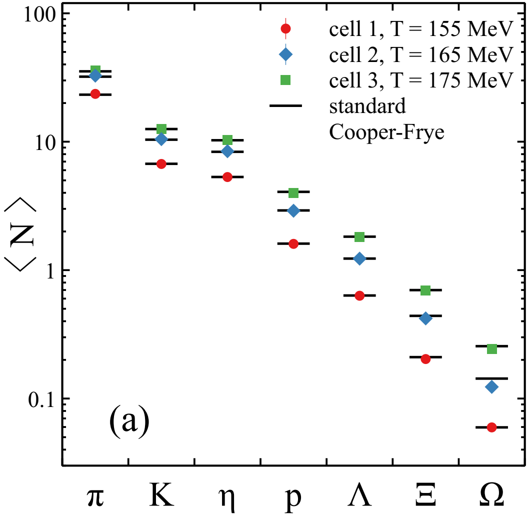


github.com/doliinychenko/microcanonical_cooper_frye

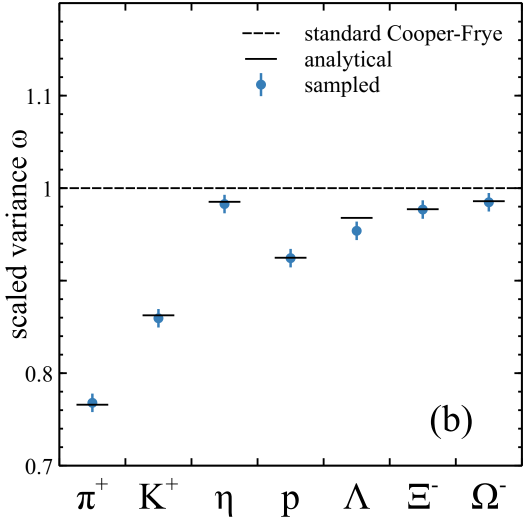
Testing the sampling II

- Patch consisting of 3 cells:
  - $d\sigma_1^\mu = (500.0, 50.0, 20.0, 30.0)\,\mathrm{fm}^3$, $d\sigma_2^\mu = (500.0, 40.0, 80.0, 30.0)\,\mathrm{fm}^3$, $d\sigma_3^\mu = (500.0, 20.0, 20.0, 20.0)\,\mathrm{fm}^3$
  - $\vec{v}_1 = (0.2, 0.3, 0.4)$, $\vec{v}_2 = (0.1, 0.5, 0.5)$, $\vec{v}_3 = (0.3, 0.4, 0.2)$
  - $T_1 = 0.155$ GeV, $T_2 = 0.165$ GeV, $T_3 = 0.175$ GeV
- Total energy of the patch 1268.2 GeV
- 416 different hadronic species generated ($m < 2.5$ GeV)
- Total energy, momentum, B, S, Q conserved
- Preserving local variations of T, μ, u
- Check local means, scaled variance $\omega \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$ of total multiplicities

Testing the sampling: several cells per patch



Testing the sampling: several cells per patch



Analytical: M. Hauer, V. V. Begun and M. I. Gorenstein, Eur. Phys. J. C **58**, 83 (2008)

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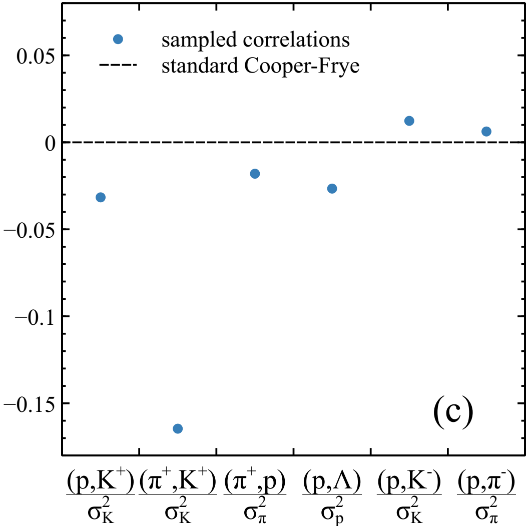
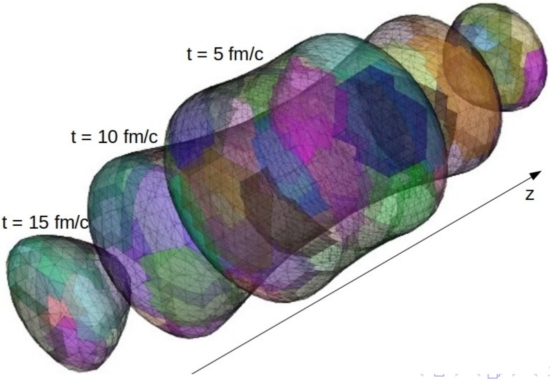


Conclusion so far: sampling works as intended

Partitioning hypersurface into patches

- How big should the patch be?
  - Not too small
  - Contain > 1 particle $\implies$ $> 100 - 1000$ cells per patch
- How to split hypersurface into patches?
- What physics remains after splitting into patches defined by an ad hoc algorithm?



Patch splitting

- Start with particular non-clustered cell, e.g. with smallest τ or η
- Define distance, add closest cells until total rest frame energy E_{patch} reached
- Start new patch

Different algorithms:

- (a) starting with t_{min}, distance $\Delta t^2 + \Delta r^2$
- (b) starting with η_{max}, distance $\Delta t^2 + \Delta r^2$
- (c) starting with η_{max}, distance $\Delta\eta$
- (d) starting with E_{max}, distance $\Delta r^2/d_0^2 + (\Delta T/\sigma_T)^2 + (\Delta\mu_B/\sigma_{\mu_B})^2$
- Additional adjustments to keep patch charges integer

How much do these ad hoc details influence results?

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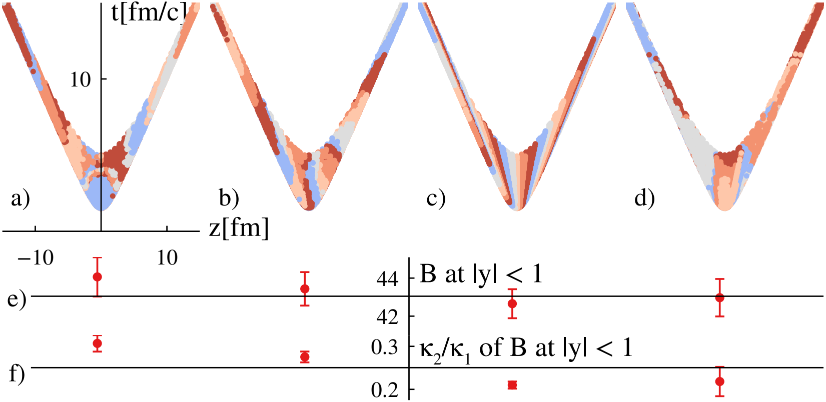
Patch energy E_{patch} – physical parameter, algorithm – systematic error

Effects from splitting algorithm



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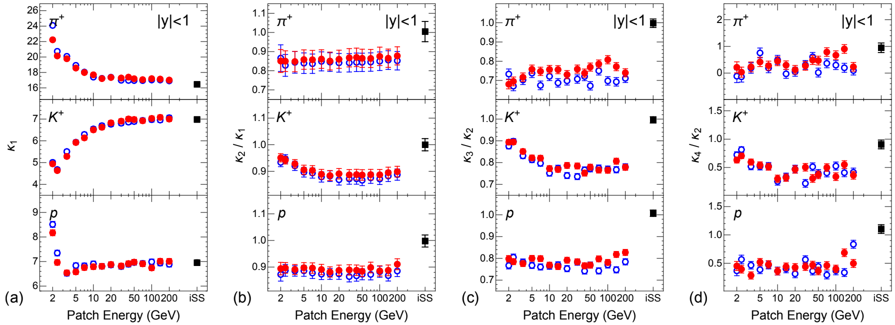
Particle cumulants



Red points – algorithm (d)

Blue points – algorithm (a)

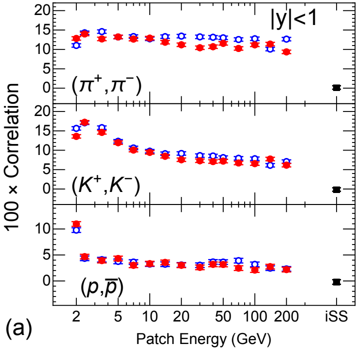
Black points – grand-canonical sampler

Systematic error due to algorithm is tolerable

(Micro-)canonical effects clearly seen even after rapidity cut

Microcanonical sampling: effects and applications

Example: AuAu at 19.6 GeV, 30-40% centrality



- Correlations and fluctuations
- Chiral effects
- Small systems
- Quark-Gluon Plasma in droplets

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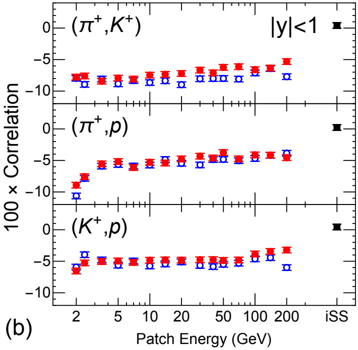

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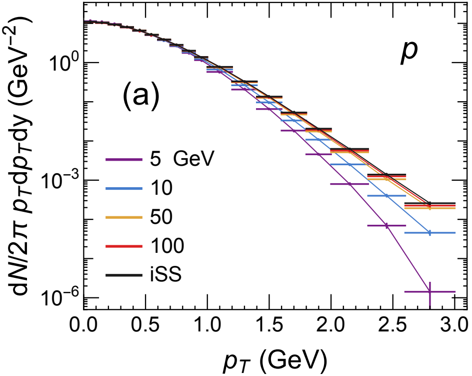

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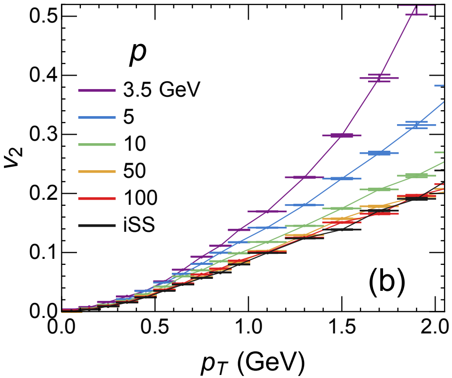

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Summary

- Standard sampling neglects event-by-event conservation laws
- This obfuscates fluctuations
- Reasonably fast method exists to include conservation laws
  - Split hypersurface into patches and conserve on every patch
  - Using Markov chain reminiscent of $2 \leftrightarrow 3$ stochastic collisions to thermalize
  - Passes non-trivial test cases
  - Code publically available github.com/doliinychenko/microcanonical_cooper_frye
- Patch splitting
  - Contains certain degree of arbitrariness
  - To which physics is not too sensitive
  - Patch size is a physical parameter, and it matters
- Microcanonical effects:
  - high-p_T suppression
  - v_2 enhancement
  - Non-trivial correlations
  - Suppression of fluctuations compared to grand-canonical

Outlook

- Try to reproduce STAR data on correlations [Adam:2019xmk]
- Apply to hydrodynamics simulations of p+Pb and p+p
- Check background for Chiral Magnetic Effect
- Use with fluctuating hydrodynamics

Correlations and fluctuations: from hydro to particles

$B_H(i)$ – baryon number from hydro cell i
$B_S(i)$ – baryon number sampled from cell i
typically $B_S(i) = Poisson(B_H(i))$
$\delta B(i) = B_S(i) - B_H(i)$
$\langle \delta B(i) \rangle = 0$

$\langle \ldots \rangle$ – average over samples
$\overline{\ldots}$ – average over hydro events
$\langle\langle \ldots \rangle\rangle$ – average over samples and hydro events

$$\langle\langle B_S(i) B_S(j) \rangle\rangle - \langle\langle B_S(i) \rangle\rangle \langle\langle B_S(j) \rangle\rangle = \overline{B_H(i) B_H(j)} + \langle\langle \delta B(i) \delta B(j) \rangle\rangle - \overline{B_H(i)}\, \overline{B_H(j)}$$
$$\langle\langle \delta B(i) \delta B(j) \rangle\rangle = \delta_{ij} \overline{B_H(i)}$$

- Standard sampling
  - Preserves correlations
  - Increases fluctuations
- Sampling with local conservation laws: $\langle\langle \delta B(i) \delta B(j) \rangle\rangle = 0$

Correlations and fluctuations

Cooper-Frye formula tells nothing about

- correlations between charges, momenta, energies, ...
- fluctuations of B, S, Q, $< p_T >$, ...

They are determined by a sampling algorithm:

- Standard choice: all particles independent
- UrQMD hybrid: attempt to account for conservation laws "mode sampling" Huovinen, Petersen Eur.Phys.J. A48 (2012) 171
- Bozek/Broniowski: always sample particle and antiparticle relies on $\mu = 0$, Phys.Rev.Lett. 109 (2012) 062301
- SPREW: reject particles driving conserved quantities in wrong direction
  SER: canonical rejection

  C. Schwarz, DO, L.-G. Pang, S. Ryu, H. Petersen, J Phys G 45 (2018), 015001

First formulate mathematical problem, then algorithm!

Metropolis algorithm: general

- Random walk (Markov chain) with many steps
- One step t:
  - in state ξ propose new state ξ', probability $T(\xi \to \xi')$
  - Accept this proposal with probability $A(\xi \to \xi')$
  - $w(\xi \to \xi') = T(\xi \to \xi') A(\xi \to \xi')$
- After many steps reach stationary distribution $P(\xi)$
- $P(\xi)$ should be the desired distribution

$$P^{t+1}(\xi) - P^t(\xi) = \sum_{\xi'} [w(\xi' \to \xi) P^t(\xi') - w(\xi \to \xi') P^t(\xi)]$$

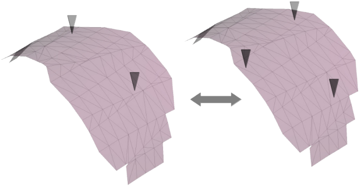
Sufficient condition for stationary distribution (detailed balance):

$$\frac{P(\xi')}{P(\xi)} = \frac{w(\xi \to \xi')}{w(\xi' \to \xi)} \implies \frac{A(\xi \to \xi')}{A(\xi' \to \xi)} = \frac{P(\xi')\, T(\xi' \to \xi)}{P(\xi)\, T(\xi \to \xi')}$$

Common choice:

$$a \equiv A(\xi \to \xi') = \min\left(1, \frac{P(\xi')\, T(\xi' \to \xi)}{P(\xi)\, T(\xi \to \xi')}\right)$$

Proposal function



1. With 50% probability choose a $2 \to 3$ or $3 \to 2$ transition.
2. Select the "incoming" particles by uniformly picking one of all possible pairs or triples.
3. Select the outgoing channel democratically with probability $1/N^{ch}$, N^{ch} – number of possible channels, satisfying quantum number and energy-momentum conservation.
4. For the selected channel sample the "collision" kinematics uniformly from the available phase space with probability $\frac{dR_n}{R_n}$, $n = 2$ or 3.
$dR_n(\sqrt{s}, m_1, m_2, \dots, m_n) = \frac{(2\pi)^4}{(2\pi)^{3n}} \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \dots \frac{d^3p_n}{2E_n} \delta^{(4)}(P^\mu_{tot} - \sum P^\mu_i)$
5. Choose a cell for each of the outgoing particles uniformly from all cells in the patch.

Properties of proposal function

- Never changes total energy, momentum, or quantum numbers
- Generates proposal probabilities:

$$T(2 \to 3) = \frac{1}{2} \frac{G_2^{ch}}{G_2} \frac{1}{N_3^{ch}} \frac{dR_3^{ch}}{R_3^{ch}} \frac{1}{N_{cells}^3}$$

$$T(3 \to 2) = \frac{1}{2} \frac{G_3^{ch}}{G_3} \frac{1}{N_2^{ch}} \frac{dR_2^{ch}}{R_2^{ch}} \frac{1}{N_{cells}^2}$$

$G_2 = \frac{N(N-1)}{2!}$, $G_3 = \frac{N(N-1)(N-2)}{3!}$
total numbers of incoming pairs/triplets of any species
G_2^{ch}, G_3^{ch} – numbers of ways to select given incoming species
N_2^{ch}, N_3^{ch} – numbers of channels with necessary quantum numbers

Acceptance probability

$$a_{n\to m} = \frac{N_m^{ch} R_m}{N_n^{ch} R_n} \frac{N!}{(N+m-n)!} \frac{m!}{n!} \frac{k_m^{id}!}{k_n^{id}!} \times$$

$$\left(\frac{2N_{cells}}{\hbar^3}\right)^{m-n} \frac{\prod_{i=1}^{m} g_i\, f_i(\mu_i - p_i^\alpha u_\alpha, T)\, p_i^\mu d\sigma_\mu}{\prod_{j=1}^{n} g_j\, f_j(\mu_j - p_j^\alpha u_\alpha, T)\, p_j^\mu d\sigma_\mu}$$

T, μ, u are taken at positions of the incoming/outgoing particles