Dynamical magnetic field in heavy ion collisions

Anping Huang
Indiana University

JinFeng Liao (Indiana University)

Dmitri Kharzeev (Stony Brook U. and BNL)

Shuzhe Shi (McGill University)

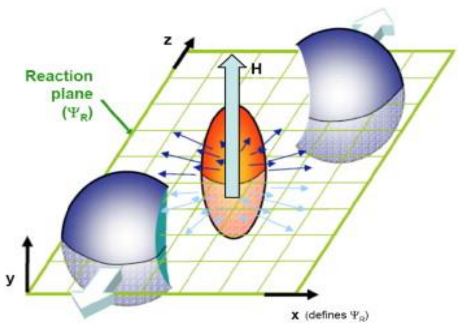
Duan She (CCNU)

Outline

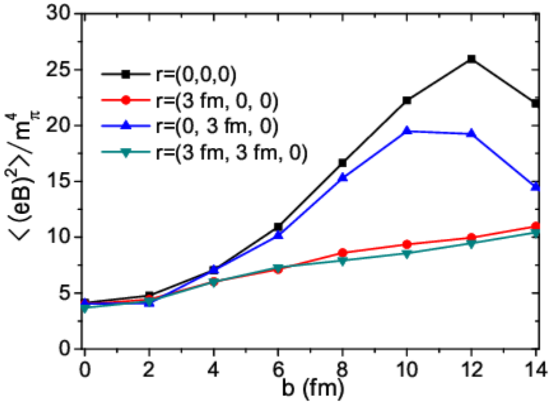
- Motivation
- Dynamical magnetic field in different methods
- Framework of our work
- Maxwell equation in Milne space
- Numerical results
- Summary and outlook

Motivation

Strong B field created in heavy ion collisions

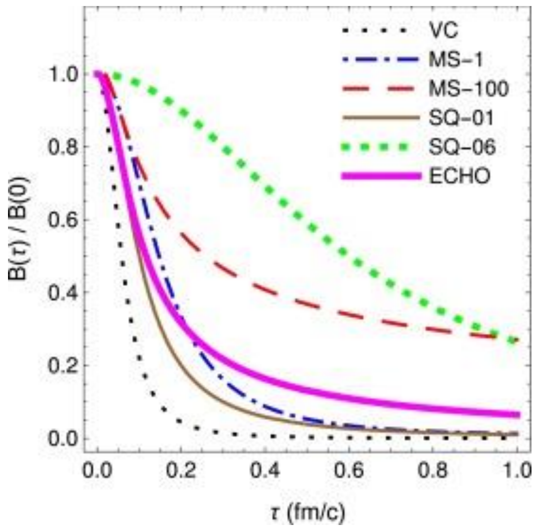


Kharzeev



Bloczynski-Huang-Zhang-Liao

Life time of B in heavy ion collisions



1. The dynamical magnetic field is very important to many corresponding physics in HIC, especially CME.
2. AVFD code need dynamical magnetic field.

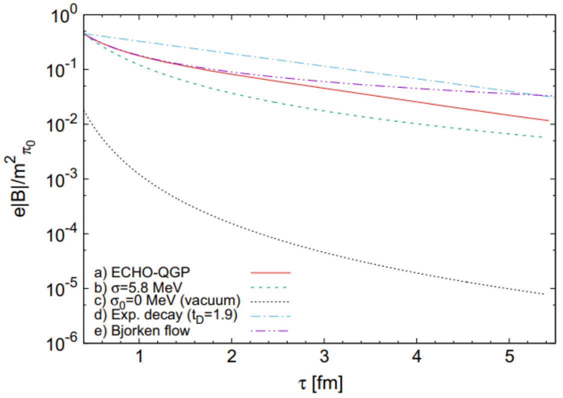
$\tau_B \sim \frac{R_A}{\gamma}$ ~0.06fm for vacuum

Dynamical magnetic field in different methods

1. MHD method:
   medium and B influence each other.

$$B(\tau) = B_0 \frac{\tau_0}{\tau},$$

1) Idea MHD, $\sigma \to \infty$, Bjorken scenario (Kord-Moghaddam-Ghaani,……)
2) Finite σ, but special E^μ, B^μ just have y component, Bjorken scenario. (Siddique-Wang-Pu-Wang)
3) Inghirami-Zanna-Beraudo



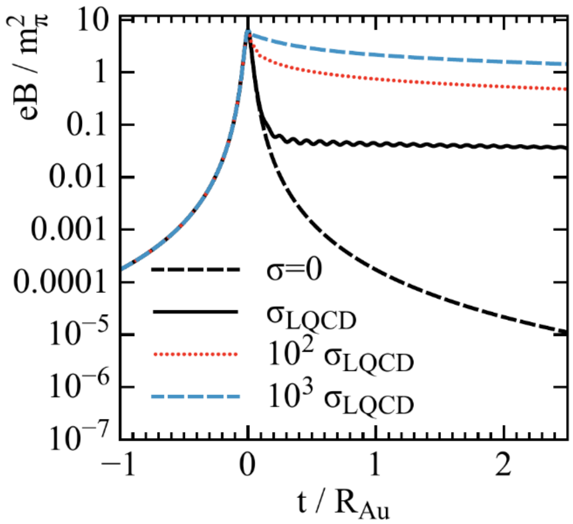
ECHO-QGP
1)Inghirami-Zanna-Beraudo-Moghaddam-Becattini-Bleicher
2) Inghirami-Mace-Hirono-Zanna-Kharzeev-Bleicher

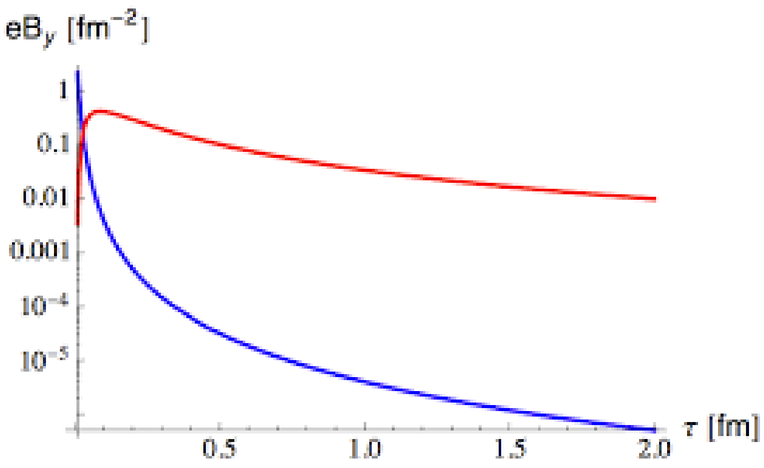
$$B_y(t, \mathbf{0}) = \frac{t_0}{t} e^{-\frac{c_s^2}{2a_x^2}(t^2 - t_0^2)} B_y^0(\mathbf{0}).$$

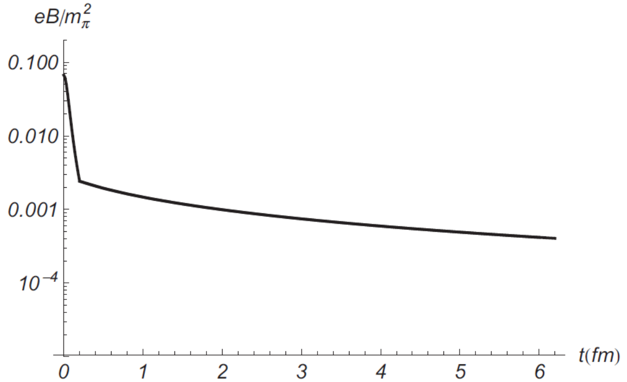
1) ideally conducting limit, $\sigma \to \infty$, but transverse expansion (Deng-Huang)

Dynamical magnetic field in different methods

2. Weak field method:
the effect of B on medium is too weak, but medium effect on B cannot be ignored.



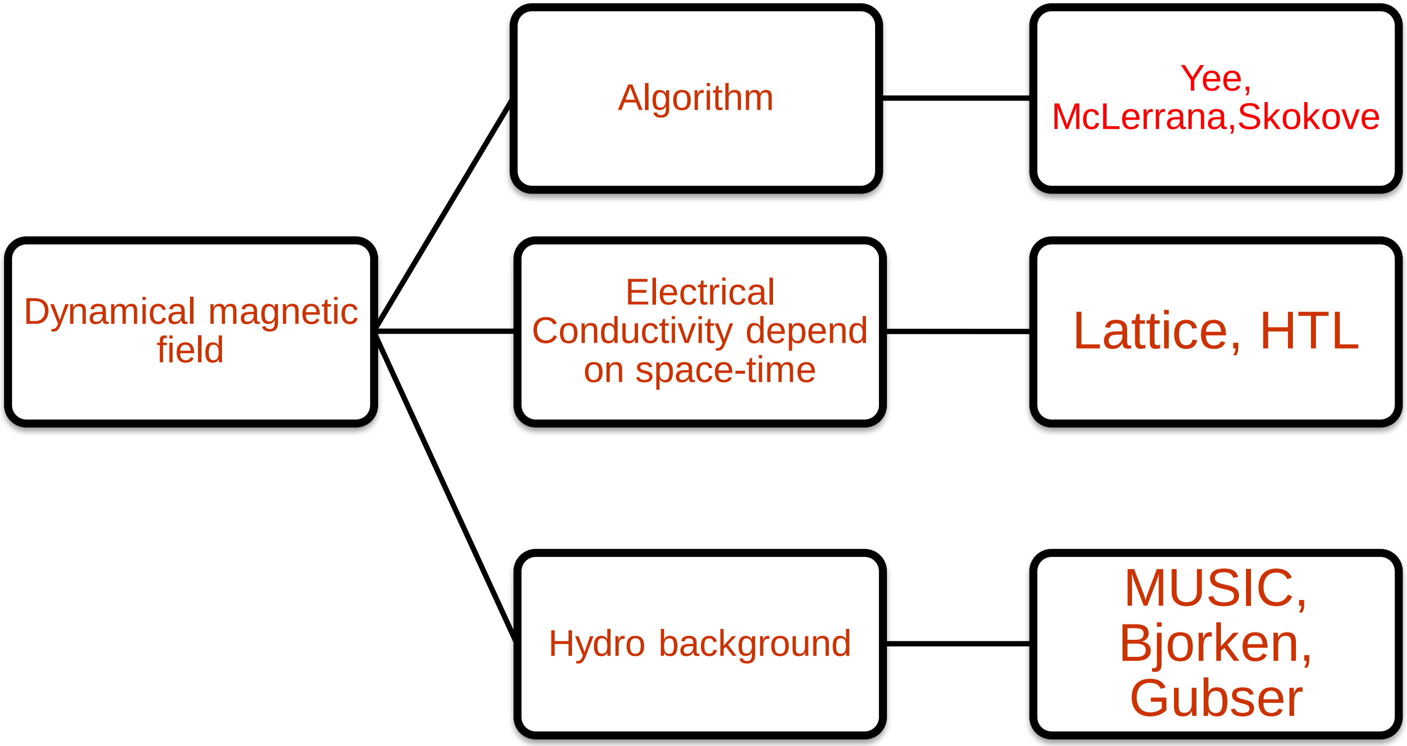




1. McLerrana,Skokove
   numerical method, static medium
2. Gursoy-Kharzeev-Rajagopal, Umut
   Gursoy-Kharzeev-Marcus-
   Rajagopal-Shen.
   theoretical method, constant σ
3. Tuchin
   theoretical method, constant σ

Framework of our work

1. Assume: B effect on medium is weak, but medium effect on B is considerable.
2. Numerical simulation to Maxwell equation.



Maxwell equation in Milne space

The Maxwell equation in Milne space: $\widetilde{E}^i = F_M^{i0}, \quad \widetilde{B}^i = \widetilde{F}_M^{i0}.$

$$\hat{D}_\mu F_M^{\mu\nu} = J^\nu$$

$$\partial_x \widetilde{E}_x + \partial_y \widetilde{E}_y + \partial_\eta \widetilde{E}_z = J_\tau,$$
$$\partial_\tau(\tau\,\widetilde{E}_x) = \partial_y(\tau^2 \widetilde{B}_z) - \partial_\eta \widetilde{B}_y - \tau\,J_x,$$
$$\partial_\tau(\tau\,\widetilde{E}_y) = -\partial_x(\tau^2 \widetilde{B}_z) + \partial_\eta \widetilde{B}_x - \tau\,J_y,$$
$$\partial_\tau(\tau\,\widetilde{E}_z) = \partial_x \widetilde{B}_y - \partial_y \widetilde{B}_x - \tau\,J_\eta.$$

$$\hat{D}_\mu \widetilde{F}_M^{\mu\nu} = 0$$

$$\partial_x \widetilde{B}_x + \partial_y \widetilde{B}_y + \partial_\eta \widetilde{B}_z = 0,$$
$$\partial_\tau(\tau\,\widetilde{B}_x) = -\partial_y(\tau^2 \widetilde{E}_z) + \partial_\eta \widetilde{E}_y,$$
$$\partial_\tau(\tau\,\widetilde{B}_y) = \partial_x(\tau^2 \widetilde{E}_z) - \partial_\eta \widetilde{E}_x,$$
$$\partial_\tau(\tau\,\widetilde{B}_z) = -\partial_x \widetilde{E}_y + \partial_y \widetilde{E}_x.$$

$$J^\mu = n u_M^\mu + d^\mu + \sigma\,F_M^{\mu\nu} u_\nu + \sigma_\chi \widetilde{F}_M^{\mu\nu} u_\nu,$$
$$J_\tau = n u_\tau + d_\tau + \sigma\left(\widetilde{E}_x u_x + \widetilde{E}_y u_y + \tau^2 \widetilde{E}_z u_\eta\right) + \sigma_\chi\left(\widetilde{B}_x u_x + \widetilde{B}_y u_y + \tau^2 \widetilde{B}_z u_\eta\right),$$
$$J_x = n u_x + d_x + \sigma\left(\widetilde{E}_x u_\tau + \tau \widetilde{B}_z u_y - \tau \widetilde{B}_y u_\eta\right) + \sigma_\chi\left(\widetilde{B}_x u_\tau - \tau \widetilde{E}_z u_y + \tau \widetilde{E}_y u_\eta\right),$$
$$J_y = n u_y + d_y + \sigma\left(\widetilde{E}_y u_\tau - \tau \widetilde{B}_z u_x + \tau \widetilde{B}_x u_\eta\right) + \sigma_\chi\left(\widetilde{B}_y u_\tau + \tau \widetilde{E}_z u_x - \tau \widetilde{E}_x u_\eta\right),$$
$$J_\eta = n u_\eta + d_\eta + \sigma\left(\widetilde{E}_z u_\tau + \frac{\widetilde{B}_y}{\tau} u_x - \frac{\widetilde{B}_x}{\tau} u_y\right) + \sigma_\chi\left(\widetilde{B}_z u_\tau - \frac{\widetilde{E}_y}{\tau} u_x + \frac{\widetilde{E}_x}{\tau} u_y\right).$$

Maxwell equation in Milne space

The relations between Minkowski and Milne space

$$E_x = \cosh\eta\,\widetilde{E}_x + \sinh\eta\,\widetilde{B}_y, \quad E_y = \cosh\eta\,\widetilde{E}_y - \sinh\eta\,\widetilde{B}_x, \quad E_z = \tau\widetilde{E}_z,$$
$$B_x = \cosh\eta\,\widetilde{B}_x - \sinh\eta\,\widetilde{E}_y, \quad B_y = \cosh\eta\,\widetilde{B}_y + \sinh\eta\,\widetilde{E}_x, \quad B_z = \tau\widetilde{B}_z.$$

The corresponding velocity in Milne space

$$u^\mu_M = R^\mu{}_\nu u^\nu = \begin{cases} \left(\cosh\eta, 0, 0, -\frac{\sinh\eta}{\tau}\right), & \text{for static case: } u^\mu = (1,0,0,0), \\ (1,0,0,0), & \text{for Bjorken flow: } u^\mu = (\frac{t}{\tau}, 0, 0, \frac{z}{\tau}), \\ \left(u^\tau, u^\perp\frac{x}{x_\perp}, u^\perp\frac{y}{x_\perp}, 0\right), & \text{for Gubser flow: } u^\mu = (u^\tau\cosh\eta, u^\perp\frac{x}{x_\perp}, u^\perp\frac{y}{x_\perp}, u^\tau\sinh\eta). \end{cases}$$

1. Turn off n, d and σ_χ in the following results.

Maxwell equation in Milne space

The physical picture: Three stages

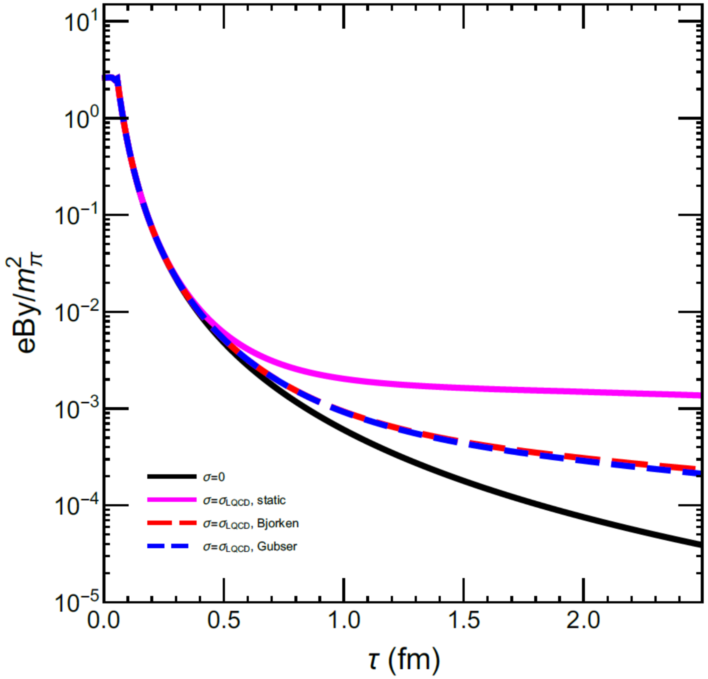
1. Initial stage ($\tau = 0 - 0.1$ fm): $\sigma = 0$, no medium effect

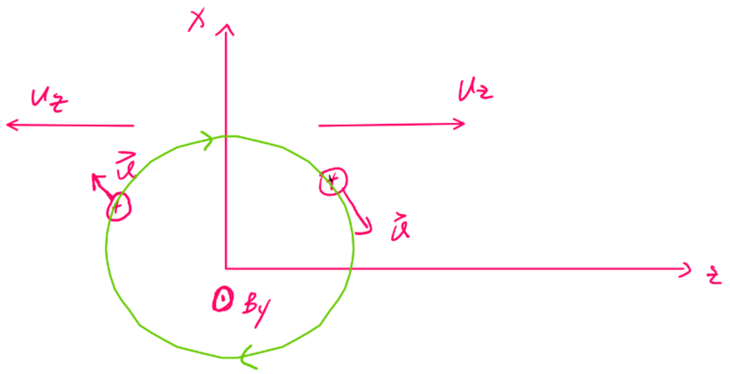
2. Pre-equilibrium stage ($\tau = 0.1 - 0.4$ fm): (Bjorken expansion)
   1) zero model: $\sigma(\tau, \vec{x}) = 0$,
   2) constant model: $\sigma(\tau, \vec{x}) = \sigma(\tau = 0.4, \vec{x})$
   3) linear model: $\sigma(\tau, \vec{x})$ is linear increase, ($\tau = 0.1, 0.4$) leads to a linear function.

3. Hydro stage ($\tau \geq 0.4$ fm): (Bjorken, Gubser, MUSIC)
   $\sigma = \sigma_{LQCD}$, 100 σ_{LQCD}; $\sigma = 0.1$T, 100T;
   $\sigma_{LQCD} = 5.8$ Mev
   This will be determined finally by LQCD, HTL, kinetic theory and other theory.

Numerical results

1. The longitudinal expansion will depress the magnetic field.



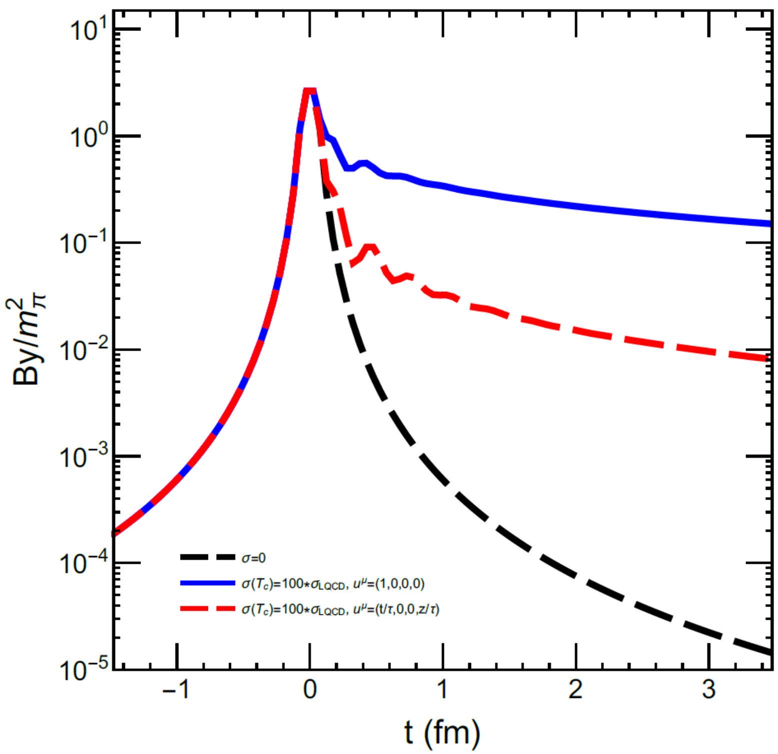


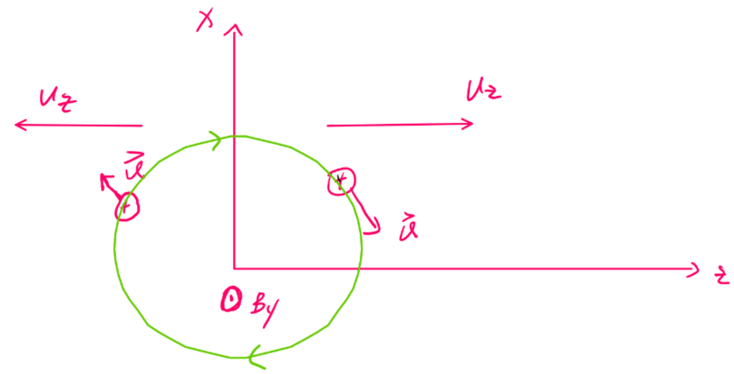
The Lorentz force induced by the longitudinal expansion depress the magnetic field.

$$u^\mu_M = R^\mu{}_\nu u^\nu = \begin{cases} \left(\cosh\eta, 0, 0, -\dfrac{\sinh\eta}{\tau}\right), & \text{for static case: } u \\ (1,0,0,0)\,, & \text{for Bjorken flow} \\ \left(u^\tau, u^\perp \dfrac{x}{x_\perp}, u^\perp \dfrac{y}{x_\perp}, 0\right), & \text{for Gubser flow:} \end{cases}$$

Numerical results

The depression effect also exist in Minkowski space

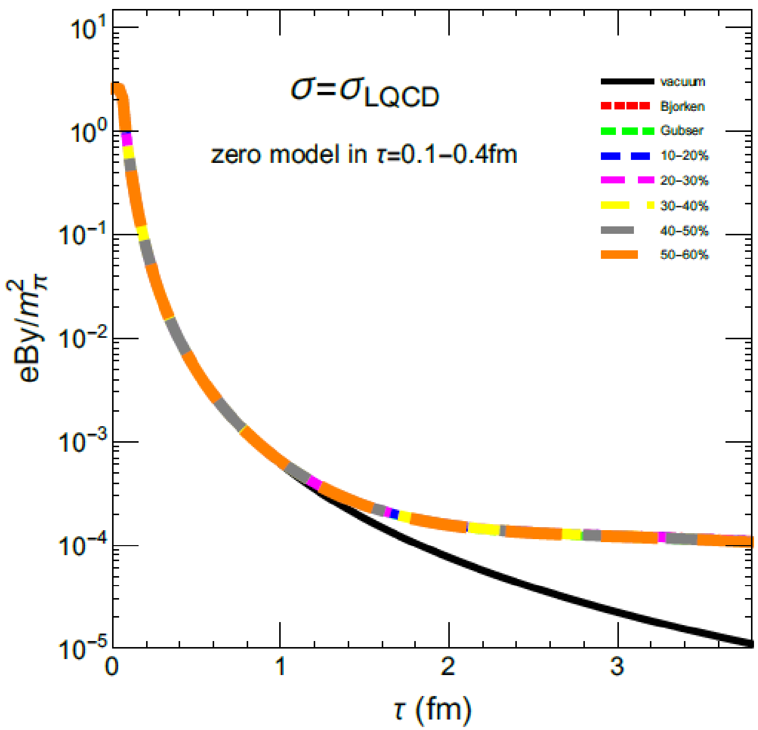


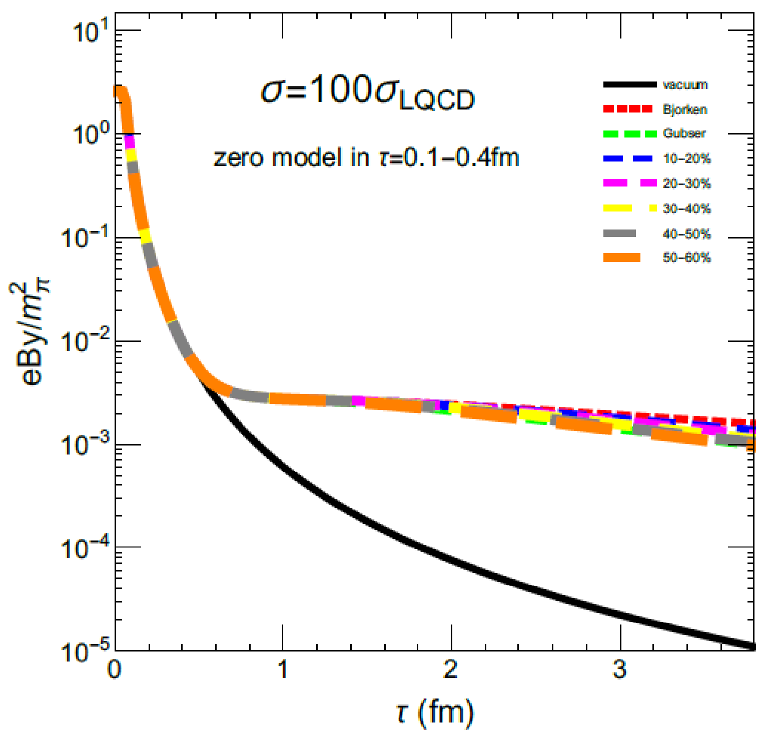


The Lorentz force induced by the longitudinal expansion depress the magnetic field.

Numerical results

2. B is sensitive to electrical conductivity in hydro stage ($\tau \geq 0.4$ fm)
   ($\sigma = 100\sigma_{LQCD}$ is just for comparison and shows stable)

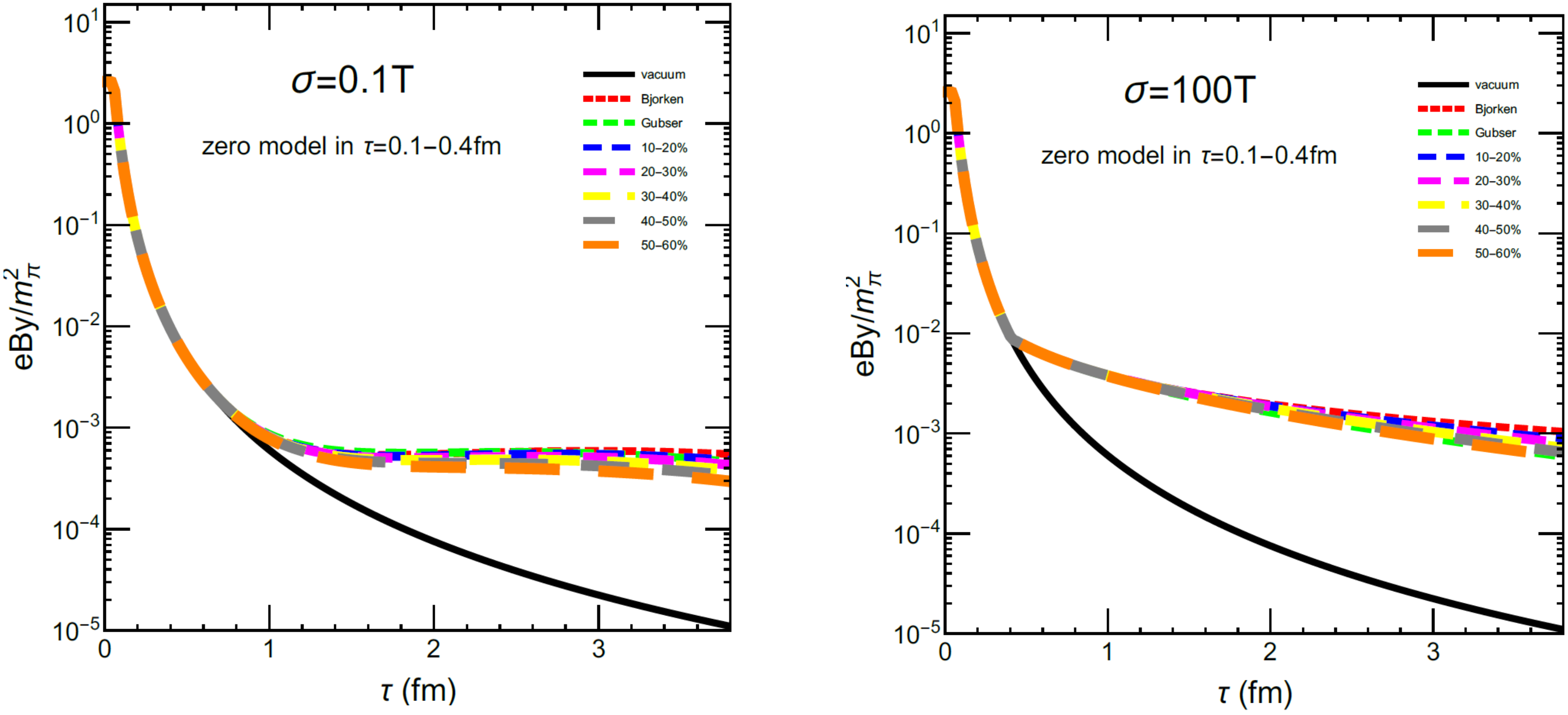




The fitted function: $$eB_y = eB_{y0}\,\mathrm{Exp}\left[a[0]\,e^{\frac{b[0]+b[1]\tau+b[2]\tau^2}{c[0]+c[1]\tau+c[2]\tau^2+c[3]\tau^3}}\right]$$

Numerical results

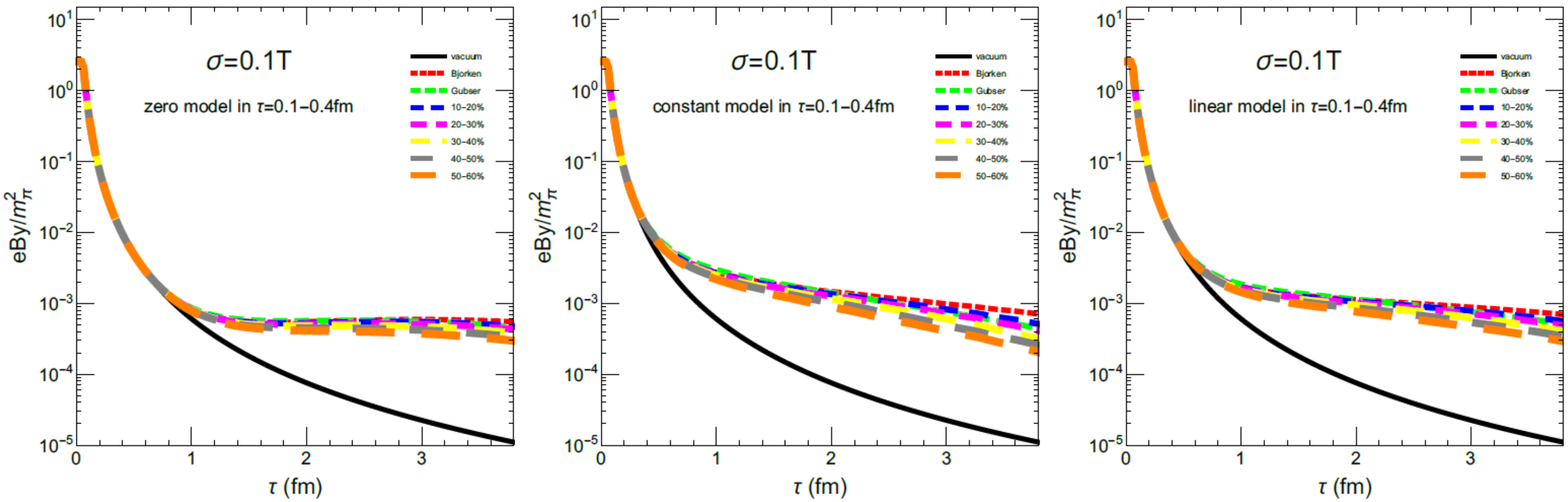
2. B is sensitive to electrical conductivity in hydro stage ($\tau \geq 0.4$ fm)
   ($\sigma = 100$T is just for comparison and shows stable)



The fitted function: $$eB_y = eB_{y0}\,\mathrm{Exp}\left[a[0]\,e^{\frac{b[0]+b[1]\tau+b[2]\tau^2}{c[0]+c[1]\tau+c[2]\tau^2+c[3]\tau^3}}\right]$$

Numerical results

3. Electrical conductivity model in pre-equilibrium stage ($\tau = 0.1 - 0.4$ fm) is very important to B.

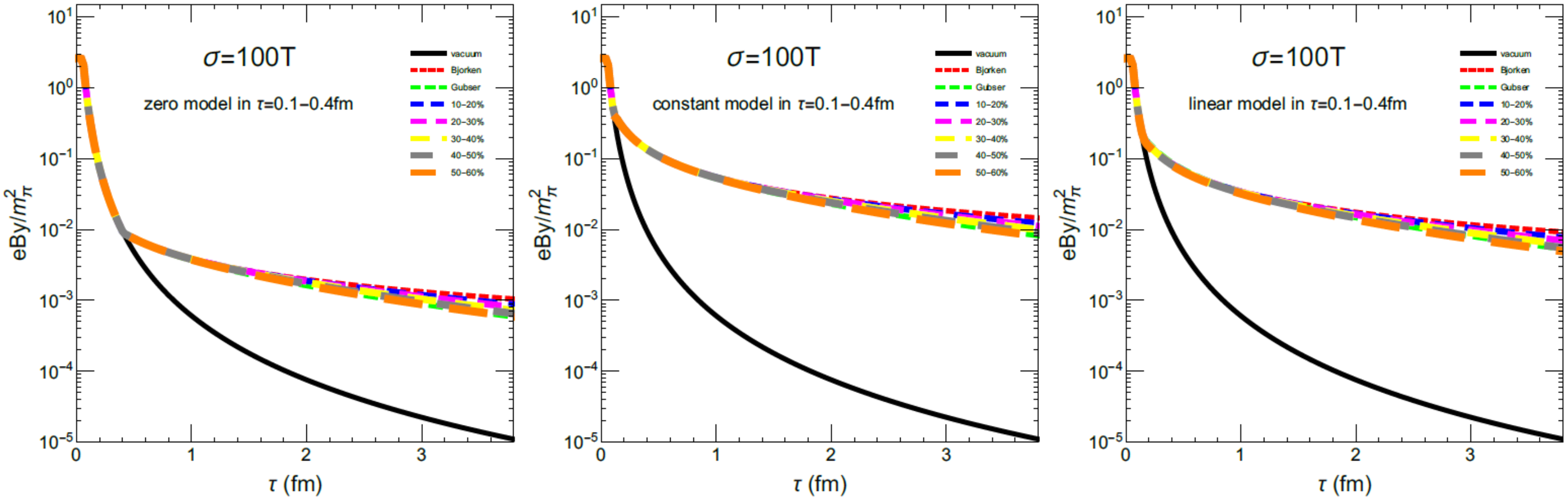


The fitted function:

$$eB_y = eB_{y0}\,\mathrm{Exp}\left[a[0]\, e^{\frac{b[0]+b[1]\tau+b[2]\tau^2}{c[0]+c[1]\tau+c[2]\tau^2+c[3]\tau^3}}\right]$$

Numerical results

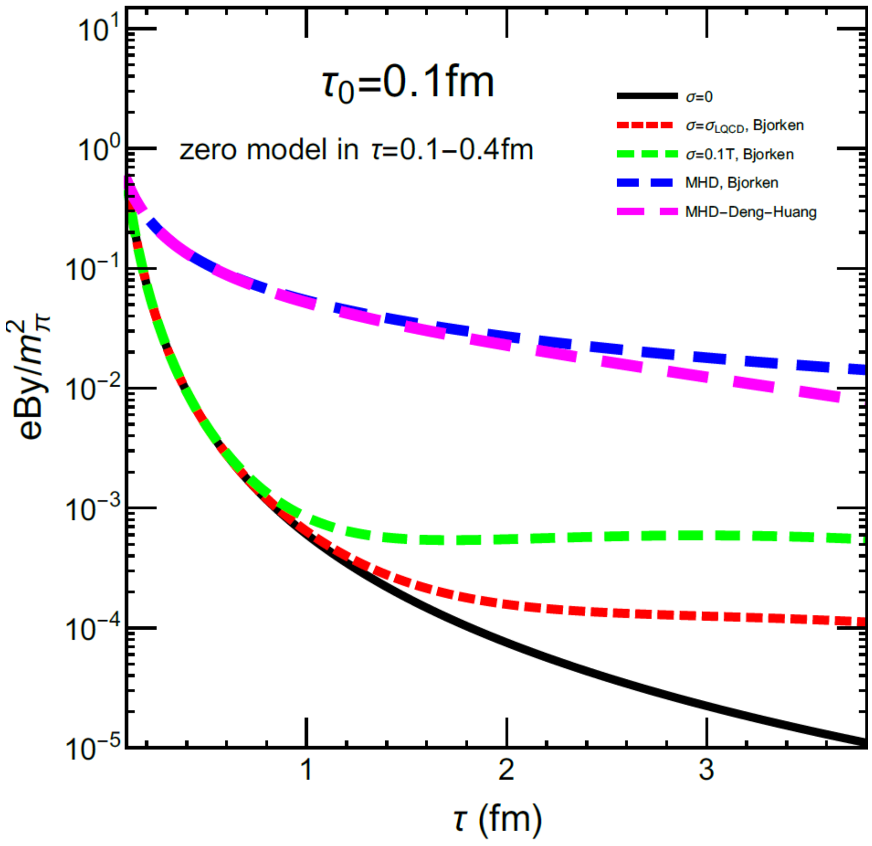
3. Electrical conductivity in pre-equilibrium stage ($\tau = 0.1 - 0.4$ fm) is very important to B. ($\sigma = 100$T is just for comparison and shows stable)

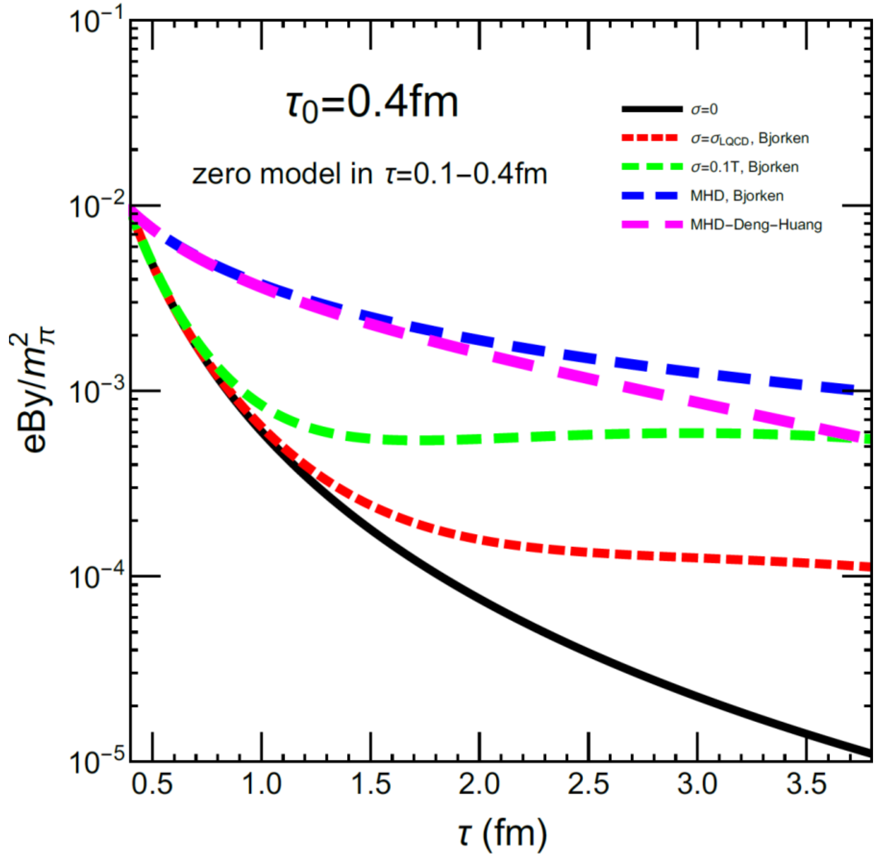


The fitted function: $$eB_y = eB_{y0}\,\mathrm{Exp}\left[a[0]\,e^{\frac{b[0]+b[1]\tau+b[2]\tau^2}{c[0]+c[1]\tau+c[2]\tau^2+c[3]\tau^3}}\right]$$

Numerical results

4. Comparison with other methods





Summary and outlook

1. Summary
   1.1 The numerical calculation of Maxwell equation are more realistic and stable.
   1.2 There some sensitive factors to the evolution of magnetic field.
      1) longitudinal expansion will depress the magnetic field
      2) the evolution of magnetic field is very sensitive to electrical conductivity model in pre-equilibrium stage.
      3) electrical conductivity in hydro stage is very important.
   1.3 Comparison with other methods.

Summary and outlook

2. Outlook
   1.1 determine a reasonable value of the electrical conductivity in hydro stage by Lattice and HTL.
   1.2 get a reasonable model for the electrical conductivity in pre-equilibrium stage.
   1.3 combine with AVFD code.

Thank You!