Extraction of PDF with the Hybrid Renormalization Scheme

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TMD Collaboration

Based on 2008.03886, in collaboration with X. Ji, Y.-Z. Liu, A. Schaefer, W. Wang, Y.-B. Yang and J.-H. Zhang.

Outline

- The Hybrid Renormalization Scheme
  - Review of existing renormalization schemes
  - Hybrid renormalization scheme
- Large-Momentum and Coordinate-Space Factorizations
- Application to Lattice Data

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LaMET calculation of the PDF

Bare quasi light-front correlation (QLFC):

$$\tilde{h}(\lambda, P^z, a) = \langle P \,|\, \bar{\psi}_0(z)\gamma^0 W_0[z,0]\psi_0(0) \,|\, P\rangle/(2P^0)$$

[Ji, 1305.1539]

Quasi light-cone distance: $\lambda = zP^z$

Lattice renormalization:

$$\tilde{h}_X(\lambda, P^z, \mu_R) = \lim_{a\to 0} Z_X^{-1}(z, a, \mu_R)\ \tilde{h}(\lambda, P^z, a)$$

Quasi-PDF from Fourier Transform:

$$\tilde{f}_X(y, P^z, \mu_R) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi}\ e^{iy\lambda}\ \tilde{h}_X(\lambda, P^z, \mu_R)$$

Matching to the PDF through large-momentum expansion:

$$\tilde{f}_X(y, P^z, \mu_R) = \int_{-1}^{1} \frac{dx}{|x|} C_X\left(\frac{y}{x}, \frac{\mu}{xP^z}, \frac{\mu_R}{\mu}\right) f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2}, \frac{\Lambda_{QCD}^2}{((1-y)P^z)^2}\right)$$

$$f = C^{-1} \otimes \tilde{f}$$

[Ji, 1305.1539, 1404.6680]
[Xiong, Ji, Zhang and YZ, 1310.7471]
[Ma and Qiu, 1404.6860]
[Izubuchi, YZ et al., 1801.03917]
[Ji, Liu, Schaefer et al., 2008.03886]

Renormalization schemes

$$O_B^\Gamma(z,a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0) = e^{\delta m|z|}\, Z_{j_1}(a) Z_{j_2}(a) O_R^\Gamma(z)$$

[Ji, Zhang and YZ, 1706.08962; Ishikawa, Ma, Qiu and Yoshida, 1707.03107, Green, Jansen and Steffens, 1707.07152.]

Mass subtraction: $Z_X = e^{\delta m|z|} Z_{j_1} Z_{j_2}$

[Musch et al., 1011.1213;
Ishikawa, Ma, Qiu and Yoshida, 1609.02018;
Chen, Ji and Zhang, 1609.08102;
Green, Jansen and Steffens, 1707.07152.]

- δm: includes linear divergence, can be determined from e.g. static $q\bar{q}$ potential, etc.;
- Z_j: Renormalization of the "heavy-to-light" current, independent of z;
- Corresponds to the MSbar scheme.

RI/MOM: $Z_X = \langle q \,|\, O^\Gamma(z) \,|\, q \rangle$

[Constantinou and Panagopoulos, 1705.11193;
I. Stewart and YZ, 1709.04933;
C. Alexandrou et al., 1706.00265;
Chen et al., 1706.01295.]

- Perturbative window: $\Lambda_{\rm QCD}^2 \ll -q^2 \ll 1/a^2$
- Still introduces nonperturabtive z-dependence as $z \gtrsim \Lambda_{\rm QCD}^{-1}$

Ratio schemes: $Z_X = \langle P_0^z = 0 \,|\, O^\Gamma(z) \,|\, P_0^z = 0 \rangle$ [A. Radyushkin, 1705.01488; K. Orginos, et al., 1706.05373]

$Z_X = \langle \Omega \,|\, O^\Gamma(z) \,|\, \Omega \rangle$

$P_0^z \neq 0$ considered by [X. Gao, et al. (BNL/SBU/THU), 2007.06590].

[Braun, Vladimirov and Zhang, 1810.00048;
Li, Ma and Qiu, 2006.12370.]

- Introduces higher-twist effects as $z \gtrsim \Lambda_{\rm QCD}^{-1}$.

Renormalization schemes

Scheme	Short-distance	Long-distance
Mass subtraction	Not the expected $\ln z^2$ behavior due to discretization effects when z~a, e.g., the function $\ln\left[(z^2+a^2)/a^2\right]$	Works well except for the $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity in the determination of Wilson line mass correction.
RIMOM	$\ln z^2$ dependence cancelled out, therefore the discretization error is cancelled to a large degree	Uncontrolled nonperturbative z-dependence
Ratios	$\ln z^2$ dependence cancelled out, therefore the discretization error is cancelled to a large degree	Uncontrolled higher-twist effects

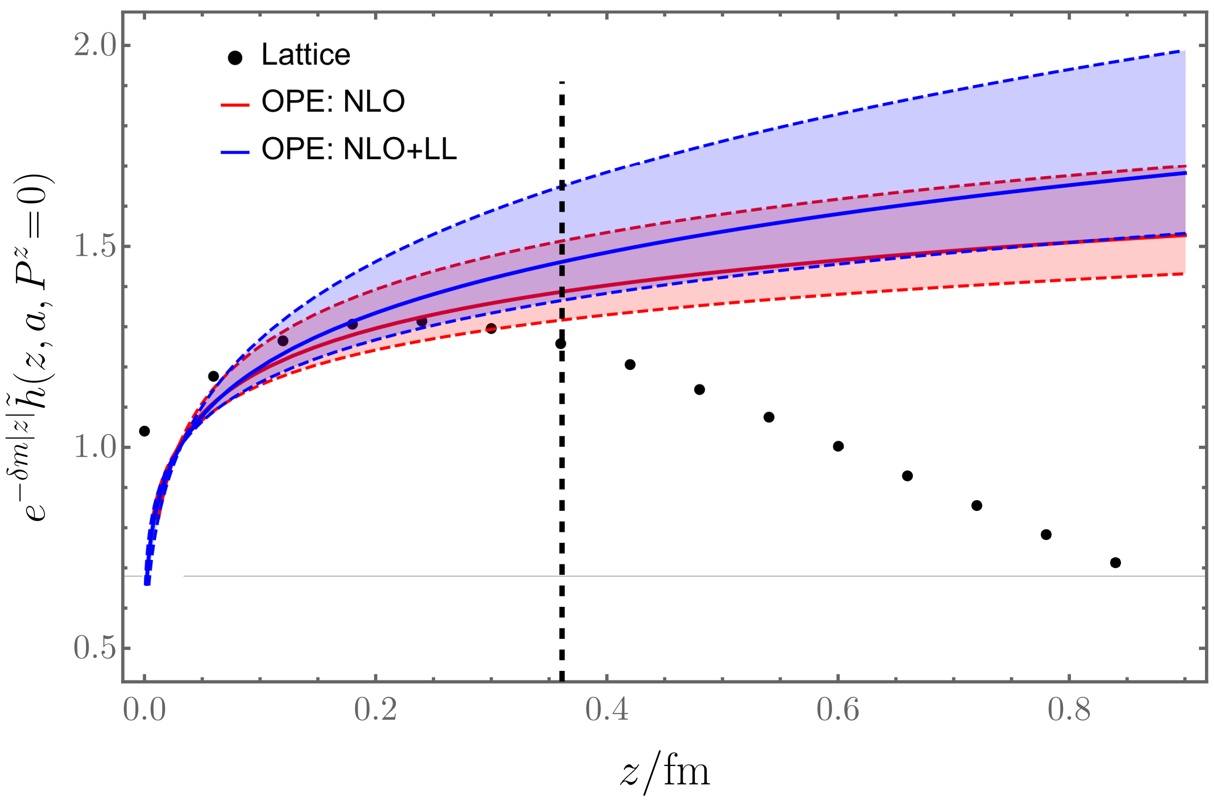
A cancellation of higher-twist effects in the ratio? Cannot be quantified.

Hybrid renormalization scheme

- For short distance $0 \leq z \leq z_S$, use RI/MOM or ratio schemes:

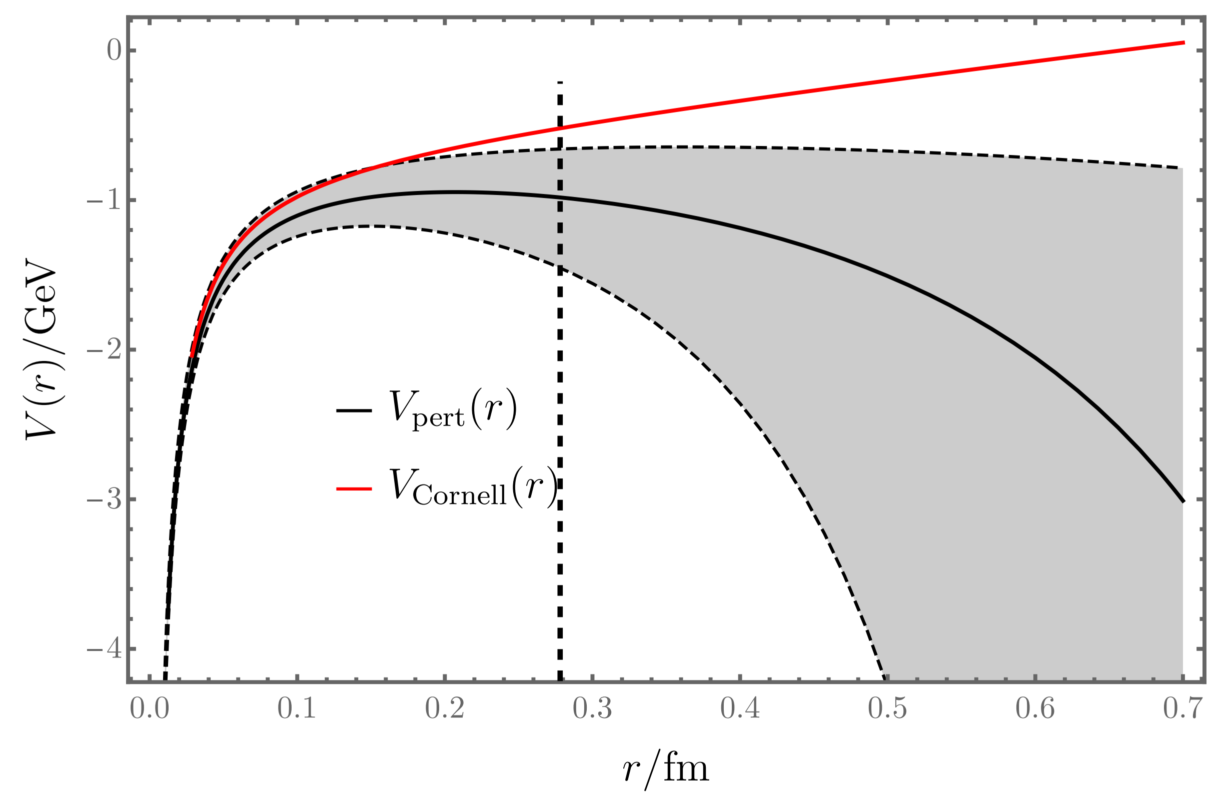
How short should z_S be?

$z_S \leq z_{\rm LT}, \qquad z_{\rm LT} \sim 0.3\ {\rm fm}$



Comparison of $P^z = 0$ lattice matrix element [X. Gao, et al. (BNL/SBU/THU), 2007.06590] (a=0.06 fm) after δm subtraction with one-loop OPE

[Ji, Liu, Schaefer and YZ et al., 2008.03886]



Comparison of 3-loop perturbative prediction of the $q\bar{q}$ potential with calculation from lattice QCD

[G.S. Bali et al., 0001312; C. Aubin et al., 0402030]

Hybrid renormalization scheme

- For long distance $z_S < z \le z_L$, use Wilson-line mass subtraction scheme:

z_L is limited by lattice volume and acceptable uncertainty.

$$O_R^\Gamma(z, \mu_R) = Z_{\text{hybrid}}(a, \mu_R) e^{-\delta m |z|} O_B^\Gamma(z, a)$$

$$\delta m = \frac{m_{-1}}{a} + m_0$$

Z_{hybrid}: independent of z

Determine δm by fitting $e^{\delta m |z|}$ to:

- $\langle P | O_B^\Gamma(z, a) | P \rangle$
- $\langle q | O_B^\Gamma(z, a) | q \rangle$ [Izubuchi et al., 1905.06349; Huo and Sun, 1912.06056.]
- $\langle \Omega | O_B^\Gamma(z_S, a) | \Omega \rangle$
- The gauge-invariant Polyakov loop
- $\langle \Omega | W[z, a] | \Omega \rangle$ in a fixed gauge

[Green, Jansen and Steffens, 1707.07152, 2002.09408.]

m_{-1} should be universal for different methods, whereas m_0 has an uncertainty of $\mathcal{O}(\Lambda_{\text{QCD}})$.

$$\tilde{f}'(z, P^z) = e^{-\delta m_0 |z|} \tilde{f}(z, P^z)$$

$$\longrightarrow |\tilde{f}(y, P^z) - f(y, P^z)| \sim \frac{\delta m_0}{P^z}$$

See J.-H. Zhang's talk for further discussion.

Hybrid renormalization scheme

- Matching the renormalized matrix elements at $z = z_S$:

$$Z_{\rm hybrid} e^{-\delta m |z_S|} \langle P | O_B^\Gamma(z_S, a) | P \rangle = \frac{\langle P | O_B^\Gamma(z_S, a) | P \rangle}{Z_X(z_S, a)},$$

$$\longrightarrow Z_{\rm hybrid}(z_S, a) = e^{\delta m |z_S|} / Z_X(z_S, a)$$

For example, for $Z_X(z_S, a) = \langle P_0^z = 0 | O_B^\Gamma(z, a) | P_0^z = 0 \rangle$, the matching coefficient for the corresponding quasi-PDF is

$$C_{\rm hybrid}(\xi, \mu^2/p_z^2, z_S^2\mu^2) = C_{\rm ratio}(\xi, \mu^2/p_z^2) + \frac{\alpha_s C_F}{2\pi} \frac{3}{2} \left[-\frac{1}{|1-\xi|_+} + \frac{2{\rm Si}((1-\xi)\lambda_S)}{\pi(1-\xi)} \right]$$

$$\frac{1}{|1-\xi|_+} \equiv \lim_{\beta \to 0^+} \left[\frac{\theta(|1-\xi| - \beta)}{|1-\xi|} + 2\delta(1-\xi)\ln\beta \right] \qquad \xi = \frac{y}{x}, \quad \lambda_s = z_S p^z, \quad p^z = x P^z$$

[Ji, Liu, Schaefer and YZ et al., 2008.03886]

Hybrid renormalization scheme

- When z_L is larger than the correlation length of the spacelike correlator ($\propto P^z$), the latter falls close to zero. The uncertainty from truncated Fourier transform (FT) at z_L will be negligible;

- Otherwise, for $z > z_L$, one can extrapolate to $z = \infty$ to do the FT:

  - The contribution from $z > z_L$ affects the small-x region;

  - The extrapolation can be achieved by using models inspired by the Regge-behavior;

  - The extrapolation **does not** provide prediction for small-x PDF, but it changes the small-x behavior in the right direction.

See J. Zhang and J. Hua's talks for further discussions.

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Large-Momentum V.S. Coordinate-Space Factorizations

- Equivalent when $P^z \to \infty$;

[Ji, Zhang and YZ, 1706.07416; Izubuchi, Ji, Jin, Stewart and YZ, 1801.03917]

- However, when P^z is finite, the two approaches are practically different.

Large-Momentum Expansion:

$$\tilde{f}_X(y, P^z) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-y)P^z)^2}\right)$$

- Quasi-PDF is obtained by integrating over $-\infty < z < \infty$;
- Due to power corrections, a sub region $[x_{\rm min}, x_{\rm max}]$ is under systematic control for finite P^z, which can be improved with larger momenta;
- Direct computation of the x-dependence of PDF.

Large-Momentum V.S. Coordinate-Space Factorizations

- Equivalent when $P^z \to \infty$;

[Ji, Zhang and YZ, 1706.07416; Izubuchi, Ji, Jin, Stewart and YZ, 1801.03917]

- However, when P^z is finite, the two approaches are practically different.

Coordinate-Space Factorization (CSF):

$$\tilde{h}(\lambda, z^2\mu^2) = \int_0^1 d\alpha\ \mathscr{C}(\alpha, z^2\mu^2)\ h(\alpha\lambda, \mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

$$\tilde{h}(\lambda, z^2) = \sum_{n=0}^{\infty} \frac{(-i\lambda)^n}{n!} C_n\left(z^2\mu^2\right) a_n(\mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

"Ioffe-time distribution", [Radyushkin, 1705.01488; Orginos, et al., 1706.05373]
Current-current correlator, [Braun and Mueller, 0709.1348; Ma and Qiu, 1709.03018]

- The requirement for $z \le z_{\rm LT} \sim 0.3\ \text{fm}$ severely limits $\lambda_{\rm max}$ for available P^z, which makes FT unreliable;
- One has to parametrize the PDF with models, which are not unique and usually have broad parameter space; [Izubuchi, et al., 1905.06349; Sufian, 2001.04960]
- For finite $\lambda_{\rm max}$, only sensitive to several moments due to suppression by $\lambda^n/n!$.

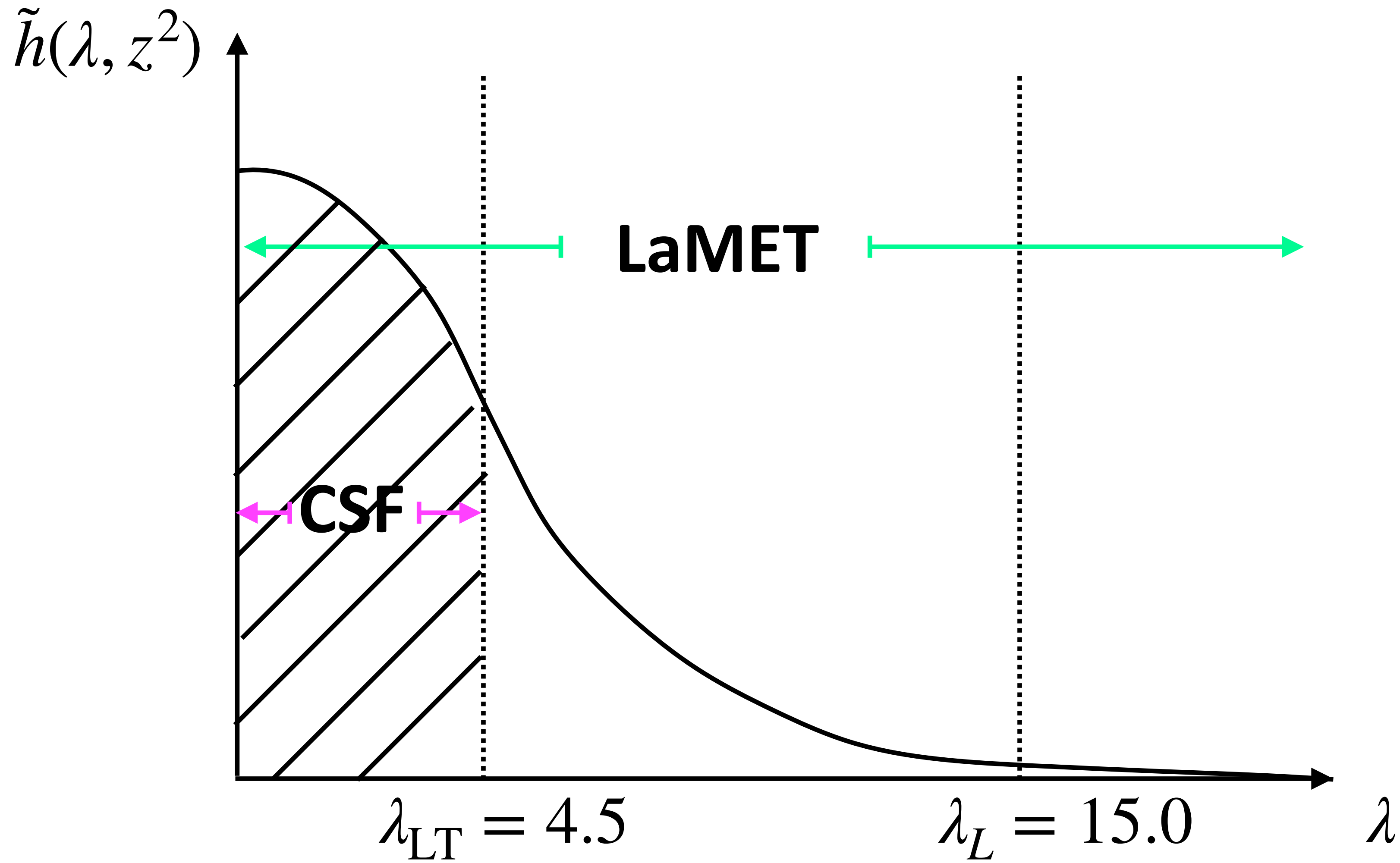
Large-Momentum V.S. Coordinate-Space Factorizations

e.g.,

$z_{\rm LT} = 0.3$ fm, $z_L = 1.0$ fm

$P^z = 3.0$ GeV

$\lambda_{\rm LT} = 4.5$, $\lambda_L = 15.0$

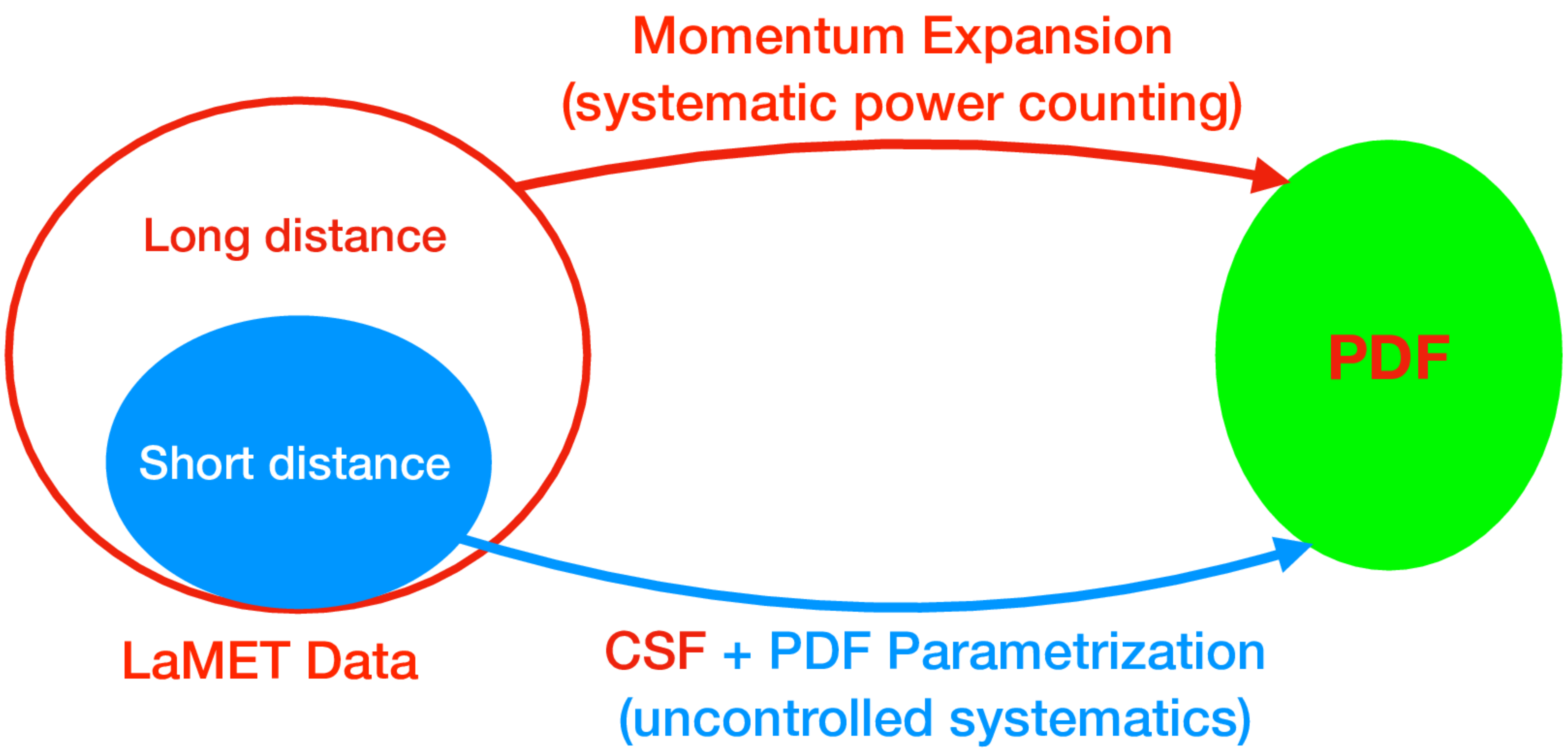


In recent CSF applications:

- $z_{\rm max} = 1.0$ fm • by B. Joo, J. Karpie, et al.,1909.08517;
- 0.72 fm • by B. Joo, J. Karpie, et al., 2004.01687;
- 0.72 fm • by X. Gao et al., 2007.06590;
- 0.75 fm • by M. Bhat, K. Cichy et al., 2005.02102.

Beyond NLO matching, it requires running the coupling from μ to $\sim 1/z$, which could hit the Landau pole if z is too large.

Large-Momentum V.S. Coordinate-Space Factorizations



[X. Ji, 2007.06613.]

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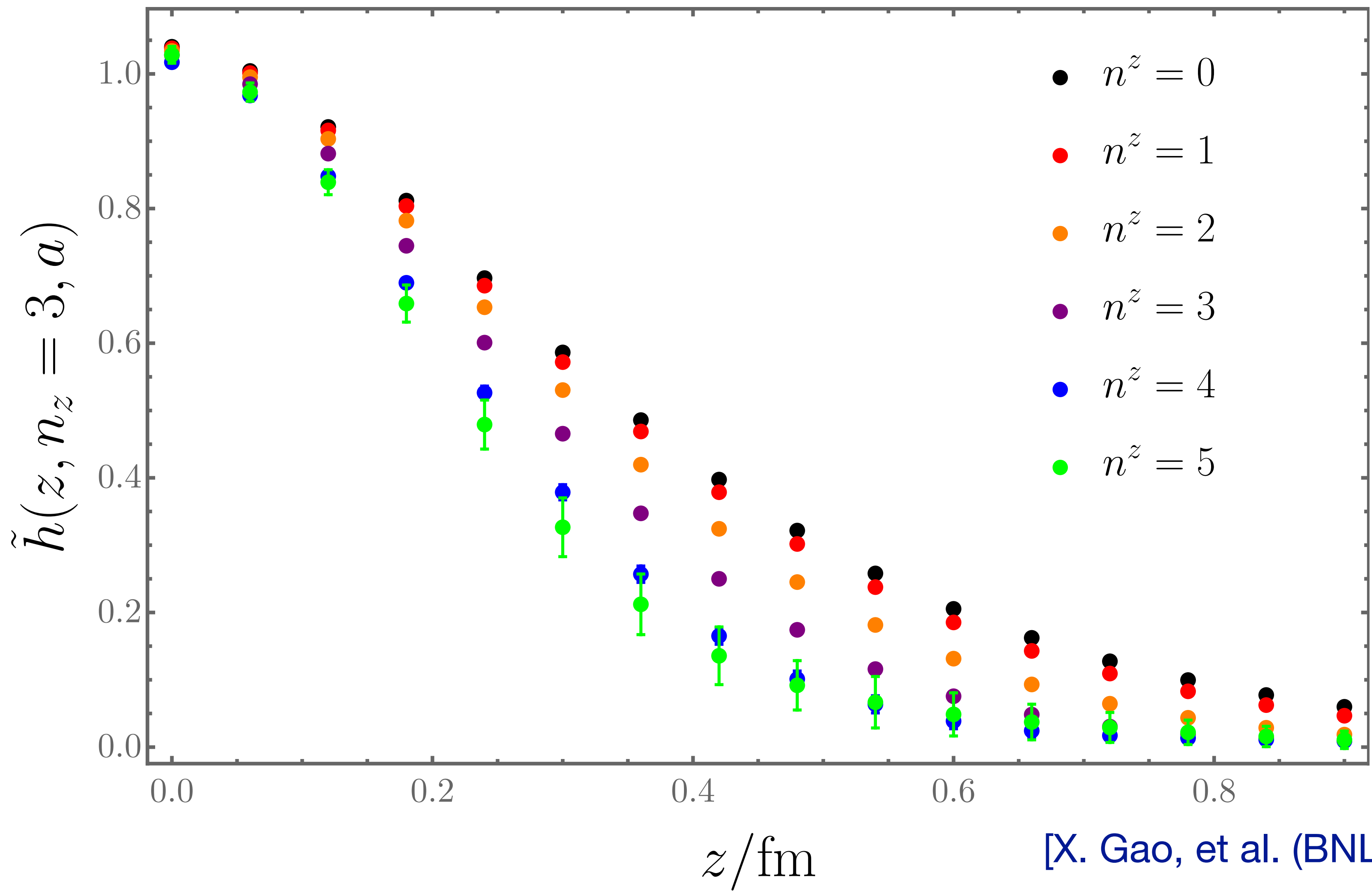
Preliminary!

Lattice matrix elements

- Bare matrix elements for pion valence quasi-PDF:

$$a = 0.06\ \text{fm}\,, \quad L = 48a\,, \quad m_\pi^{\text{v}} = 300\ \text{MeV}\,, \quad m_\pi^{\text{sea}} = 160\text{MeV}\,.$$

$$n^z = \{0,1,2,3,4,5\}\,, \quad P^z = n^z\frac{2\pi}{L} = \{0,\ 0.43,\ 0.86,\ 1.29,\ 1.72,\ 2.15\}\text{GeV}$$

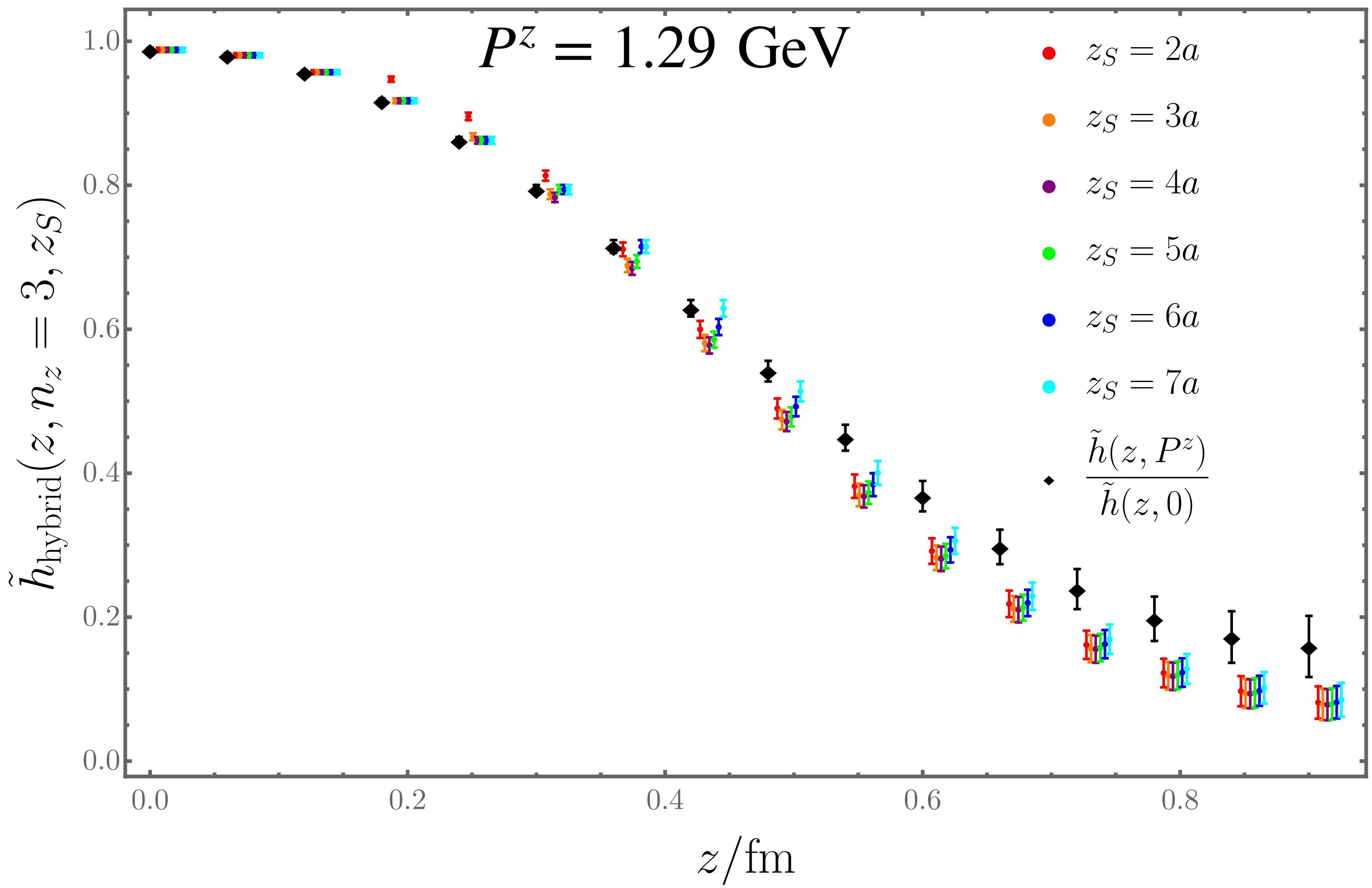


[X. Gao, et al. (BNL/SBU/THU), 2007.06590]

Lattice matrix elements

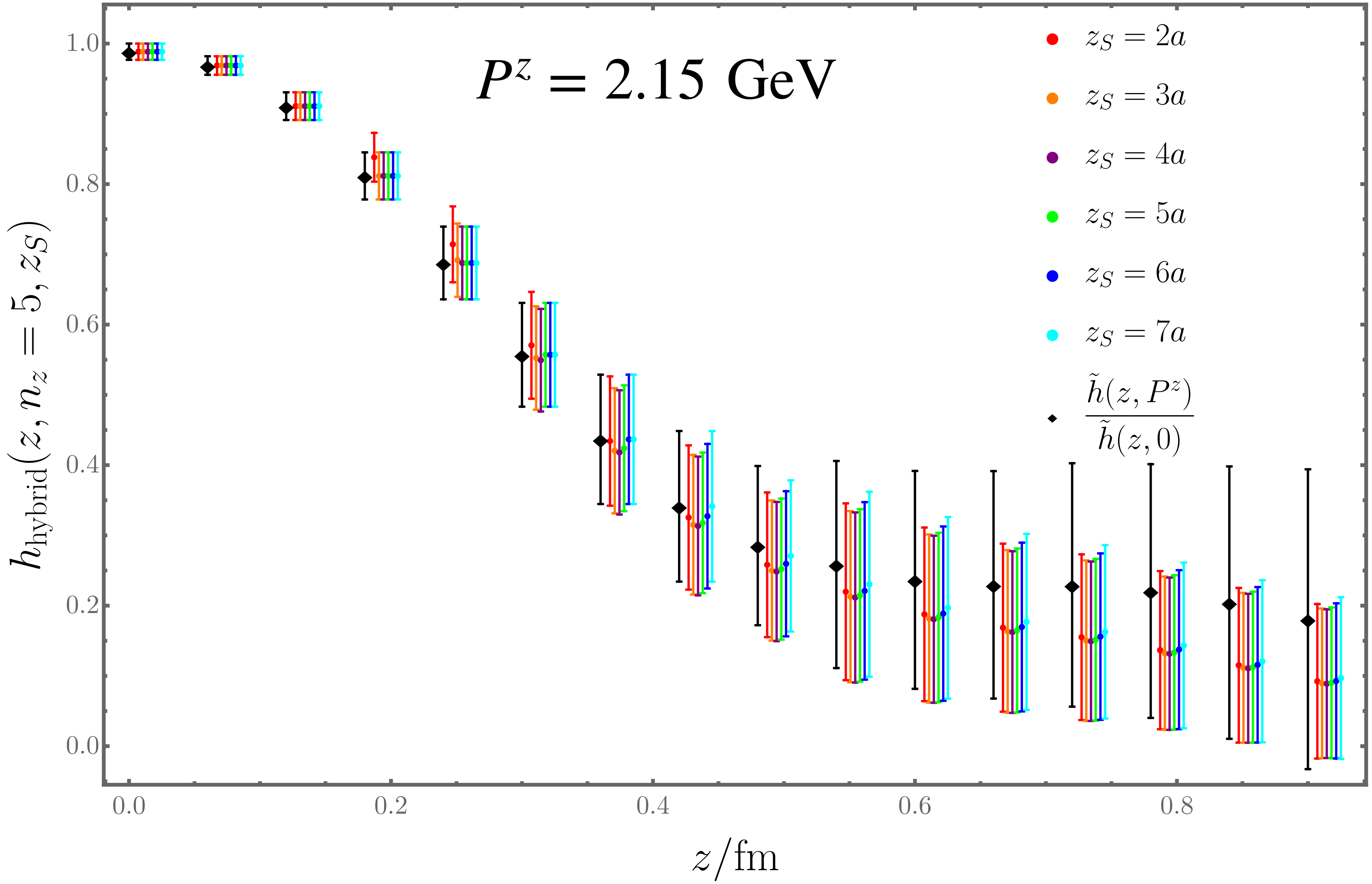
- Renormalized matrix element in the hybrid scheme

$\delta m a = 0.1586$ from static $q\bar{q}$ potential [Izubuchi, et al., 1905.06349]



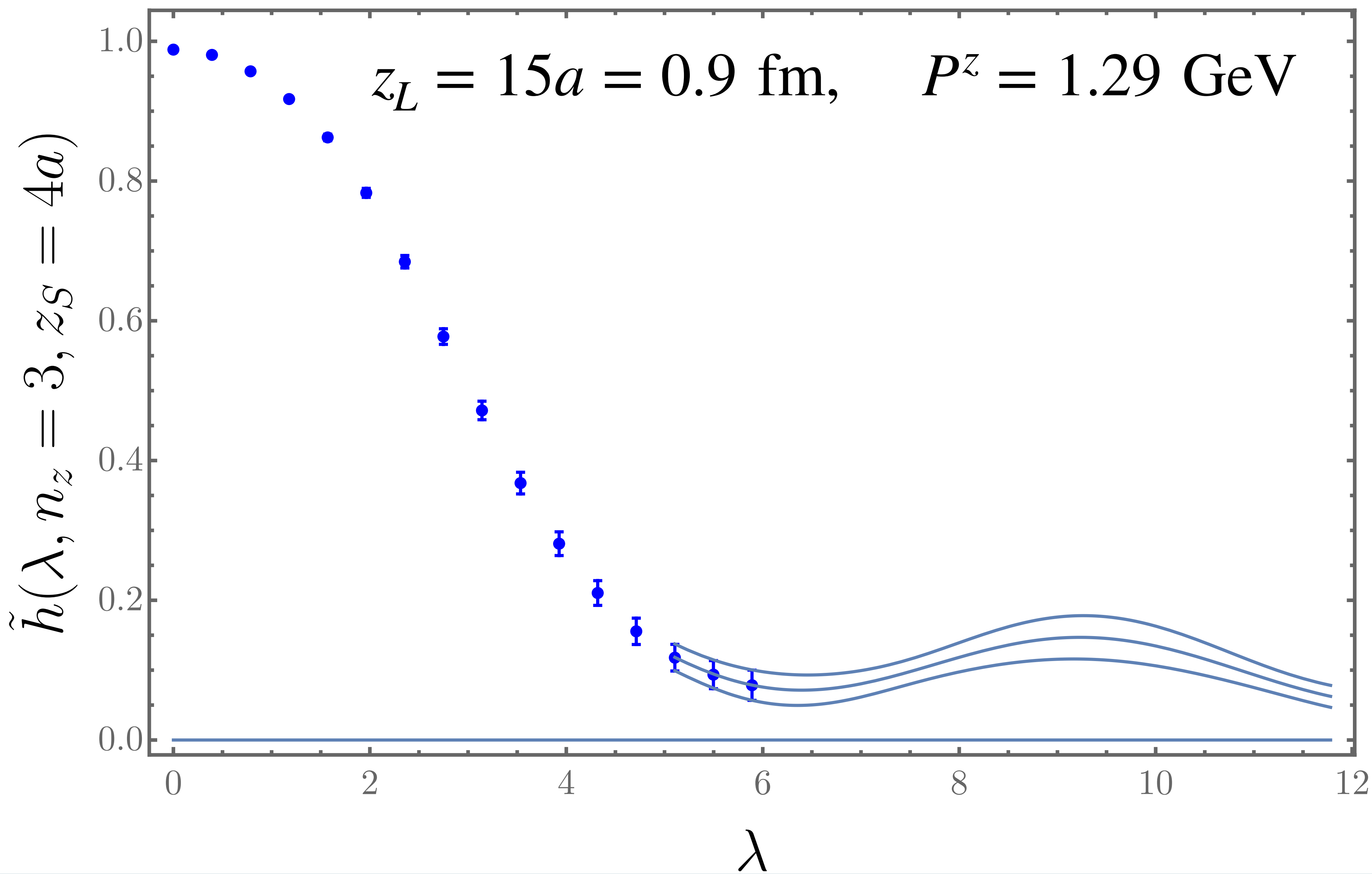
Lattice matrix elements

- Renormalized matrix element in the hybrid scheme



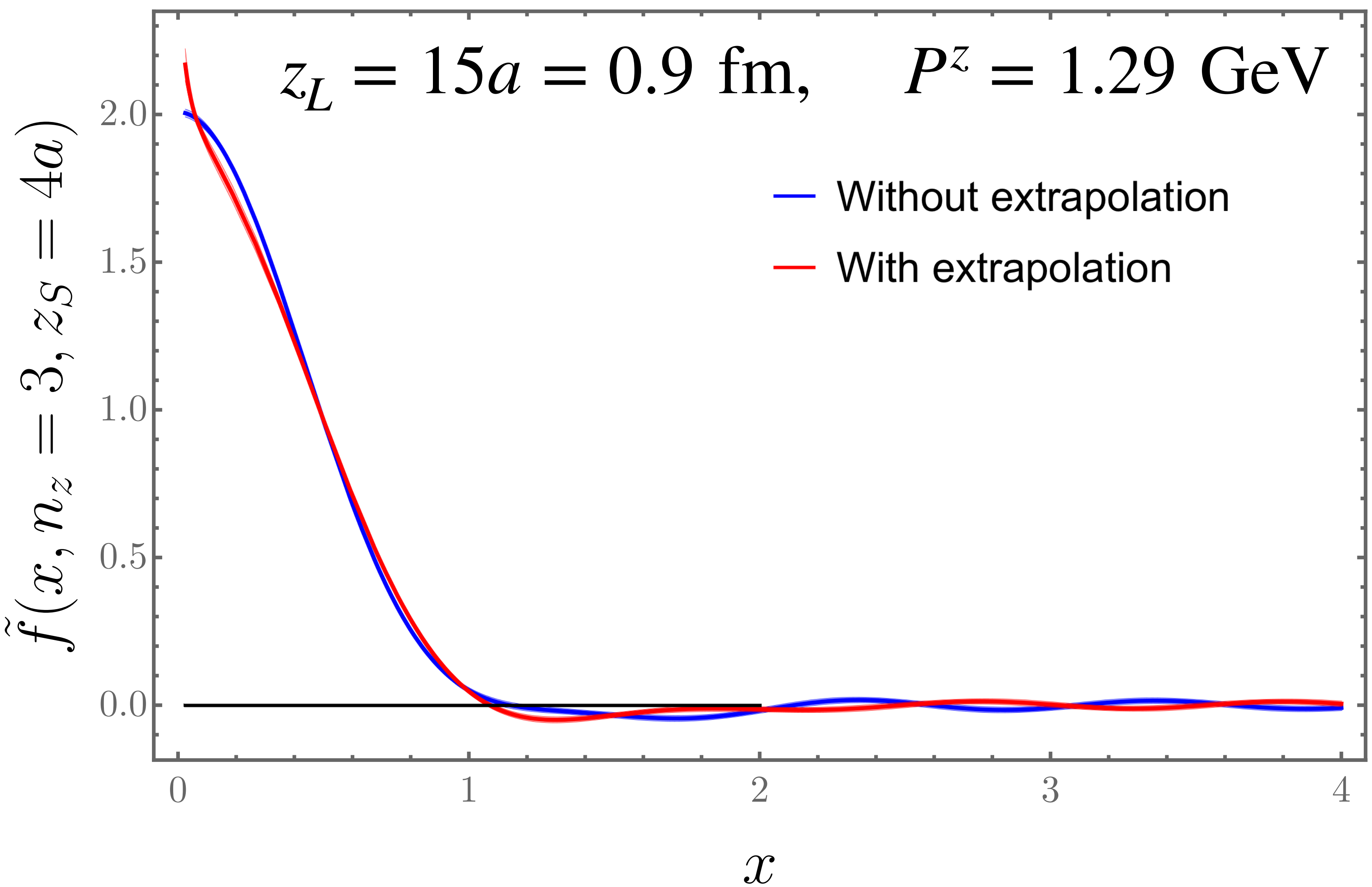
Quasi-PDF from the Fourier transform

- Extrapolation beyond z_L: $$\tilde{h}(\lambda) = \frac{c_1 + c_2 \cos(\lambda)}{|\lambda|^d}$$



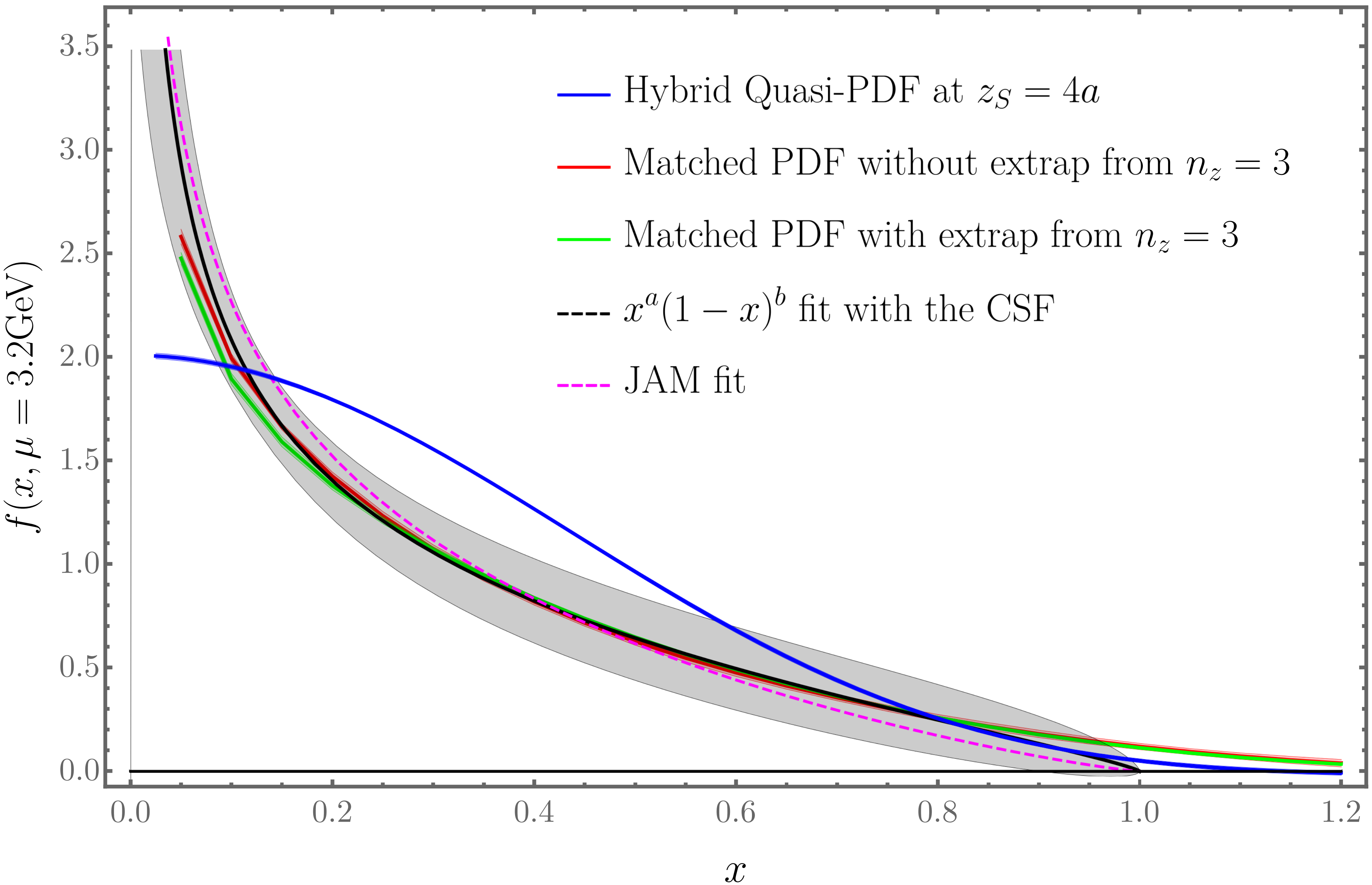
Quasi-PDF from the Fourier transform

- Fourier transform (FT): Discrete FT for $z \leq z_L$ and analytical FT for $z > z_L$.



Perturbative matching

$z_L = 15a, \quad P^z = 1.29\ \text{GeV}$



Perturbative matching

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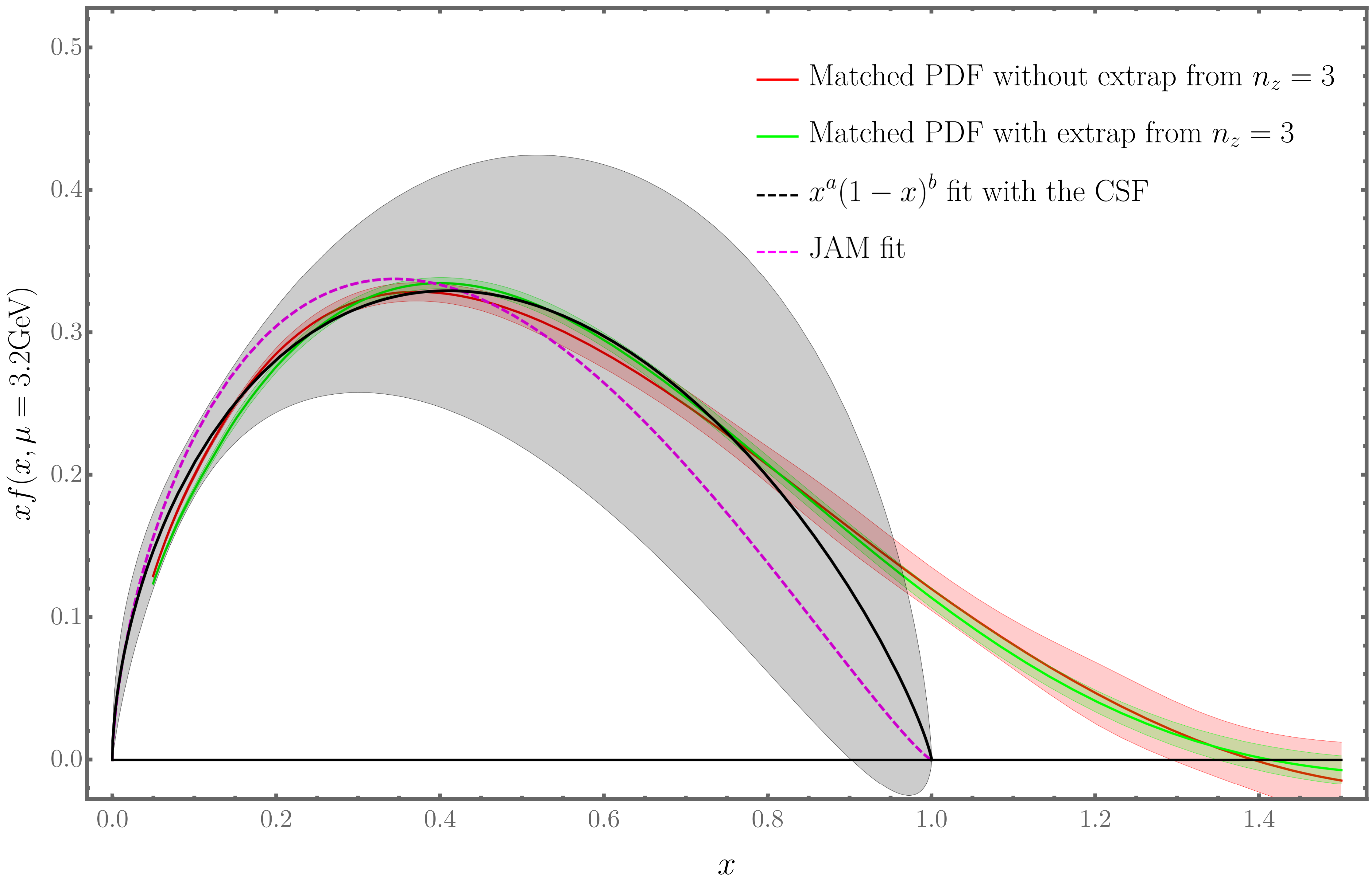
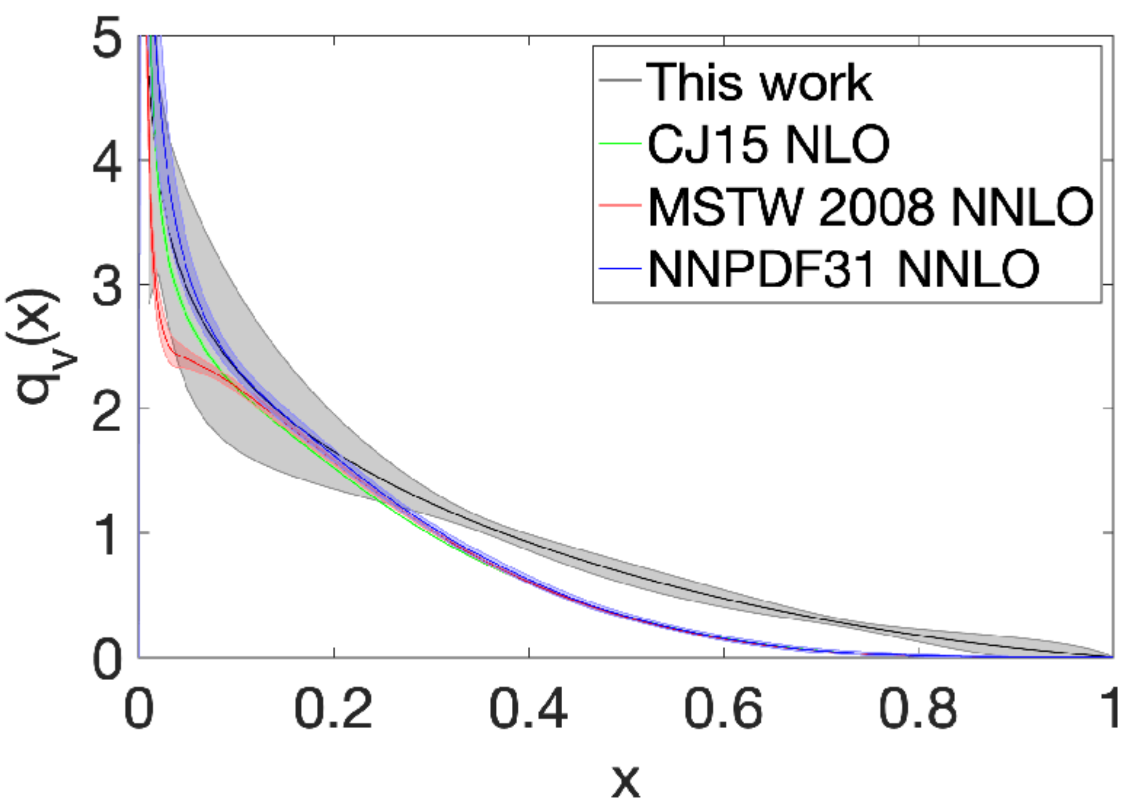


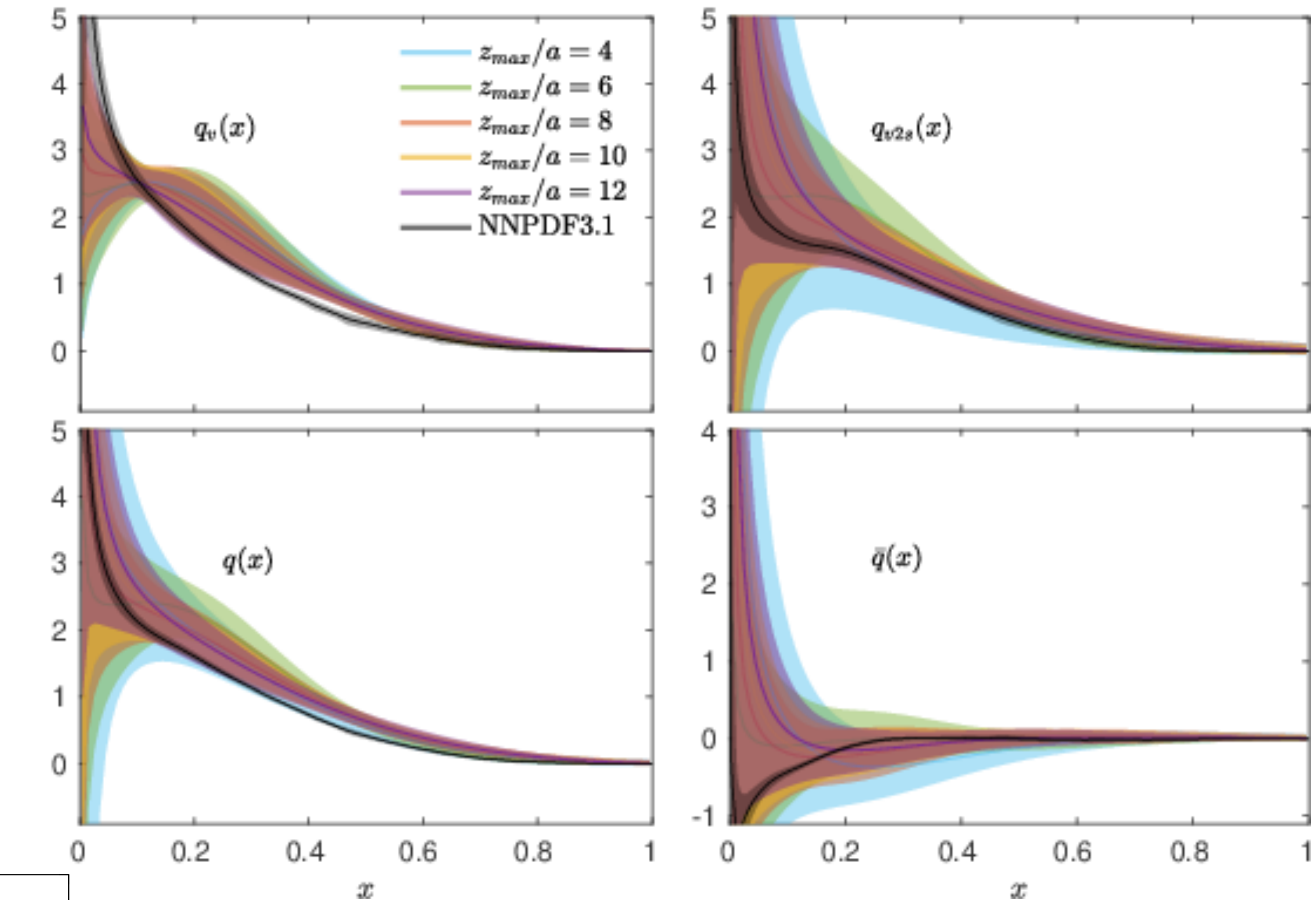
Conclusion

- The hybrid renormalization scheme has been introduced for renormalizing QLFC at short and large distances and for a smooth FT;
- The large-momentum expansion can be used to directly obtain the x-dependence in an effective range with systematic control, whereas CSF approaches relies on the model assumption of the PDF;
- A preliminary application of the hybrid scheme shows that it meets the expectation.

Backup Slides



[B. Joo, J. Karpie, et al., 2004.01687]



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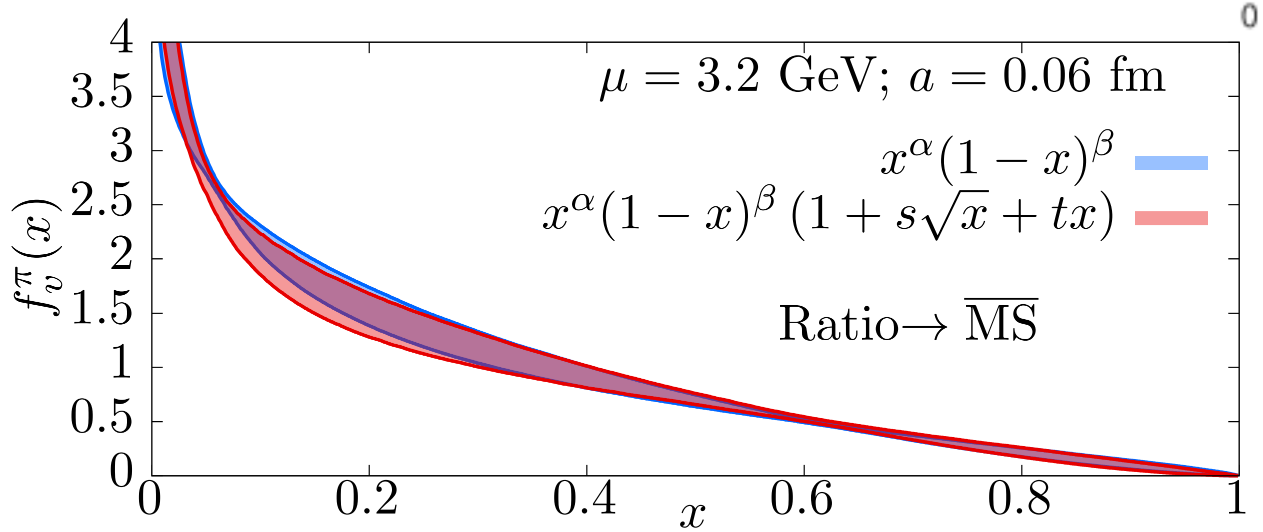

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