Precision tests of the Standard Model

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Effective field theory approach

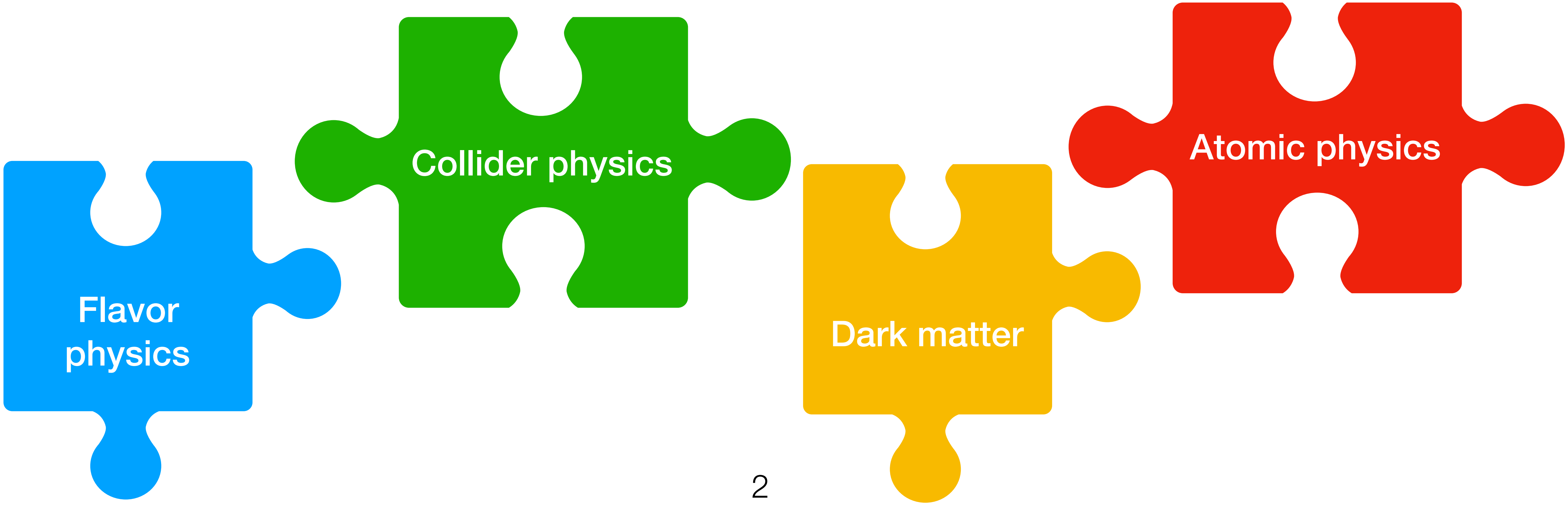
High energy ↔ **Low energy**

Soft-collinear EFT

Allows to describe radiation from energetic particles in a systematic way

Potential non-relativistic EFT

Deals with slowly moving particles and bound states



My personal list of topics

eV

MeV

GeV

TeV

- **Lamb shift**
- **Bound electron g-factor**
- **Muon g-2**
- **CLFV and bound muon decay**
- **QED effects in flavor physics with SCET**
- **Power corrections with SCET and Higgs threshold production**
- **Dark Matter**

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Today I want to focus on these topics, but feel free to ask me about any other

Spectroscopy of Hydrogen

Transition frequencies of hydrogen are among the most precisely measured observables

$$\nu_H(1S_{1/2} - 2S_{1/2}) = 2\,466\,061\,413\,187.035(10)\ \mathrm{kHz}$$

Parthey et al., 2011

Relative uncertainty 4.2×10^{-15}

What can we use it for?

$$\nu_{ij} = \varepsilon_j - \varepsilon_i$$

Radiative and recoil corrections

S. Karshenboim, A. Ozawa, V. Shelyuto, R.S., V. Ivanov, 2019

Transitions are related to a difference of Hydrogen energies

$$R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$$

$$\varepsilon_i = -\frac{R_\infty c}{n_i^2}(1 + \delta_i)$$

Rydberg constant can be used to determine the fine structure constant — test bound state QED and SM!

eV

Determination of α

Determined from spectroscopic measurements

$$R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$$

Rydberg constant

$$\varepsilon_i = -\frac{R_\infty c}{n_i^2}(1+\delta_i)$$

Need to know δ_i and

$$\frac{\hbar}{m_e} = \frac{u}{m_e}\frac{M_X}{u}\frac{\hbar}{M_X}$$

$\frac{u}{m_e}$ **determined from the g-factor of a bound electron** ★

$\frac{\hbar}{M_X}$ **recoil velocity of Rb/Cs atom** $v_r = \frac{\hbar k}{M_{\mathrm{Rb}}}$

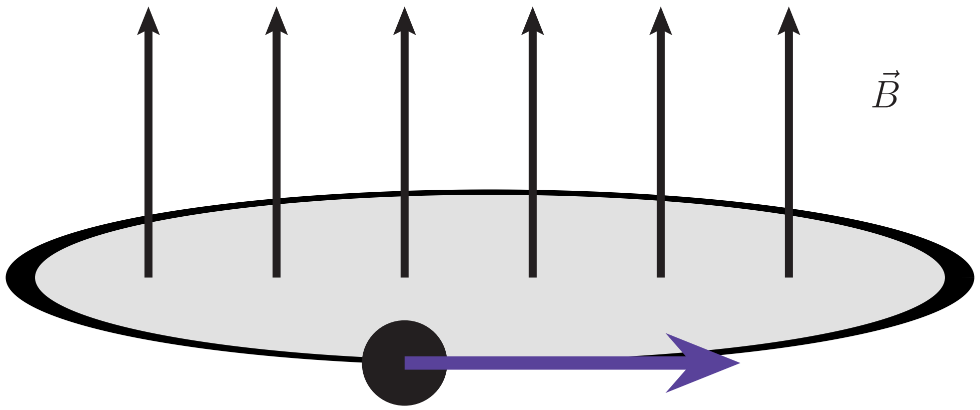
Parker et al., 2018

Electron in a magnetic field

Cyclotron motion: Landau levels

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

$$\omega_c = \frac{|q|}{m}B$$

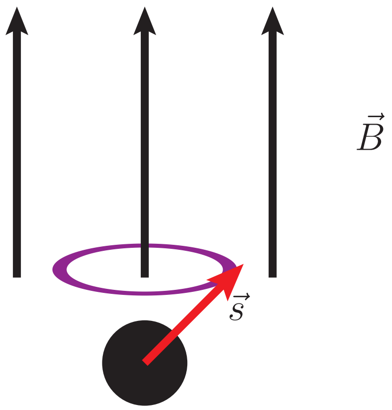


Spin part: two levels separated by an interval

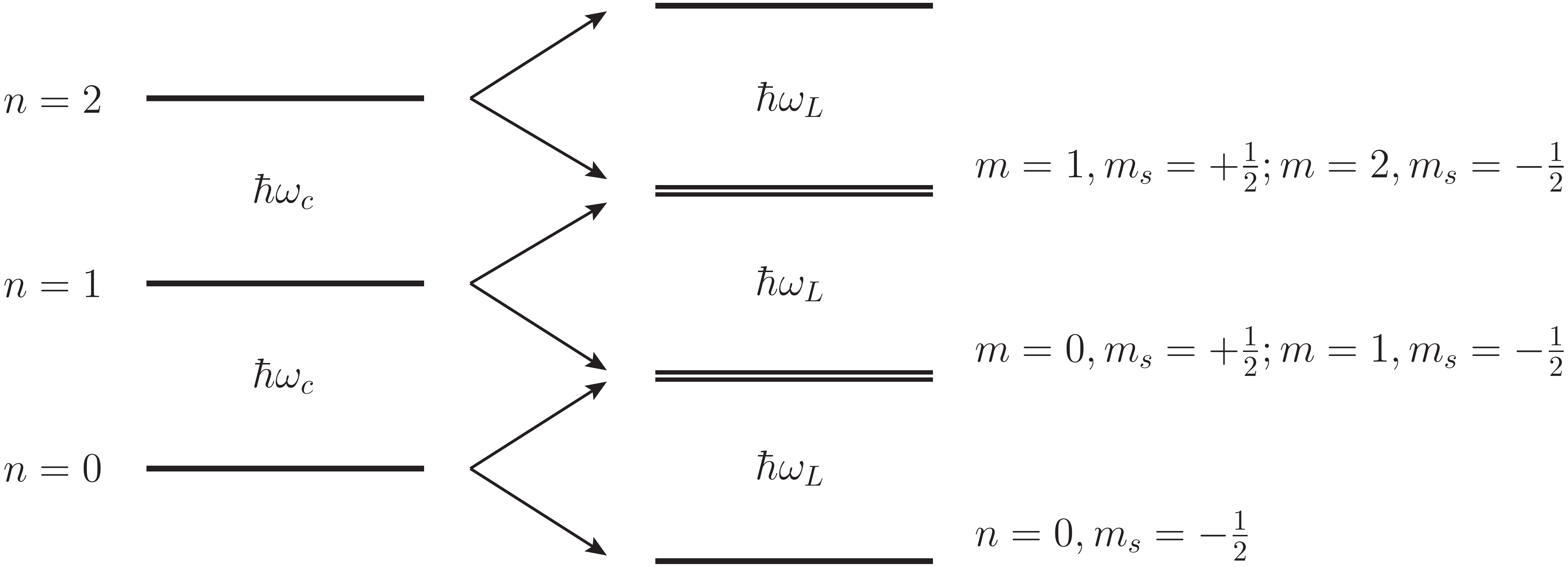
$$\Delta E = \hbar\omega_L$$

with Larmor frequency

$$\omega_L = 2 \times \frac{|q|}{2m}B$$



Electron g-2



$n = 2$

$n = 1$

$n = 0$

$\hbar\omega_c$

$\hbar\omega_c$

$\hbar\omega_L$

$\hbar\omega_L$

$\hbar\omega_L$

$m = 1, m_s = +\frac{1}{2}; m = 2, m_s = -\frac{1}{2}$

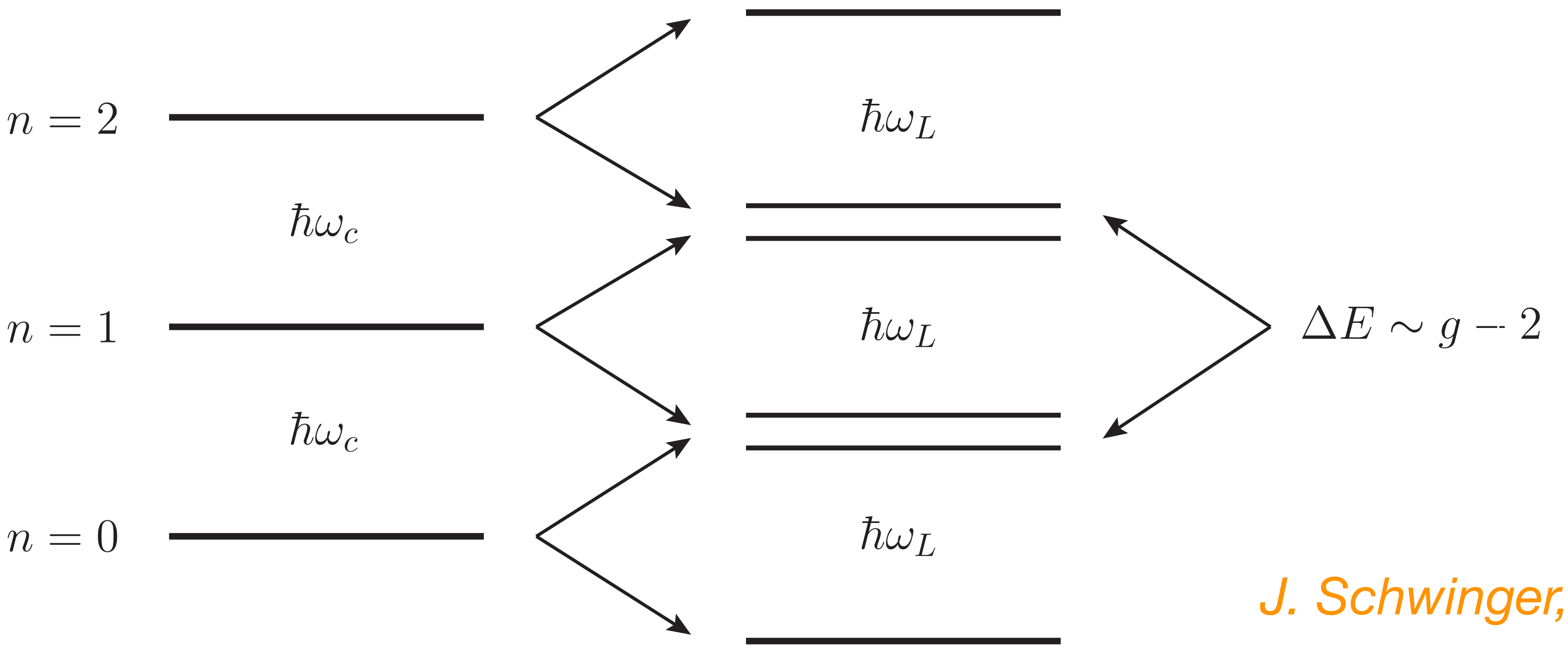
$m = 0, m_s = +\frac{1}{2}; m = 1, m_s = -\frac{1}{2}$

$n = 0, m_s = -\frac{1}{2}$

Dirac equation predicts $\omega_c = \omega_L$

$$g = 2\frac{\omega_L}{\omega_c} = 2$$

Electron g-2



J. Schwinger, 1948

But in reality we see $\omega_c \neq \omega_L$ $\quad g = 2 + \frac{\alpha}{\pi} + \ldots$

$$\Delta a_e = a_e^{\mathrm{exp}} - a_e^{\mathrm{th}} = (-88 \pm 36) \times 10^{-14}$$

R. Parker et al., 2018
T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, 2017
D. Hanneke, S. Fogwell, G. Gabrielse, 2008

Bound electron g-factor

Larmor frequency

$$\omega_L = \frac{g}{2}\frac{e}{m_e}$$

Now includes binding corrections

$$g = \frac{2}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) \approx 2 - \frac{2}{3}(Z\alpha)^2$$

Breit, 1928

Cyclotron frequency of the ion

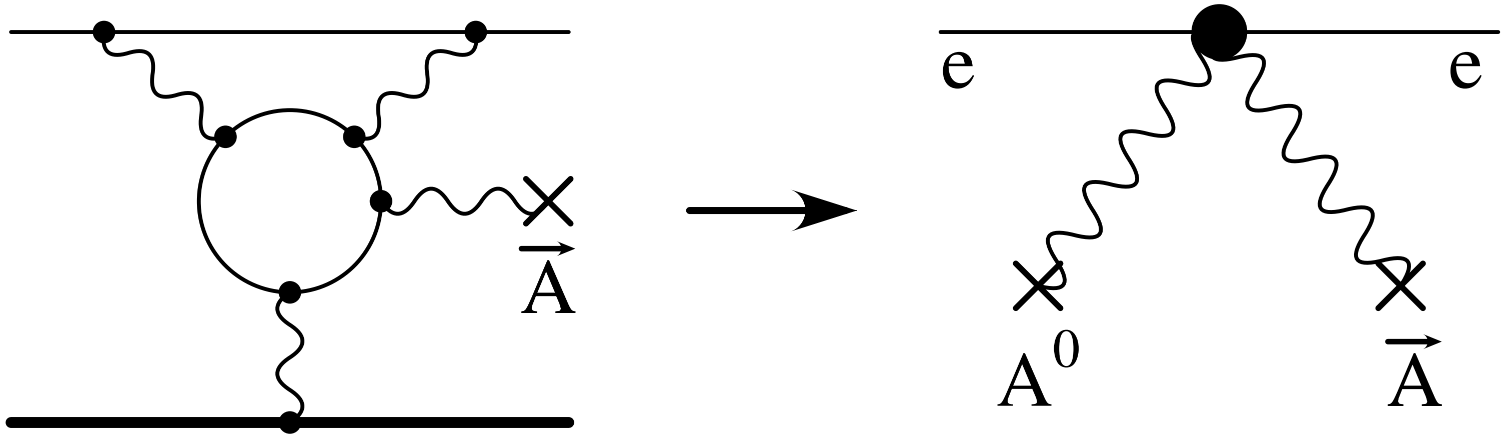
$$\omega_c = \frac{Q}{M}B, \qquad Q = (Z-1)e$$

$$m_e = \frac{g}{2}\frac{e}{Q}\frac{\omega_c}{\omega_L}M$$

Currently the most precise source of electron mass determination

Note that now we measure g and not g-2!

We need a good handle on theory



A. Czarnecki, R.S., 2016

EFT allows to disentangle short- and long-distance contribution

$$\mathcal{L}_{\mathrm{NRQED}} \supset \frac{\psi^\dagger(\vec{\sigma}\cdot\vec{B})(\vec{\nabla}\cdot\vec{E})\psi}{m_e^3}$$

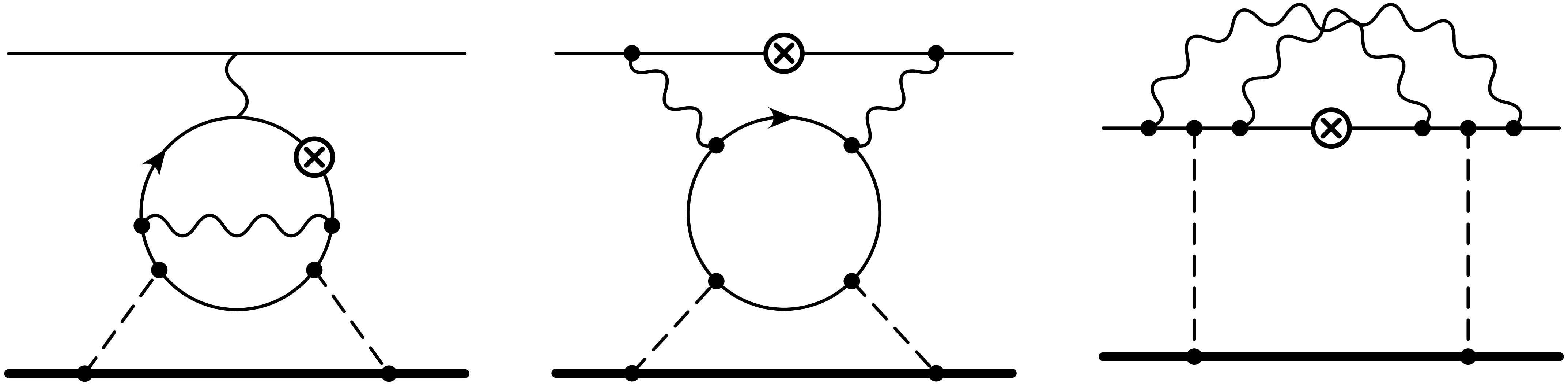
$$\delta g_e = (Z\alpha)^4\left(\frac{\alpha}{\pi}\right)^2\frac{16-19\pi^2}{108}$$

eV

We need a good handle on theory

Many corrections have been computed, with the most recent ones $\left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^5$

A. Czarnecki, M. Dowling, J. Piclum, R.S., 2017



Contribution	$^4\mathrm{He}^+$	$^{12}\mathrm{C}^{5+}$	$^{28}\mathrm{Si}^{13+}$
Dirac/Breit value	1.999 857 988 825 37(7)	1.998 721 354 392 0(6)	1.993 023 571 557(3)
+ other known corrections	2.002 177 406 711 41(55)	2.001 041 590 168 6(12)	1.995 348 957 825 (39)
g^{SE}	0.000 000 000 000 02	0.000 000 000 005 0	0.000 000 000 348
g^{LBL}	-0.000 000 000 000 01	-0.000 000 000 001 5	-0.000 000 000 102
g^{ML}	0.000 000 000 000 00	0.000 000 000 000 6	0.000 000 000 038
$\alpha^2(Z\alpha)^6 \ln^3(Z\alpha)$	0.000 000 000 000 00(3)	0.000 000 000 000 0(93)	0.000 000 000 000(590)
Total	2.002 177 406 711 42(55)	2.001 041 590 172 7(94)	1.995 348 958 109 (591)

Electron mass

$$m_e = 0.000\ 548\ 579\ 909\ 065(16)u$$

Sturm et al. 2014; Zatorski et al. 2017

Uncertainty is dominated by the experimental measurement but improvement by an order of magnitude is expected soon

New independent source of α!

- Combine Li-like H-like ions to cancel dependence on the finite nuclear size effects
- Use different nuclei to cancel dependence on the free electron g – 2
- Use heavy ions Z » 1 and construct a function of g-factors for different energy levels such that the finite nuclear size effects cancel and α dependence is enhanced by Z

Future prospects

Experiments designed to provide tests of bound state QED

- Mainz g-factor experiment
- ALPHATRAP (MPI-K Heidelberg)
- HITRAP (GSI Darmstadt)

Note also prospects for muon g-2: Fermilab and J-PARC

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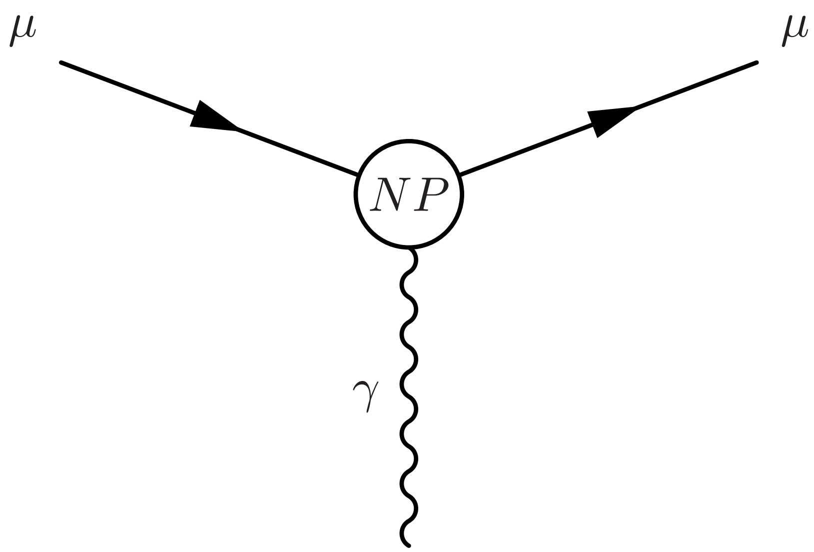
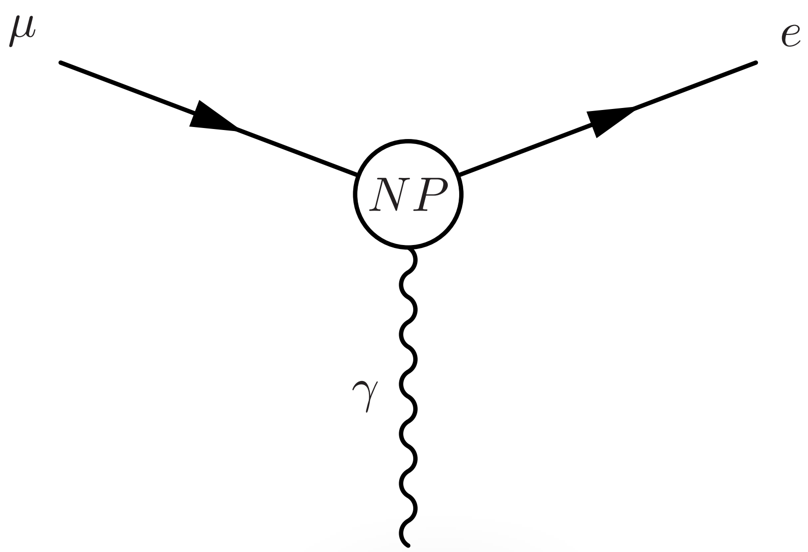
Theory of bound electron should be further improved!

MeV

Searches for CLFV

If there is New Physics in muon sector, maybe we can also see it in rare processes with muons



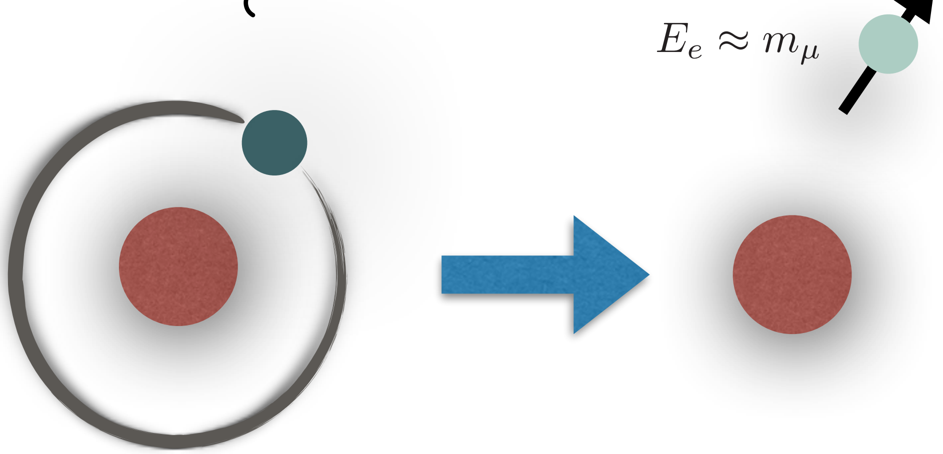


Flagship processes:

$$\mu \to e\gamma$$

$$\mu \to eee$$

$$\mu N \to eN$$ **Muon-electron conversion**



New experiments: **Mu2e (Fermilab), COMET (J-PARC)**

Flavor physics

Observation of CLFV may be a hint of New Physics in the quark flavor sector

Several discrepancies in B-meson decays hint at a violation of lepton flavor universality

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)}{\mathcal{B}(B \to K^{(*)}ee)} \qquad R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})}\bigg|_{\ell=e,\mu}$$

But QED corrections are also not flavor universal

$$\ln\frac{m_\mu}{\Delta E} \sim 2.5; \qquad \ln\frac{m_B}{m_\mu} \sim 4; \qquad \ln\frac{m_B}{\Lambda_{\text{QCD}}} \sim 3; \qquad \ldots$$

Expansion parameter is $\frac{\alpha_{\text{em}}}{\pi} \times \log^2$ rather than just $\frac{\alpha_{\text{em}}}{\pi}$

$$\ln\frac{m_\mu}{m_e} \sim 5$$

GeV

Ultra-soft photons

- Typically we can impose a cut on the photon energy forcing it to be ultra-soft ΔE
- This leads to simple classification
  - Ultra-soft photons: based on eikonal approximation, well understood, under the assumption that

$$\Delta E \ll \Lambda_{\mathrm{QCD}}$$

  - Non-universal, structure dependent correction

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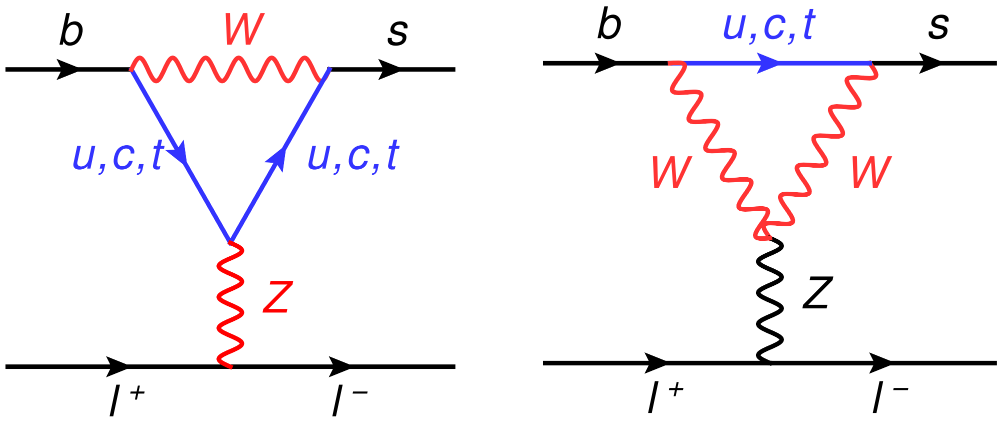
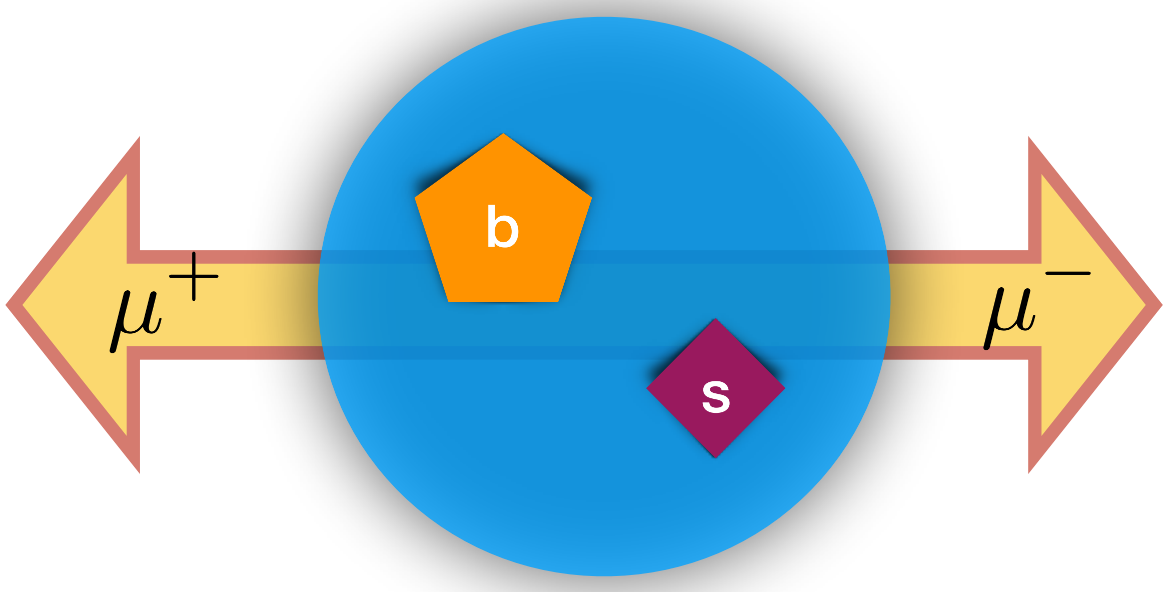
  - Non-universal, structure dependent correction

Both effects are important - even with a cut on real photons ΔE the virtual photons can resolve the structure of the meson! Virtual photons can couple to initial and final state and may have wave-lengths smaller than the typical meson size

$B_s \to \mu^+\mu^-$



- Loop suppressed (FCNC)
- Helicity suppressed (scalar meson decaying into energetic muons through vector interactions)
- Precisely known in SM thanks to purely leptonic final state

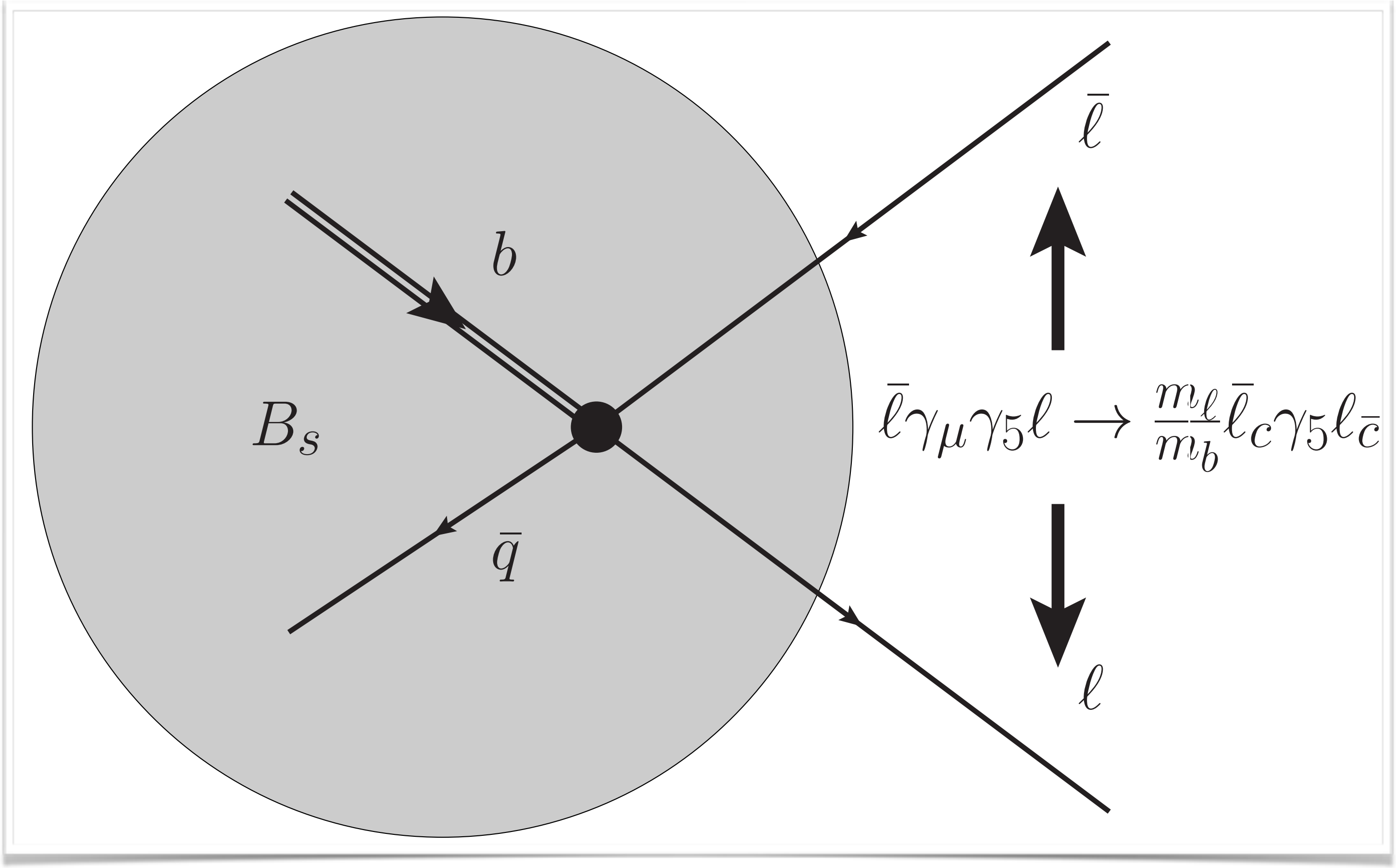


$$\langle 0|\, \bar{q}(0)\gamma^\mu\gamma_5 b(0)\, |\bar{B}_q(p)\rangle = i f_{B_q} p^\mu$$

$$\bar{\ell}_L\gamma^\mu\ell_L \pm \bar{\ell}_R\gamma^\mu\ell_R$$

$$Br(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}} \times \left|\frac{2m_\mu}{m_{B_s}} C_{10}\right|^2$$

Can the helicity suppression be relaxed?

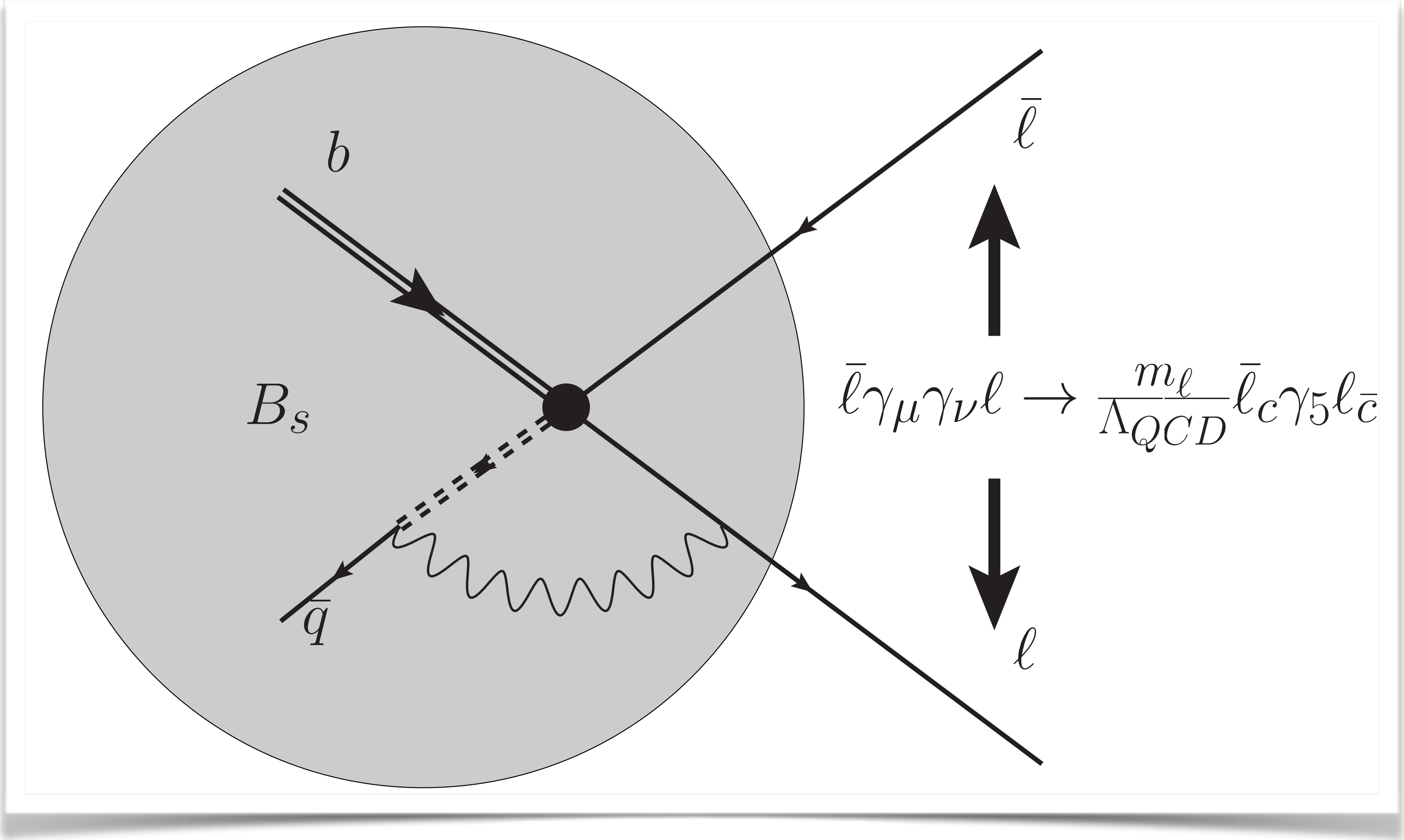


$$u(p_\ell) = u_c(p_\ell) + \mathcal{O}\left(\frac{m_\ell}{E_\ell}\right)$$

Annihilation and helicity flip take place at the same point $r \lesssim \frac{1}{m_b}$

Can the helicity suppression be relaxed?

M. Beneke, C. Bobeth, R.S., 2017

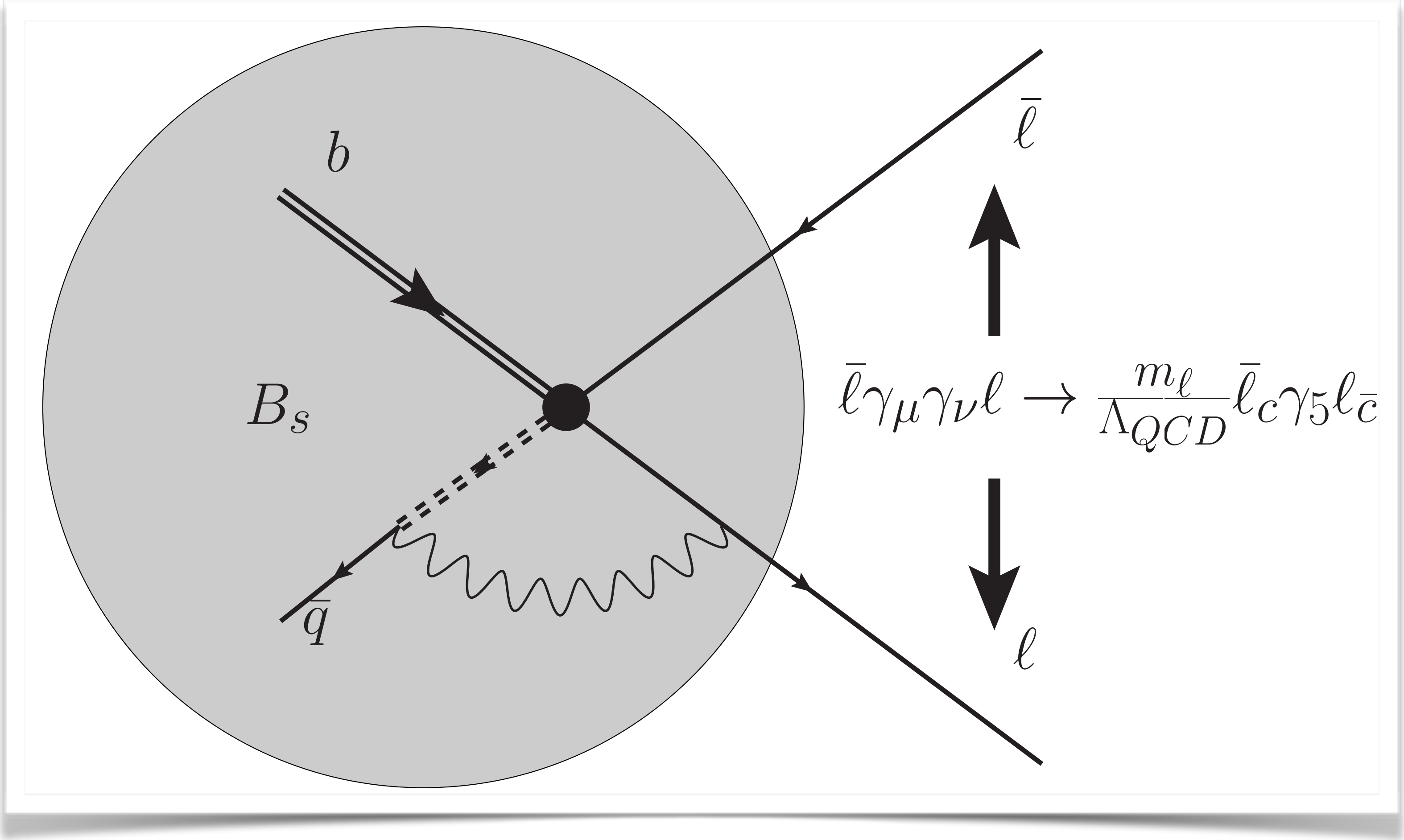


Annihilation and helicity flip can be separated by $r \sim 1/\sqrt{m_b \Lambda_{\text{QCD}}}$

It is still a short distance effect since the size of the meson is $r \sim 1/\Lambda_{\text{QCD}}$

Can the helicity suppression be relaxed?

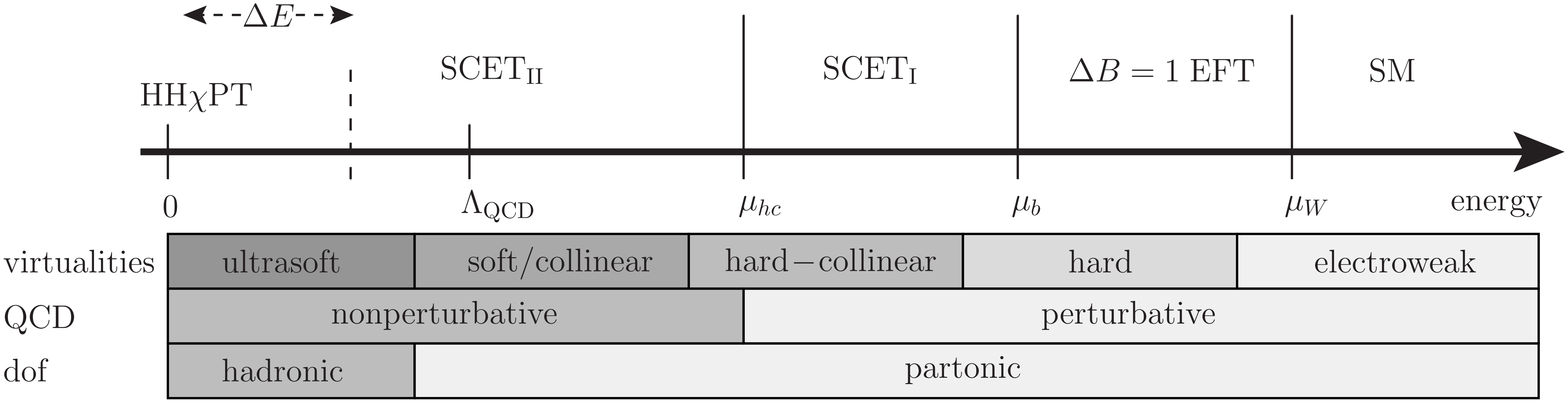
M. Beneke, C. Bobeth, R.S., 2017



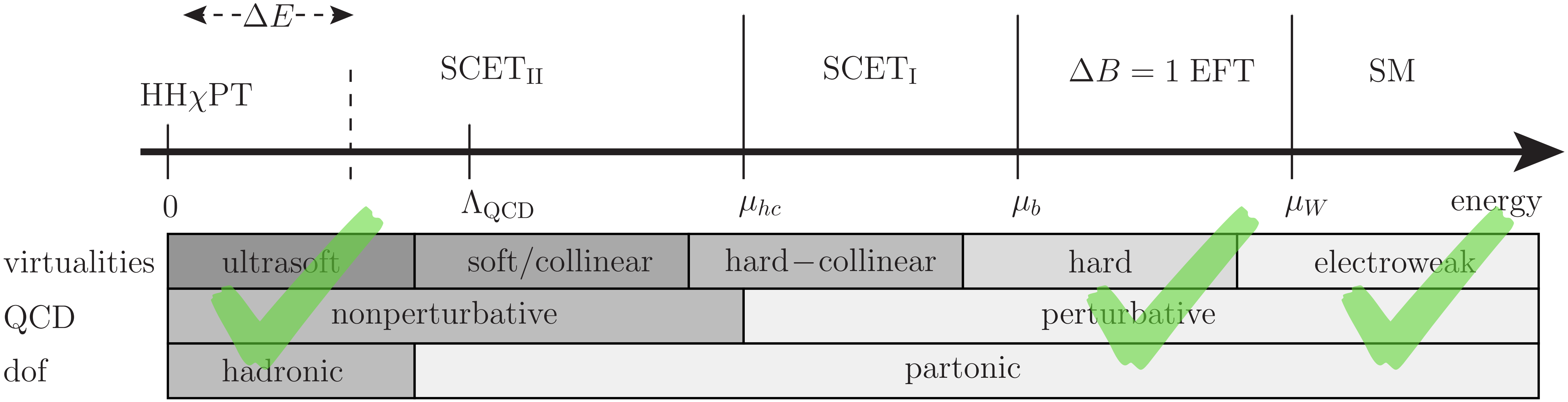
Annihilation and helicity flip can be separated by $r \sim 1/\sqrt{m_b \Lambda_{\text{QCD}}}$

Non-local annihilation

Tower of EFTs …



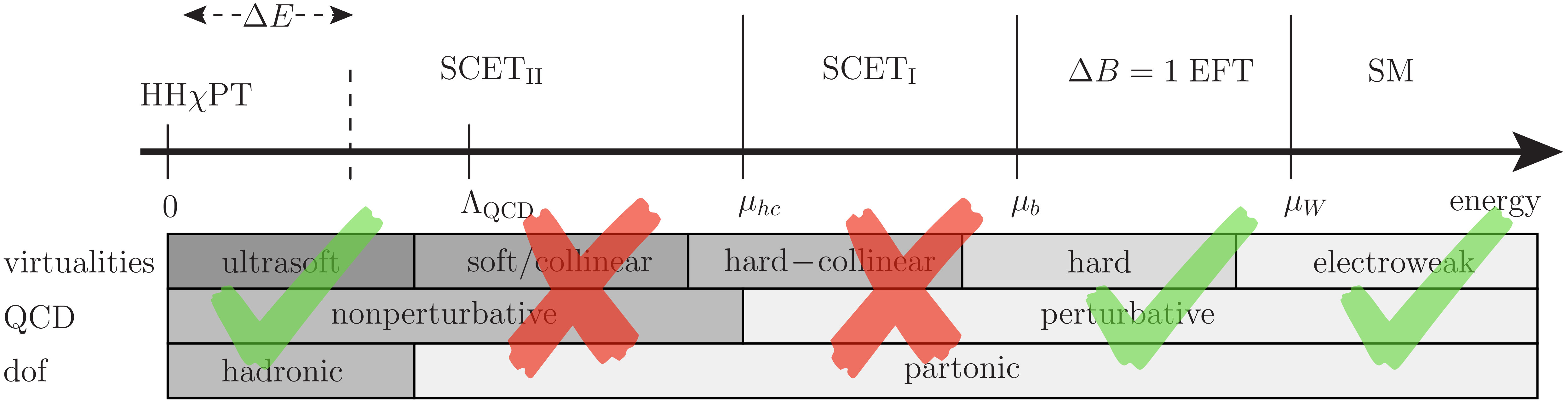
Tower of EFTs ...



We understand quite well the long-distance physics — when mesons look point-like

Short distance physics is also under good control — pQCD works well

Tower of EFTs ...



We understand quite well the long-distance physics — when mesons look point-like

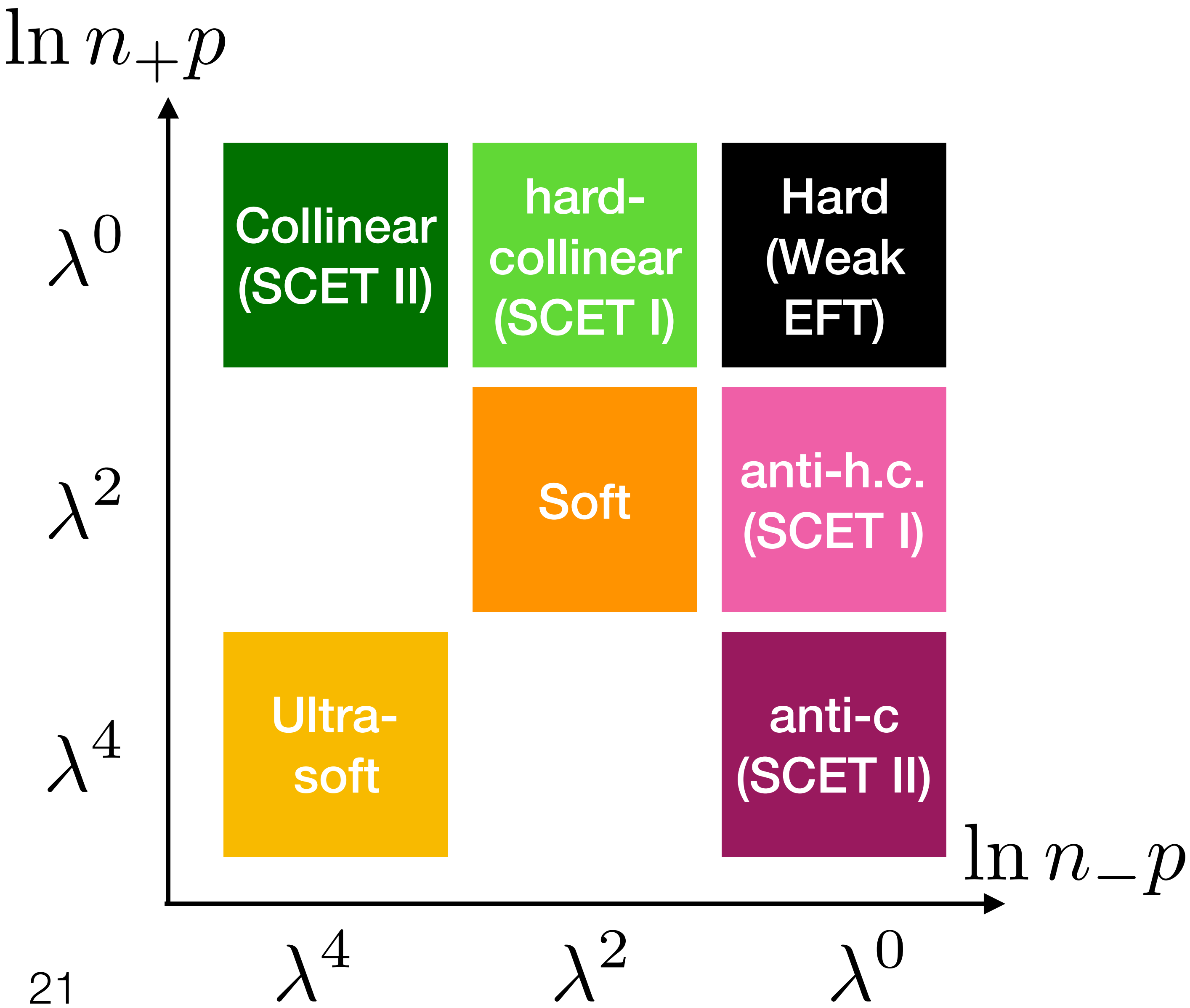
Short distance physics is also under good control — pQCD works well

If we want to understand structure dependent QED corrections we need to also understand intermediate scales

... SCET ...

- Soft-collinear effective field theory is designed to describe long-distance physics associated with energetic particles
- Different modes are represented by different fields

$$n_+^\mu = (1,0,0,1)$$
$$n_-^\mu = (1,0,0,-1)$$
$$n_+^2 = n_-^2 = 0$$
$$n_+ \cdot n_- = 2$$
$$p^\mu = (n_+p)\frac{n_-^\mu}{2} + p_\perp^\mu + (n_-p)\frac{n_+^\mu}{2}$$
$$p^2 = n_+p\, n_-p + p_\perp^2$$

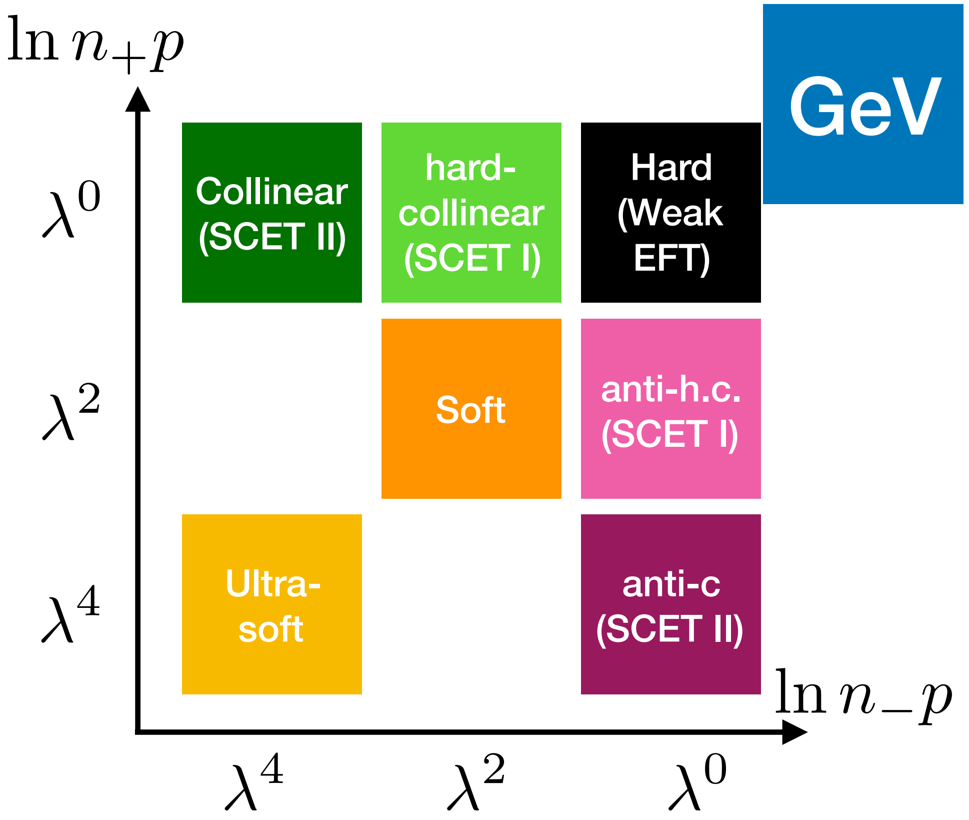


$\ln n_+p$			
λ^0	Collinear (SCET II)	hard-collinear (SCET I)	Hard (Weak EFT)
λ^2		Soft	anti-h.c. (SCET I)
λ^4	Ultra-soft		anti-c (SCET II)
	λ^4	λ^2	λ^0

$\ln n_-p$

GeV

…and modes



$$\lambda^2 = \frac{\Lambda_{\mathrm{QCD}}}{m_b} \sim \frac{m_\mu}{m_b}$$

$$k = (n_+ k, k_\perp, n_- k)$$

mode	relative scaling	absolute scaling	virtuality k^2
hard	$(1,1,1)$	(m_b, m_b, m_b)	m_b^2
hard-collinear	$(1, \lambda, \lambda^2)$	$(m_b, \sqrt{m_b \Lambda_{\mathrm{QCD}}}, \Lambda_{\mathrm{QCD}})$	$m_b \Lambda_{\mathrm{QCD}}$
anti-hard-collinear	$(\lambda^2, \lambda, 1)$	$(\Lambda_{\mathrm{QCD}}, \sqrt{m_b \Lambda_{\mathrm{QCD}}}, m_b)$	$m_b \Lambda_{\mathrm{QCD}}$
collinear	$(1, \lambda^2, \lambda^4)$	$(m_b, m_\mu, m_\mu^2/m_b)$	m_μ^2
anticollinear	$(\lambda^4, \lambda^2, 1)$	$(m_\mu^2/m_b, m_\mu, m_b)$	m_μ^2
soft	$(\lambda^2, \lambda^2, \lambda^2)$	$(\Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}})$	Λ_{QCD}^2

M. Beneke, T. Feldmann, 2003

GeV

...and modes

$$\lambda^2 = \frac{\Lambda_{\rm QCD}}{m_b} \sim \frac{m_\mu}{m_b}$$

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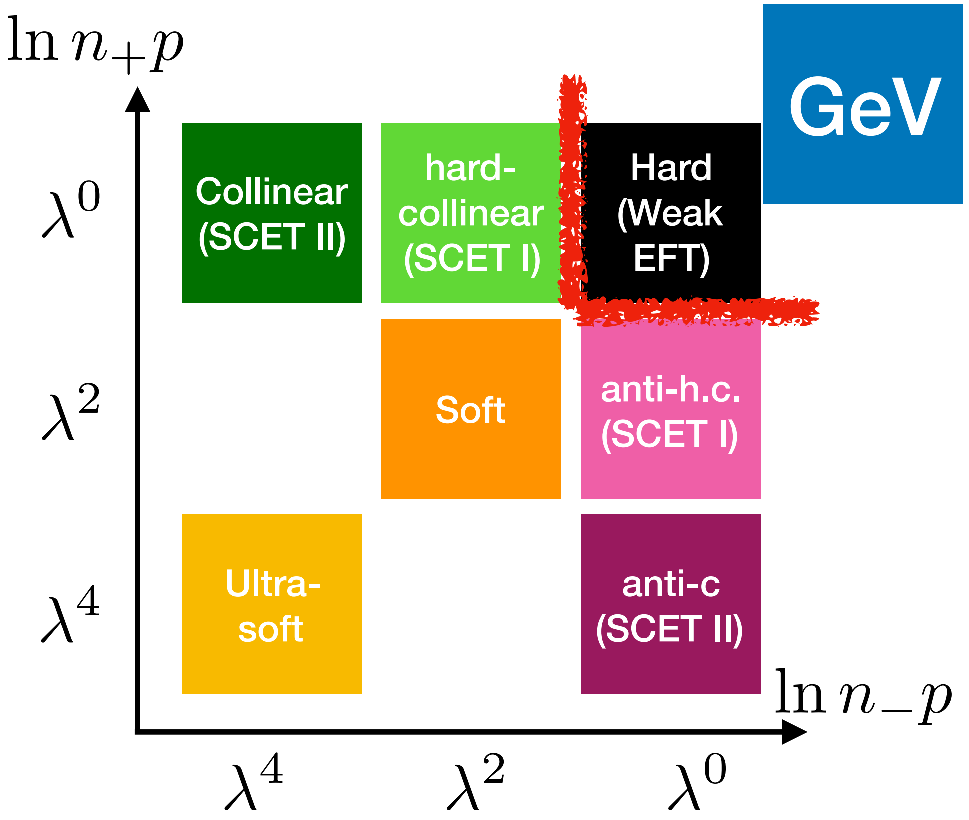

	mode	relative scaling	absolute scaling	virtuality k^2
	hard	$(1,1,1)$	(m_b, m_b, m_b)	m_b^2
SCET I	hard-collinear	$(1,\lambda,\lambda^2)$	$(m_b, \sqrt{m_b\Lambda_{\rm QCD}}, \Lambda_{\rm QCD})$	$m_b\Lambda_{\rm QCD}$
	anti-hard-collinear	$(\lambda^2,\lambda,1)$	$(\Lambda_{\rm QCD}, \sqrt{m_b\Lambda_{\rm QCD}}, m_b)$	$m_b\Lambda_{\rm QCD}$
	collinear	$(1,\lambda^2,\lambda^4)$	$(m_b, m_\mu, m_\mu^2/m_b)$	m_μ^2
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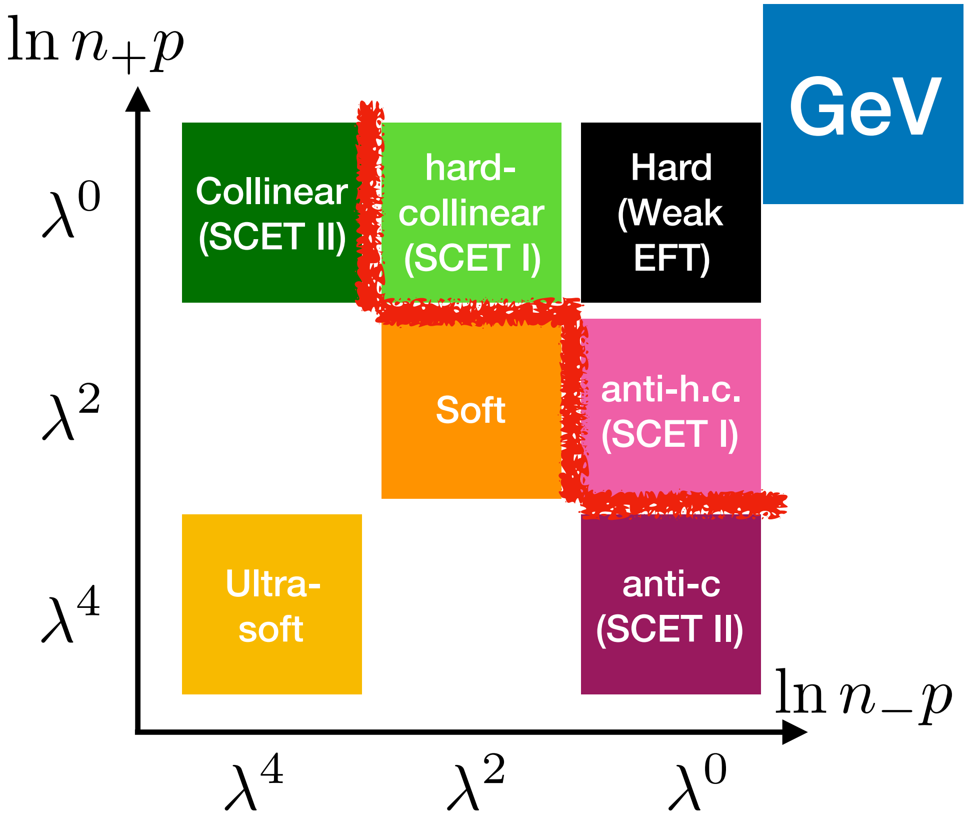

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SCET II	collinear	$(1,\lambda^2,\lambda^4)$	$(m_b, m_\mu, m_\mu^2/m_b)$	m_μ^2
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	soft	$(\lambda^2,\lambda^2,\lambda^2)$	$(\Lambda_{\rm QCD}, \Lambda_{\rm QCD}, \Lambda_{\rm QCD})$	$\Lambda_{\rm QCD}^2$

M. Beneke, T. Feldmann, 2003

SCET beyond LP

- Allows for systematic expansion in λ
- Permits to parameterize non-perturbative physics in terms of matrix elements of specific operators

Two sources of power-suppression

- Subleading Lagrangian interaction (within a single collinear sector)
- Subleading currents (connects different sector after integrating out hard modes)

$$\mathcal{L} = \mathcal{L}_C^{(0)} + \mathcal{L}_{\overline{C}}^{(0)} + \mathcal{L}_C^{(1)} + \mathcal{L}_{\overline{C}}^{(1)} \ldots + J^{(0)} + \ldots$$

$$\mathcal{L}_C^{(0)} = \bar{\xi}_C \left[in_- D + i\not{D}_\perp \frac{1}{in_+ D} i\not{D}_\perp \right] \frac{\not{n}_+}{2} \xi_C$$

Subleading currents

- Constructed from collinear gauge invariant building blocks and soft gauge covariant

- Each field carries extra suppression

	Hard-collinear modes	**Collinear modes**	**Soft modes**
$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i(t_i n_{i+})$	$\sim \lambda$	λ^2	$q_s \sim \lambda^3$
$\mathcal{A}^\mu_{\perp i}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W]$	$\sim \lambda$	λ^2	$F_s^{\mu\nu} \sim \lambda^4$

For renormalization of the operator basis see

M. Beneke, M. Garny, R.S., J. Wang, 2017, 2018, 2019

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Example

LP

$$J_i^{A0} = \chi_i(t_i n_{i+})$$

NLP

$$J_i^{A1} = \partial_{\perp i} \chi_i(t_i n_{i+})$$

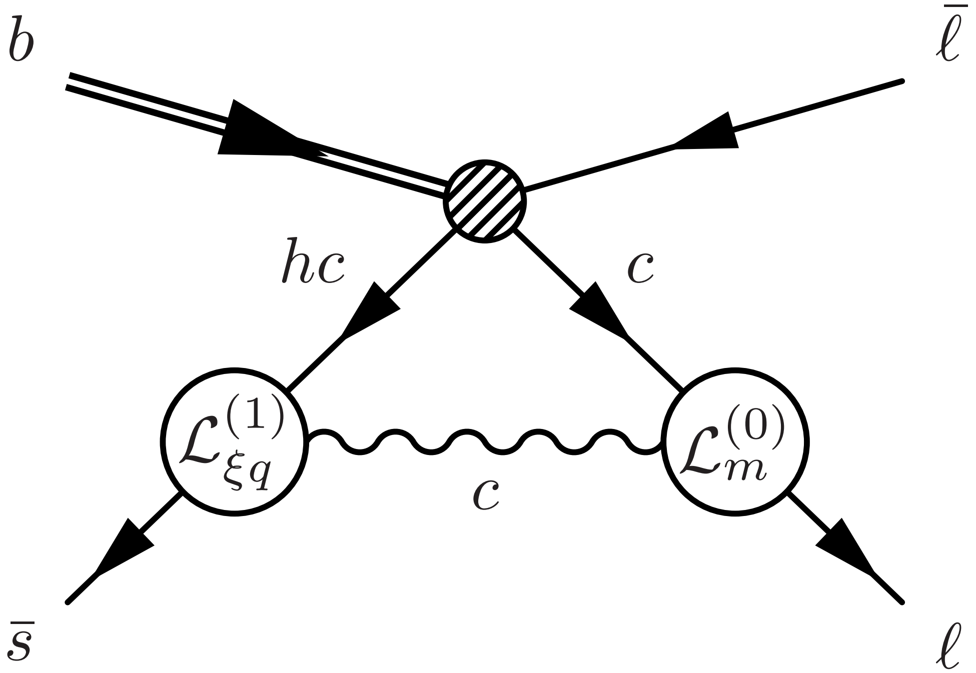
$$J_i^{B1} = \mathcal{A}_{\perp i}(t_i' n_{i+}) \chi_i(t_i n_{i+})$$

For renormalization of the operator basis see *M. Beneke, M. Garny, R.S., J. Wang, 2017, 2018, 2019*

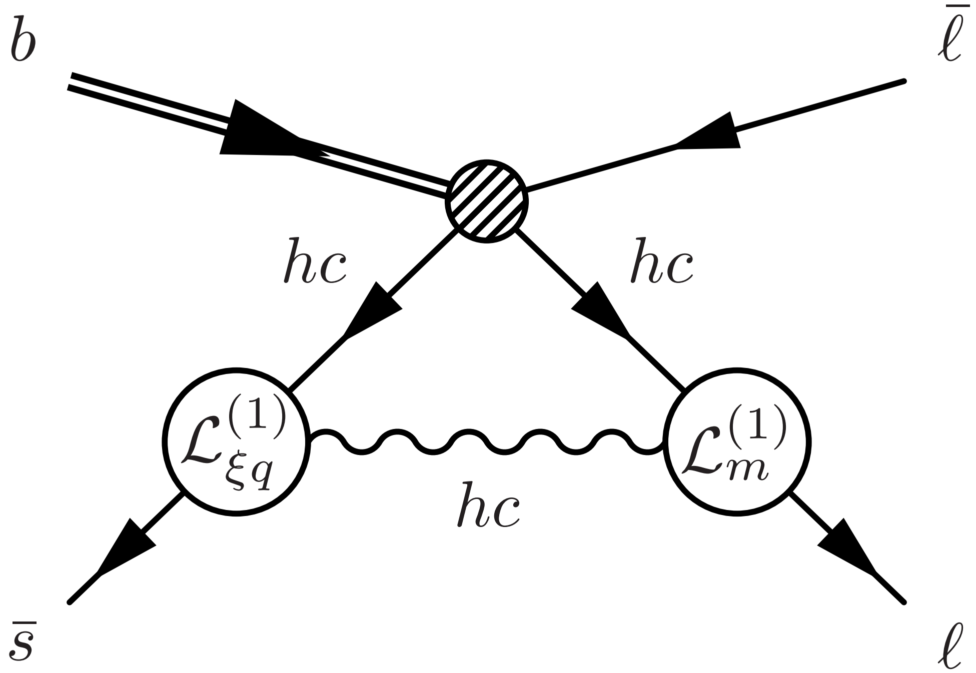
Regions and EFT interpretation

M. Beneke, C. Bobeth, R.S., 2019

Collinear



Hard-Collinear



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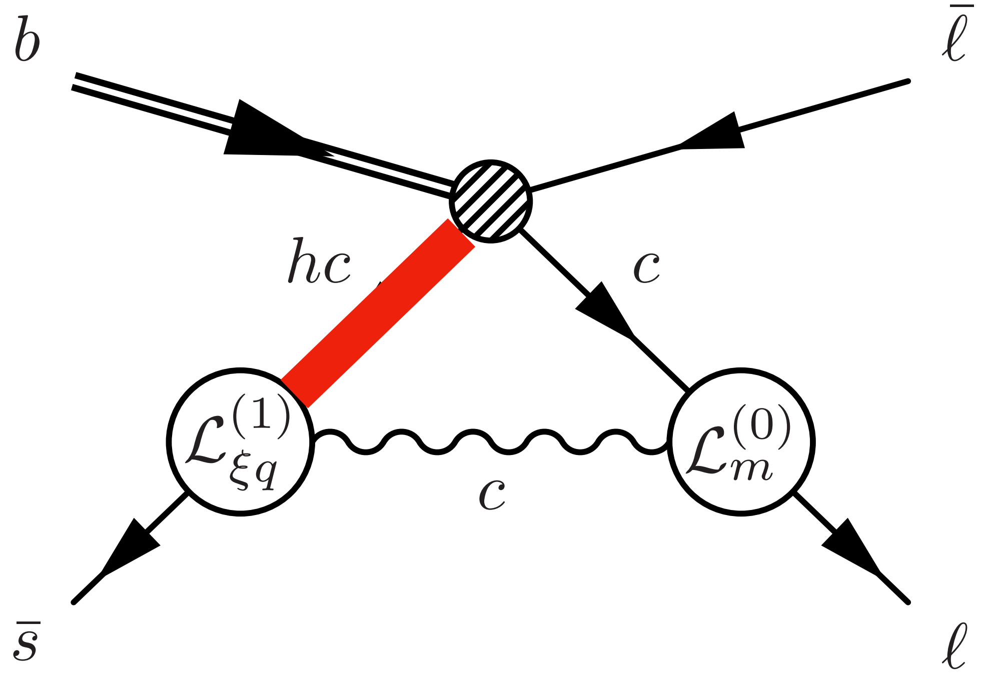
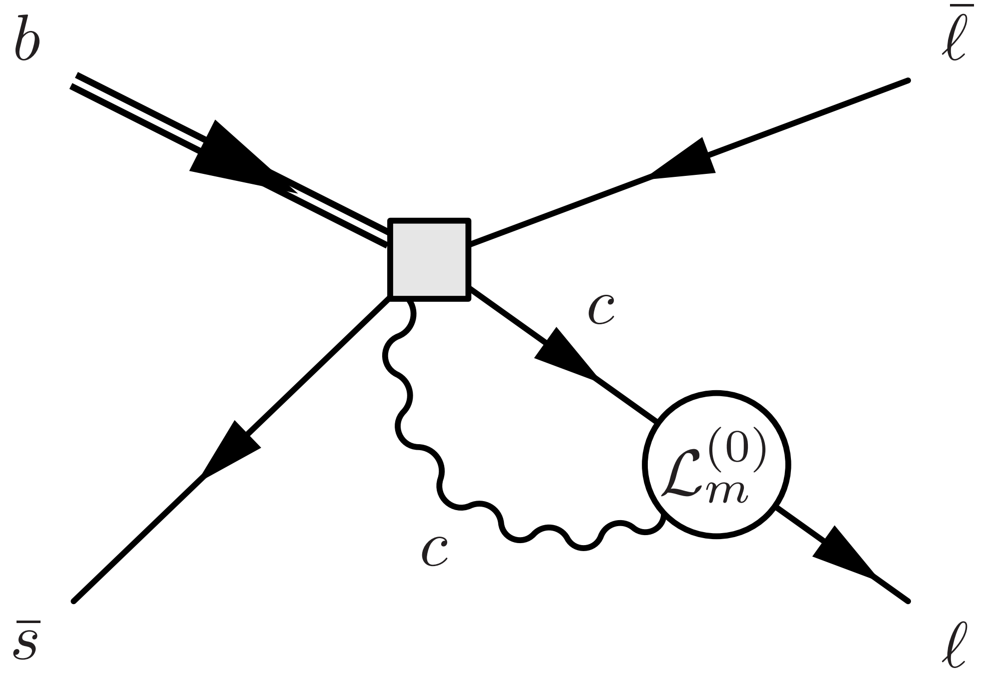
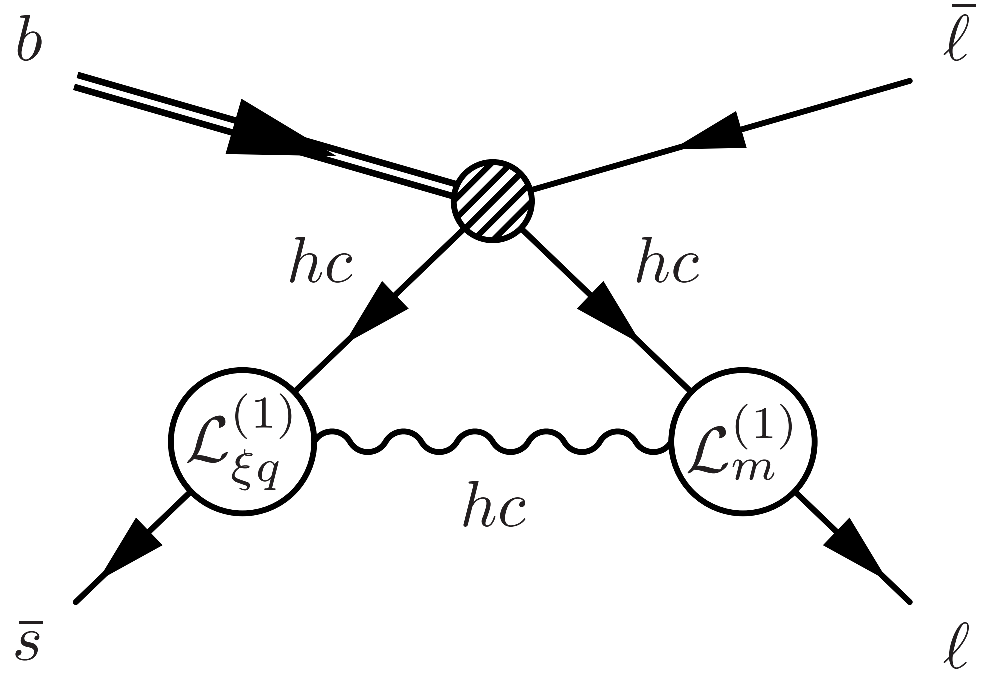
Collinear





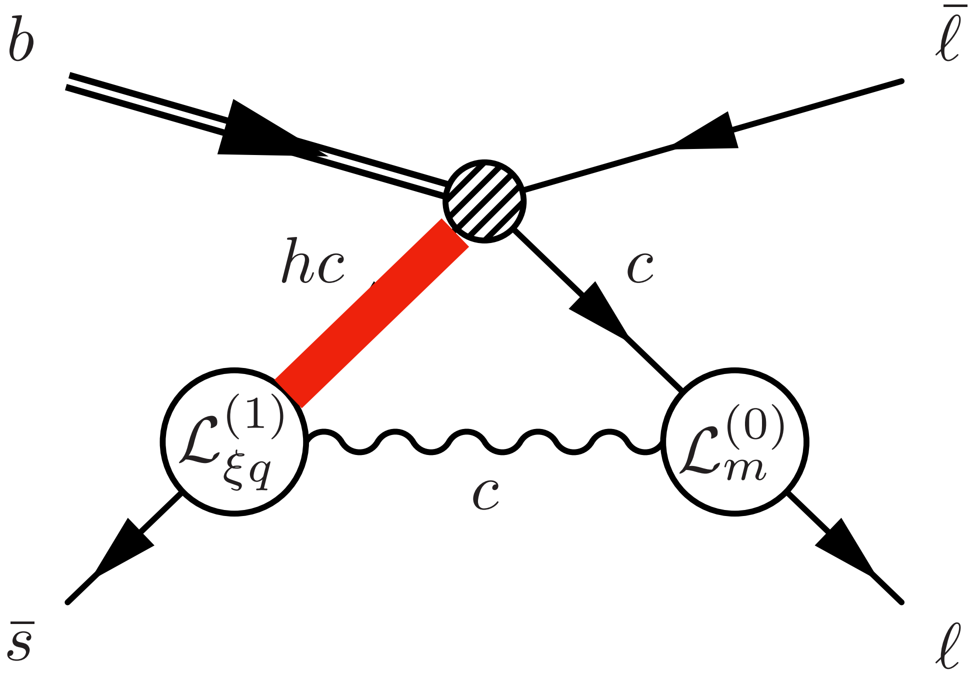
Hard-Collinear

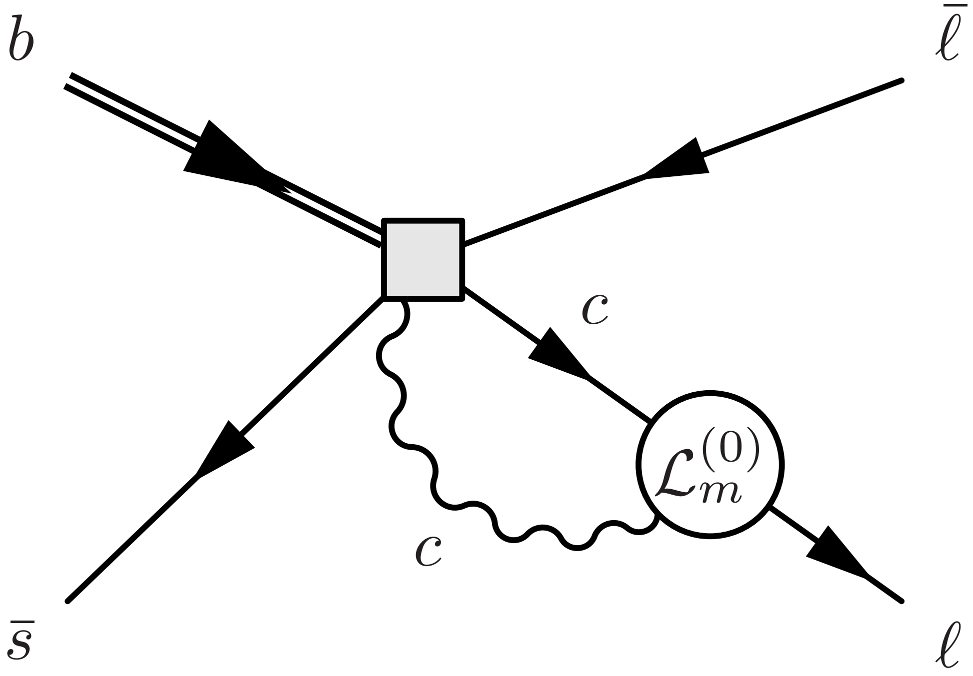


Regions and EFT interpretation

M. Beneke, C. Bobeth, R.S., 2019

Collinear





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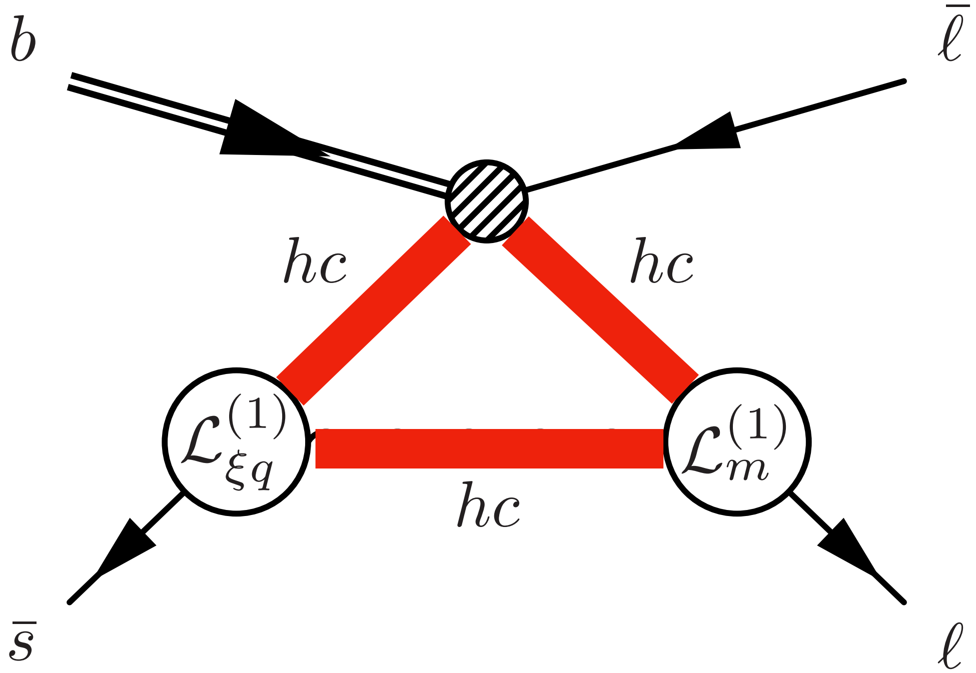
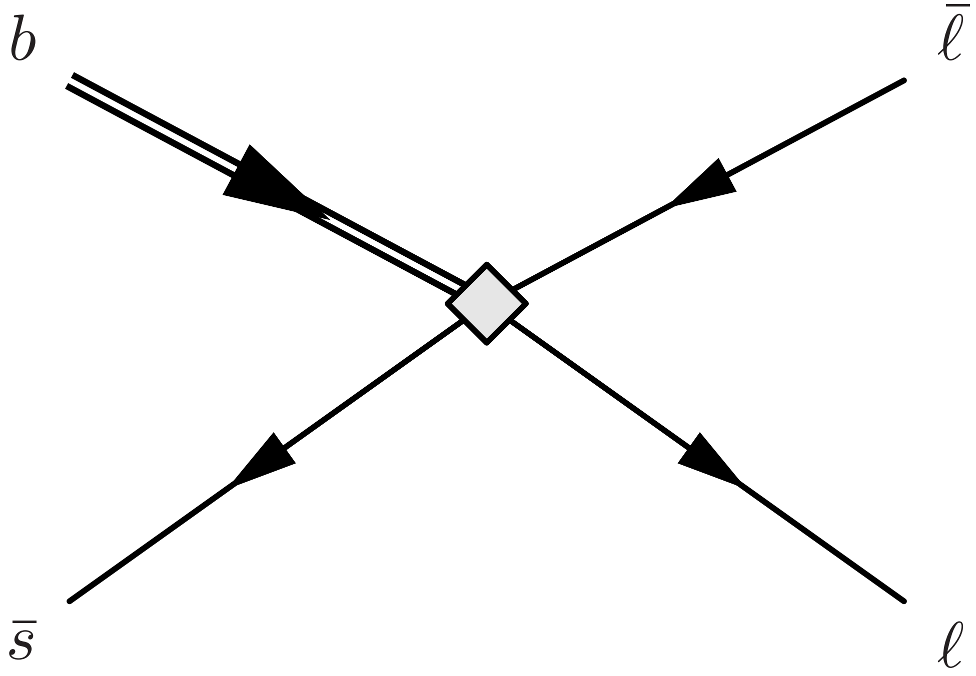
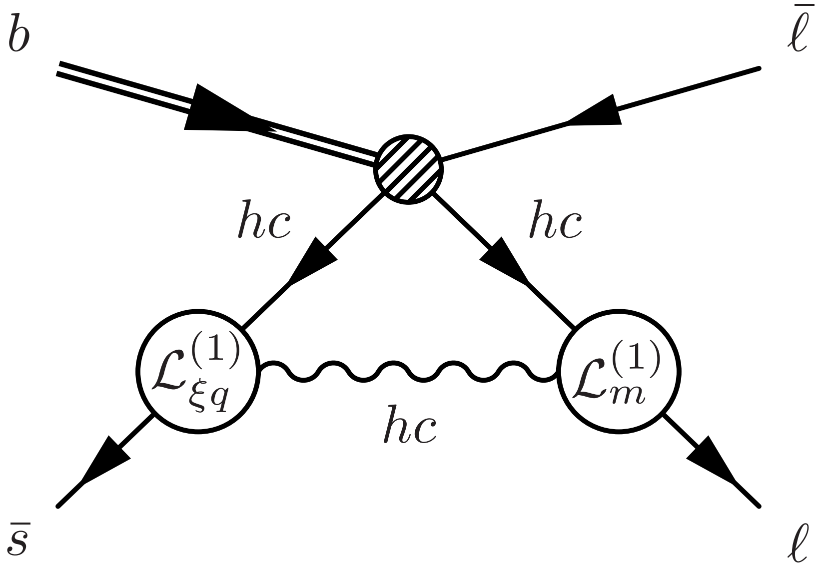


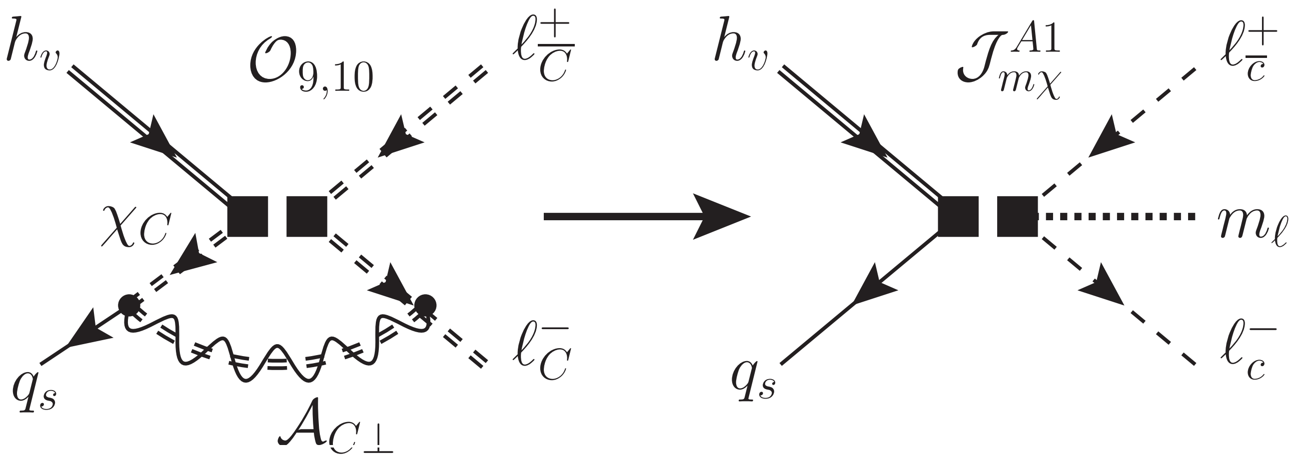


GeV

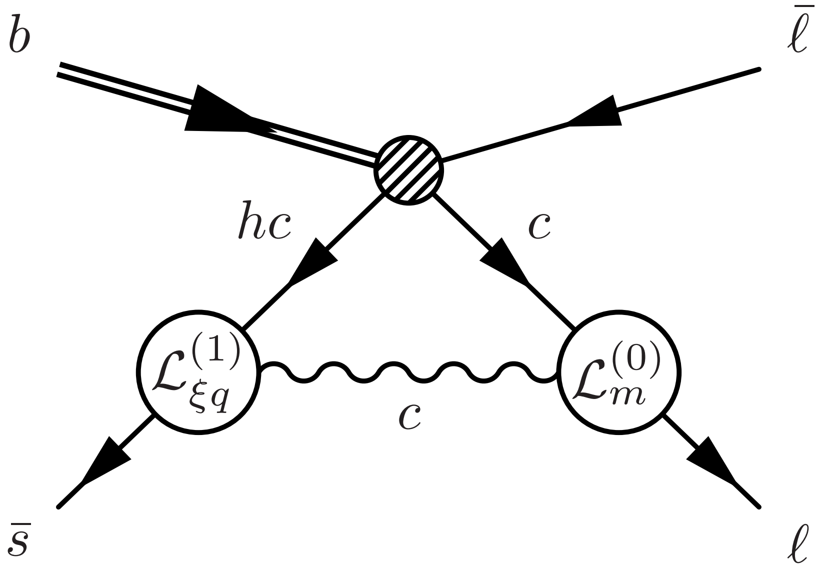
Hard-collinear matching

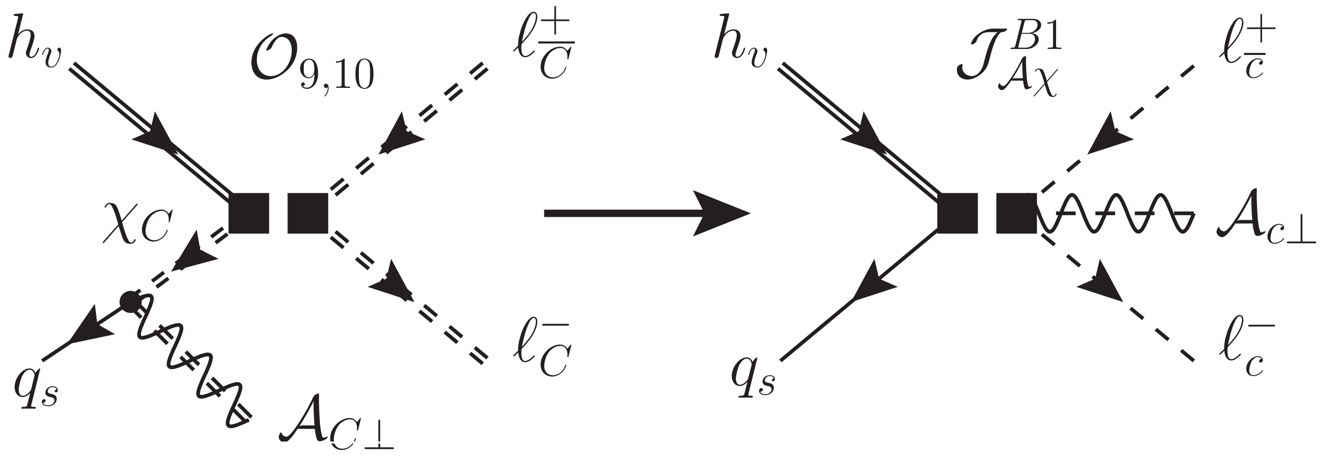
Hard-collinear region ↔ **1-loop matching**



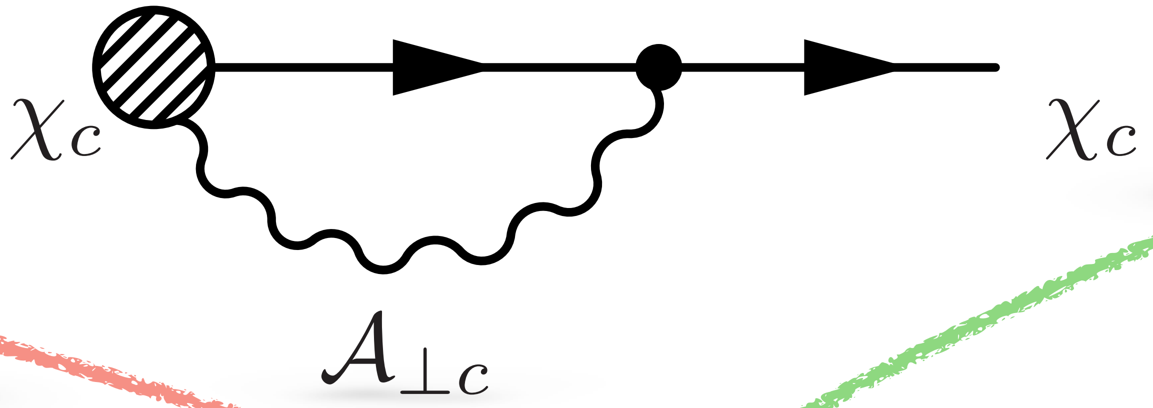


Collinear region ↔ **Tree-level matching**





B to A mixing



B-type current

Single fermion matrix element — A type

No mixing for massless fermions

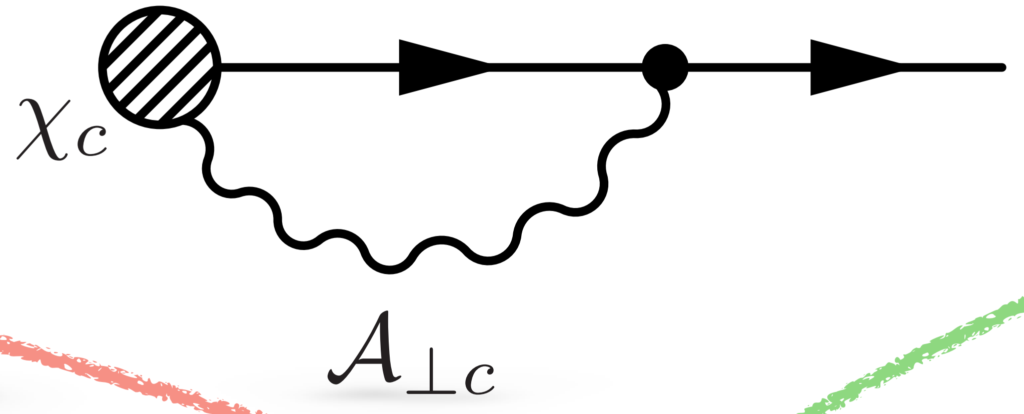
M. Beneke, M. Garny, R.S., J. Wang, 2018

$$n_+p \int \frac{dt}{2\pi}\, e^{-i\overline{w}t\,(n_+p)} \left\langle \ell(p) \middle| \bar{\ell}_c(0)\, \mathcal{A}^{\mu}_{c\perp}(tn_+) \middle| 0 \right\rangle$$

$$= -\frac{\alpha_{\rm em}}{4\pi} Q_\ell\, \overline{w} \left[\frac{1}{\epsilon} + \ln \frac{\mu^2}{\overline{w}\,(m_\ell^2 - p^2 w)} \right] m_\ell\, \bar{u}_c(p) \gamma_\perp^\mu$$

Off-shell to regularize IR divergence

B to A mixing



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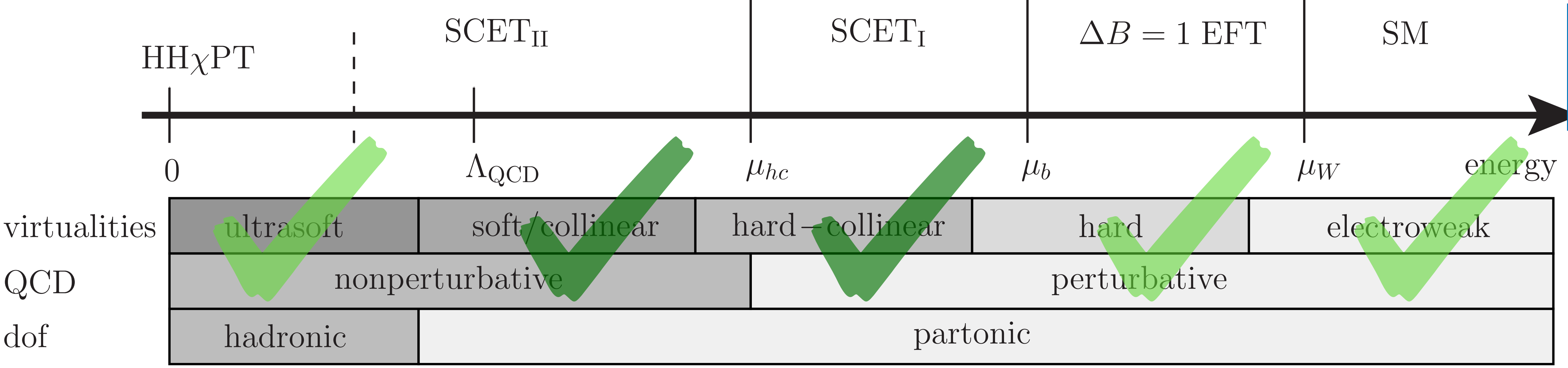
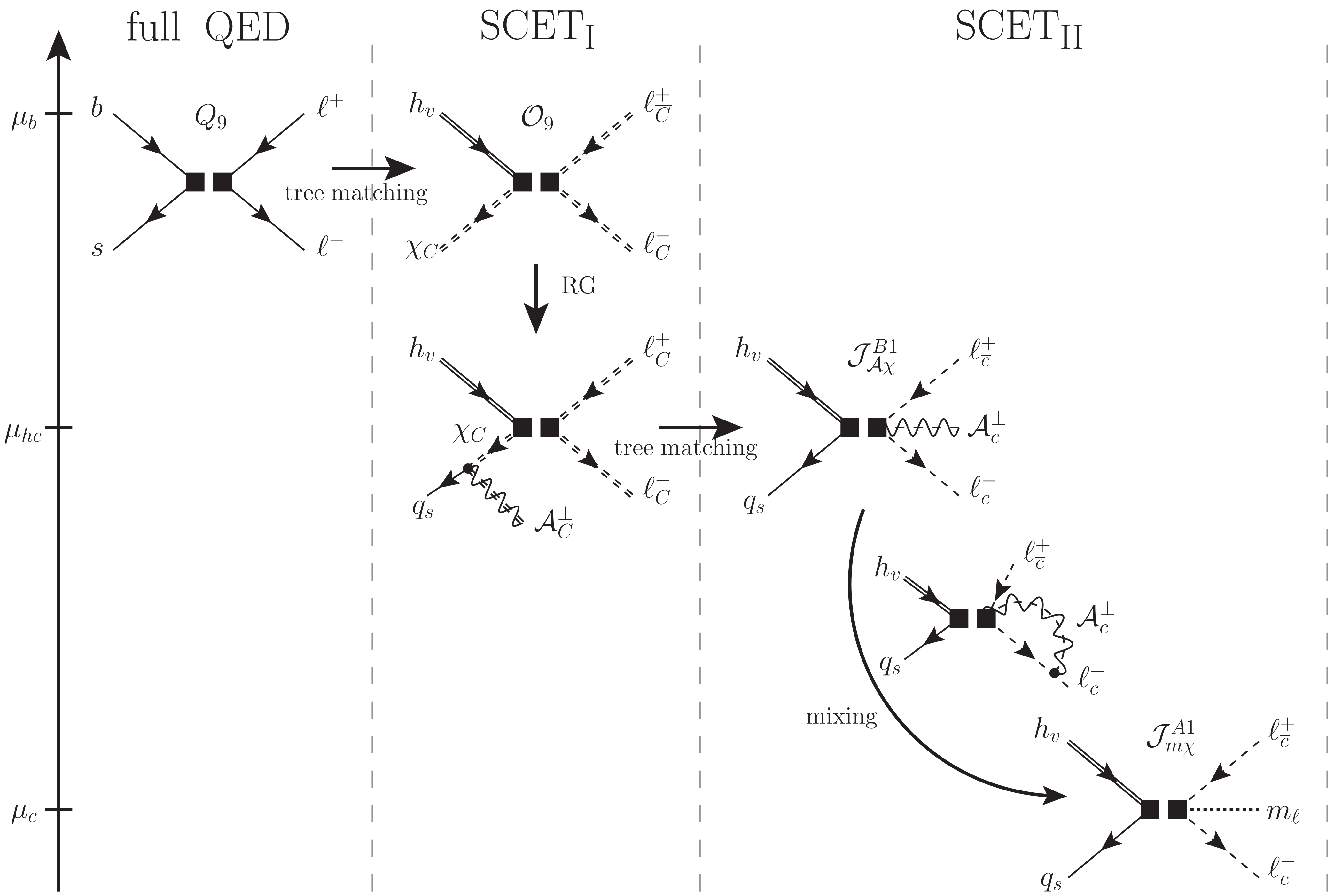
UV pole

Off-shell to regularize IR divergence


HHχPT
SCET$_{\mathrm{II}}$
SCET$_{\mathrm{I}}$
$\Delta B = 1$ EFT
SM
0
Λ_{QCD}
μ_{hc}
μ_b
μ_W
energy
virtualities
ultrasoft
soft/collinear
hard − collinear
hard
electroweak
QCD
nonperturbative
perturbative
dof
hadronic
partonic


full QED
SCET$_{\mathrm{I}}$
SCET$_{\mathrm{II}}$
μ_b
μ_{hc}
μ_c
b
s
Q_9
ℓ^+
ℓ^-
tree matching
h_v
$\mathcal{O}_9$
ℓ^+_C
ℓ^-_C
χ_C
RG
h_v
ℓ^+_C
χ_C
q_s
$\mathcal{A}^\perp_C$
ℓ^-_C
tree matching
h_v
$\mathcal{J}^{B1}_{\mathcal{A}\chi}$
ℓ^+_c
$\mathcal{A}^\perp_c$
ℓ^-_c
q_s
h_v
ℓ^+_c
$\mathcal{A}^\perp_c$
q_s
ℓ^-_c
mixing
h_v
$\mathcal{J}^{A1}_{m\chi}$
ℓ^+_c
m_ℓ
ℓ^-_c
q_s

Results

We computed the structure dependent corrections and performed factorization using soft-collinear effective field theory

$$\overline{\mathrm{Br}}^{(0)}_{s\mu} = 3.677 \cdot 10^{-9} \times \big(1 - 0.0166\, S_9 + 0.0105\, S_7\big) = 3.660 \cdot 10^{-9}$$

$$\overline{\mathrm{Br}}^{(0)}_{d\mu} = 1.031 \cdot 10^{-10} \times \big(1 - 0.0155\, S_9 + 0.0103\, S_7\big) = 1.027 \cdot 10^{-10}$$

$S_{9,7}$ contain effects of the resummation, i.e. higher order QED and QCD corrections

Neglecting QED resummation, but using QCD resummation $S_9 = S_7$

μ_{hc} [GeV]	S_9 QCD+QED	only QCD
1.0	0.815	0.817
1.5	0.815	0.817
2.0	0.769	0.769

QCD resummation is important! QED can be safely neglected

M. Beneke, C. Bobeth, R.S., 2019

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[GeV]	QCD+QED	only QCD

Theory has to be prepared for the upcoming era of precision flavor physics

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M. Beneke, C. Bobeth, R.S., 2019

Higgs physics

LHC can perform more and more accurate measurements

Higgs sector of the standard model is the best example

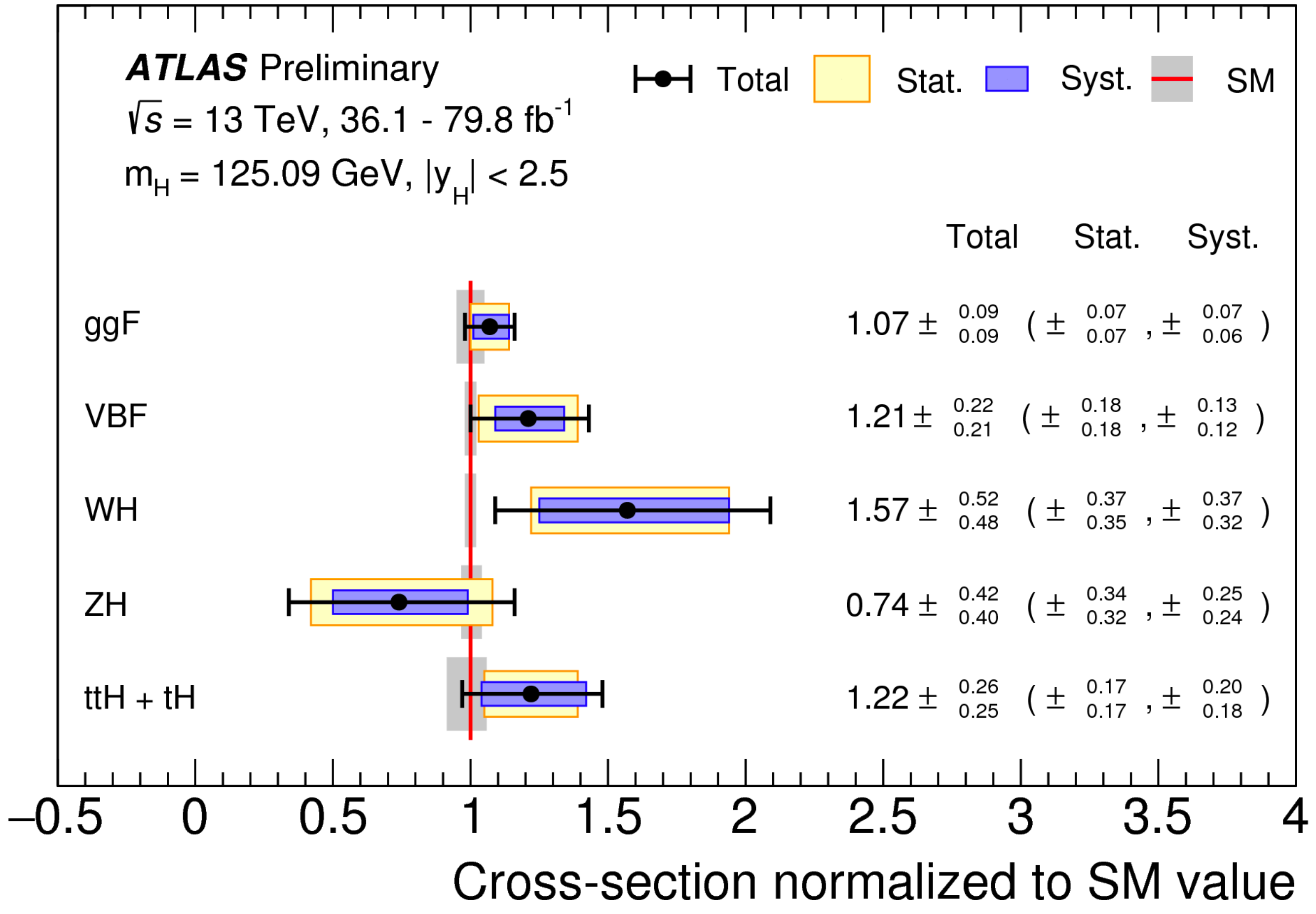


Image: ATLAS Collaboration/CERN

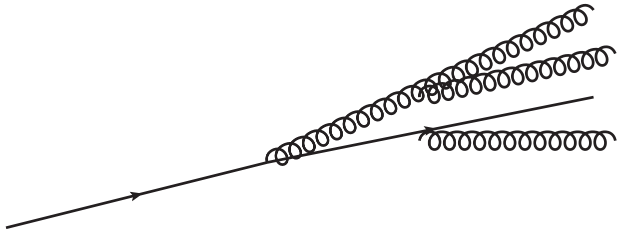
Very accurate measurements at the LHC demand precise theoretical predictions – large QCD corrections must be under control

In recent years, many processes have been computed to two or even three loop order

But sometimes perturbation theory breaks down

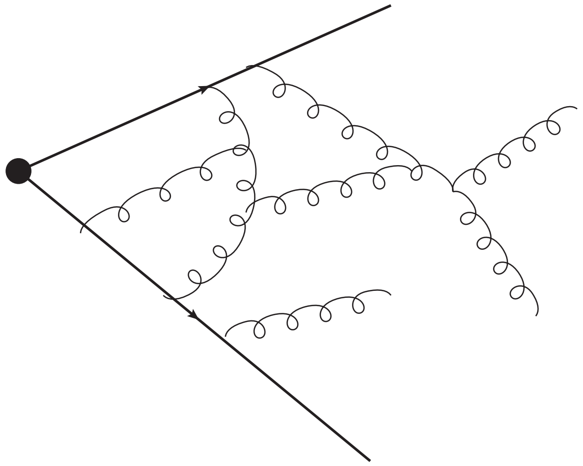
A quest for precision in collider physics

Collinear



$$p_i \cdot p_j \ll Q^2$$

Soft



$$E_s \ll Q$$

QCD singular limits lead to the appearance of large logarithms of a ratio of different scales

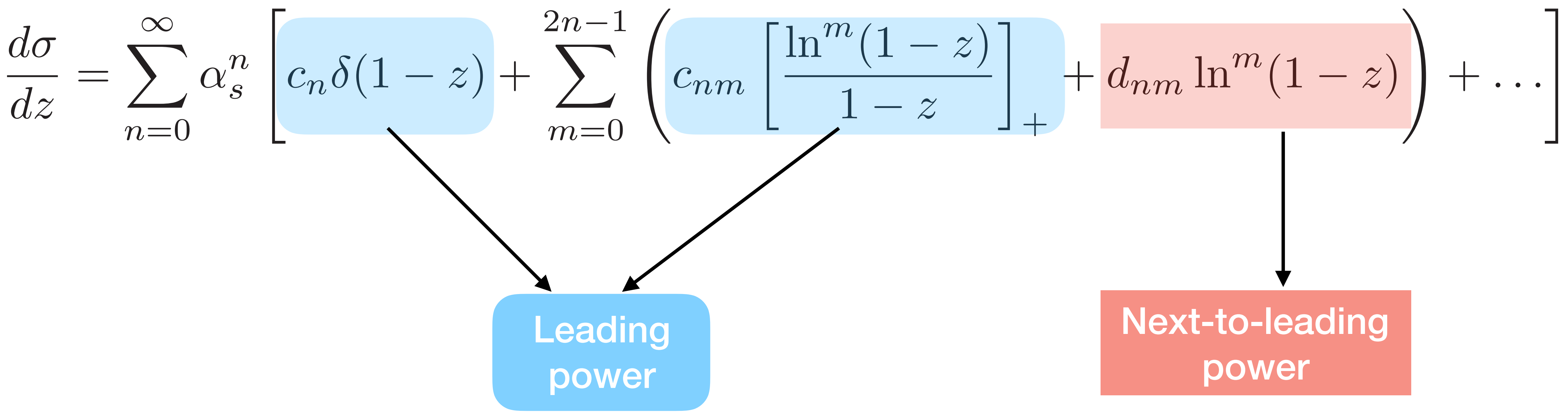
Just like in the QED case, the expansion parameter is not $\frac{\alpha_s}{4\pi}$

but $\frac{\alpha_s}{4\pi} \ln^2 \frac{Q^2}{\mu_{c,s}^2}$

Threshold expansion

$z = \frac{m_H^2}{\hat{s}}$ **partonic threshold variable, for Higgs production cross-section**

In the limit $z \to 1$ **logs of 1-z must be resummed**



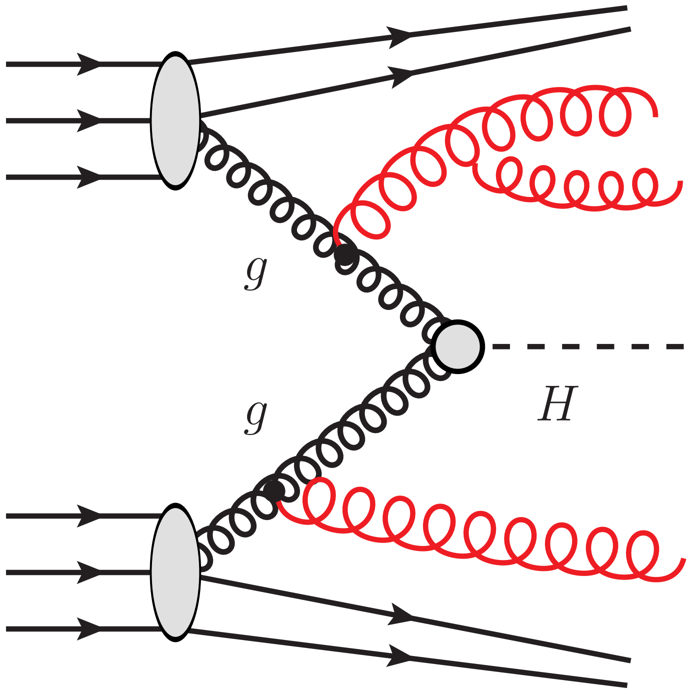
$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

Leading power

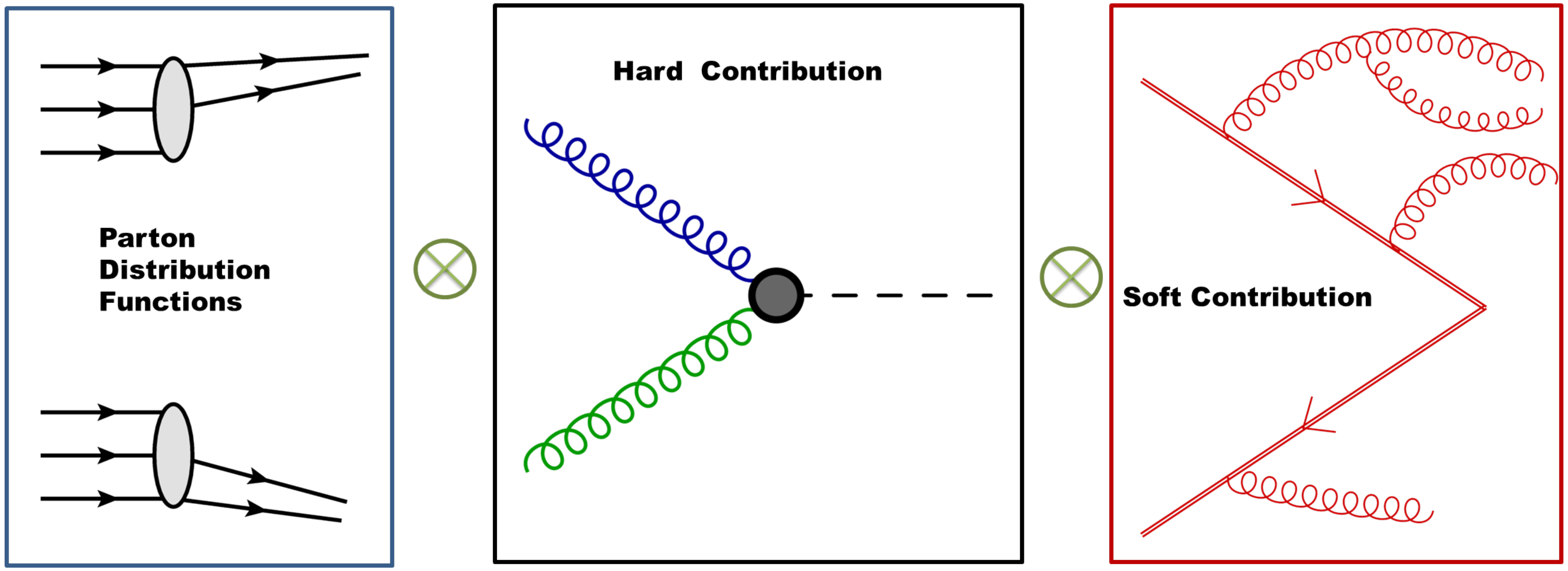
Next-to-leading power

Factorization

Similar structure at NLP but more complicated — new operators, LL is generated by operator mixing like in the case QED corrections



To resum large logarithmic corrections we must separate the scales — this is known as hard-collinear factorization



Resummed cross-section at NLP

$$\Delta^{\mathrm{LL}}_{\mathrm{NLP}}(z,\mu) = \left[\frac{\beta\big(\alpha_s(\mu)\big)}{\alpha_s^2(\mu)}\,\frac{\alpha_s^2(\mu_t)}{\beta\big(\alpha_s(\mu_t)\big)}\right]^2 C_t^2(m_t,\mu_t)$$
$$\times\ \exp\left[4S^{\mathrm{LL}}(\mu_h,\mu) - 4S^{\mathrm{LL}}(\mu_s,\mu)\right]\ \frac{-8C_A}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}\,\theta(1-z)$$

$$\Delta(z) = \frac{\hat{\sigma}(z)}{z}$$

Fixed order expansion in perfect agreement with previously known results

$$\begin{aligned}
\Delta^{\mathrm{LL}}_{\mathrm{NLP}}(z,\mu) = &-\theta(1-z)\bigg\{4C_A\frac{\alpha_s}{\pi}\Big[\ln(1-z) - L_\mu\Big] \\
&+ 8C_A^2\left(\frac{\alpha_s}{\pi}\right)^2\Big[\ln^3(1-z) - 3L_\mu\ln^2(1-z) + 2L_\mu^2\ln(1-z)\Big] \\
&+ 8C_A^3\left(\frac{\alpha_s}{\pi}\right)^3\Big[\ln^5(1-z) - 5L_\mu\ln^4(1-z) + 8L_\mu^2\ln^3(1-z) - 4L_\mu^3\ln^2(1-z)\Big] \\
&+ \frac{16}{3}C_A^4\left(\frac{\alpha_s}{\pi}\right)^4\Big[\ln^7(1-z) - 7L_\mu\ln^6(1-z) + 18L_\mu^2\ln^5(1-z) - 20L_\mu^3\ln^4(1-z) \\
&\qquad + 8L_\mu^4\ln^3(1-z)\Big] \\
&+ \frac{8}{3}C_A^5\left(\frac{\alpha_s}{\pi}\right)^5\Big[\ln^9(1-z) - 9L_\mu\ln^8(1-z) + 32L_\mu^2\ln^7(1-z) - 56L_\mu^3\ln^6(1-z) \\
&\qquad + 48L_\mu^4\ln^5(1-z) - 16L_\mu^5\ln^4(1-z)\Big]\bigg\} + \mathcal{O}(\alpha_s^6\times(\log)^{11})
\end{aligned}$$

Results

$$\text{A)} \quad S_{\text{NLP}}\big(m_H(1-z),\mu_s\big) = 0, \qquad S^{\text{ad}}_{x0}\big(m_H(1-z),\mu_s\big) = 1,$$

$$\text{B)} \quad S_{\text{NLP}}\big(m_H(1-z),\mu_s\big) = -4C_A\frac{\alpha_s(\mu_s)}{2\pi}\ln\frac{m_H^2(1-z)^2}{\mu_s^2}, \qquad S^{\text{ad}}_{x0}\big(m_H(1-z),\mu_s\big) = 1,$$

σ (pb)	$\mu_s = \mu_s^{\text{dyn}}$	
	$\mu_h^2 = m_H^2$	$\mu_h^2 = -m_H^2$
$\sigma_{\text{LP}}^{\text{NNLL}}$	24.12	28.04
$\sigma_{\text{LP}}^{\text{NNLO}}$	28.93	
$\sigma_{\text{LP}}^{\text{N}^3\text{LO}}$	29.24	
$\sigma_{\text{NLP}}^{\text{LL}}$ (A)	7.18	12.76
$\sigma_{\text{NLP}}^{\text{LL}}$ (B)	8.82	15.68
$\sigma_{\text{non LP}}^{\text{NNLO}}$	11.90	
$\sigma_{\text{non LP}}^{\text{N}^3\text{LO}}$	16.27	
$\sigma_{\text{LP}}^{\text{NNLL}} + \sigma_{\text{NLP}}^{\text{LL}}$ (A)	31.30	40.80
$\sigma_{\text{LP}}^{\text{NNLL}} + \sigma_{\text{NLP}}^{\text{LL}}$ (B)	32.94	43.72
σ^{NNLO}	40.82	
$\sigma^{\text{N}^3\text{LO}}$	45.52	

Resummation at NLP gives seizable corrections

M. Beneke, M. Garny, S. Jaskiewicz, R.S., L. Vernazza, J. Wang, 2019

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M. Beneke, M. Garny, S. Jaskiewicz, R.S., L. Vernazza, J. Wang, 2019

Systematic investigation of NLP effects has started recently — many conceptual problems still need to be solved

Dark Matter

Precise theoretical predictions are not limited to the Standard Model

The same formalism that we use to compute corrections in the atomic physics can help us to better understand Dark Matter

We consider a fermionic triplet in addition to the SM (Wino-like DM)

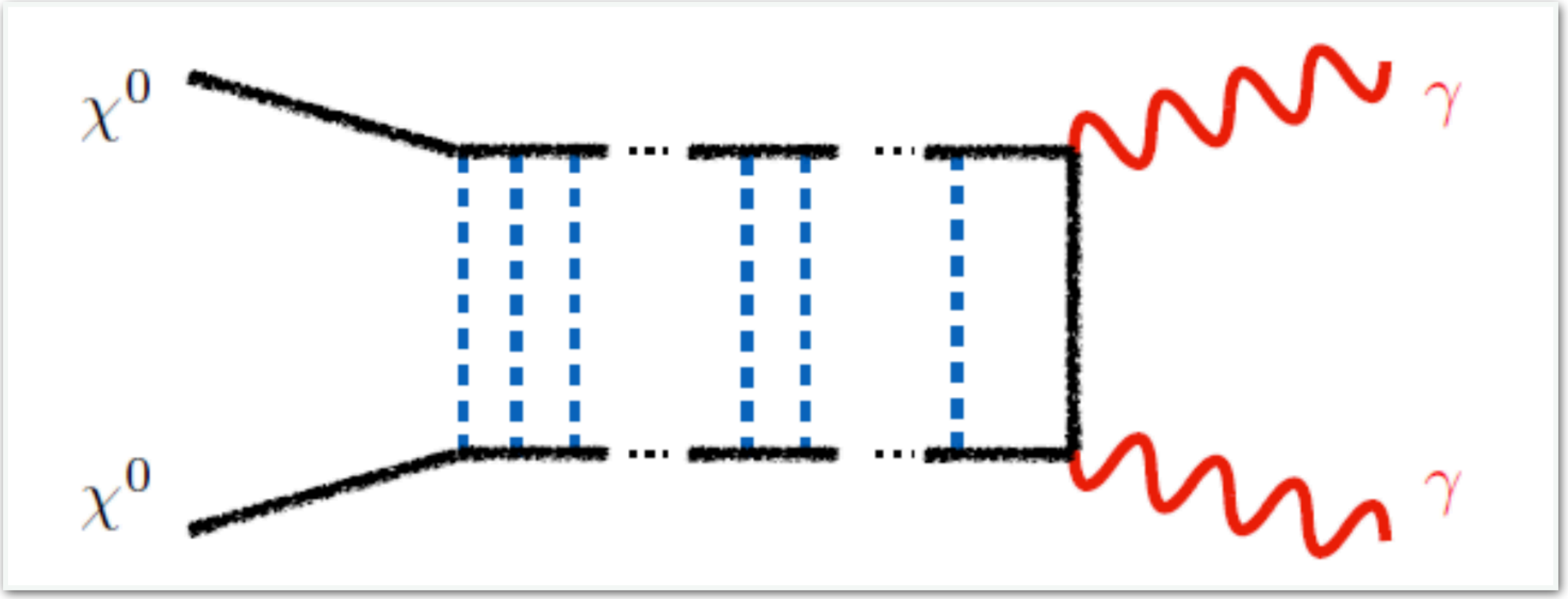
$$\mathcal{L}_{\mathrm{DM}} = \frac{1}{2}\bar{\chi}\left(i\gamma^{\mu}D_{\mu} - m_{\chi}\right)\chi$$

Simple and attractive model, gives observed relic density for

$$m_{\chi} \sim 2.8\mathrm{TeV}$$

How good are tree-level predictions?

Sommerfeld effect



$$\left(\alpha_2 \frac{m_\chi}{m_W}\right)^n$$

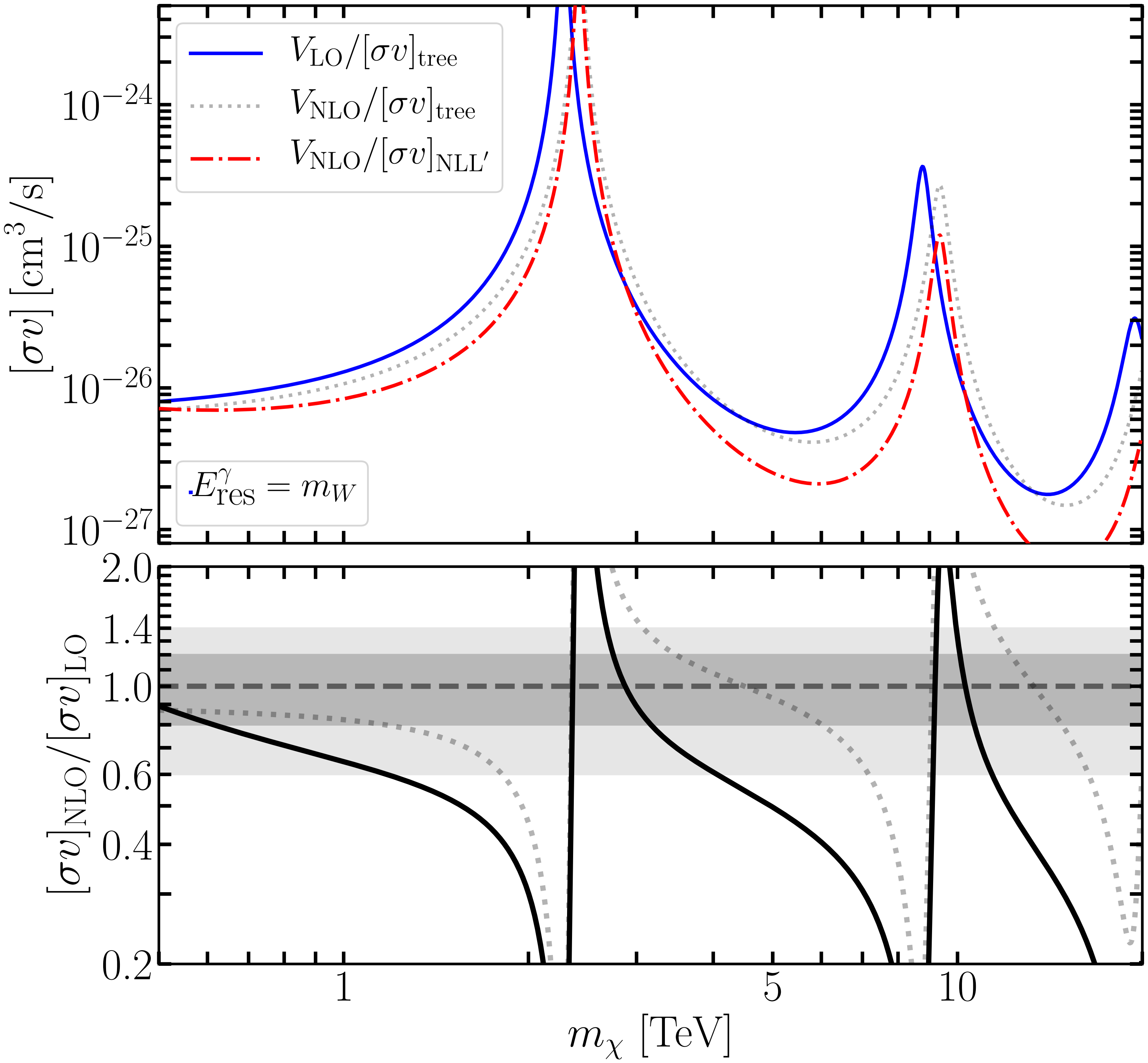
Ladder diagram with n rungs

Non-relativistic scattering — we need to solve Schrödinger equation

$$\alpha_2 \frac{m_\chi}{m_W} \sim 1$$

$$V_{\rm LO}(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2 \,\frac{e^{-m_W r}}{r} \\ -\sqrt{2}\,\alpha_2 \,\frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2\, c_W^2 \,\frac{e^{-m_Z r}}{r} \end{pmatrix}$$

Annihilation cross-section



$\chi^0\chi^0 \to \gamma + X$

Finite range $\sim 1/m_W$ cuts off “too large” states

$r \sim n^2/(m_\chi \alpha_2)$

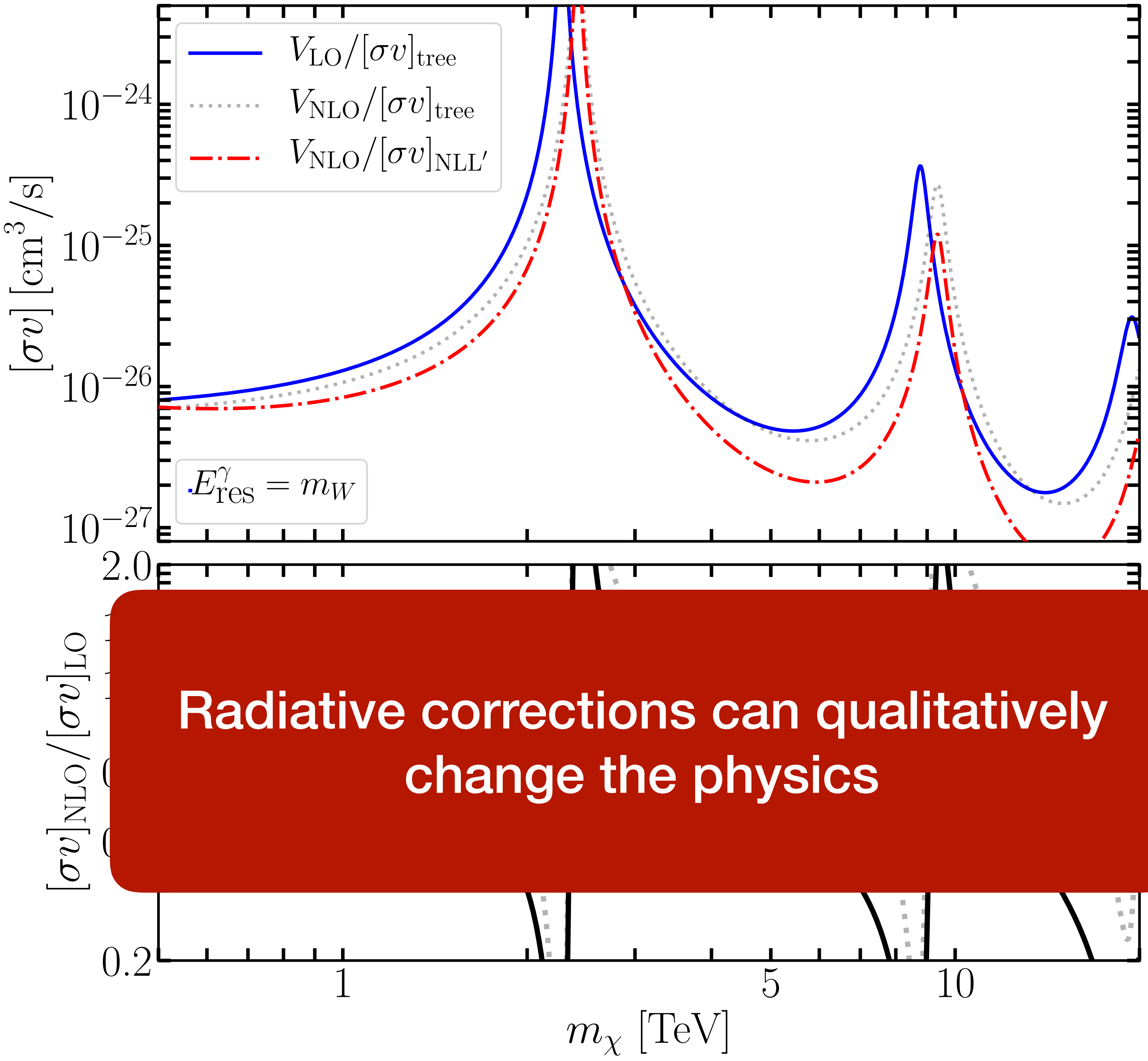
So

$$n^2_{\rm max} \sim \frac{m_\chi}{m_W}\alpha_2$$

As DM mass grows, new bound states cross the threshold

M. Beneke, R.S., K. Urban, 2019

Annihilation cross-section



Radiative corrections can qualitatively change the physics

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Tasks for the future

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Radiative corrections can qualitatively change the physics, e.g. for Dark Matter

Theory of bound electron should be further improved!

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We can achieve all this by systematically analyzing higher order effects in the framework of modern EFTs such as SCET and PNREFT

Summary

- Precision is the key to open the doors for New Physics
- Without higher order effects we could not understand Standard Model and we will not be able to find New Physics
- Effective field theories, such as SCET and PNREFT, offer systematic approach to understand complicated multi-scale problems
- Studies of SM through EFT are challenging but allow for ample opportunities

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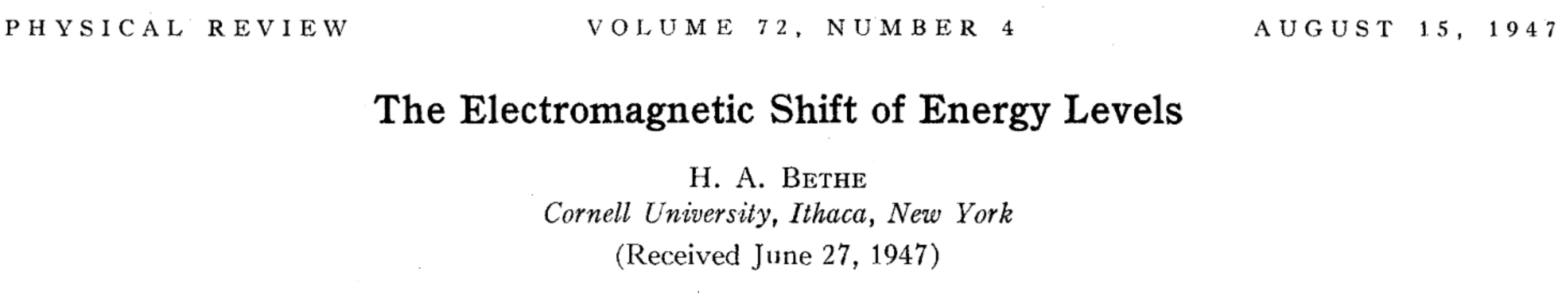
Thank you!

Auxiliary Slides

Lamb shift

Over 70 years old, one of the earliest radiative correction ever computed

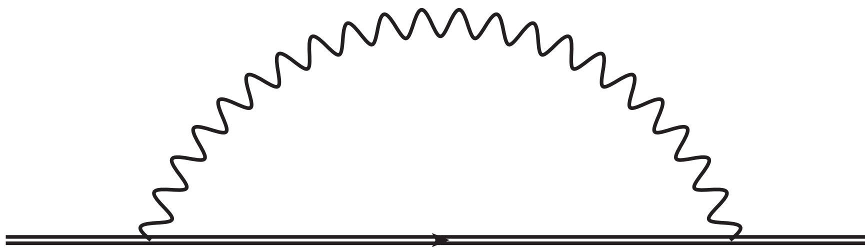


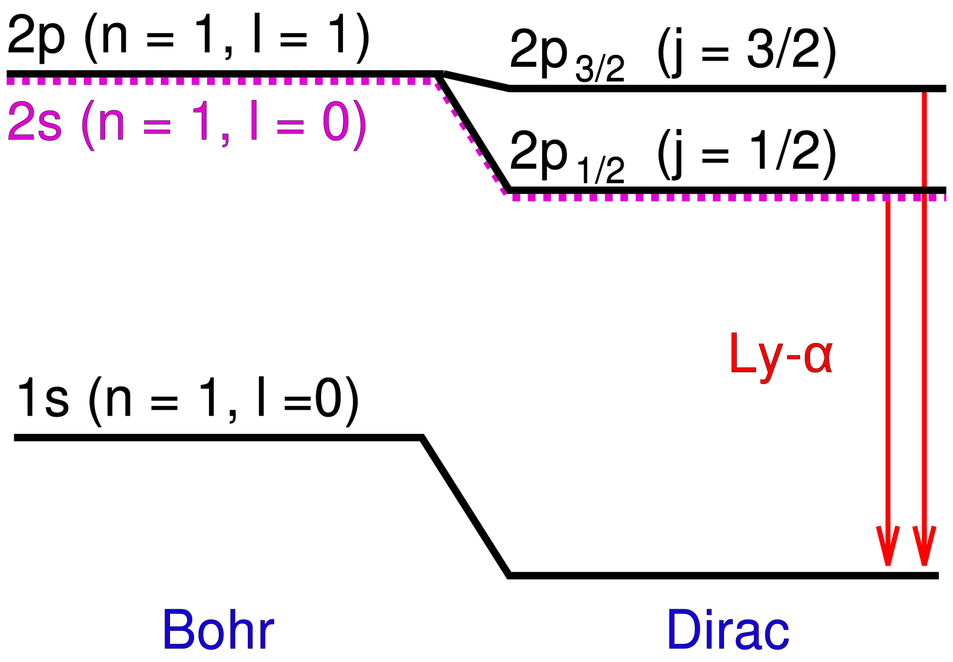
PHYSICAL REVIEW VOLUME 72, NUMBER 4 AUGUST 15, 1947

The Electromagnetic Shift of Energy Levels

H. A. BETHE
Cornell University, Ithaca, New York

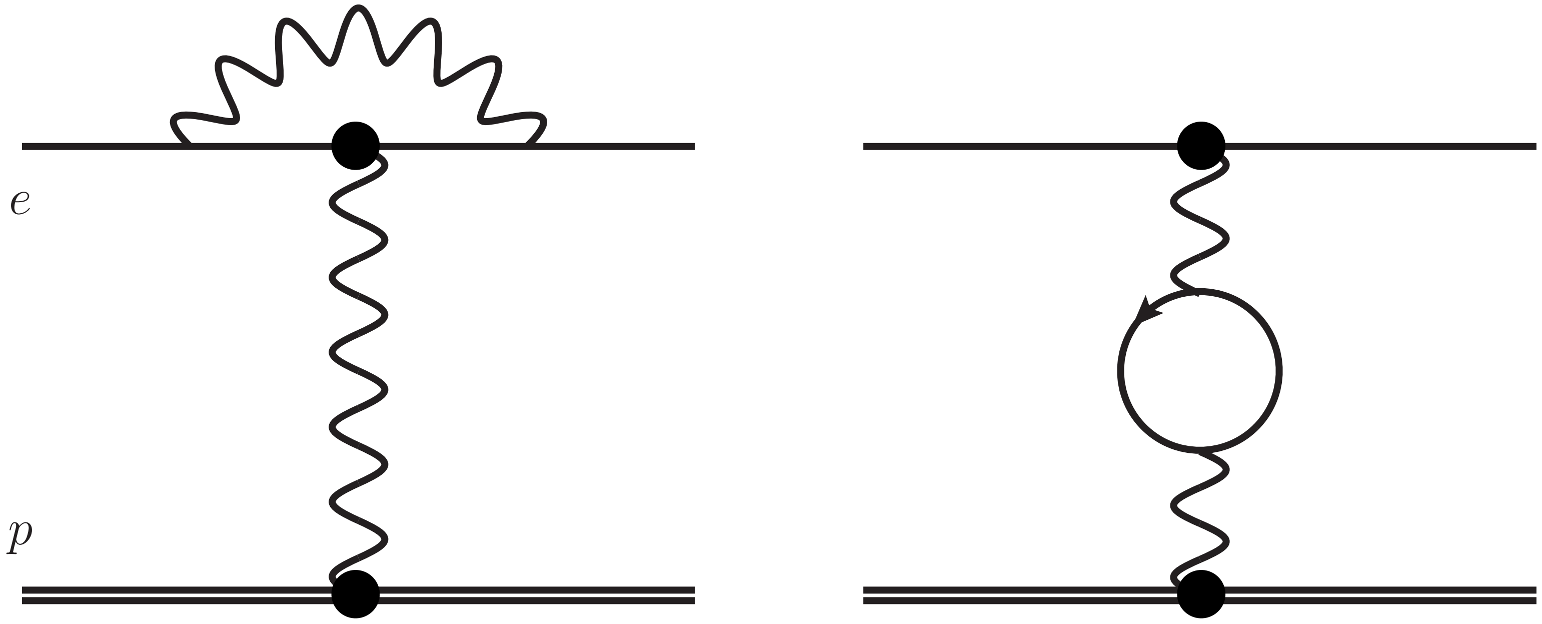
(Received June 27, 1947)





$$\Delta E_{2S-2P} \sim \frac{\alpha}{\pi}(Z\alpha)^4 \ln(Z\alpha) \sim 1057\ \mathrm{MHz}$$

Radiative corrections



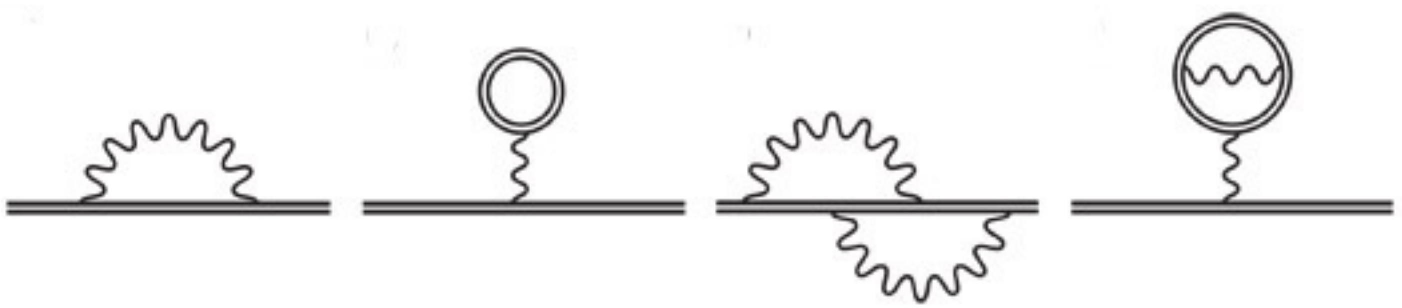
Form-factor — electron no longer point-like.

Binding is weaker — positive correction to the energy

Vacuum polarization — electron at distance $r_B \sim 1/m_e\alpha$

sees larger charge of the proton than electron at infinity. Binding is stronger — negative correction to the binding energy

Higher-order corrections



- Several higher order corrections known analytically - series expansion in Zα and α
- Numerical computations are good for medium and large values of Zα

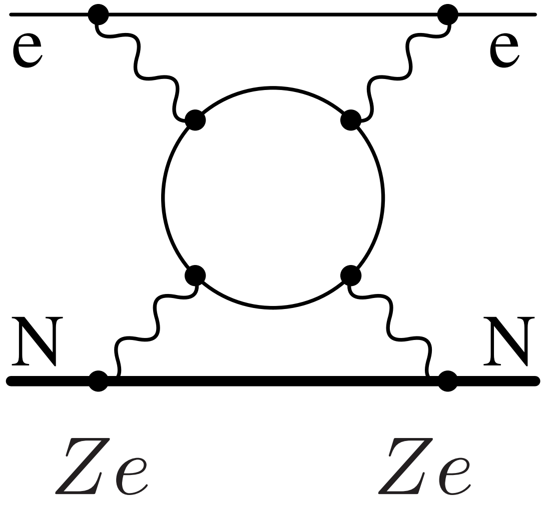
Recent progress possible thanks to application of EFT methods

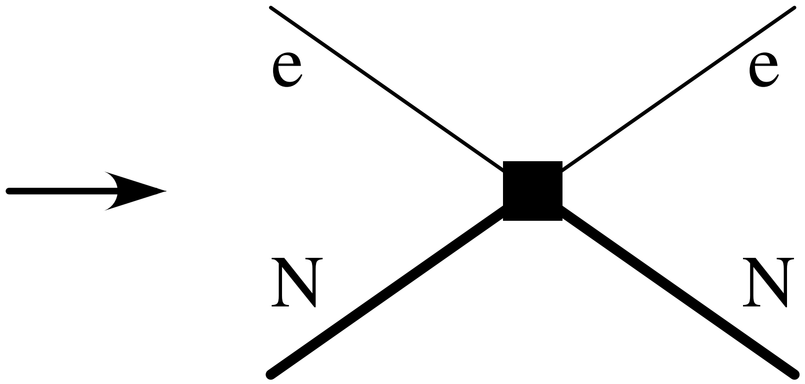
$$\Delta E = \frac{\alpha}{\pi}\left(A_{41}(Z\alpha)^4\ln(Z\alpha)^{-2} + A_{40}(Z\alpha)^4 + A_{50}(Z\alpha)^5 + \ldots\right) + \left(\frac{\alpha}{\pi}\right)^2\left(B_{40}(Z\alpha)^4 + B_{50}(Z\alpha)^5 + B_{63}(Z\alpha)^6\ln^3(Z\alpha)^{-2} + \ldots\right) + \ldots$$

EFT allows to systematically disentangle short and long distance contributions

Light-by-light corrections

Short distance part



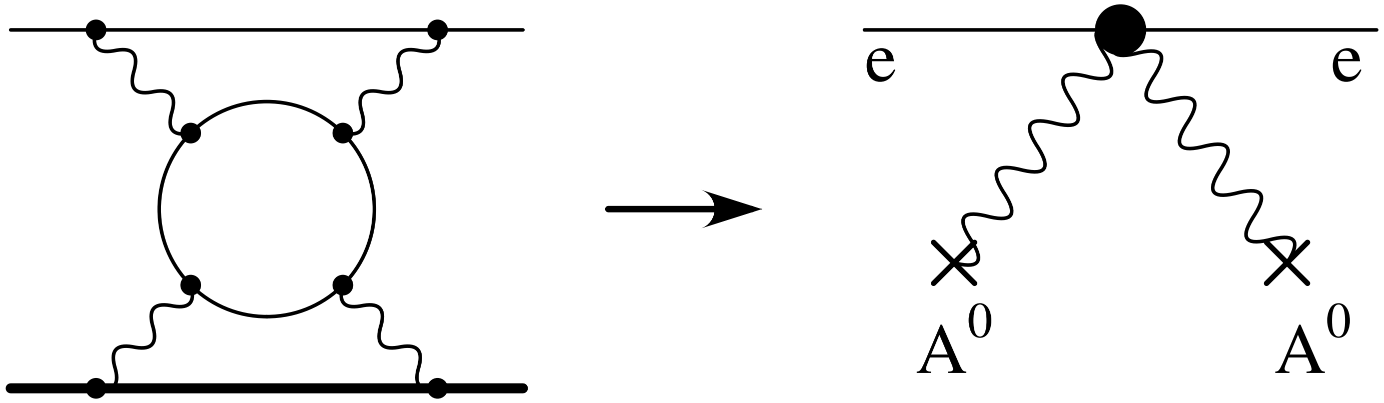


$$\sim |\psi(0)|^2 \sim (Z\alpha)^3$$

$$\sim (Z\alpha)^5$$

M. Eides, H. Grotch, P. Peble, 1994

Long distance part —logarithmically enhanced



$$\sim \vec{E}^2 \sim \left(\frac{Z\alpha}{r^2}\right)^2 \sim (Z\alpha)^6$$

A. Czarnecki, R.S., 2016
R.S., E. Korzinin, V. Shelyuto,
V. Ivanov, S. Karshenboim, 2019

$$\Delta E_{nS} \sim \left\langle \vec{E}^2 \right\rangle_{nS} \sim \frac{(Z\alpha)^6}{n^3} \ln(Z\alpha)^2$$

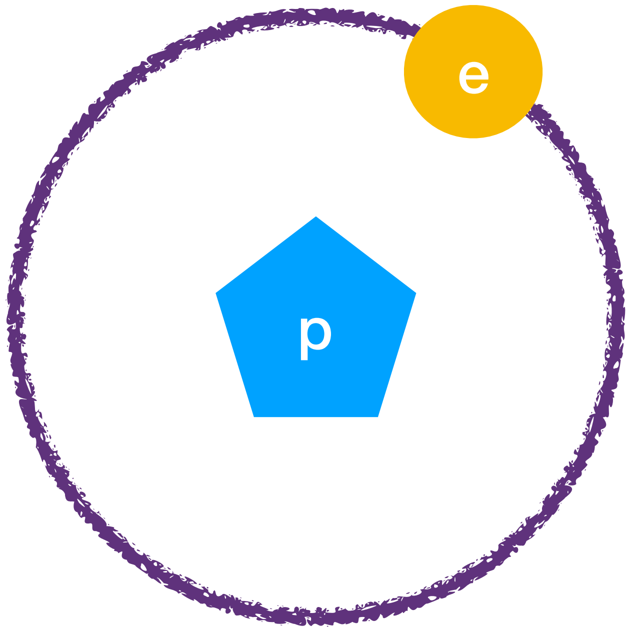
Light-by-light corrections

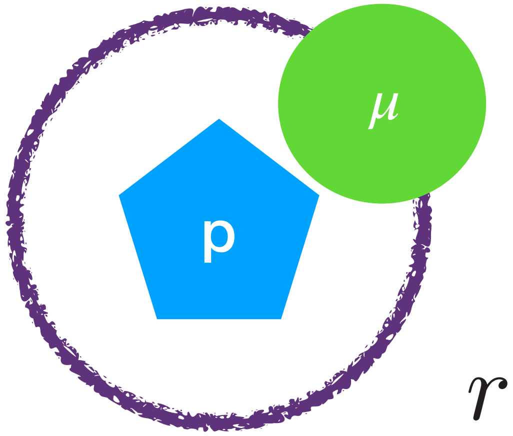
Total LBL correction decreases 1s-2s energy split by 720Hz

S. Karshenboim, A. Ozawa, V. Shelyuto, R.S., V. Ivanov, 2019

Experimental accuracy is 10Hz

Theory uncertainty is in the kHz range, and we should still consider the proton radius





$$r_b = \frac{1}{\alpha m_\mu}$$

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$$r_b = \frac{1}{\alpha m_\mu}$$

Theory of hydrogen spectrum has to be further improved and checked!

MeV

Muon g-2

Muon is expected to be more sensitive to New Physics due to its heavier mass

$$\frac{m_\mu^2}{m_e^2} \sim 40\,000$$

Comparison of the theory with BNL experiment shows famous discrepancy

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{theory}} = (31.3 \pm 7.7) \times 10^{-10} \quad (4.1\sigma)$$

F. Jegerlehner, 2017

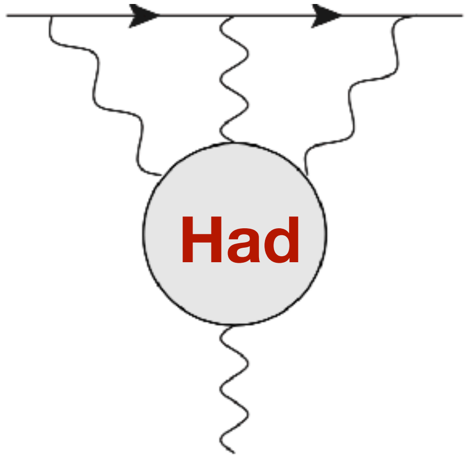
Note size and sign of the anomaly when compared with electron

$$\Delta a_e \sim \frac{m_e^2}{m_\mu^2} \Delta a_\mu \sim 7 \times 10^{-14}$$

Evaluation of muon g-2

Theory for muons is much more complicated than for electrons

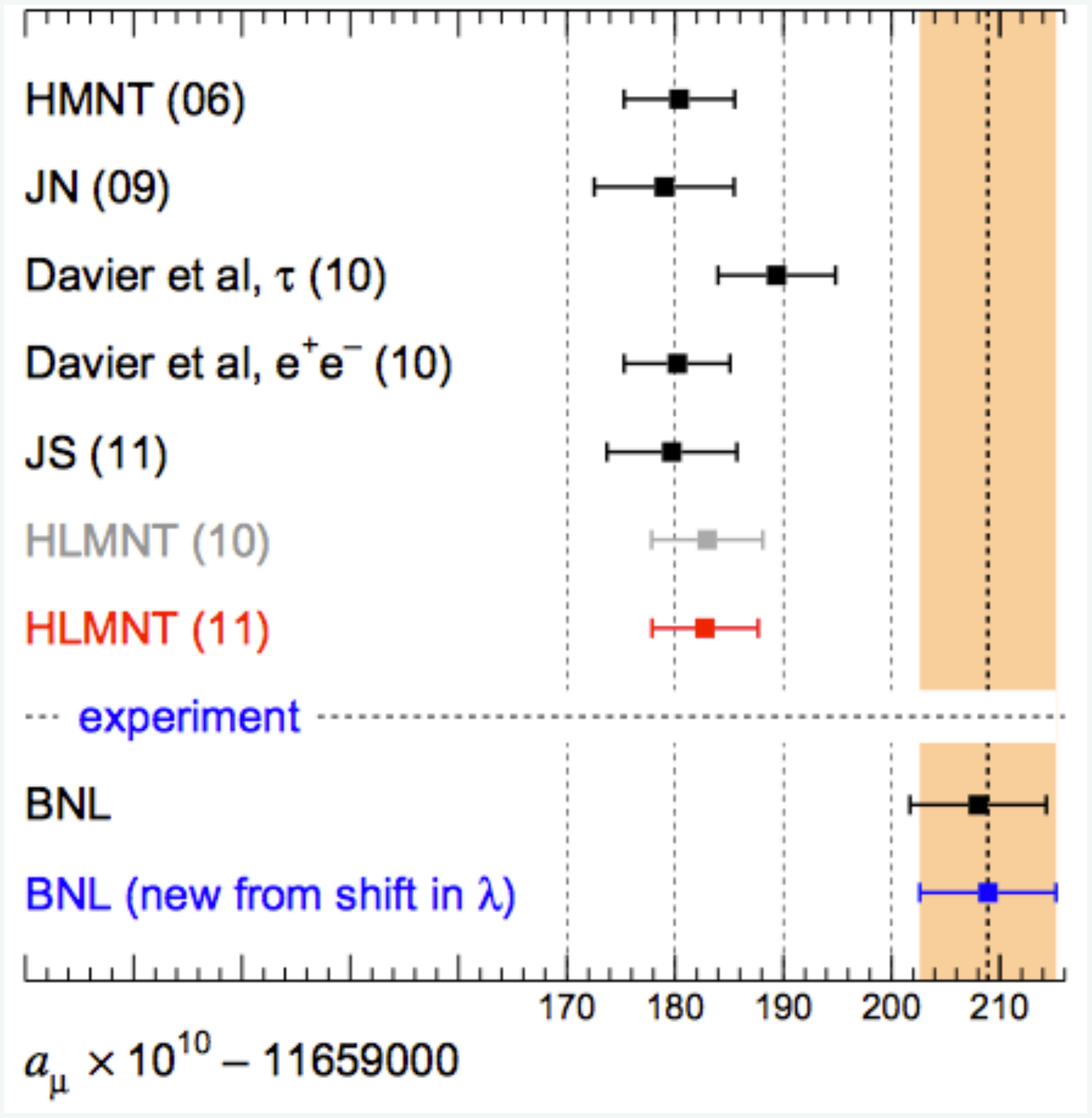
- QED corrections *Ayoma et al, 2012, S. Laporta 2017*
- Electro-Weak corrections $a^{\mu}_{\mathrm{EW}} = \quad 15.4 \times 10^{-10}$
  A. Czarnecki, B. Krause, W. Marciano, 1996
- Hadronic corrections *F. Jegerlehner, 2018*
  - Hadronic VP $a^{\mu}_{\mathrm{VP}} \approx \quad (689.5 \pm 3.3) \times 10^{-10}$
    - Pion part $a^{\mu}_{\mathrm{VP}} \quad (\pi^+\pi^-) \approx (505.7 \pm 2.7) \times 10^{-10}$
  - LBL $a^{\mu}_{\mathrm{LBL}} \approx \quad (10.3 \pm 2.9) \times 10^{-10}$



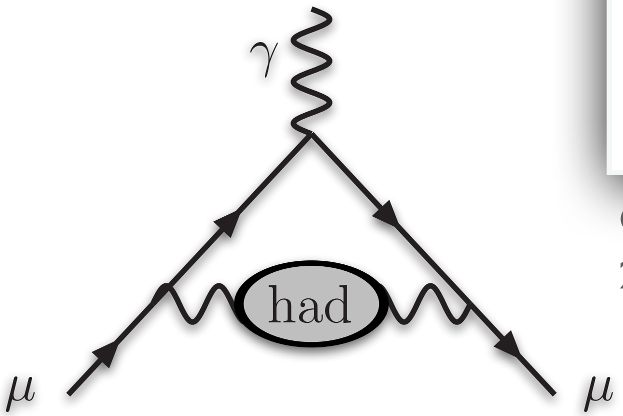
MeV

Pion contribution

- Hadronic effects needs to be further scrutinized
- Progress on experimental side (Fermilab, J-PARC)
- Lattice QCD made a huge progress in recent years



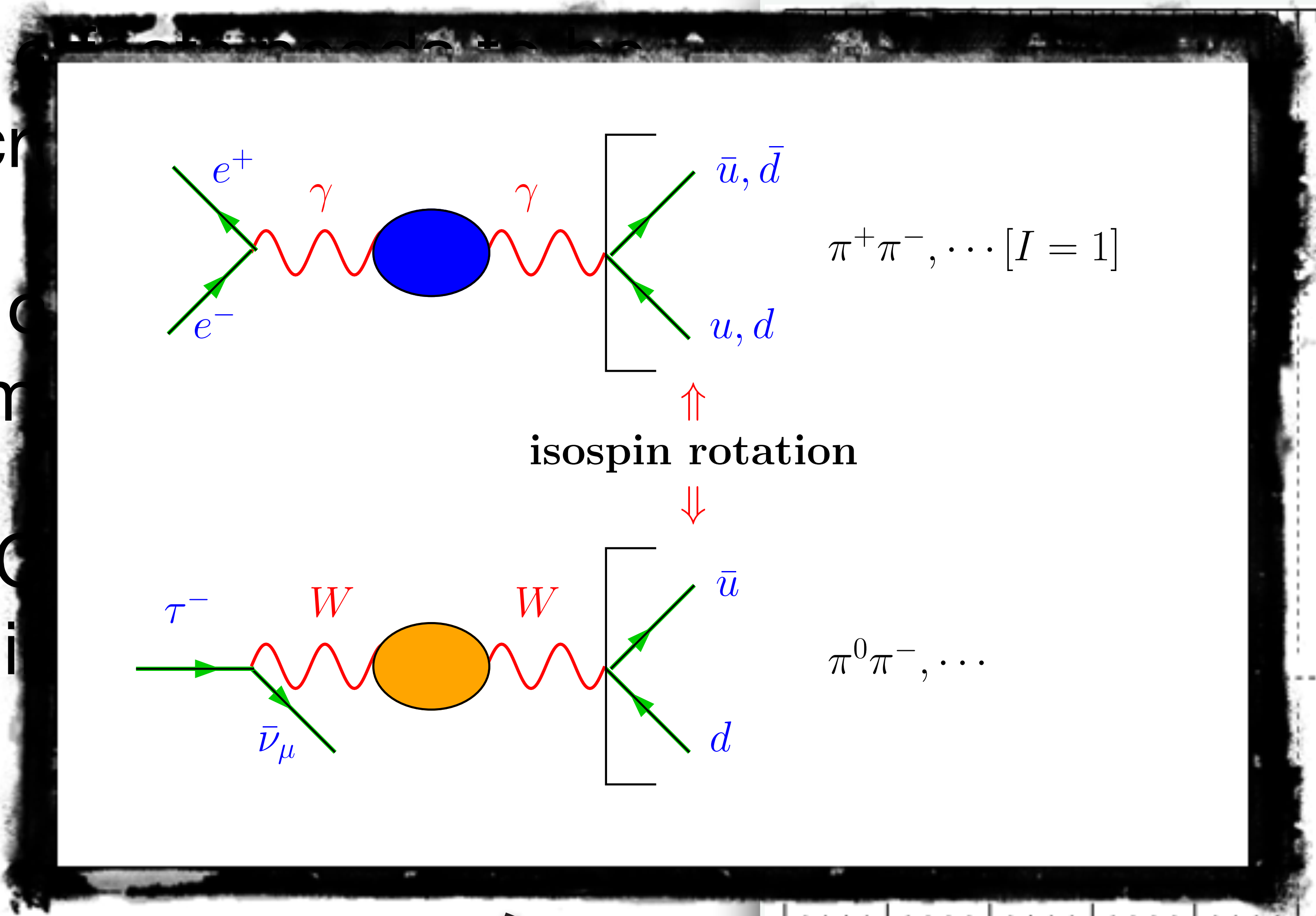
Graph from T. Teubner et al. Nucl.Phys.Proc.Suppl. 225-227 (2012) 282-287

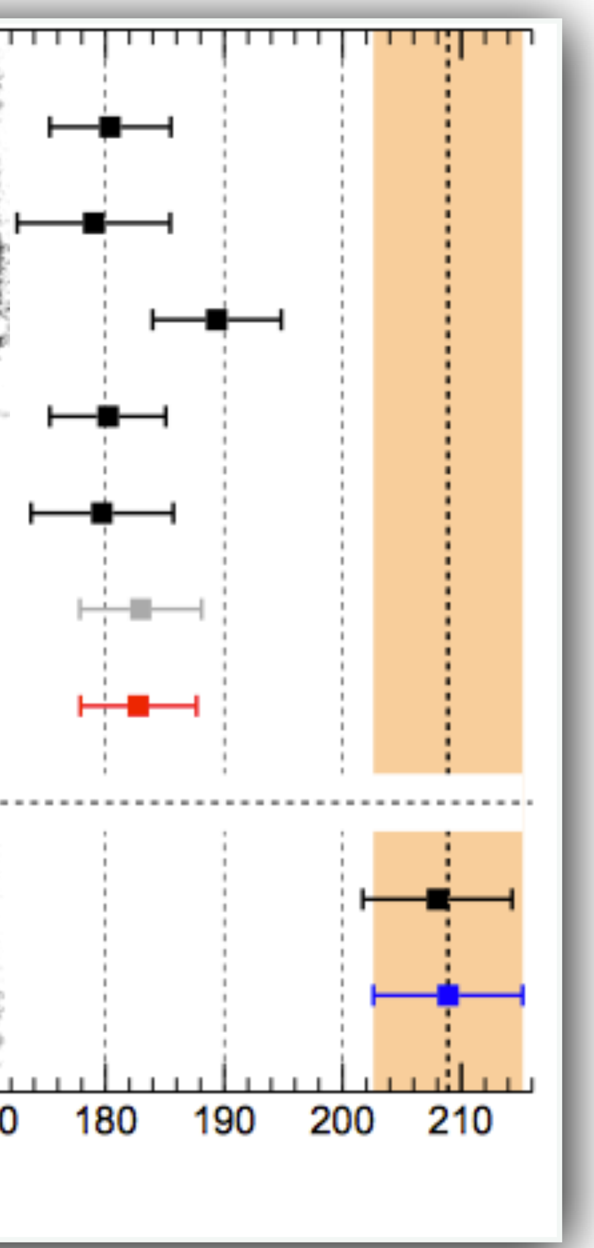


MeV

Pion contribution

- Hadronic
  further scr
- Progress c
  side (Ferm
- Lattice QC
  progress i

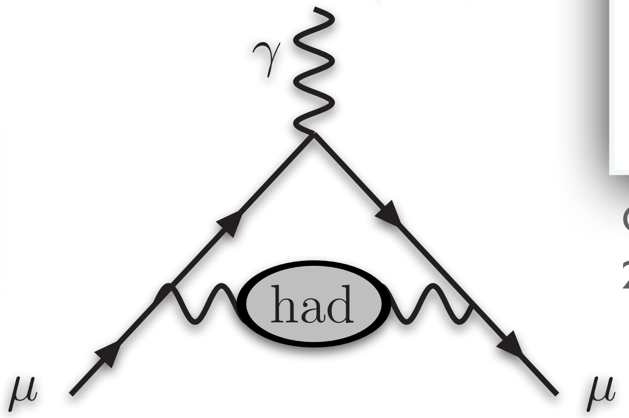




Graph from T. Teubner et al. Nucl.Phys.Proc.Suppl. 225-227 (2012) 282-287

e+e- v.s. tau problem solved

F. Jegerlehner, R.S., 2011



MeV

Pion contribution

- Hadronic
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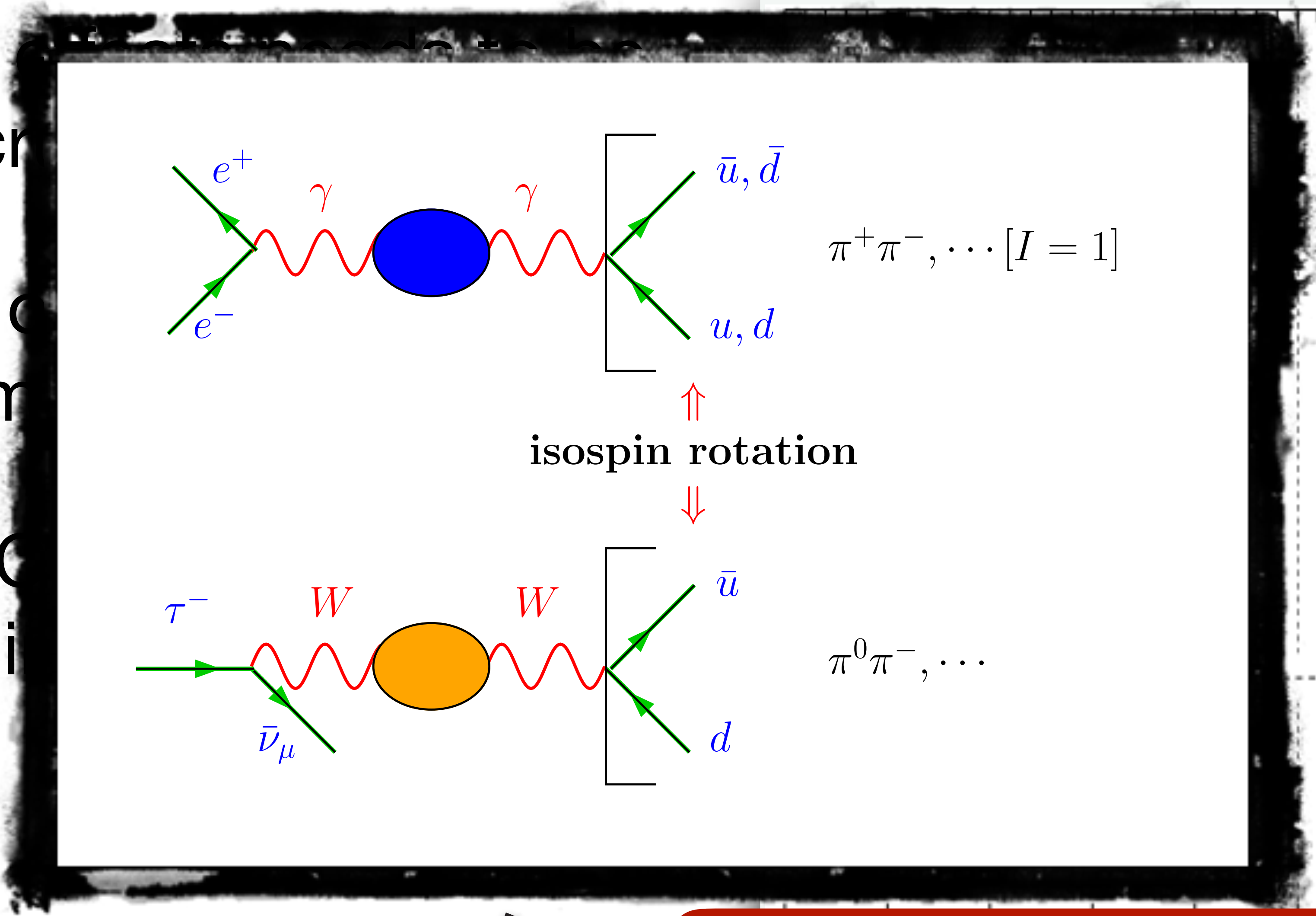

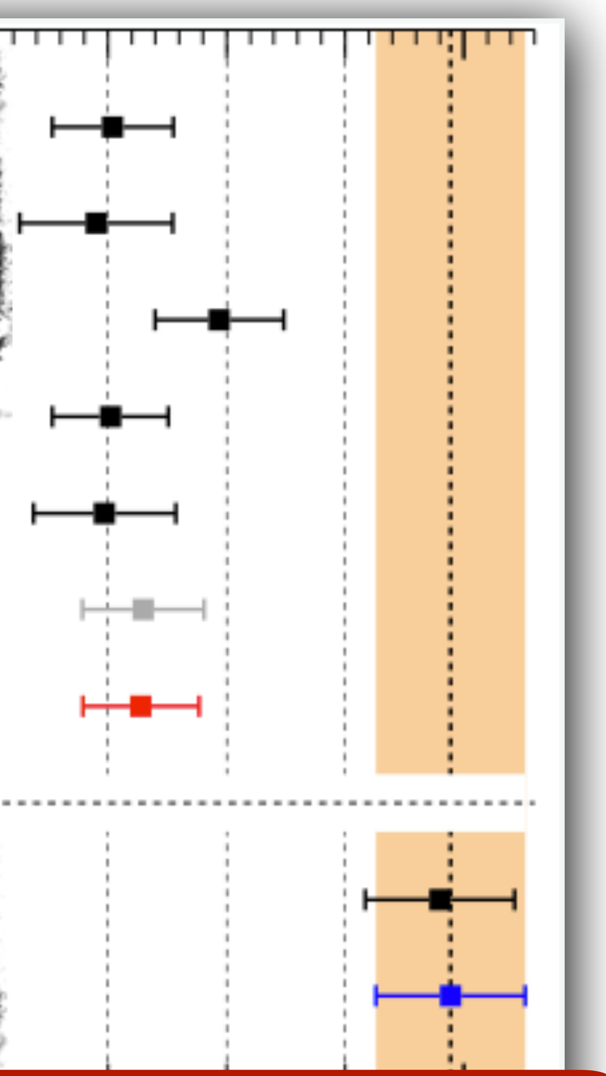

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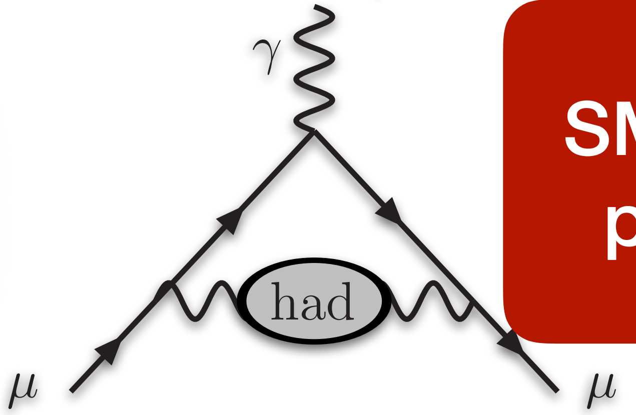
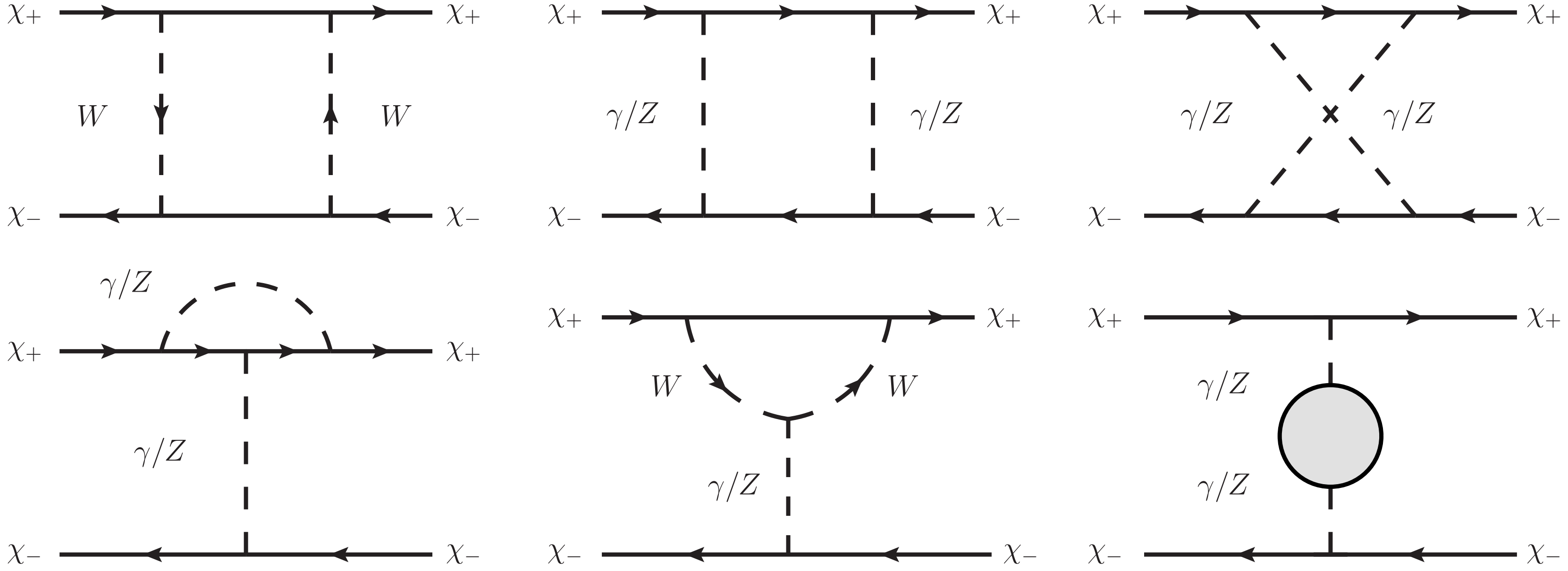
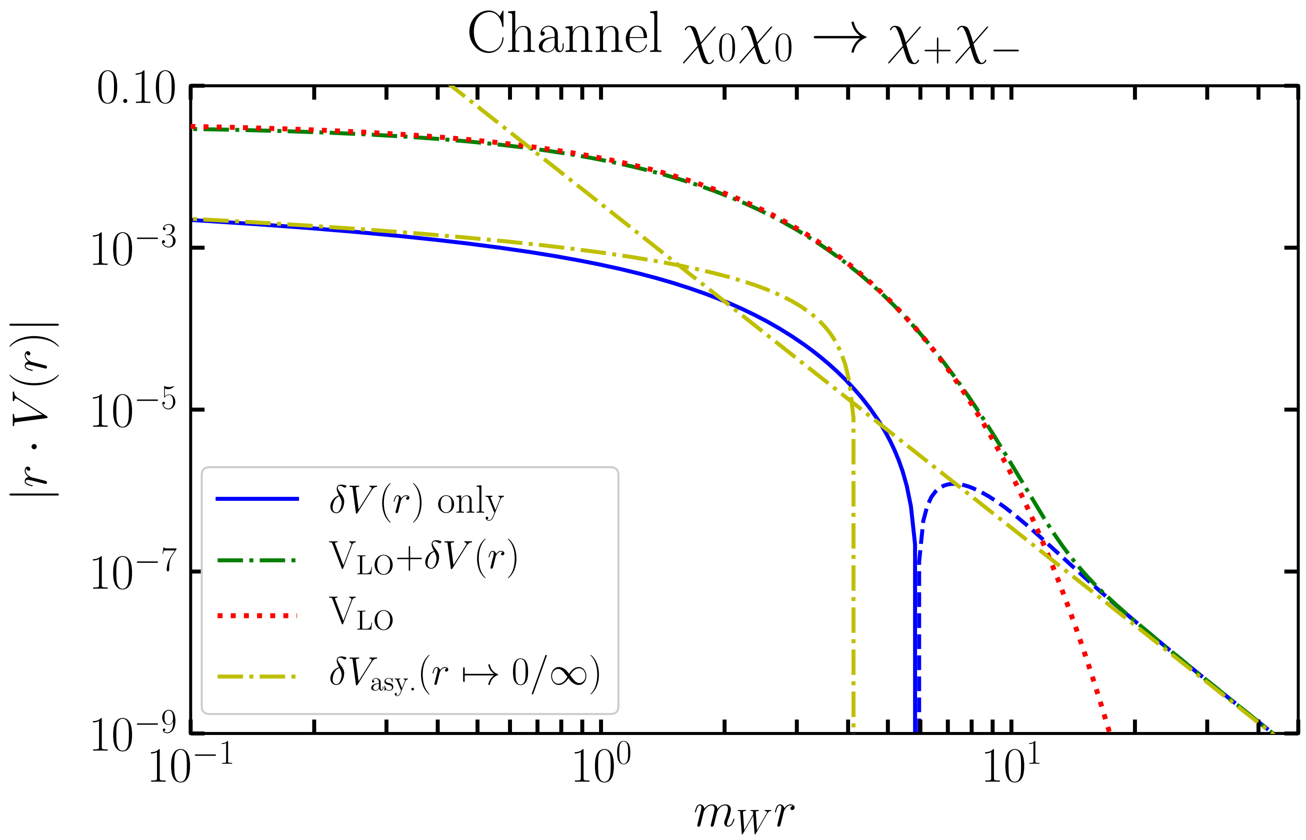


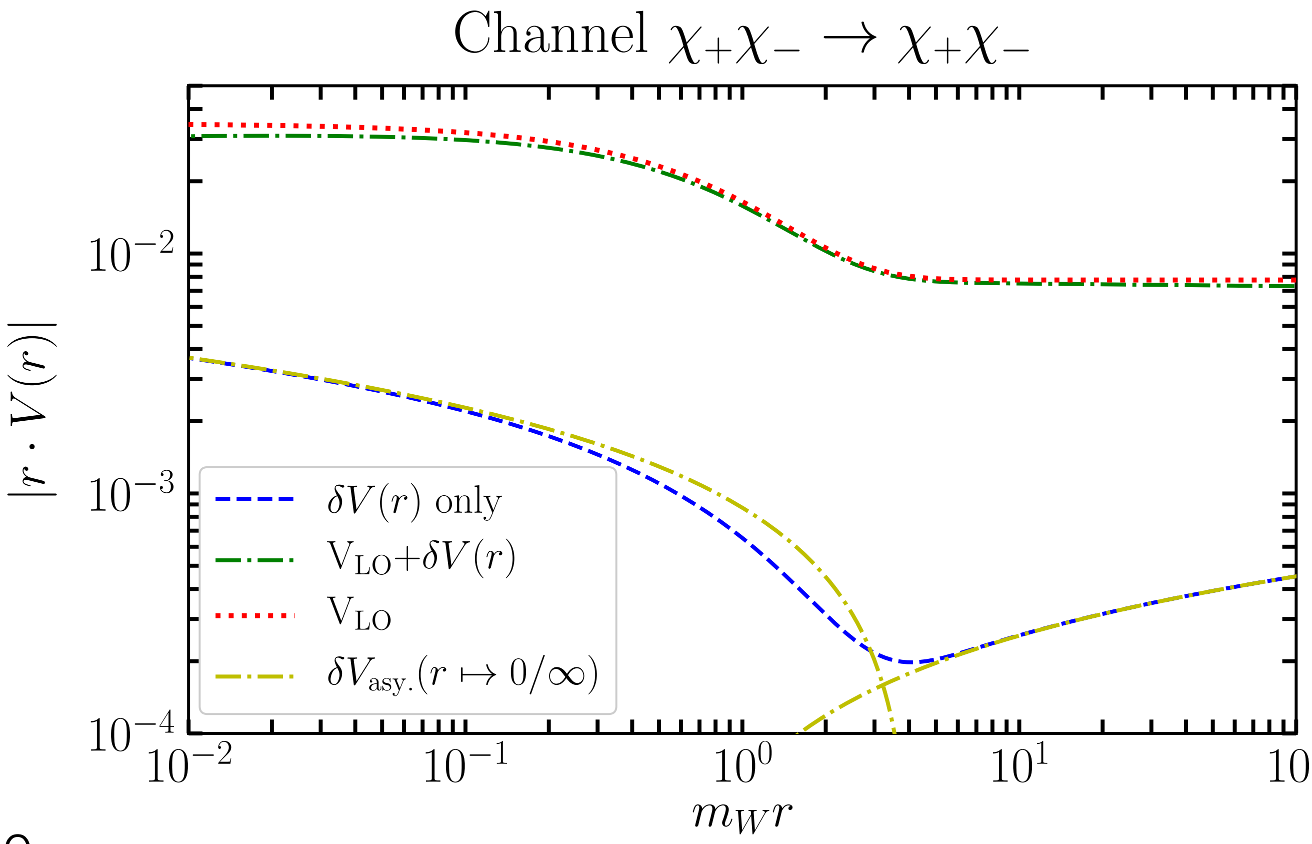
SM theory, in particular the hadronic part, has to be further scrutinized!

Corrections to the potential



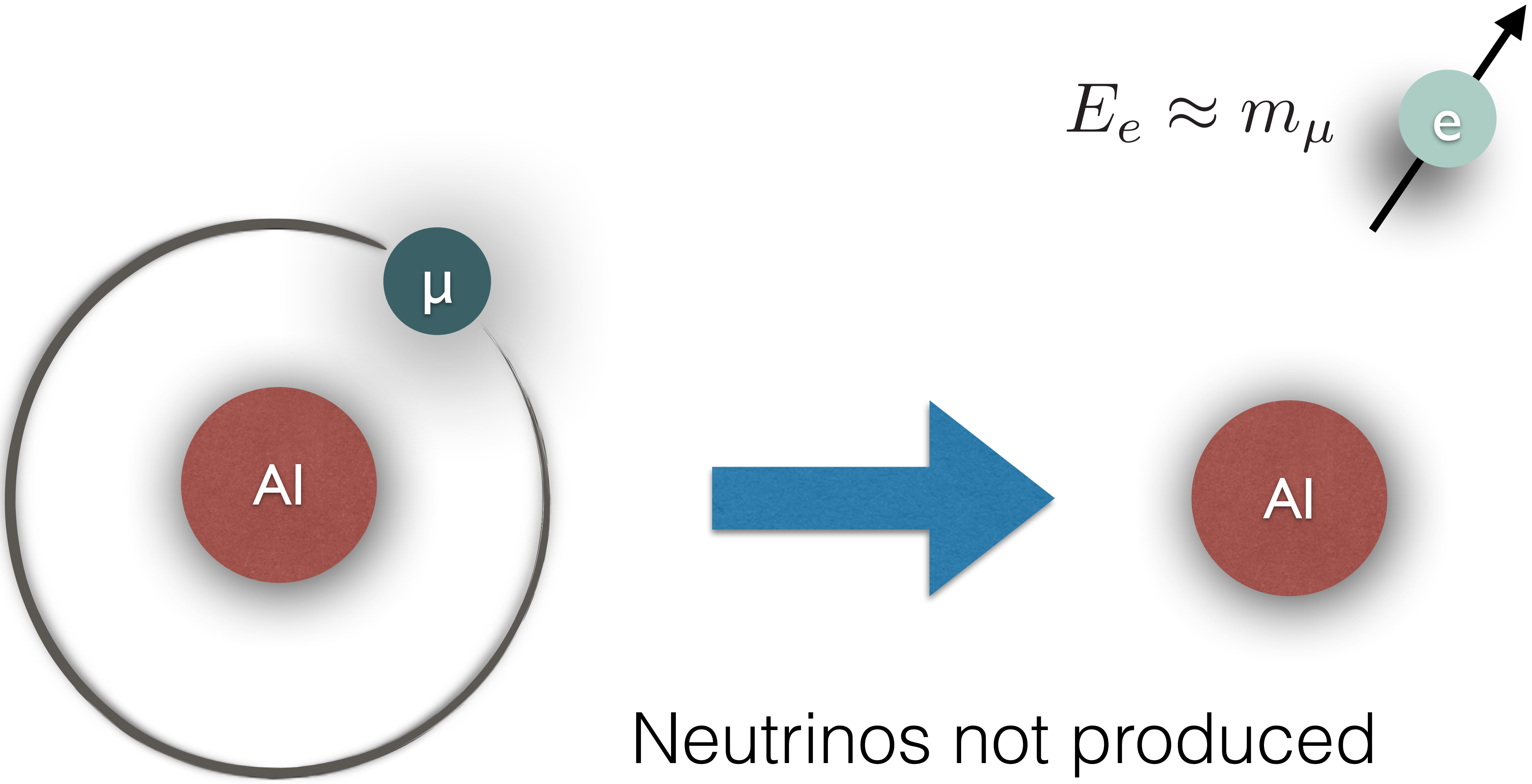
M. Beneke, R.S., K. Urban, 2019


Channel $\chi_0\chi_0 \to \chi_+\chi_-$


Channel $\chi_+\chi_- \to \chi_+\chi_-$

MeV

Muon electron coherent conversion

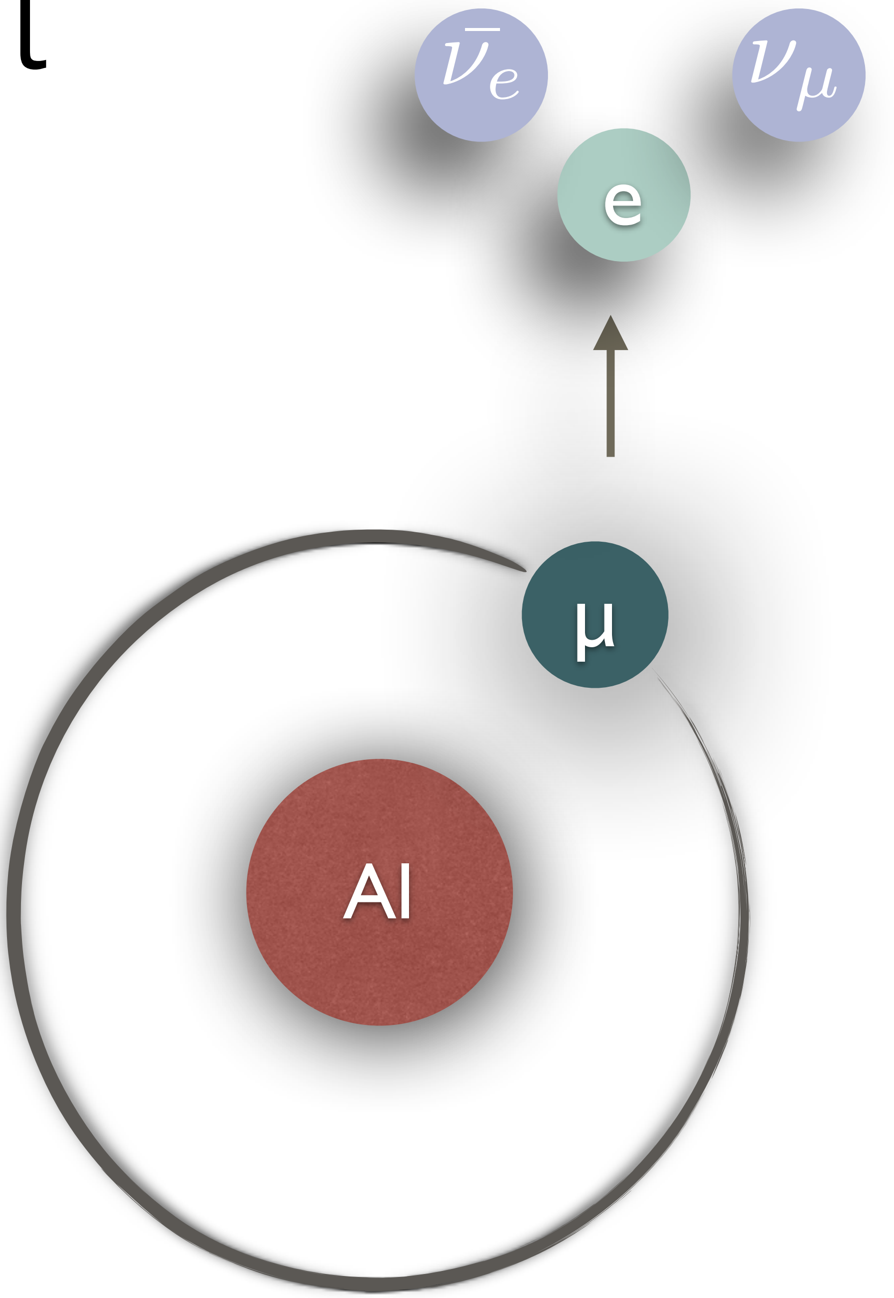


Neutrinos not produced
CLFV

Muon DIO

~39%

DIO — Decay In Orbit

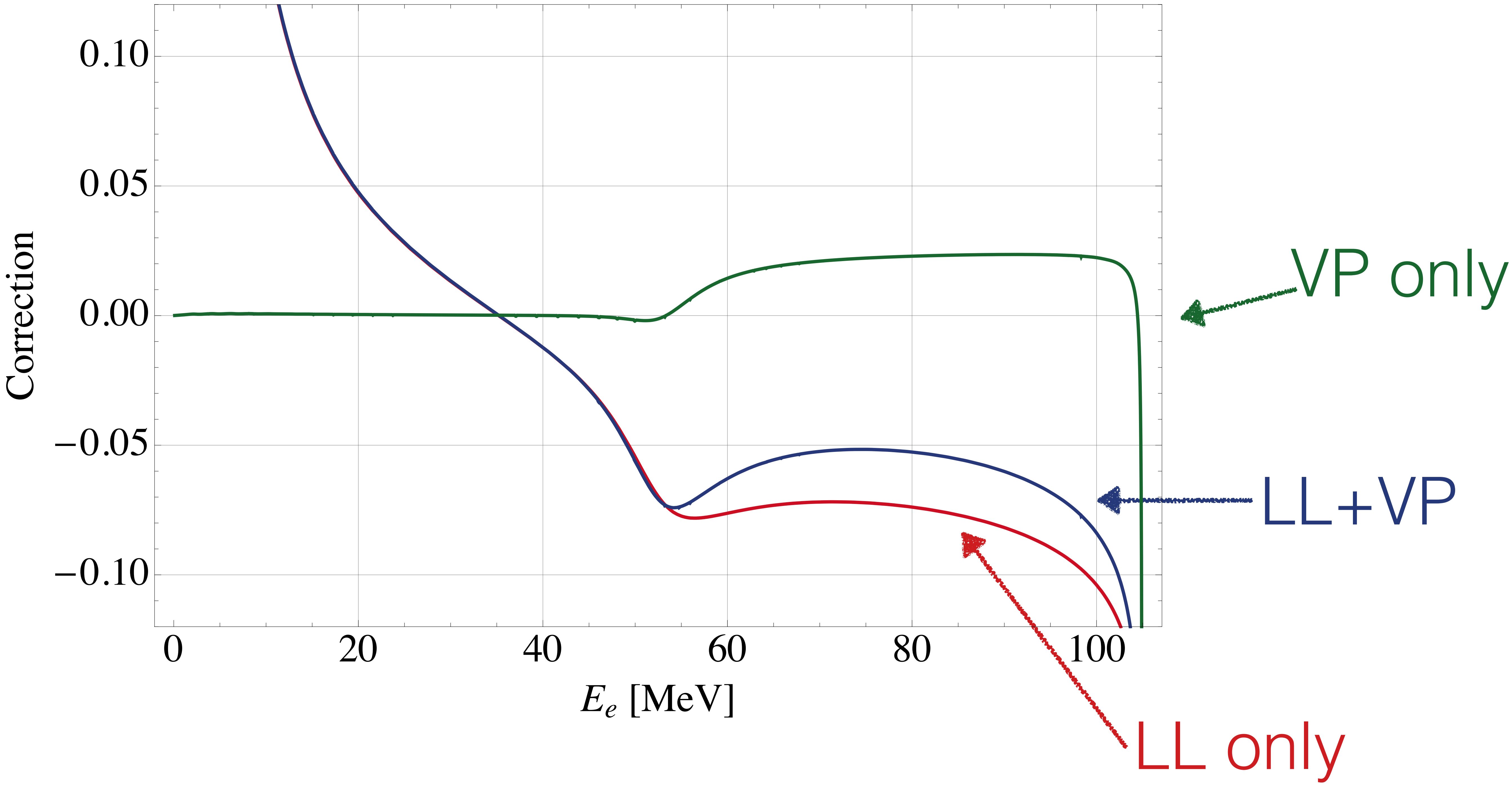


- Muon DIO: standard muon decay into an electron and two neutrinos, with the muon and a nucleus forming a bound state
- For DIO momentum can be exchanged between the nucleus and both the muon and the electron

Correction to the DIO spectrum

MeV

R.S., A. Czarnecki, 2016



Correction to the DIO spectrum

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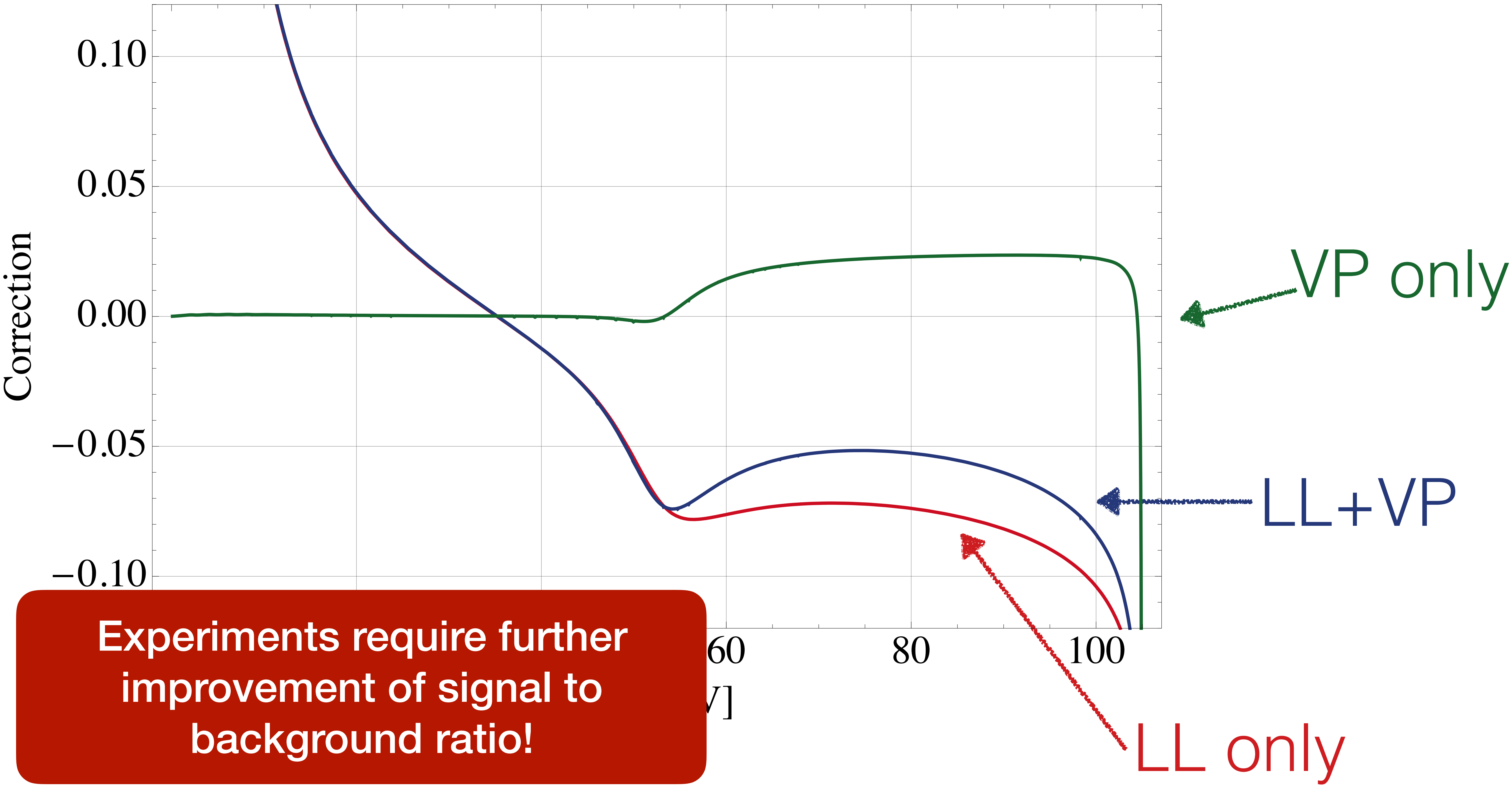
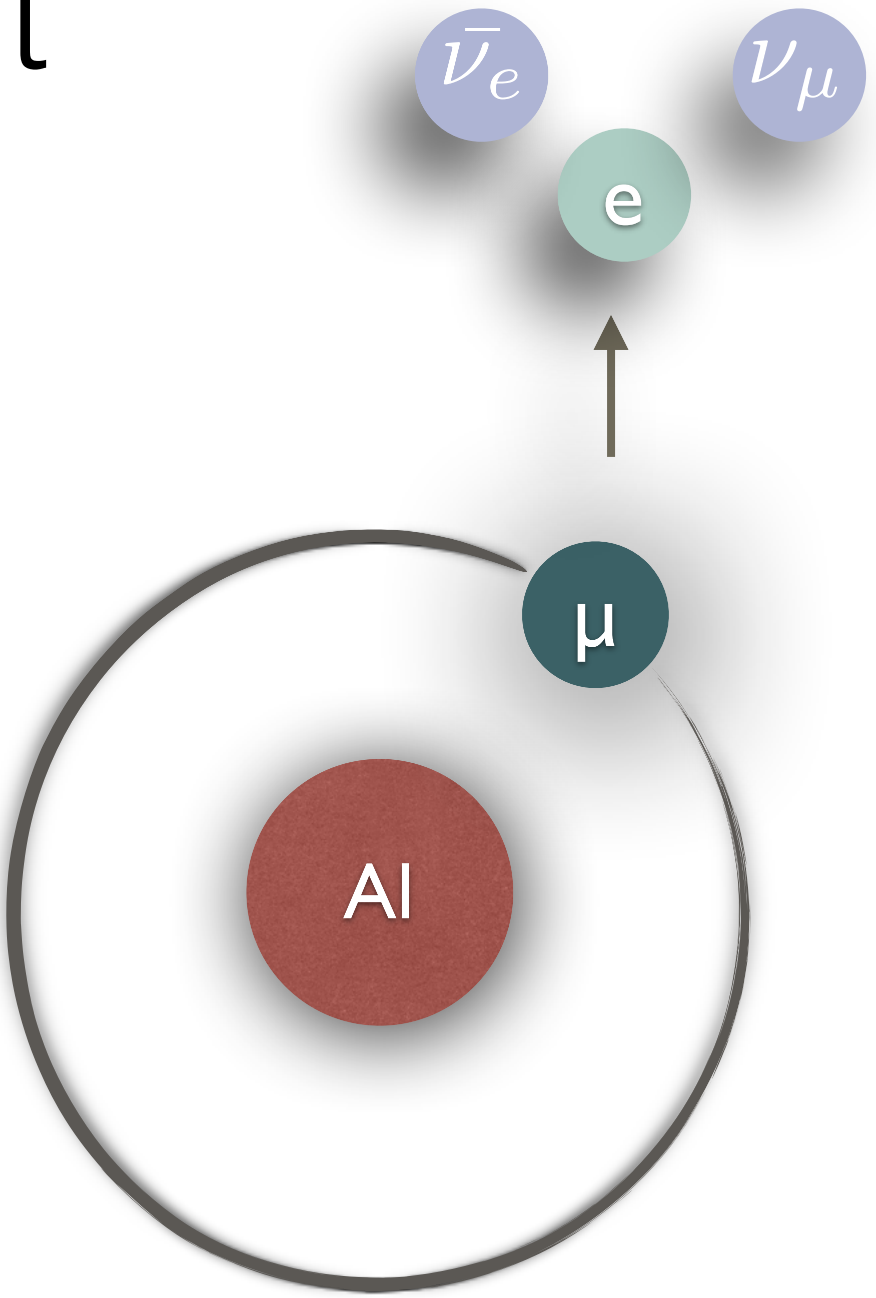


Experiments require further improvement of signal to background ratio!

Muon DIO

~39%

DIO — Decay In Orbit



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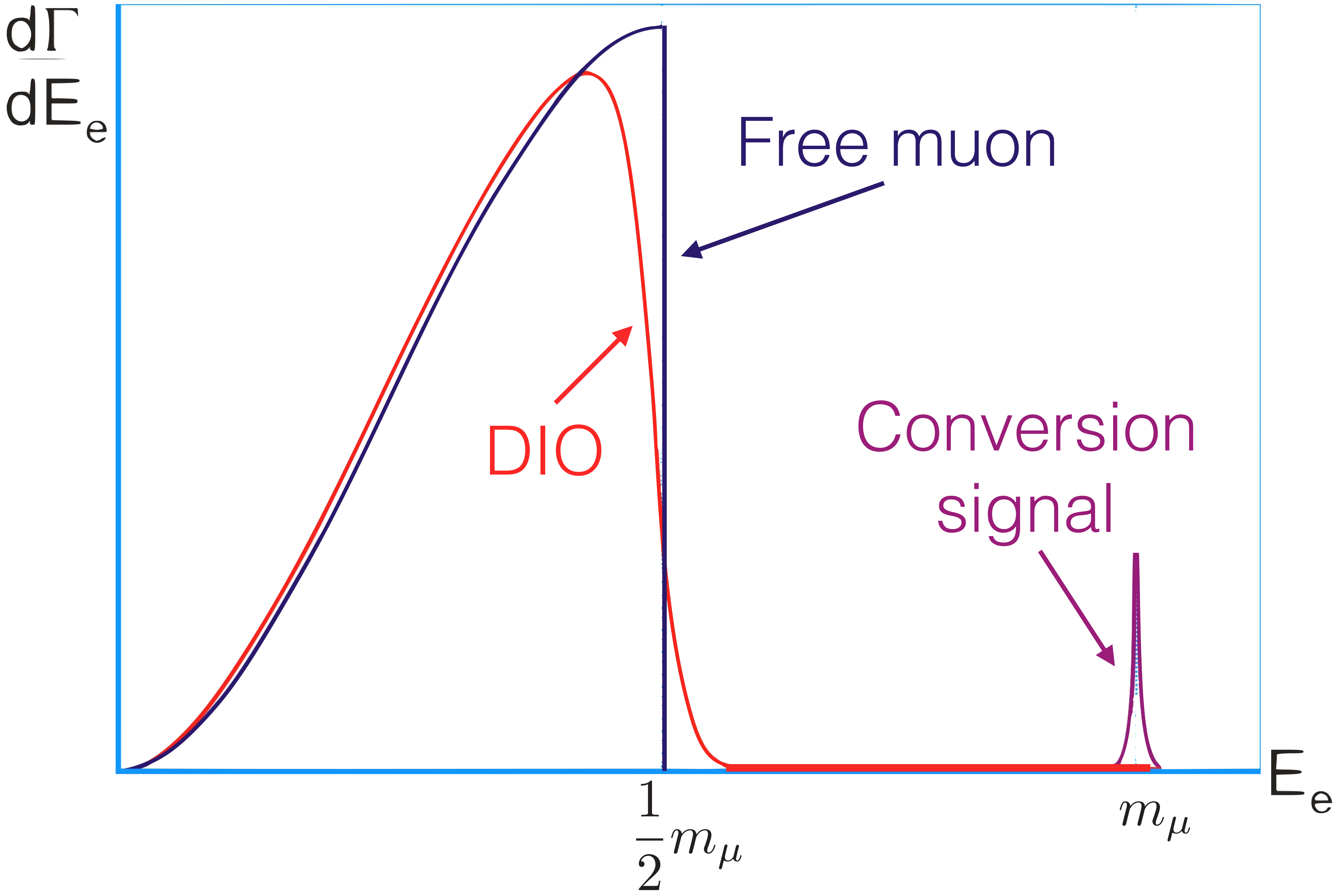
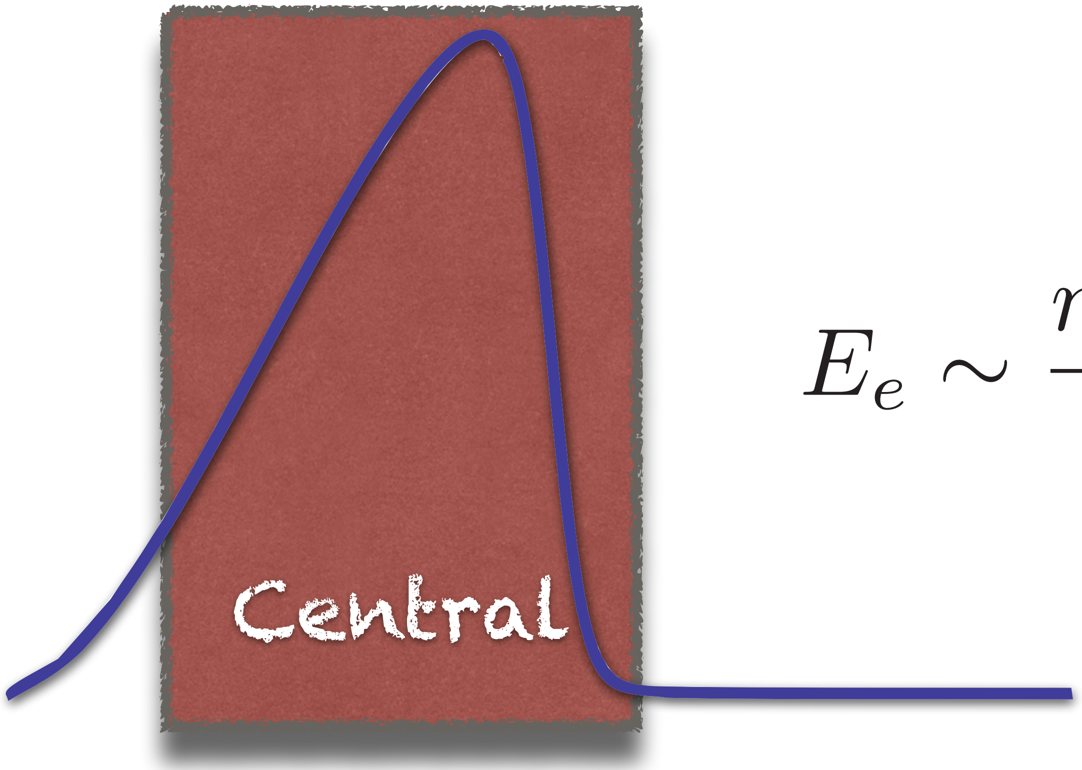
MeV

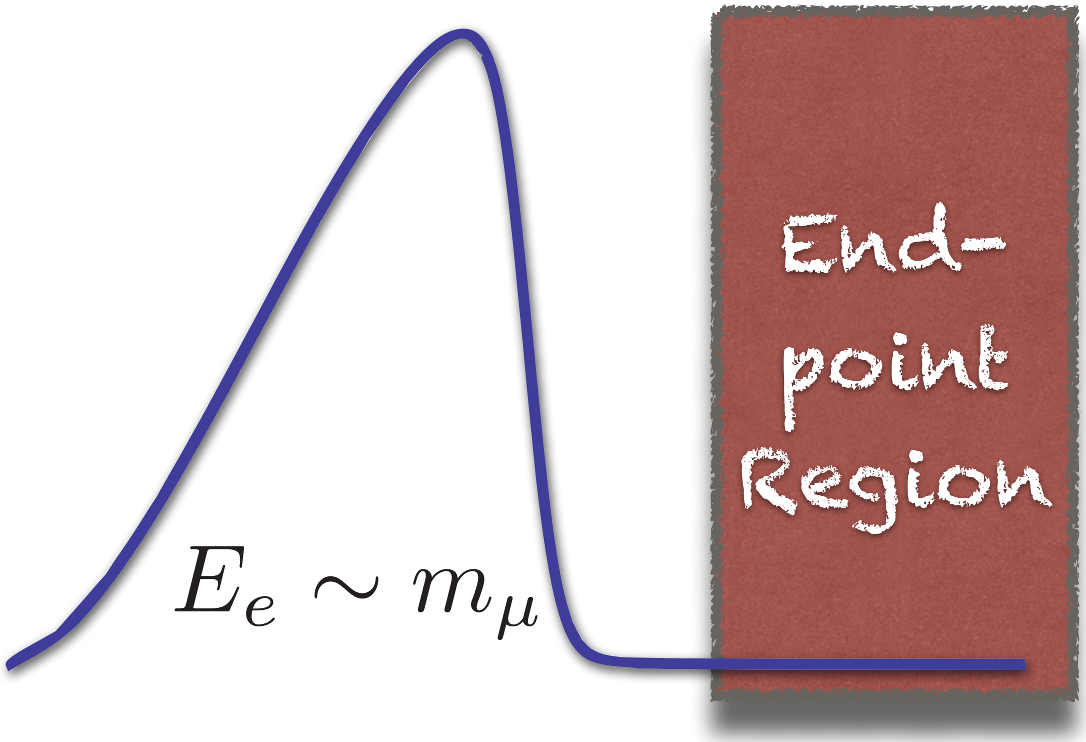
DIO Spectrum



DIO spectrum regions



$$E_e \sim \frac{m_\mu}{2}$$



- Measured by the TWIST experiment in 2009
- Muon motion dominates

- Background for the conversion experiments
- Will be measured in conversion experiments

MeV

QCD case:

Neubert 1993; Mannel, Neubert 1994; Bigi, Shifman,Uraltsev, Vainshtein, 1994

Factorization

Following QCD approach a factorization theorem can be derived

$$\frac{d\Gamma_{\text{DIO}}}{dE_e} = \frac{d\Gamma_{\text{free}}}{dE_e} \otimes S$$

Free muon spectrum
It is associated with the hard scale m_μ

QED Shape function
It is associated with the soft scale $m_\mu Z\alpha$

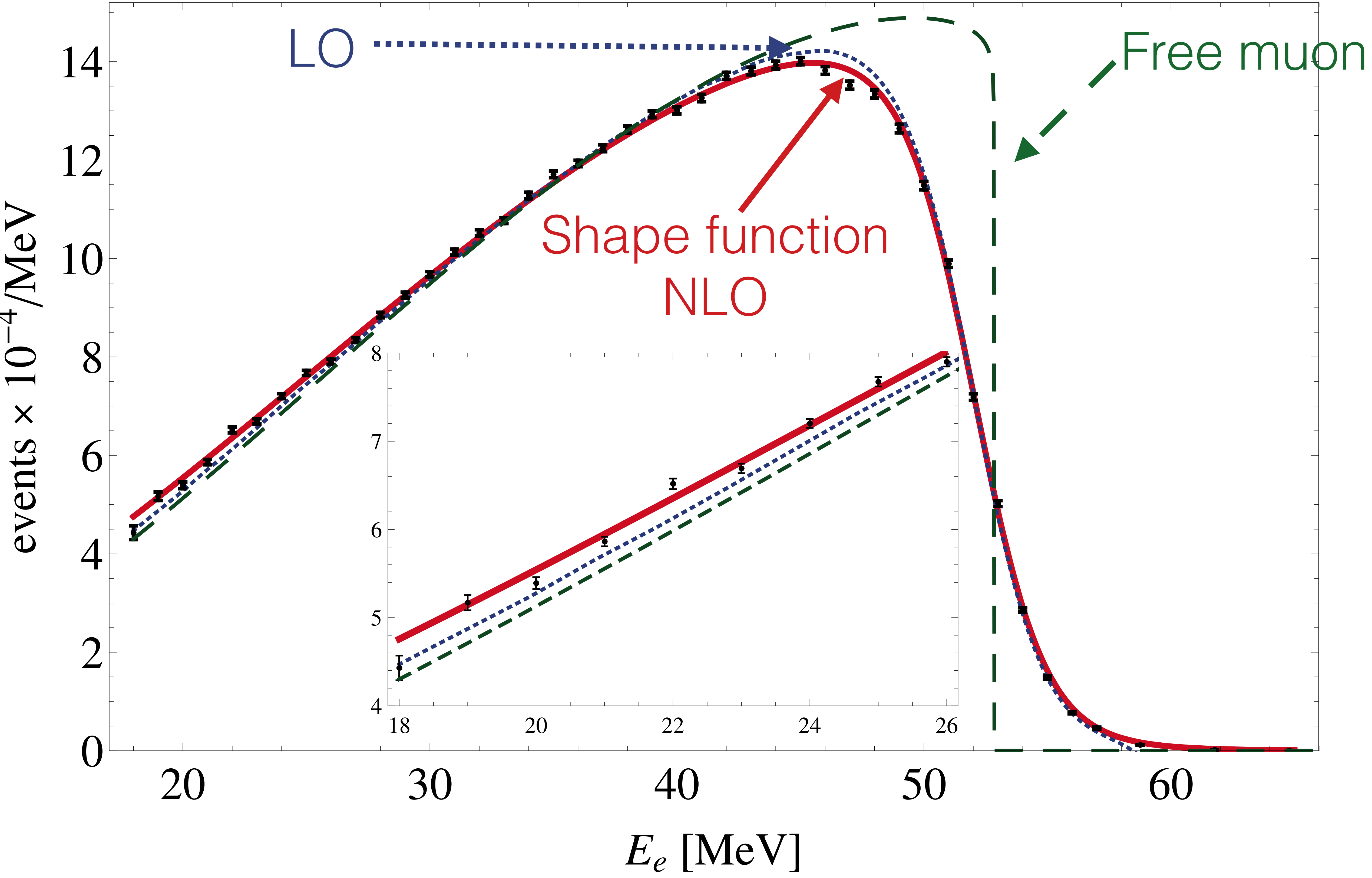
Separation of scales $m_\mu Z\alpha \ll m_\mu$

Results for Aluminum

MeV

and their relation to the TWIST data

A.Czarnecki, M. Dowling, X.Garcia i Tormo, W. Marciano, R.S., 2014

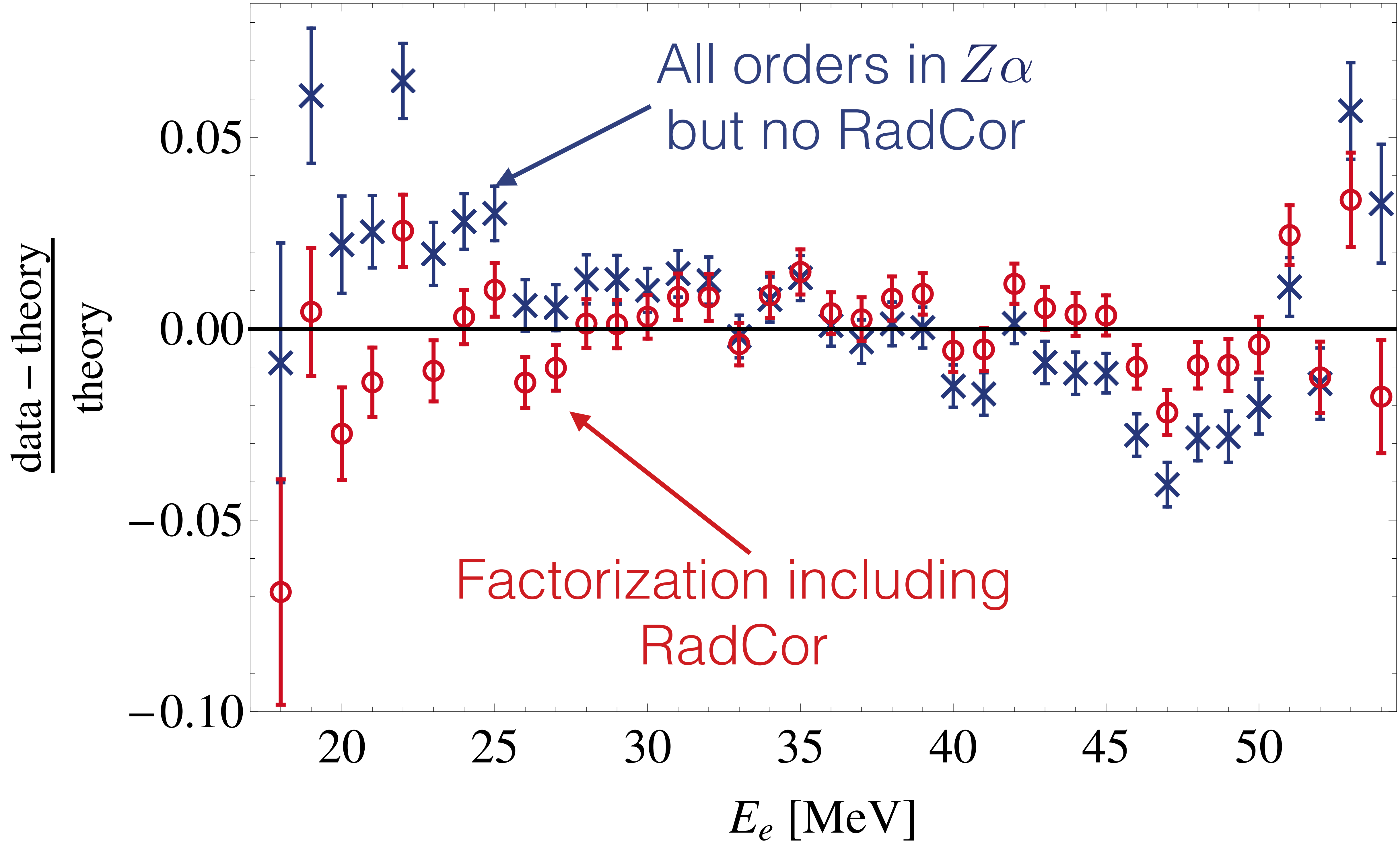


Leading Corrections

MeV

and their relation to the TWIST data

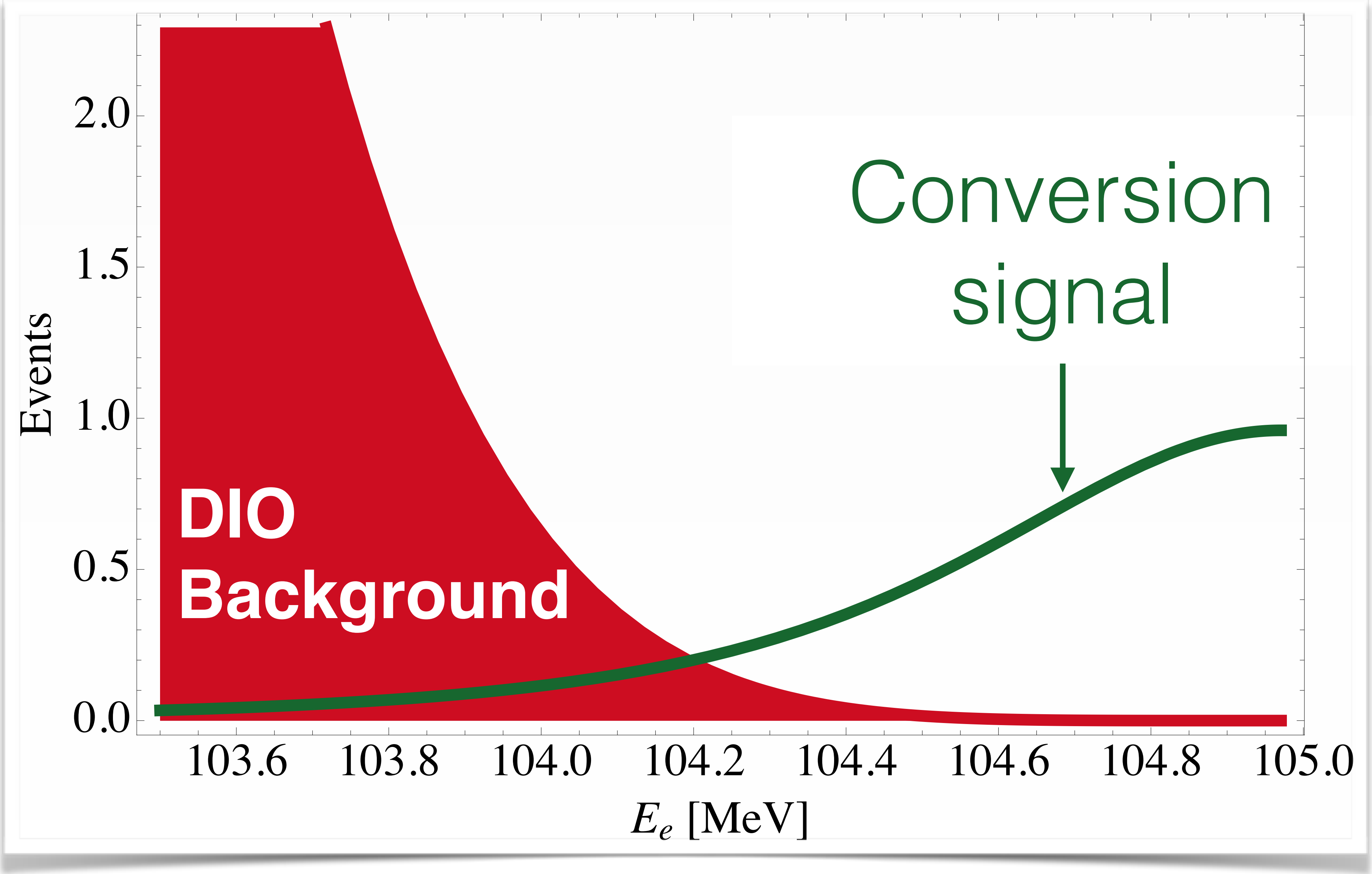
A.Czarnecki, M. Dowling, X.Garcia i Tormo, W. Marciano, R.S., 2014



DIO Spectrum

MeV

Endpoint region



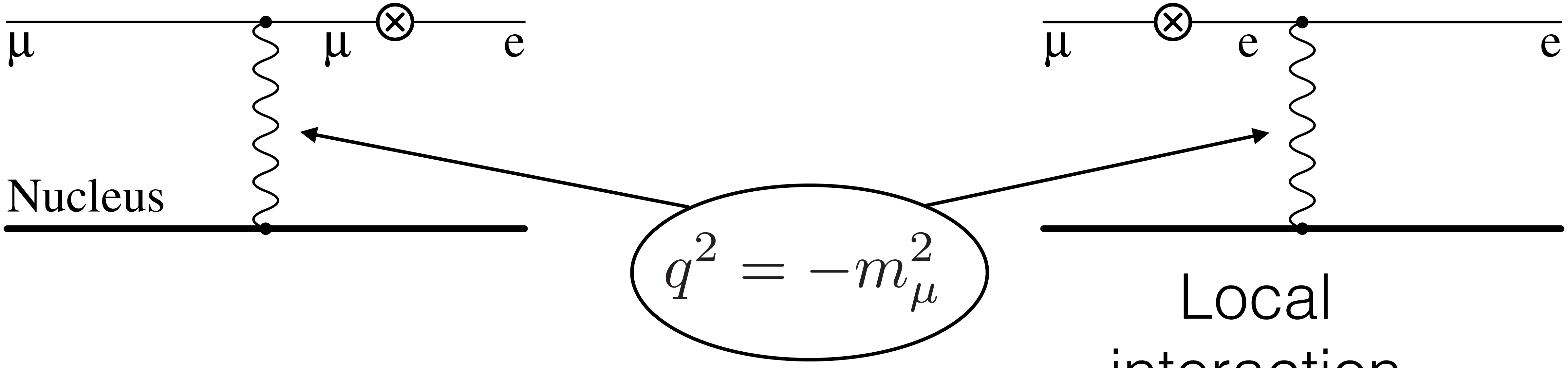
Endpoint expansion

Near the endpoint, the dominant contribution comes from the exchange of hard virtual photons.

R.S., A. Czarnecki, 2015

$$\frac{m_\mu}{\Gamma_{Free}}\frac{d\Gamma}{dE_e} \approx \frac{1024}{5\pi}(Z\alpha)^5\left(\frac{\Delta}{m_\mu}\right)^5$$

$$\Delta = E_{max} - E_e$$



$q^2 = -m_\mu^2$

Local interaction

MeV

Endpoint Radiative Correction

Background suppression ~15%

R.S., A. Czarnecki, 2015

$$\Delta = E_{max} - E_e$$

$$\frac{1}{\Gamma_{Free}}\frac{d\Gamma}{dE_e} = \Delta^5 \frac{1024}{5\pi m_\mu^6}(Z\alpha)^5 \left(\frac{\Delta}{m_\mu}\right)^{\frac{\alpha}{\pi}\delta_S} (1 + \frac{\alpha}{\pi}\delta_{VP} + \frac{\alpha}{\pi}\delta_H)$$

MeV

Endpoint Radiative Correction

- Soft vacuum polarization correction to the muon wave-function at the origin (running to $m_\mu Z\alpha$)

$$r \sim \frac{1}{m_e} \gg \frac{1}{m_\mu Z\alpha}$$

Correction range

Atom size

Background suppression ~15%

R.S., A. Czarnecki, 2015

$$\Delta = E_{max} - E_e$$

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Correction range

Atom size

- Hard vacuum polarization

Background suppression ~15%

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MeV

Endpoint Radiative Correction

- Soft vacuum polarization correction to the muon wave-function at the origin (running to $m_\mu Z\alpha$)
- Hard vacuum polarization
- Soft photon emission

$$r \sim \underset{\text{Correction range}}{\frac{1}{m_e}} \gg \underset{\text{Atom size}}{\frac{1}{m_\mu Z\alpha}}$$

Background suppression ~15%

R.S., A. Czarnecki, 2015

$$\Delta = E_{max} - E_e$$

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MeV

Endpoint Radiative Correction

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- Hard vacuum polarization
- Soft photon emission
- Hard correction

$$r \sim \underset{\text{Correction range}}{\frac{1}{m_e}} \gg \underset{\text{Atom size}}{\frac{1}{m_\mu Z\alpha}}$$

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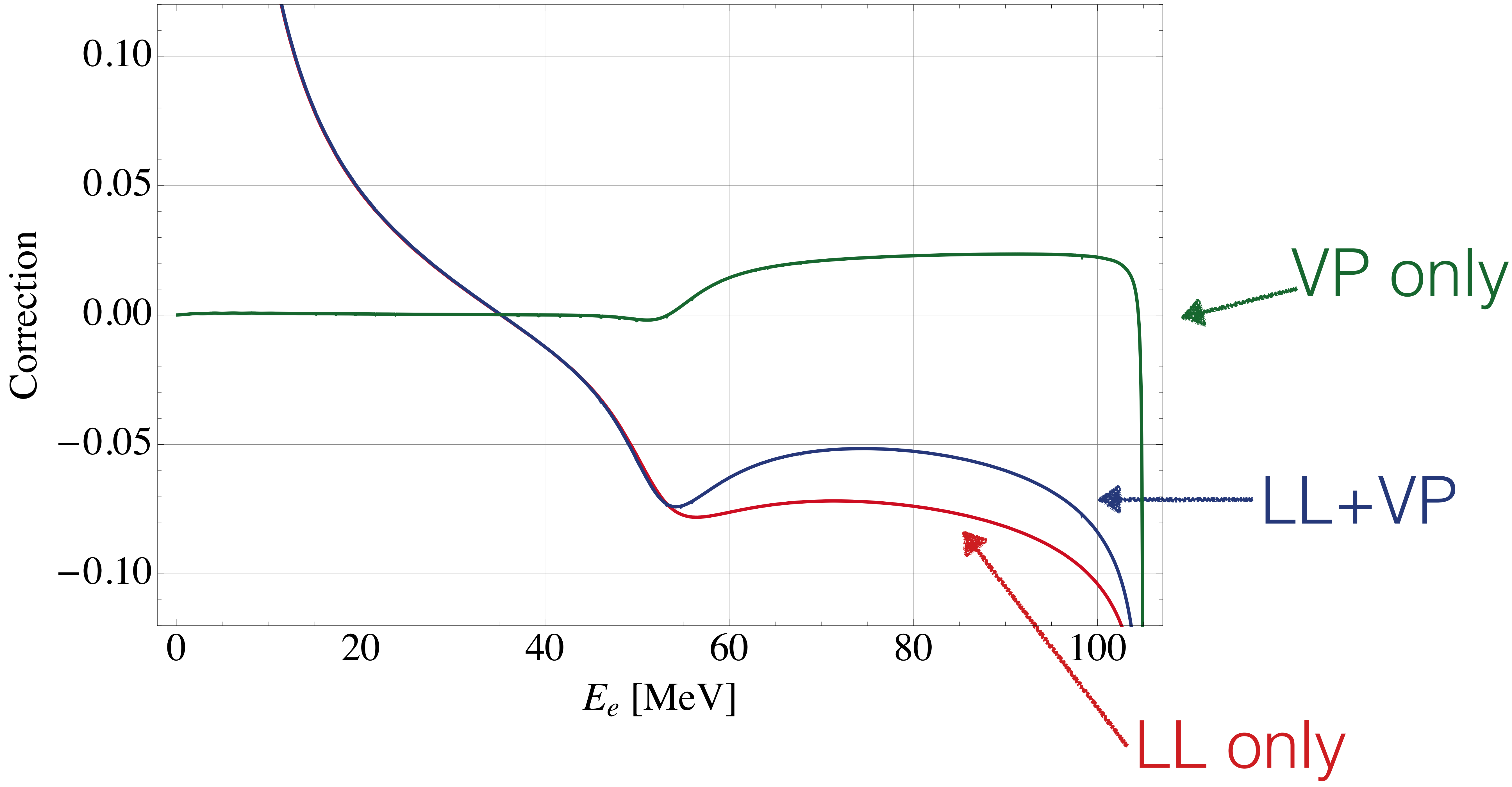
Correction to the DIO spectrum

MeV

R.S., A. Czarnecki, 2016



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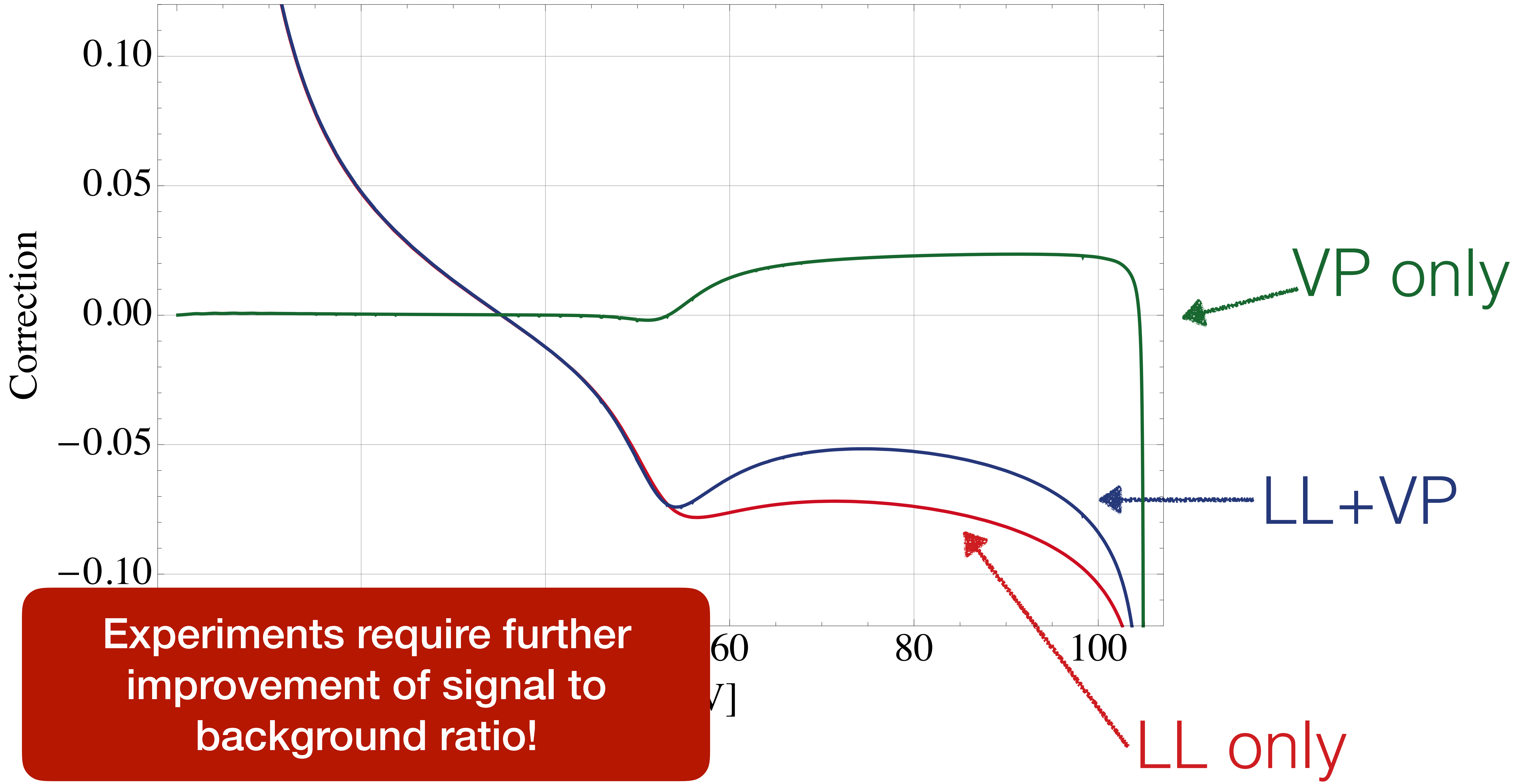

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