"From the pieces to the entire tapestry": status and future plan on critical fluctuation

"Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry." -- Feynman

Focus of the talk: collecting pieces for the entire tapestry.

Institute of Modern Physics (Lanzhou), Chinese academy of sciences

Yi Yin

BEST meeting, May.17,2020

Goal

Initial condition +BES hydro.

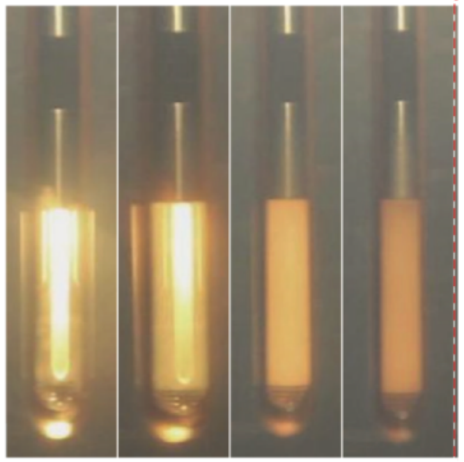
Interplay among critical fluctuations and bulk evolutions

Particlization (freezeout)

Critical dynamics

Fluctuations, correlations and criticality

"As the density fluctuations become of a size comparable to the wavelength of light, the light is scattered and causes the normally transparent liquid to appear cloudy." -- wiki



Hou et al, Journal of Chemistry 16'.

- The correlation function of the order parameter field δM (Fourier momentum Q ~ inverse of the size of the fluctuation)

$$\phi_{\rm eq}(Q) \sim \langle \delta M \delta M \rangle \sim \frac{1}{\xi^{-2} + Q^2}$$

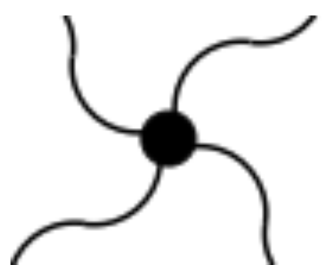
- Enhanced $\phi_{\rm eq}(Q)$ near the critical point for Q~1/ ξ ⇒ *phenomenon of the critical opalescence*.

Fluctuations near the QCD critical point

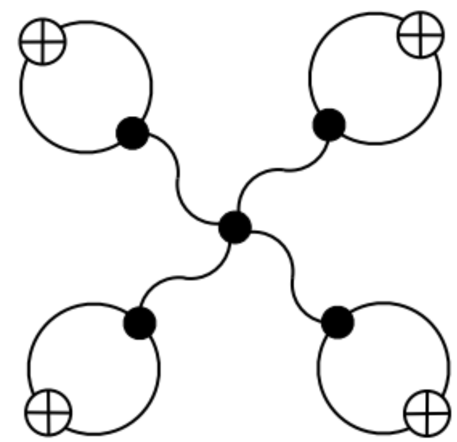
- $\phi_{eq}(Q) \Rightarrow$ the growth of non-Gaussian fluctuations of proton numbers.

Stephanov, PRL 09

$: \phi_{\rm eq}(Q)$



$: \lambda_4 \sim \xi^{-1}$

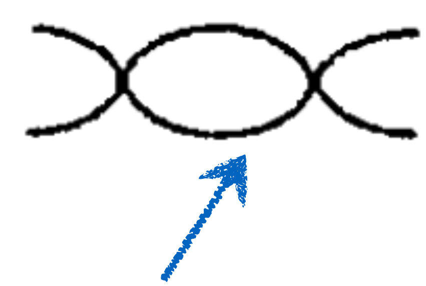


$\Rightarrow \quad \kappa_4 \sim \lambda_4 \xi^8 \sim \xi^7$

Proton distribution

($\lambda_3 = 0$ *for simplicity)*

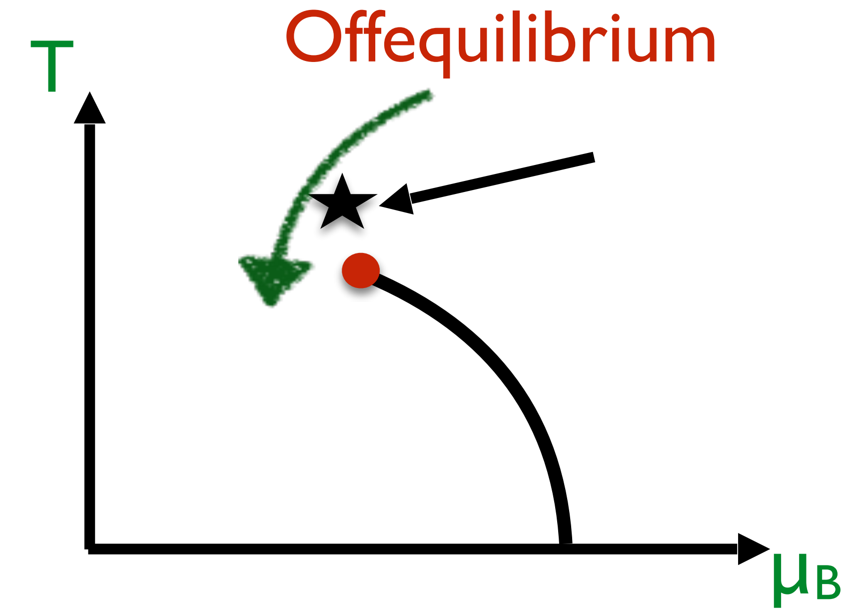
- The critical scaling of E.o.S.
  - E.o.S with Ising critical point. ✅



$\phi_{\rm eq}(Q)$

Ratti, INT talk

Real-time critical fluctuations



- Inescapably fall out of equilibrium near the critical point. ("Critical slowing down")

- Could be different from the equilibrium expectation both quantitatively and qualitatively !

e.g. S. Mukherjee, R. Venugopalan and YY, PRC15' ; see YY, 1811.06519 for a concise review on related development.

- Back-reacts on bulk evolution.

gradient of p (?) ≃ acceleration of flow

How to describe the interplay among fluctuations and bulk evolutions?

A detour: fluctuating hydro. in general

I. Stochastic hydro. approach: (adding noise to hydro. equations).

Landau-Lifshitz, Statistical Mechanics; Kapusta-Mueller-Stephanov, PRC '11;…

II. Treating off-equilibrium fluctuations as slow modes in additional to "hydro" modes.

Kawasaki, Ann. Phys. '70; Andreev, JTEP, '1971; …"hydro-kinetic", Akamatsu-Mazeliauskas-Teaney, PRC 16, PRC '18

$\Rightarrow$ Coupled deterministic equation.

III: "Effective field theory" (EFT) approach: (on Schwinger-Keldysh contour).

Kovtun-Moore-Romatschke, JHEP 14'; Glorioso-Crossley-Liu, JHEP 17'; Haehl-Loganayagam-Rangamani, 1803.11155, …

The study of hydro-kinetic equation: rapid progress

	Flow	Baryon density	Non-Conformal
Akamatsu-Mazeliauskas-Teaney, PRC 16	Bjorken	No	No
Akamatsu-Mazeliauskas-Teaney, PRC 18	Bjorken	No	Yes
Schaffer-Martinez, 1812.05279	Bjorken	Yes	No
Xin An-Basar-Stephanov-H.-U. Yee, 1902.09517	General	No	Yes
Xin An-Basar-Stephanov-H.-U. Yee, 1912.13456 .	General	Yes	Yes

The characteristic momentum scale:

Hydro. tail for Bjorken expansion.

Akamatsu-Mazeliauskas-Teaney, PRC 16'; ...

$$\frac{\langle T^{zz}\rangle}{e+p} = \Big[\underbrace{\frac{p}{e+p}}_{\sim 1} - \underbrace{\frac{4}{3}\frac{\gamma_\eta}{\tau}}_{\text{1st order}} + \underbrace{\frac{1.08318}{s\,(4\pi\gamma_\eta\tau)^{3/2}}}_{\text{3/2 order!}} + \underbrace{\frac{(\lambda_1 - \eta\tau_\pi)}{e+p}\frac{8}{9\tau^2}}_{\text{2nd order}} + \dots\Big]$$

$$p_* \sim \sqrt{\frac{\omega}{\nu_L}}, p_*^3 \to (\frac{1}{\nu_L \tau})^{3/2}$$

Recent news: finite ω and k corrections to thermal and transport properties of a liquid from EFT approaches. *Chris Lau, Hong Liu, YY, in preparation*

characteristic momentum scale: $p_* \sim \max(\sqrt{\frac{\omega}{\nu_L}}, \sqrt{\frac{c_s k}{\nu_L}})$

Sound dispersion (c.f. 2nd hydro.)

Chris Lau, Hong Liu, YY, in preparation

$$\delta\omega_+ = \pm c_s k[1 - \underbrace{\# \frac{1}{s}(\frac{k}{\nu})^{3/2}}_{\text{fluc.}} + \underbrace{\#(\nu k)^2}_{2nd.\,hydro}]$$

$$-i\nu_L k^2 [1 + \underbrace{\# \frac{1}{s\nu^3}(\nu k)^{1/2}}_{\text{fluc.}} + \underbrace{\#(\nu k)^2}_{2nd.\,hydro}]$$

Renormalization of static quantity: $\sim (\frac{p_*^3}{s})$

Ratio between phase space volume of "soundlets" and micro. d.o.f.

Renormalization of dynamic quantity: $\frac{(\nu p_*^2)^{-1}}{\nu} \times (\frac{p_*^3}{s})$

Ratio between life time of "soundlets" and micro. m.f.t .

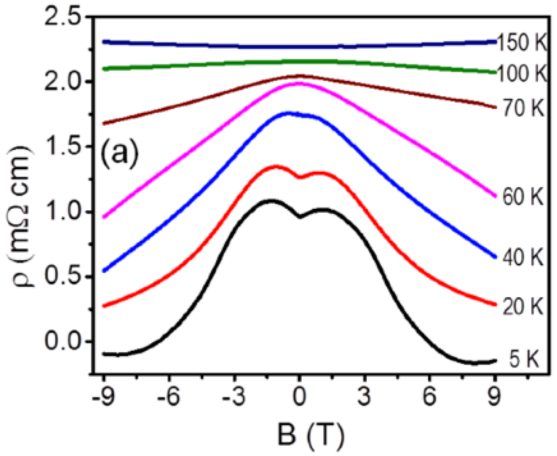
Application to hydro. with anomaly

Recent stochastic simulation.

Gui rong Liang, Jinfeng Liao, Shu Lin, Li Yan, Miao Li; 2004.04440

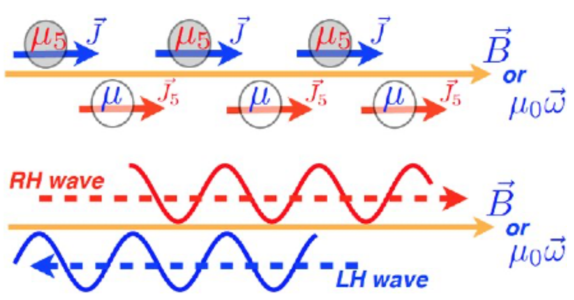
Fluctuation corrections are not only non-analytic in ω, k, but also in sound velocity c_s. Non-analytic B-dependence of magneto-resistivity from chiral magnetic wave in the loop?

Sogabe, Yamamoto, YY, in progress



from 1412.6543

+



from 1511.04050

=?

Sogabe, postdoc@IMP

Approach the critical point: the emergence of hierarchy

$$\mathscr{L}[N_{mm}] = -2\gamma_\lambda q^2(N_{mm} - N^{eq}_{mm}) + \mathscr{F}_1[N_{m(i)}]$$

$$\mathscr{L}[N_{m(i)}] = -(\gamma_\lambda + \gamma_\eta)q^2(N_{m(i)} - N^{eq}_{m(i)}) + \mathscr{F}_2[N_{mm}, N_{m(i)}, N_{(ij)}]$$

$$\mathscr{L}[N_{(ij)}] = -2\gamma_\eta q^2(N_{(ij)} - N^{eq}_{(ij)}) + \mathscr{F}_3[N_{(ij)}, N_{m(i)}]$$

$$\Gamma_\lambda \sim \xi^{-3} \gg \Gamma_\eta \sim \xi^2 \qquad (\Gamma \sim \gamma q^2 : \text{relaxation rate})$$

(taken. from Basar's talk)

Xin An-Basar-Stephanov-H.-U.Yee, 1912.13456 .

Returning to the slowest mode $\delta M \sim \delta(s/n)$ from now on.

The construction of "hydro+"

Stephanov-YY, 1712.10305

- "+": (Wigner transform of) the two point function of δM (For QCD critical point, M ~ s/n):

$$\phi(t,x;Q) = \int d\Delta x e^{-i\Delta x Q}\left\langle \delta M(t, x+\Delta x/2)\delta M(t, x-\Delta x/2)\right\rangle$$

- The evolution of "+" is modelled by relaxation rate equation.

$$u^{\mu}\,\partial_{\mu}\phi = \Gamma_{\phi}(Q)\,(\phi(Q) - \phi_{\rm eq}(e,n;Q))$$

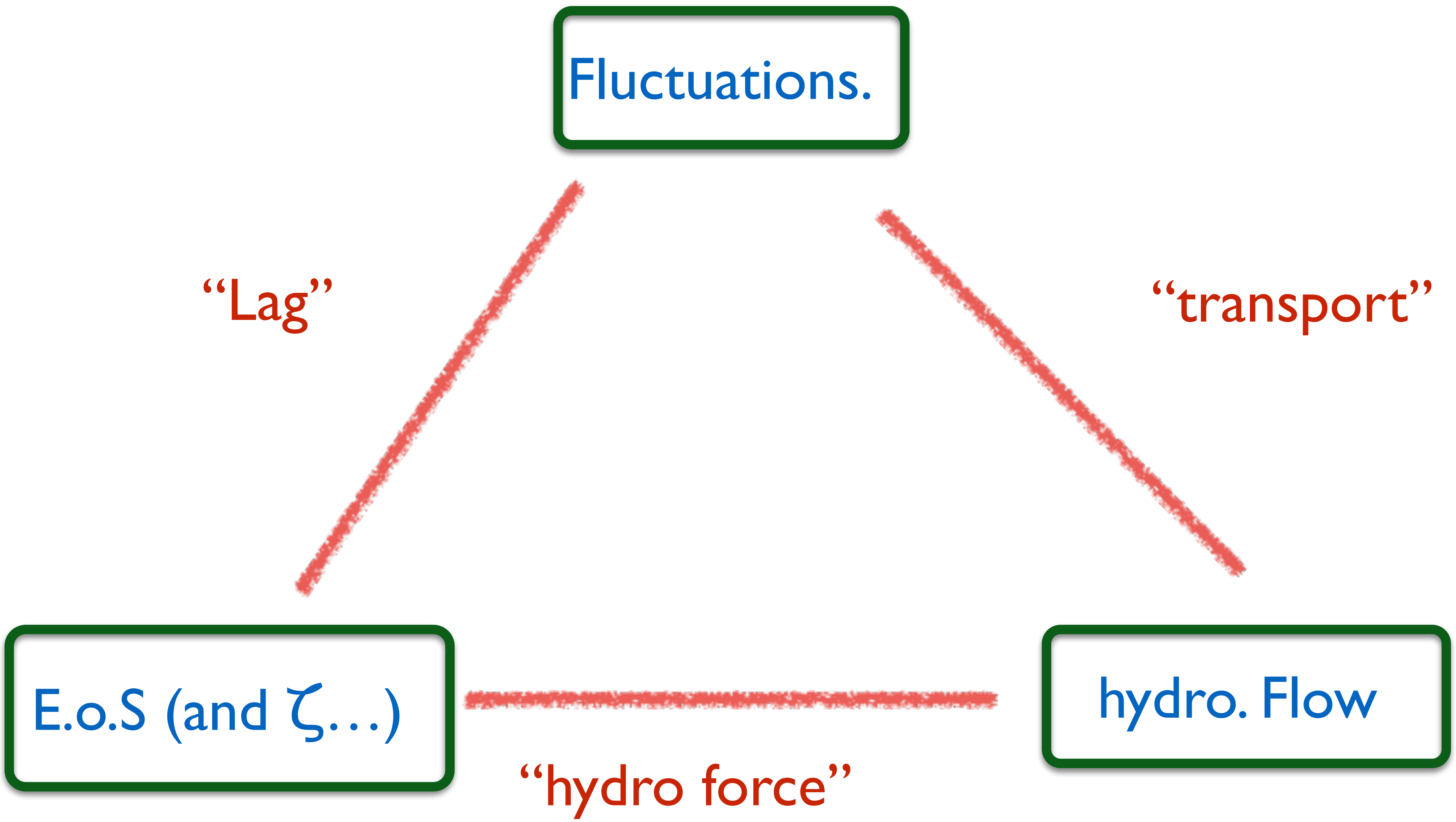
see Xin An-et al, 1902.09517 for additional "force term"

- Feedback to hydro. $\partial_{\mu}T^{\mu\nu} = 0; \partial_{\mu}J^{\mu} = 0$:

$$T^{\mu\nu} = e\,u^{\mu}\,u^{\nu} + p_{(+)}\left(g^{\mu\nu} + u^{\mu}u^{\nu}\right) + \mathcal{O}(\partial) \qquad p(e,n) \to p_{(+)}(e,n,\phi)$$

- Generalized entropy $s_{(+)}$ and generalized pressure $p_{(+)}$ can be derived, e.g.,

$$\Delta s[\phi] = \frac{1}{2}\int_{Q}\left[\log(\frac{\phi}{\phi_{eq}}) - \frac{\phi}{\phi_{eq}} + 1\right] + \ldots$$

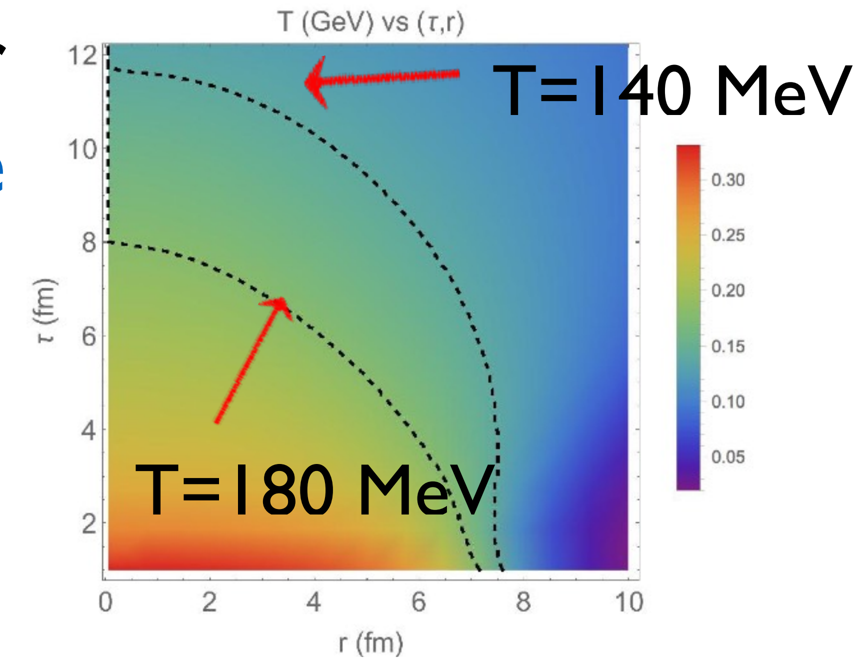


Main effects near a QCD critical point?

A tale of two simulations of hydro+

(In boost-invariant and transverse symmetric flow.)

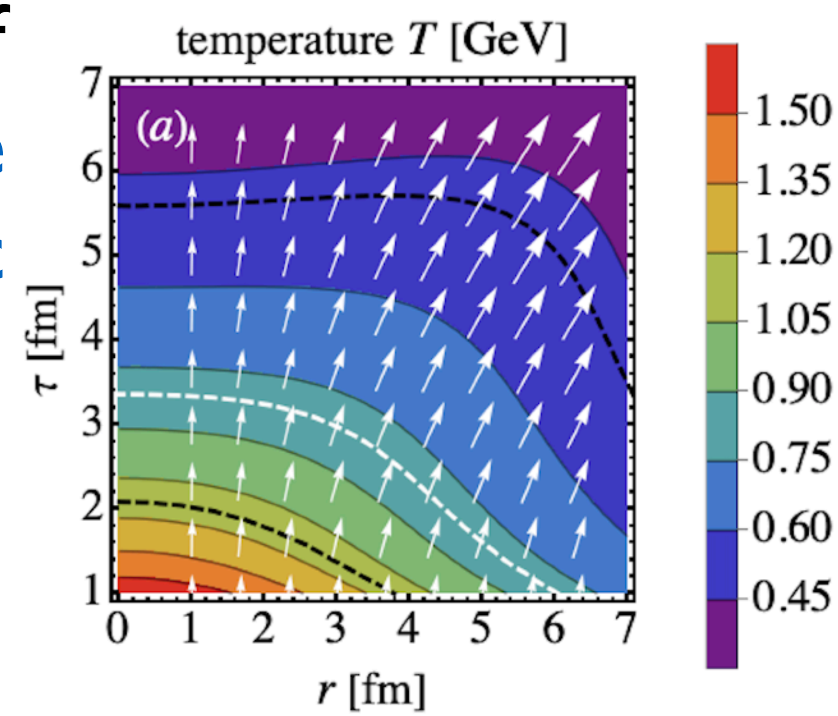
- Simulation A: placing a hypothetical C.P. near μ=0 (no eqn for baryon density): showcase the intertwined dynamics.

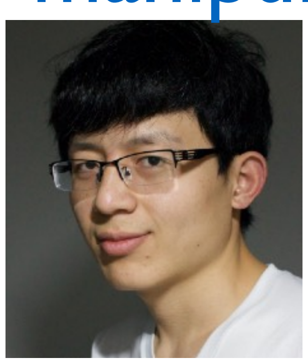


Rajagopal-Ridgway-Weller-YY, 1908.08539

Ridgway, graduate@MIT

- Simulation B: Solving eqn. for ϕ on top of Gubser flow at finite μ; "anatomy" of the intertwined dynamics by analytic manipulations.



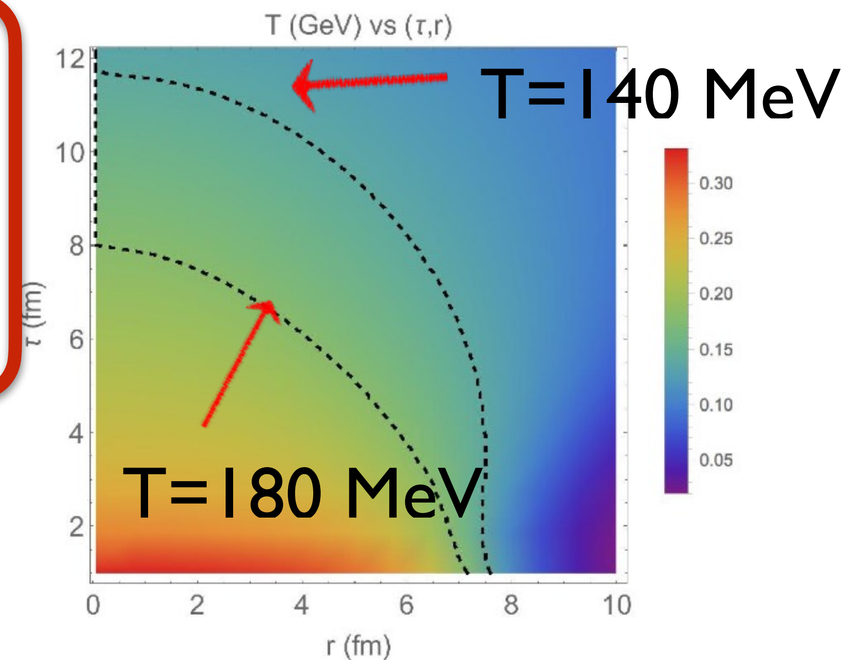


Lipei Du-Heinz-Rajagopal-YY, 2004.02719

Lipei Du, graduate@OSU

(In boost-invariant and transverse symmetric flow.)

- Simulation A: placing a hypothetical C.P. near μ=0 (no eqn for baryon density): showcase the intertwined dynamics.



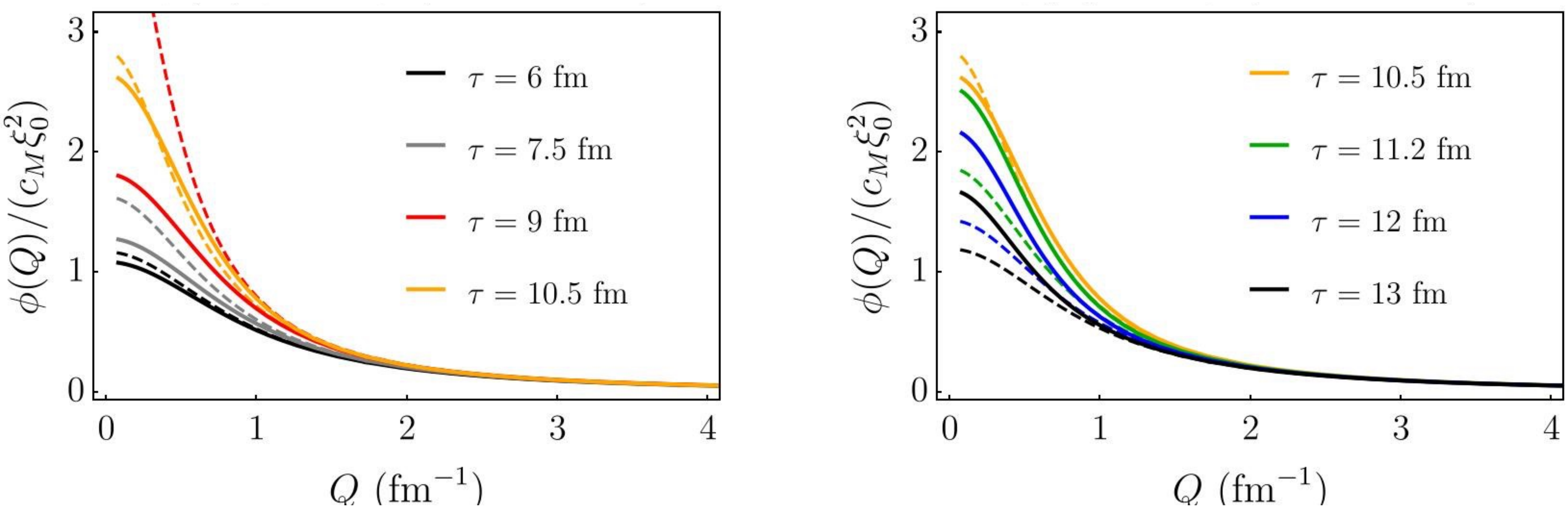
Rajagopal-Ridgway-Weller-YY, 1908.08539

Ridgway, graduate@MIT

Q-dependent off-equilibrium fluctuation at r=1fm.

See also: Berdnikov-Rajagopal ;

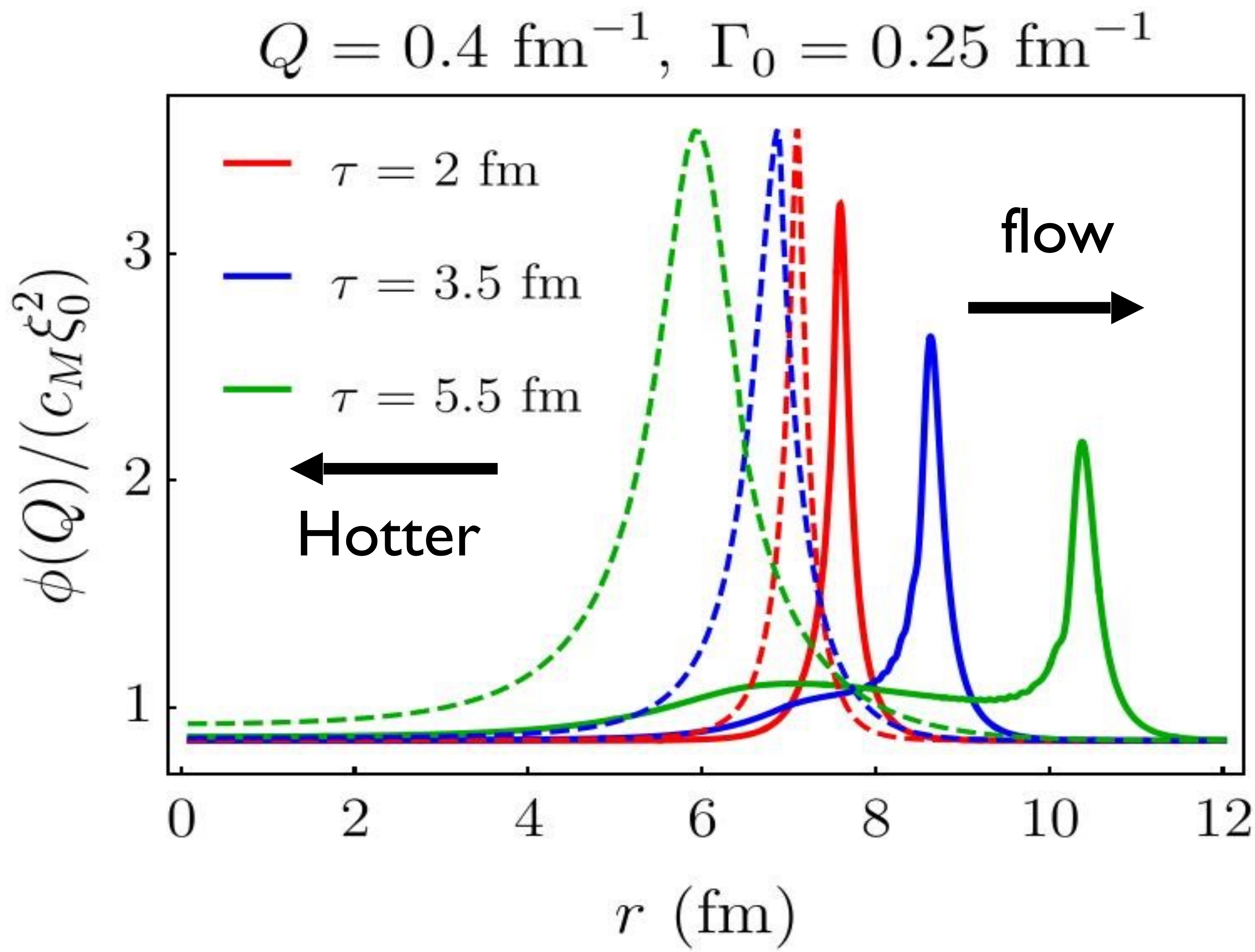
Off-equilibrium : solid ; Equilibrium: dashed.



- Large Q (shortwave length) modes are in equilibrium while small Q (long wavelength) modes are not.

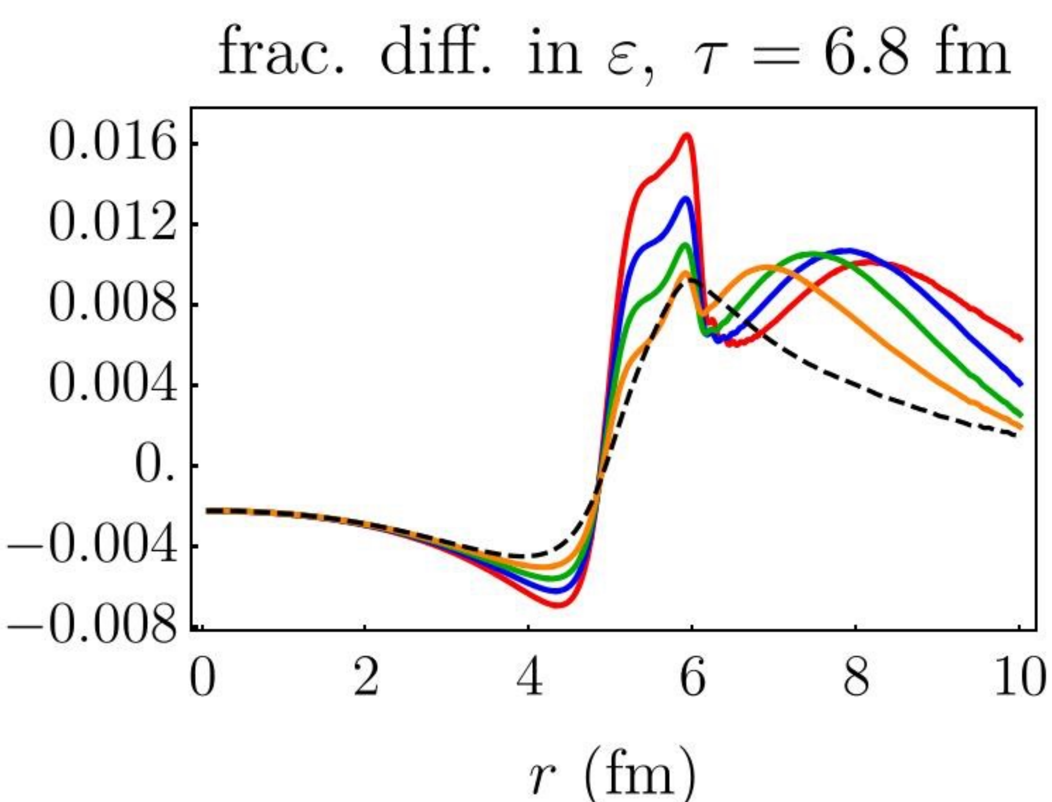
- Critical slowing down leads to the jet-lag of critical fluctuations: the information of the criticality is encoded in offequilibrium effect!

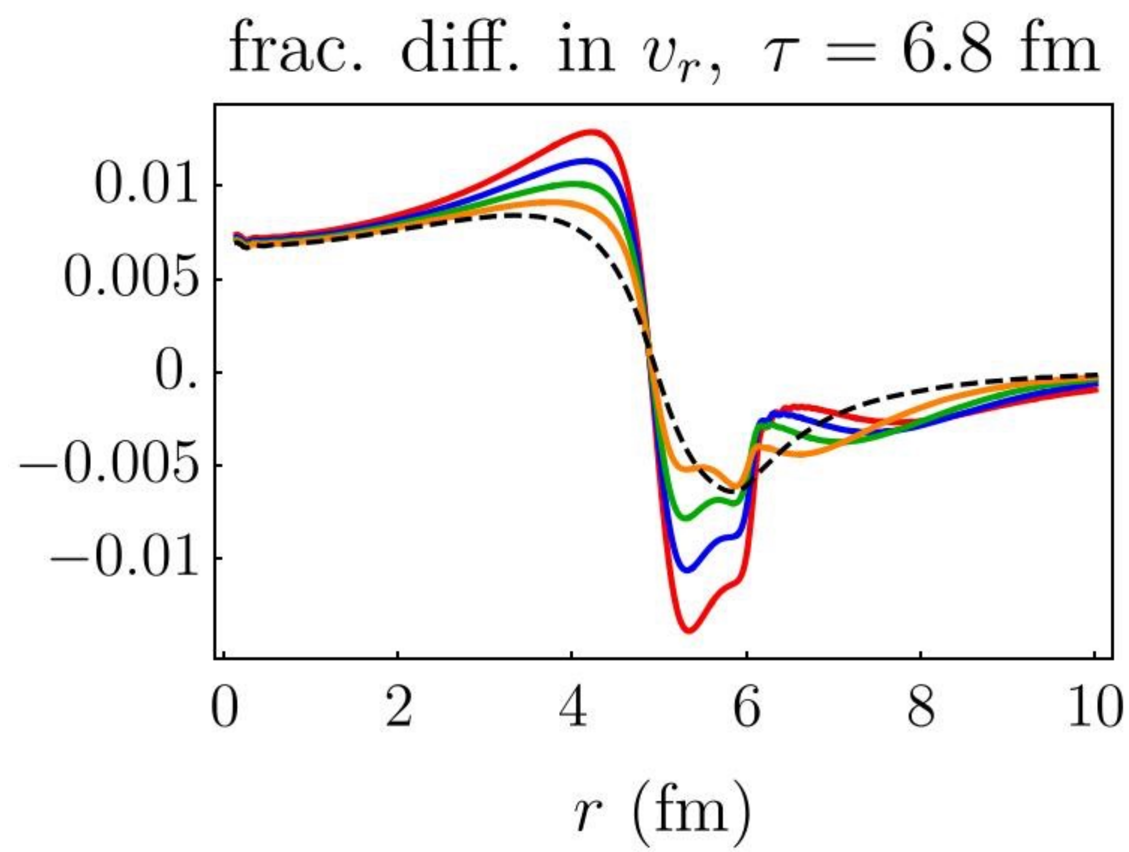
Nontrivial spatial distribution of long wavelength fluctuations.



Driven by the critical slowing down and by the advection of the flow.

Back-reaction on energy density and flow



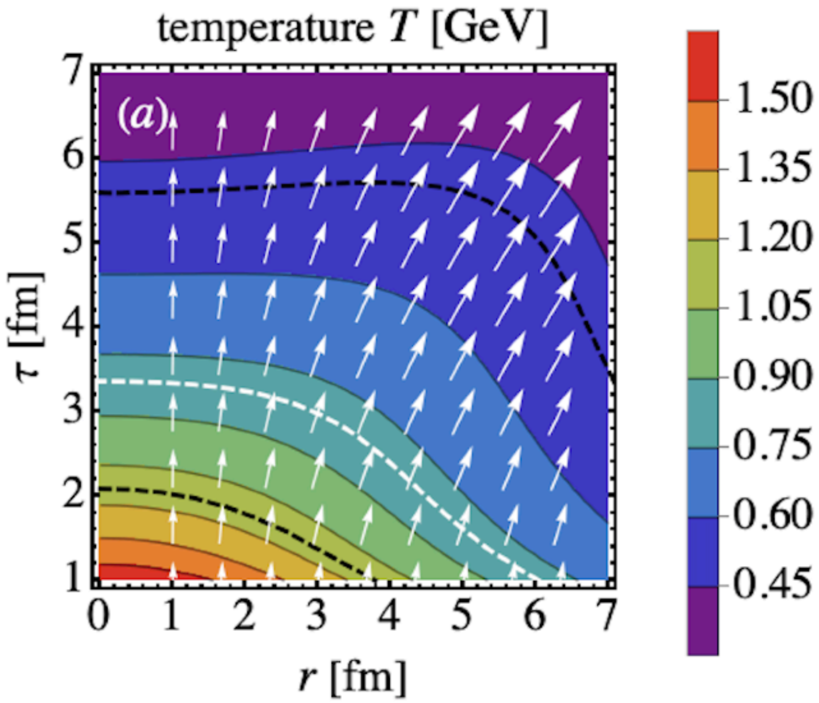


Dashed, no back-reaction.

From: red, blue, green and orange, results including back-reaction with decreasing relaxation rate.

- Within our model, the back-reaction effects are small (within 1 percent).

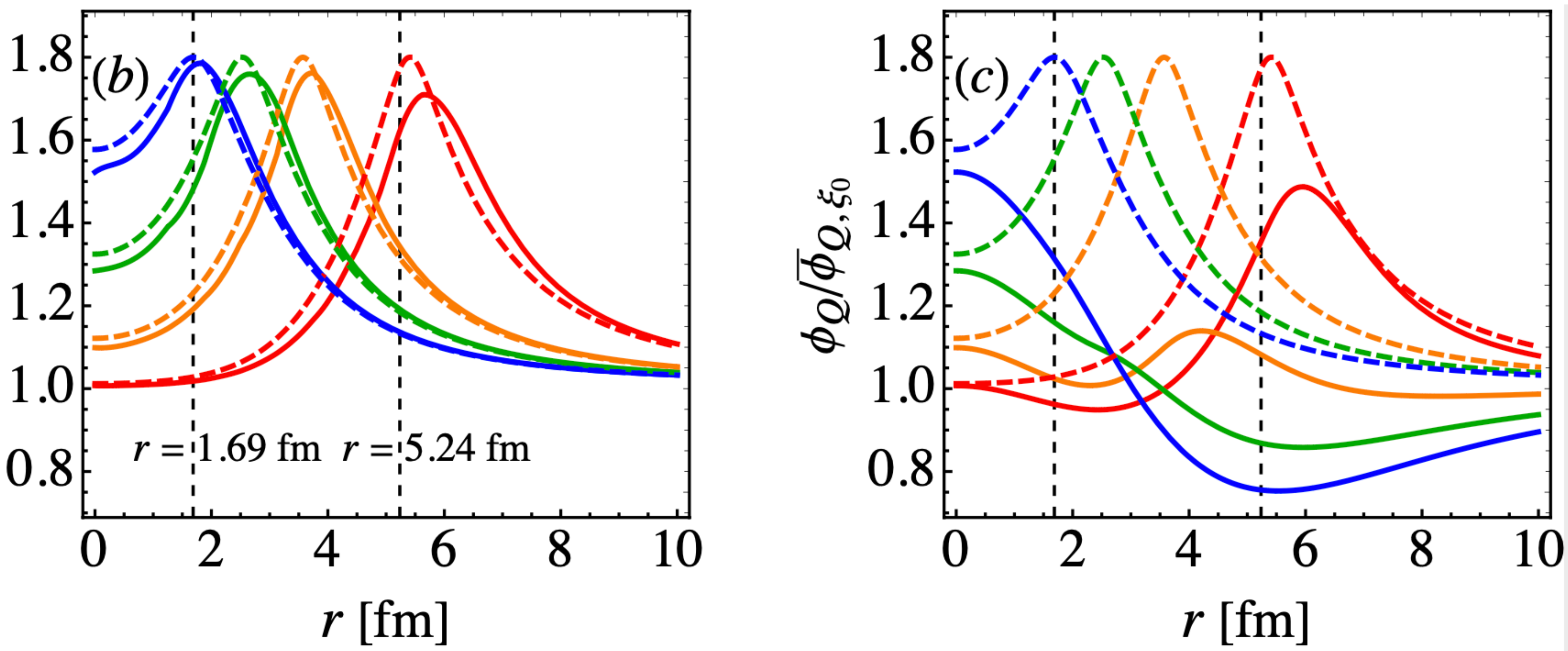
- Simulation B: Solving eqn. for ϕ on top of Gubser flow at finite μ; "anatomy" of the intertwined dynamics by analytic manipulations.



Lipei Du-Heinz-Rajagopal-YY, 2004.02719

Lipei Du, graduate@OSU

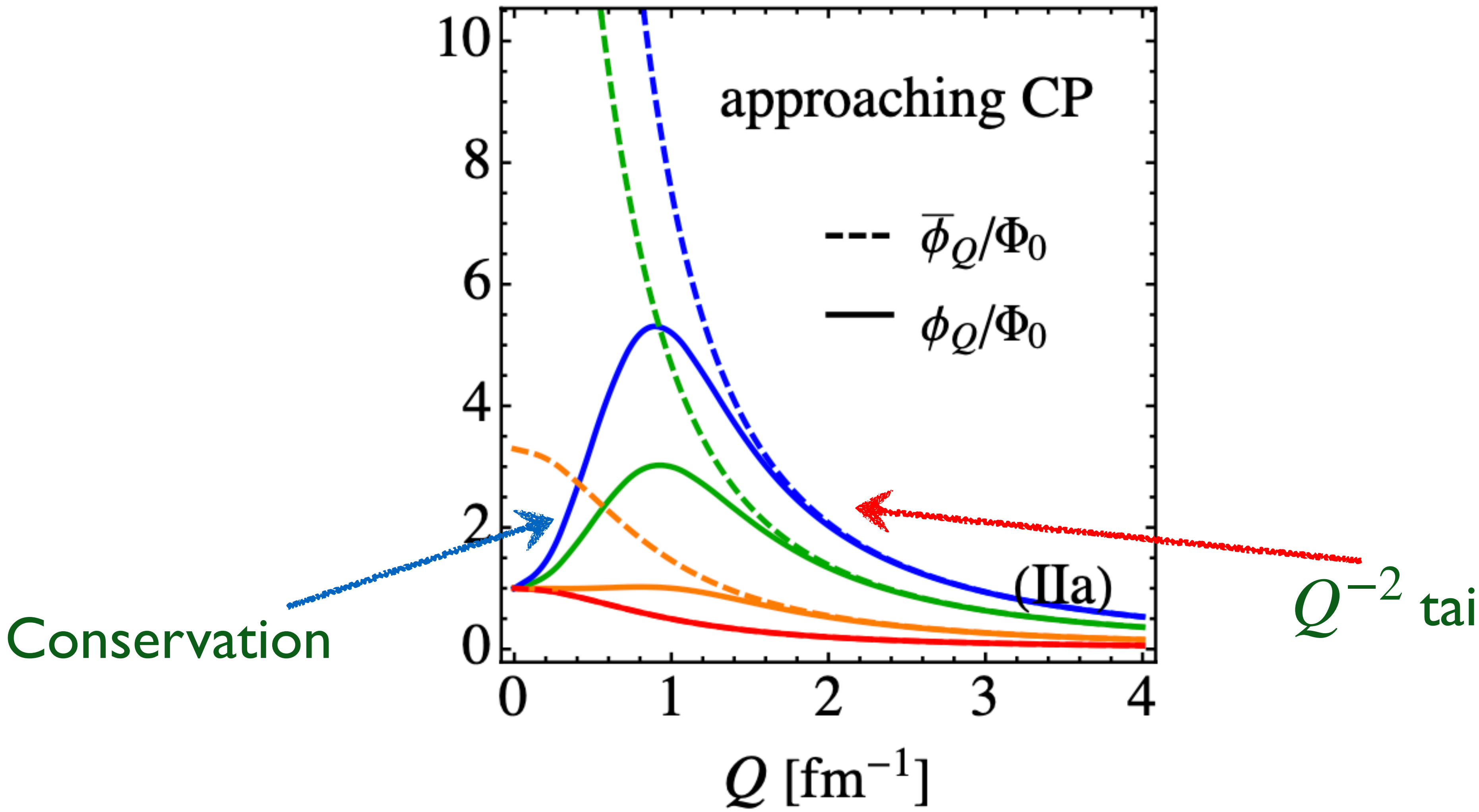
Critical slowing down and advection



Turning off radial flow

Turning on radial flow

An important difference: simulation A uses model A ($\Gamma(Q=0)\neq 0$) while simulation B uses model B ($\Gamma(Q\to 0)\sim Q^2$)



The emergence of a peak in Q. *see also Akamatsu-Teaney-Yan-Yin, PRC 19'*

- Feature of off-equilibrium evolution (related to the generalized notion of Kibble-Zurek length, see more below).

Lesson learned

The non-trivial spatial distribution of fluctuation induced by advection => simulations on realistic bulk background is imperative.

Small back-reaction => no need to track feedback from fluctuation modes (i.e. "+")

$$\text{backreaction} \propto \frac{Q^3_{\text{non-eq}}}{s} \to 10^{-3} \sim 10^{-4}$$

Ratio between phase space volume of critical modes and micro. d.o.f.

The growth of bulk viscosity is modest even though it scales as ξ^{-3} .
=> no need to include critical behavior of ζ

$$\frac{\zeta}{s} = \sin^2(\alpha_1)\left(\frac{4\pi}{s/\eta}\right)\left(\frac{\xi}{\xi_0}\right)^3 \begin{cases} 3.4\cdot 10^{-2} & r>0 \\ 2.2\cdot 10^{-1} & r<0 \end{cases}$$

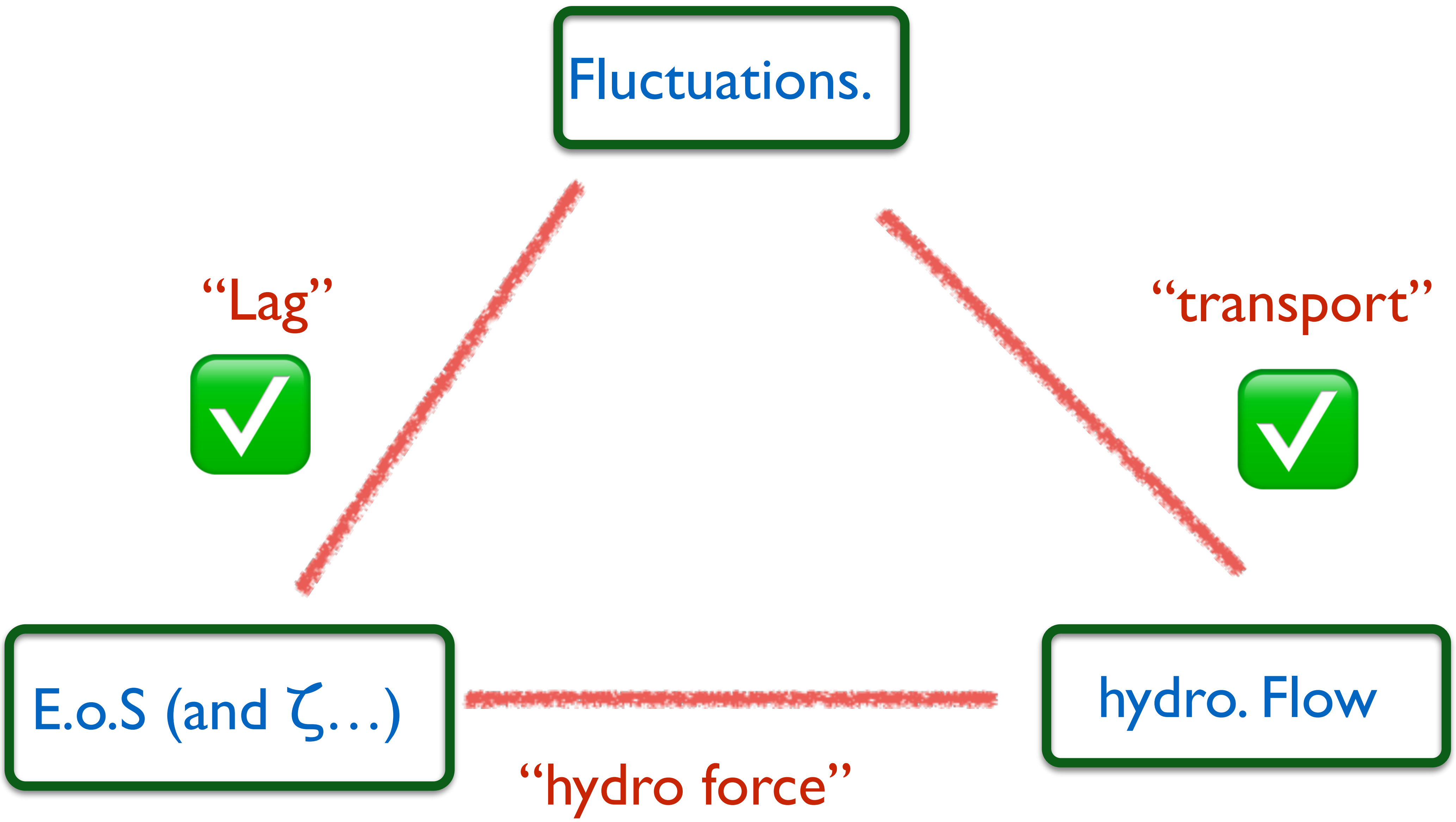
Martinez-Schafer-Skokove, 1906.11306

$\sin^2(\alpha_1) \simeq 1/4$

(taken from Schafer's INT talk)

Remaining question: how about baryon diffusion?

$$D_B \propto \frac{\lambda}{\chi_B} \qquad \lambda \sim \xi \qquad \chi_B \sim \xi^2$$


Fluctuations.
“Lag”
“transport”
E.o.S (and ζ…)
hydro. Flow
“hydro force”

Towards a quantitative description of off-equilibrium fluctuations

Initial condition +BES hydro.

Solving eqn. for $\phi(Q)$ on hydro. background;

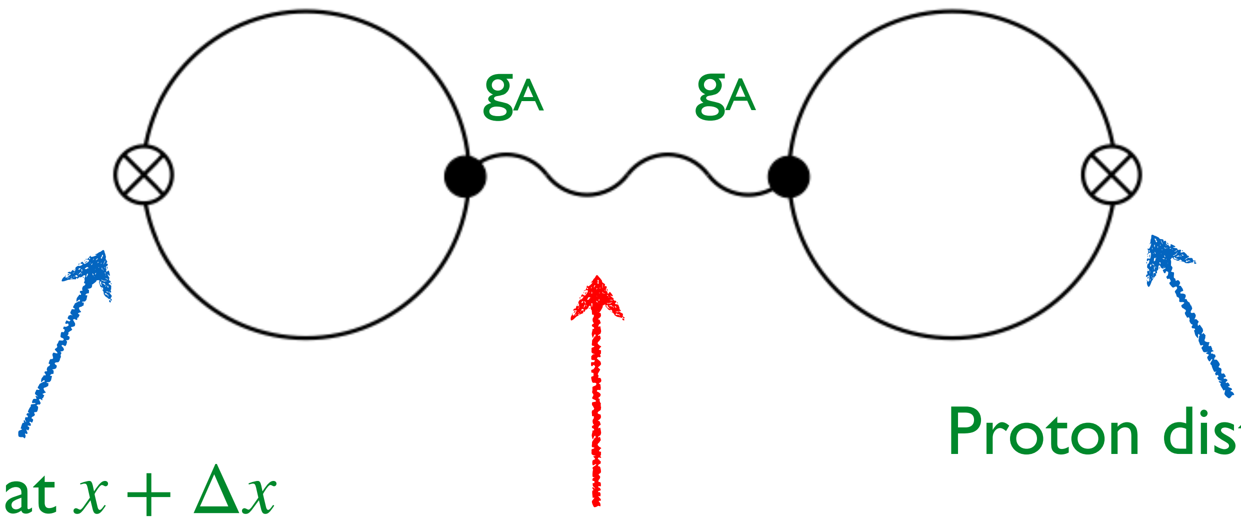
Non-Gaussian fluctuations?

Particlization (freezeout)?

Freezeout of Gaussian fluct.

Rajagopal-Stephanov-Weller-Pradeep-YY, in progress

see Pradeep's talk



Proton distri. at $x + \Delta x$

Proton distri. at $x - \Delta x$

$\phi(x, Q) \to \phi(x, \Delta x)$

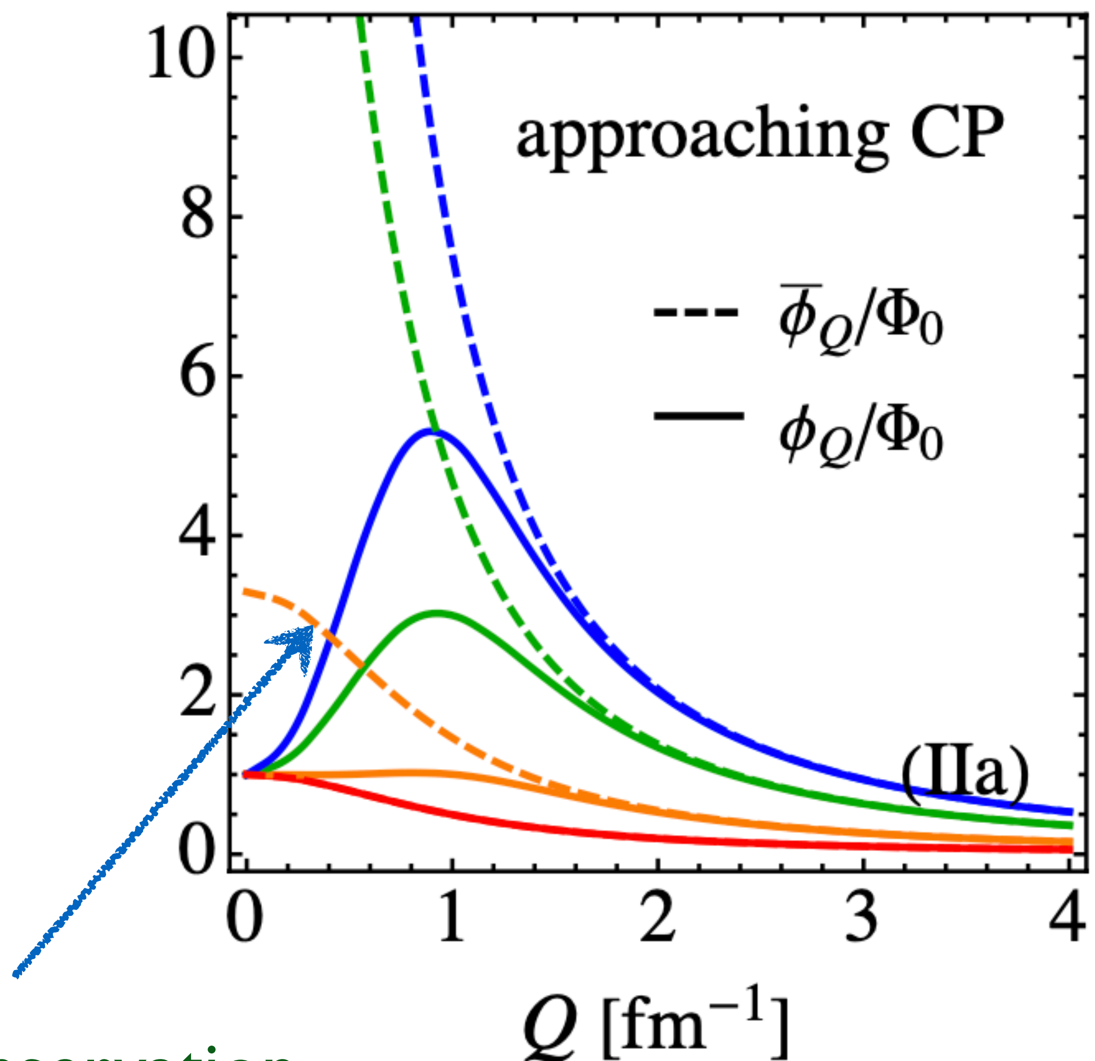
$$\langle \delta N_A^2 \rangle_\sigma = g_A^2 Z \int dS_\mu J_A^\mu(x_+) \int dS'_\nu J_A^\nu(x_-) \tilde{\phi}(x, \Delta\tilde{x})$$

(Expression for non-Gaussian cumulants is similar)

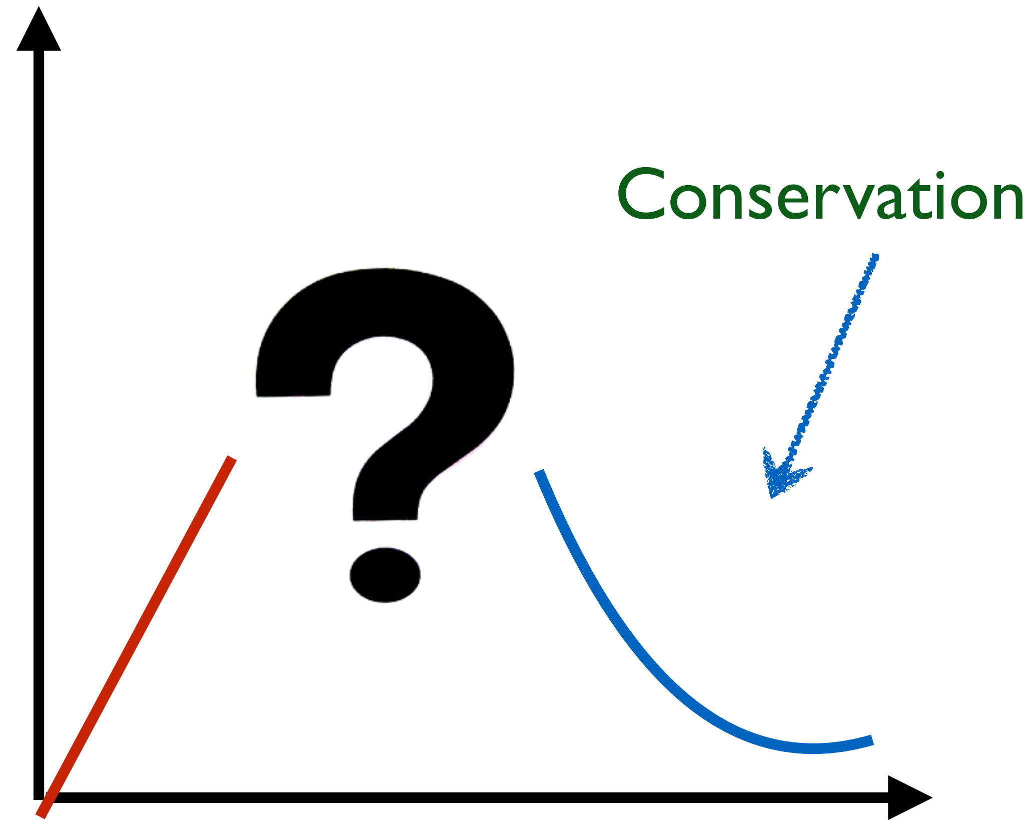
Alternative approach: constructing an ensemble of protons (hadrons) constrained by $\phi(Q)$ (and $\lambda_{3,4}$).

Particlization of non-critical fluctuations are also important.

Proton number fluct.



Conservation



(rapidity cut)

see Pradeep's talk

NB: rapidity dependence of n_B should be taken into account.

Brewer-Mukherjee-Rajagopal-YY, 1804.10215

Towards a quantitative description of off-equilibrium fluctuations

Initial condition +BES hydro.

↓

Solving eqn. for $\phi(Q)$ on hydro. background;

Non-Gaussian fluctuations?

↓

Particlization/freezeout. (Good progress.)

Off-equilibrium non-Gaussian fluctuations

Before, coupled eqn. for the Gaussian and non-Gaussian cumulants of δM at a presentative Q.

Mukherjee, Venugopalan and YY, PRC15'

Eqn for $\phi(Q)$: the evolution of Q-dependent Gaussian cumulant.

Extension to non-Gaussian cumulants:

- Generalization of Wigner transform for three, 4 pt functions?
- the resulting eqns. are expected to be very complicated.

Since the critical behavior of non-Gaussian critical fluctuations is closely related to $\phi(Q)$, how far can we go with $\phi(Q)$ at hand?

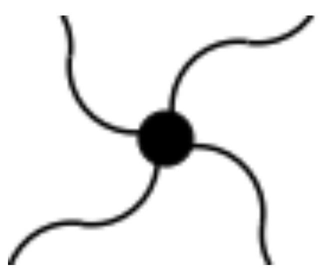
$$
\begin{aligned}
D_t C^{(1;1;1)}_{a;b;c}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3,t) &= -\nabla_1\cdot\langle \boldsymbol{j}_a(\boldsymbol{r}_1,t)\delta\rho_b(\boldsymbol{r}_2,t)\delta\rho_c(\boldsymbol{r}_3,t)\rangle' - \nabla_2\cdot\langle \boldsymbol{j}_b(\boldsymbol{r}_2,t)\delta\rho_a(\boldsymbol{r}_1,t)\delta\rho_c(\boldsymbol{r}_3,t)\rangle' \qquad (23)\\
&\quad -\nabla_3\cdot\langle \boldsymbol{j}_c(\boldsymbol{r}_3,t)\delta\rho_a(\boldsymbol{r}_1,t)\delta\rho_b(\boldsymbol{r}_2,t)\rangle'\\
&\quad +S^{(3)}_{abc}(\boldsymbol{r}_{123},t)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\delta(\boldsymbol{r}_{12}-\boldsymbol{r}_3) + S^{(2;1)}_{ab;c}(\boldsymbol{r}_{12},\boldsymbol{r}_3,t)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\\
&\quad +S^{(2;1)}_{ac;b}(\boldsymbol{r}_{13},\boldsymbol{r}_2,t)\delta(\boldsymbol{r}_1-\boldsymbol{r}_3) + S^{(2;1)}_{ab;c}(\boldsymbol{r}_{23},\boldsymbol{r}_1,t)\delta(\boldsymbol{r}_2-\boldsymbol{r}_3).\\
S^{(3)}_{abc}(\boldsymbol{r},t) &= -D_t\chi^{(3)}_{abc}(\boldsymbol{r},t) - L^{(2)}_{ab,d}S^{(2)}_{cd}(\boldsymbol{r},t)\\
&\quad -L^{(2)}_{ac,d}(\boldsymbol{r},t)S^{(2)}_{bd}(\boldsymbol{r},t) - L^{(2)}_{bc,d}(\boldsymbol{r},t)S^{(2)}_{ad}(\boldsymbol{r},t)(\boldsymbol{r},t).\\
S^{(2;1)}_{ab;c}(\boldsymbol{r},\boldsymbol{r}',t) &= -[d_t L^{(2)}_{ab,d}(\boldsymbol{r},t)]C^{(1;1)}_{d;c}(\boldsymbol{r},\boldsymbol{r}',t).
\end{aligned}
$$

Eq. for 3pt function of charge density for a non-critical system (from Pratt, 1908.01053)

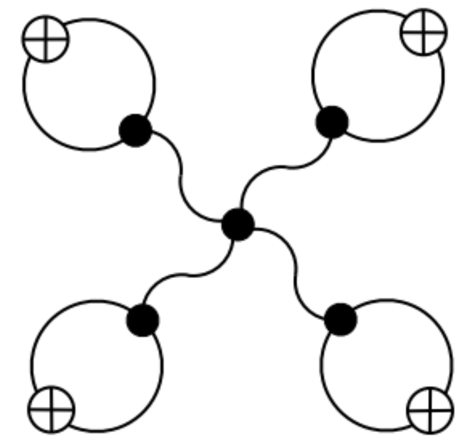
off-equilibrium Non-Gaussian cumulants at freezeout

- Replacing $\phi_{eq}(Q)$ with real time valued $\phi(Q)$

$: \phi_{\rm eq}(Q) \to \phi(Q)$



$: \lambda_4 \sim \xi^{-1}$



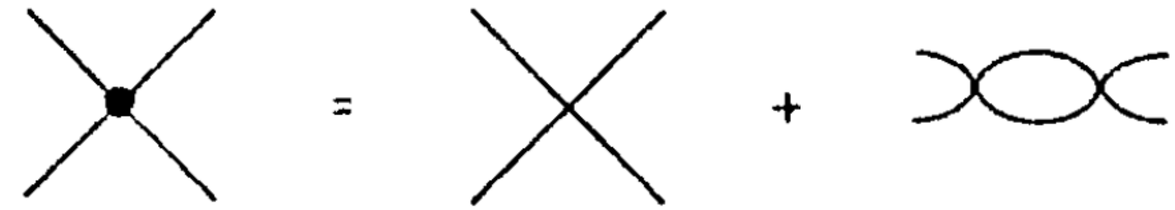
$\kappa_4 \sim ?$

(Assuming $\lambda_3 = 0$)

- How about off-equilibrium λ_3, λ_4 ?

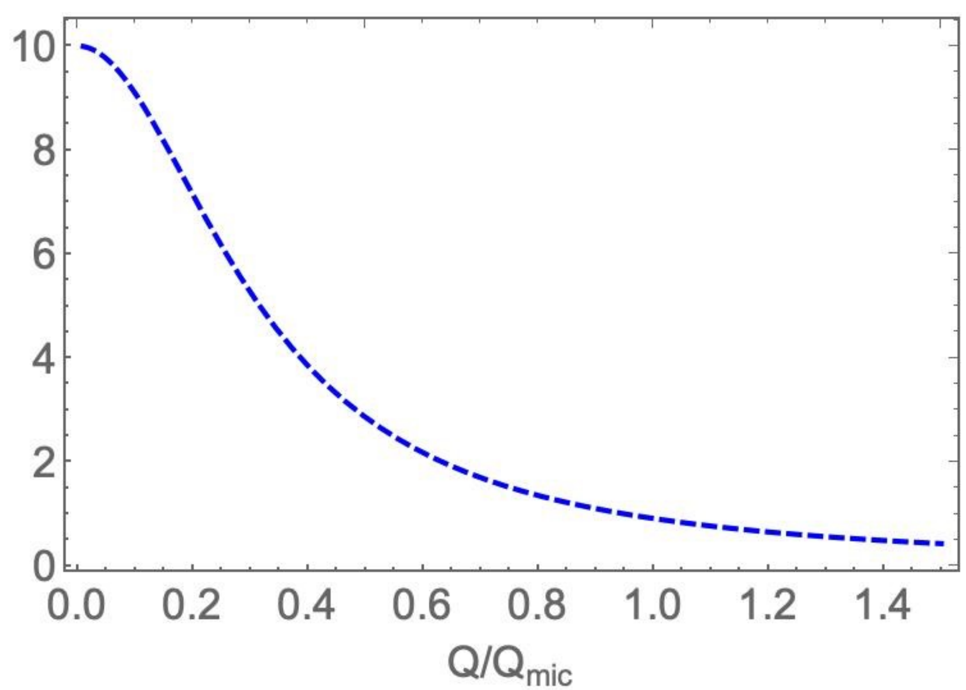
RG eqn for λ_4

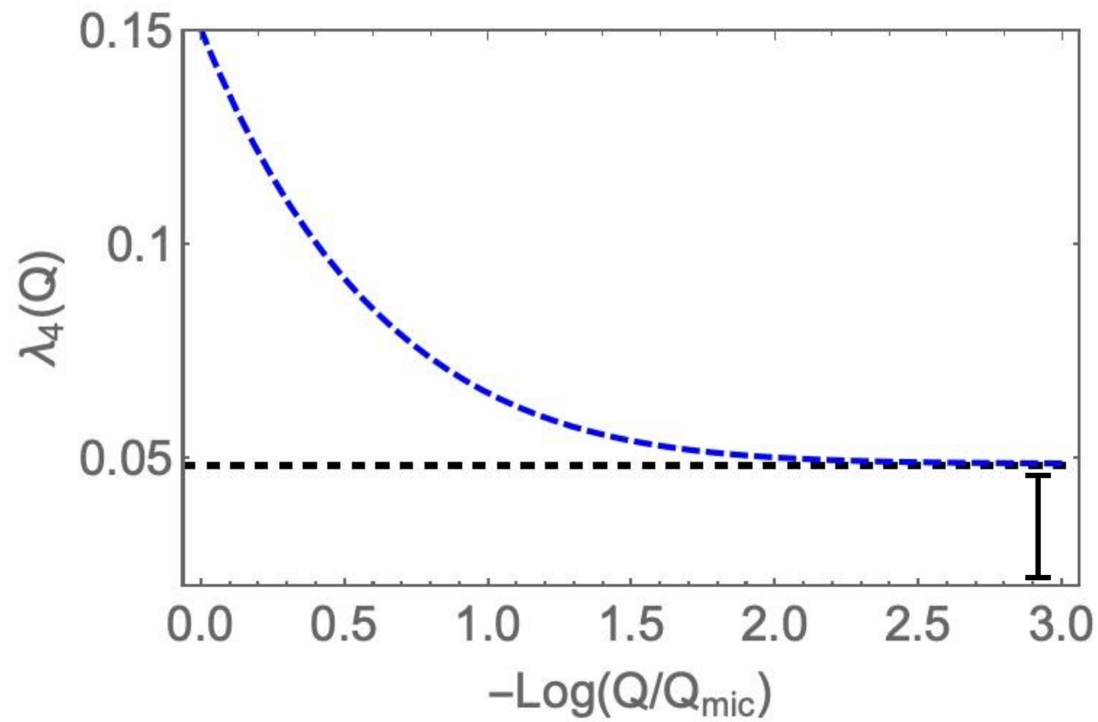
$$Q\frac{\partial g}{\partial Q} = \epsilon g - \frac{1}{9}g^2\left(Q^2\phi_{\text{eq}}(Q)\right)^2$$



$g \equiv \lambda_4/Q$

$\epsilon = 4 - d$



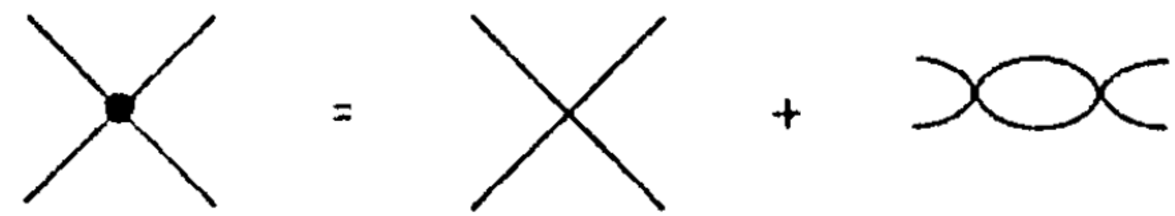


$\lambda_4^{eq} \sim \xi^{-1}$

Sol. to RG eqn

RG eqn for λ_4

$$Q\frac{\partial g}{\partial Q} = \epsilon g - \frac{1}{9}g^2\left(Q^2\phi_{\rm eq}(Q)\right)^2$$



$g \equiv \lambda_4/Q$

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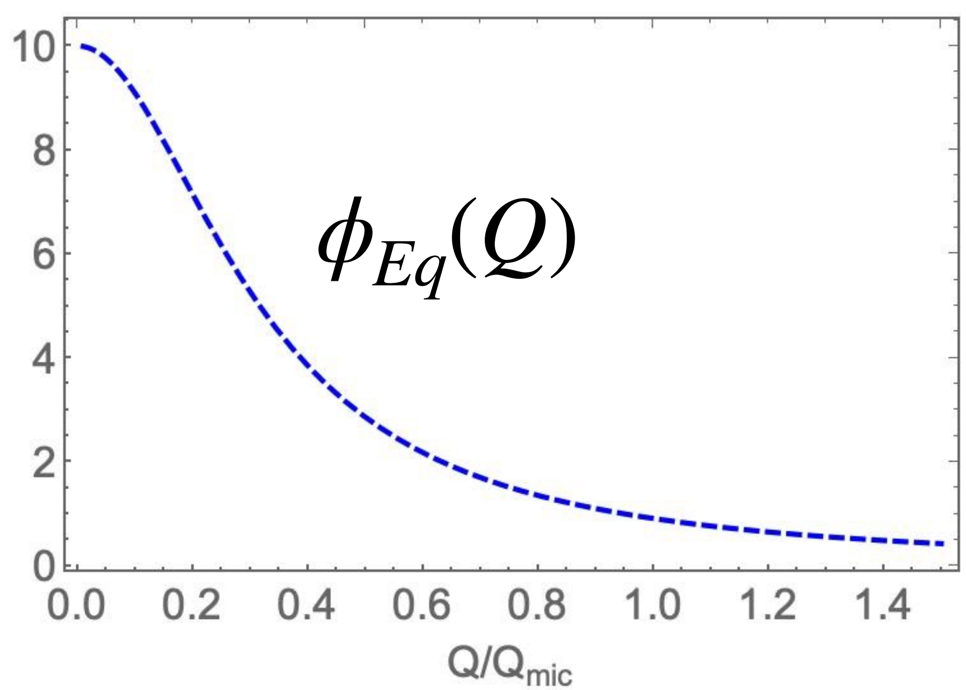
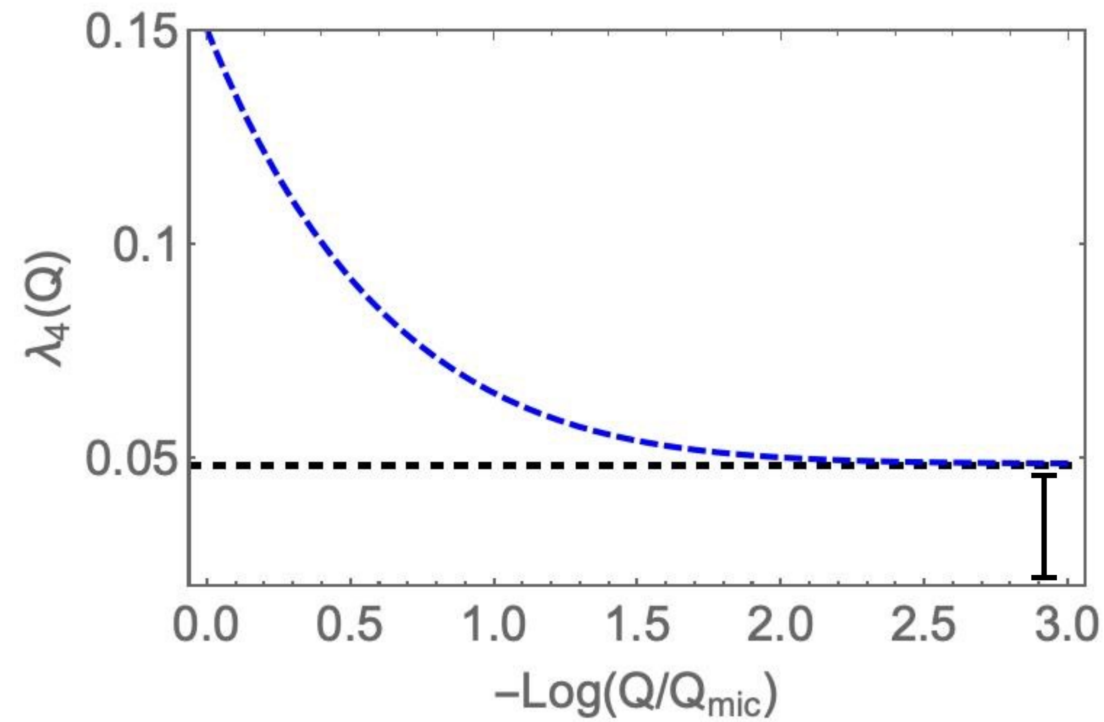




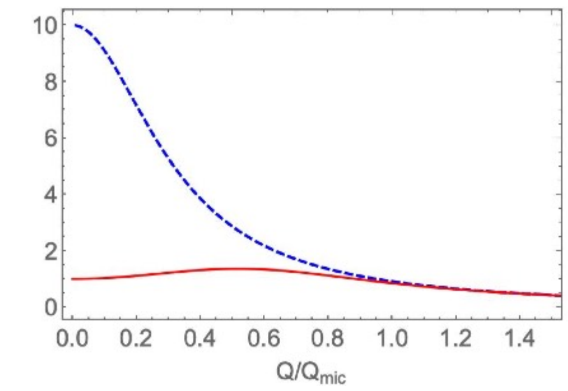
$\lambda_4^{eq} \sim \xi^{-1}$

Replacing ϕ_{eq} in RG eqn. with ϕ to determine (resolution scale-dependent) off-equilibrium $\lambda_{3,4}$!

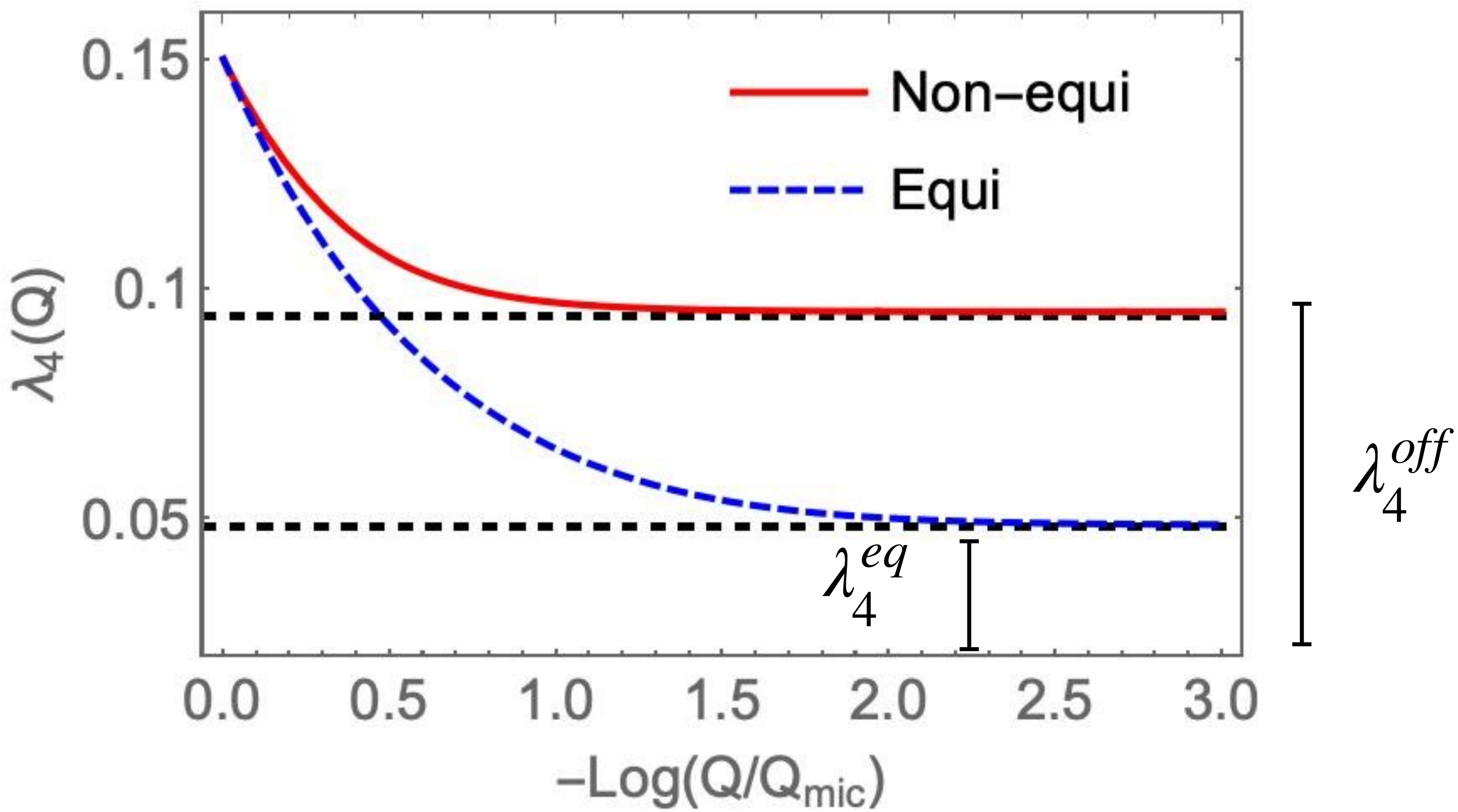
YY, in discussion with Rajagopal and Xiaojun Yao.

$$Q\frac{\partial g}{\partial Q} = g - \frac{1}{9}g^2\,\left(Q^2\phi(Q)\right)^2$$

$g \equiv \lambda_4/Q$



Sufficient to determine $\lambda_{3,4}$

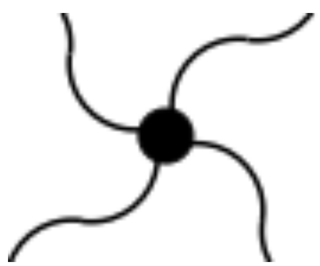


Sol. to R.G. eqn captures off-equilibrium effects

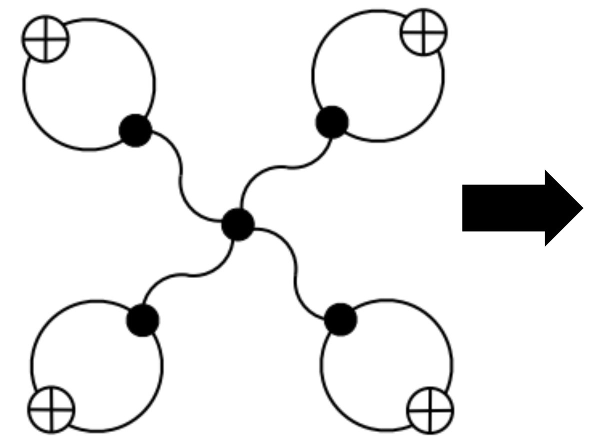
off-equilibrium Non-Gaussian cumulants at freezeout

- Replacing $\phi_{eq}(Q)$ with real time valued $\phi(Q)$

$: \phi_{\rm eq}(Q) \to \phi(Q)$



$: \lambda_4 \to \lambda_4^{off}$



$\kappa_4 \sim\ ?$

(Assuming $\lambda_3 = 0$)

- How about off-equilibrium λ_3, λ_4 ?

Towards a quantitative description of off-equilibrium fluctuations

Initial condition +BES hydro.

↓

Solving eqn. for $\phi(Q)$ on hydro. background;

Solving RG eqn. to determine off-equil. $\lambda_{3,4}$

↓

Particlization (freezeout)

BEST is one single theme with inter-correlated topics.

How to observe spin Hall effect in BESII?

Shuai Liu-YY, to appear

Shuai Liu, postdoc@IMP

Spin Hall effect

Spin Hall effect: an electric field will generate a spin current perpendicular to the direction of the electric field. *Sinova et al Rev. Mod. Phys 15'*

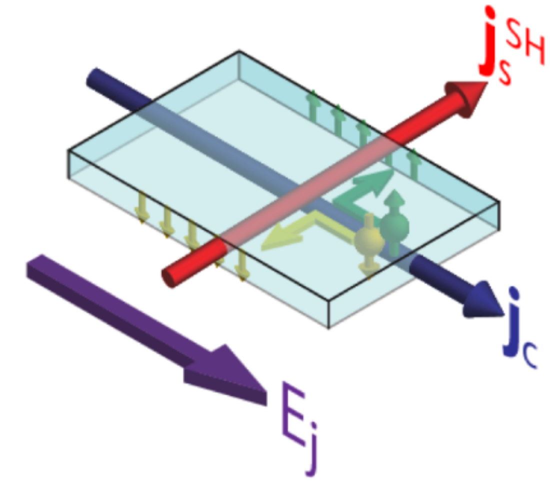


Fig. from Meyer et al, Nature Materials 17's

Observed in a number of condensed matter systems. How about hot and dense QCD matter?

Calculation based on linear response theory (± : particle-antiparticle)

$$\overrightarrow{\mathscr{P}_\pm} = \frac{1}{\epsilon_p}\left(-\frac{\partial n_\pm}{\partial \epsilon_p} + \frac{n_+ + n_-}{2\epsilon_p}\right)\hat{p} \times \nabla \mu_B$$

Shuai Liu-YY, to appear

Spin-polarization in phase space

Analogy of electric field

Observables: "directed spin flow" of Λ , anti-Λ Hyperon

$$(a^i_{1,\pm}, v^i_{1,\pm}) \equiv \int \frac{d\phi_p}{2\pi} P^i_\pm \times (\sin\phi_p, \cos\phi_p)$$

P^i : components of polarization projected along i direction, i=x,y,z

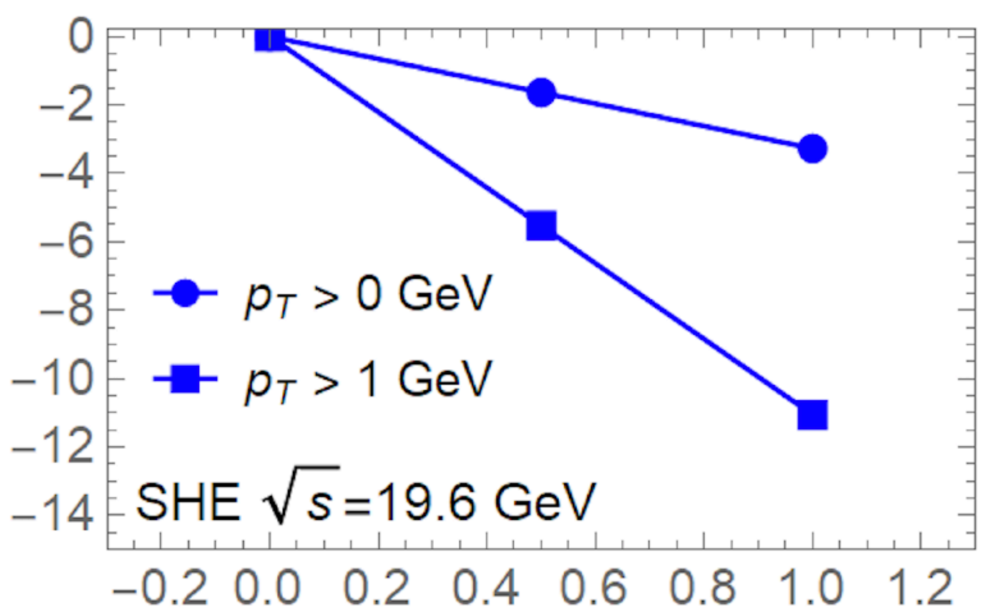
Experiment signature

Shuai Liu-YY, to appear

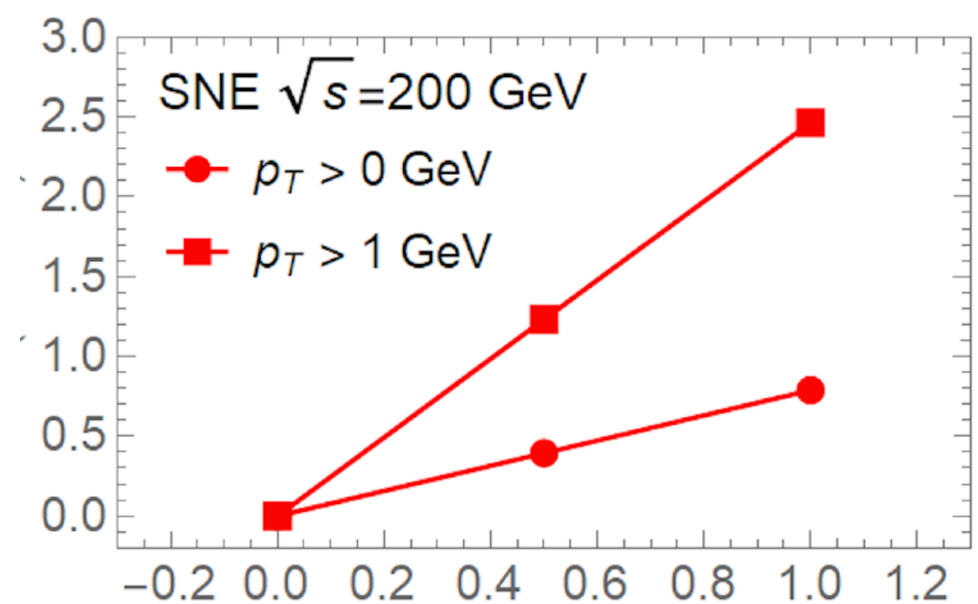
e.g. $\nabla \mu_B$ in longitudinal direction => spin current in transverse plane. (∇T leads to the cousin spin Nernst effect.)

$$\overrightarrow{\mathscr{P}_{\pm}} \propto \hat{p} \times \nabla \mu_B$$

Estimation: blast-wave+benchmark value of $\nabla \mu_B$, ∇T. (NB: SNE and SHE will be present in central collisions)



First Fourier coefficient of P^x (in 10^{-4}) vs rapidity; induced by spin Hall effect.



First Fourier coefficient of P^x (in 10^{-4}) vs rapidity; induced by spin Nernst effect

Input from BES hydro.: crucial for the precise description of Spin Hall effect.

Conclusion and discussion for the future plan

Open discussion on the near term plan

Deliverables: an integrated program to describe off-equilibrium critical fluctuations and their contributions to hadron multiplicities fluctuations.

Back-up