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Nucleon Gluon Distribution Function from 2+1+1-Flavor Lattice QCD

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Based on: **2007.16113**, in collaboration with

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LAMET 2020

09/11/2020

Outline

- Background
- Lattice setup and matrix element
- Results
  - Extrapolation to physical pion mass
  - Reduced and evolved ITDs fit
  - Comparison with global fit gluon PDFs
- Conclusion and outlook

Parton distribution function

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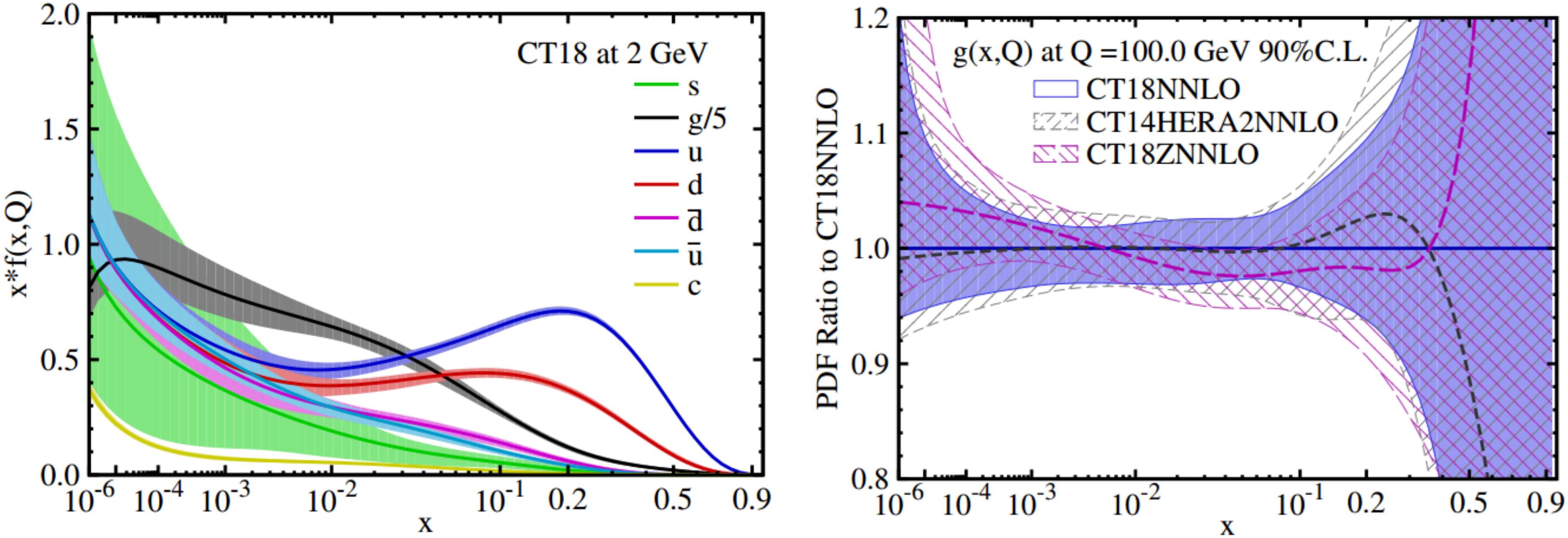
Parton Distribution Functions (PDF) represents the probability densities to find a parton carrying a momentum fraction x at energy scale Q.

arXiv:1912.10053v2



Gluon PDF dominates and its relatively error is large at large x region

To improve the gluon PDF,

- Global fit with more data from experimental runs
- Theoretical calculation

Large Momentum Effective Theory

Large Momentum Effective Theory (LaMET) [**PRL 110.26 (2013): 262002**]. is a direct approach to compute parton physics on a Euclidean lattice through Lorentz boost.

The unpolarized gluon distribution in the nucleon in the light-cone coordinates

$$g(x,\mu^2) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ix\xi^- P^+} \left\langle P \left| F_\mu^+(\xi^-) \mathcal{P} \exp\left(-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right) F^{\mu+}(0) \right| P \right\rangle$$

The unpolarized gluon quasi-PDF is defined as

$$\tilde{g}(x,\mu^2,P^z) = \int \frac{dz}{2\pi x P^z} e^{ixzP^z} \left\langle P \left| F_\mu^z(z) \mathcal{P} \exp\left(-ig \int_0^{z} d\eta A^z(\eta)\right) F^{\mu z}(0) \right| P \right\rangle$$

The quasi-PDF can be related to the P_z-independent light-front PDF through,

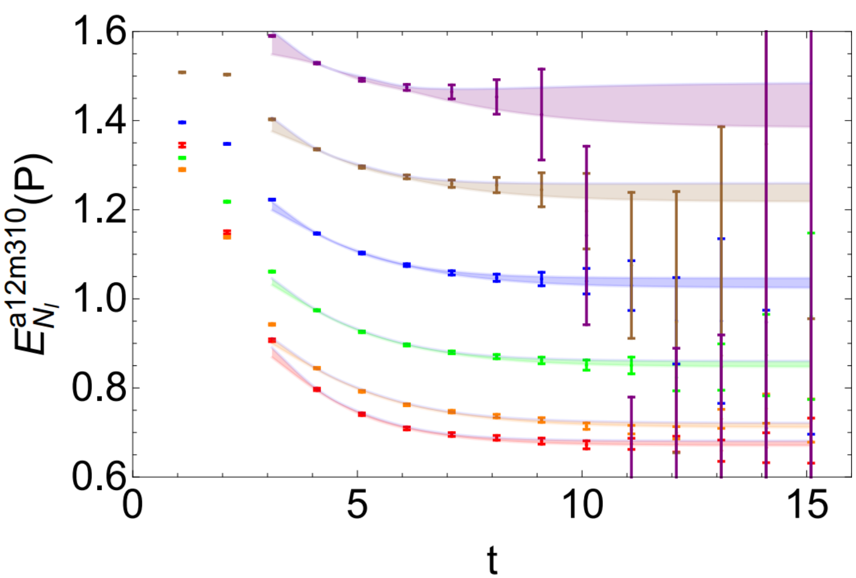
$$\tilde{g}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) g(y,\mu^2) + \boldsymbol{\mathcal{O}}(\left(\frac{M}{P^z}\right)^2, \left(\frac{\Lambda_{QCD}}{P^z}\right)^2)$$

Numerical setup

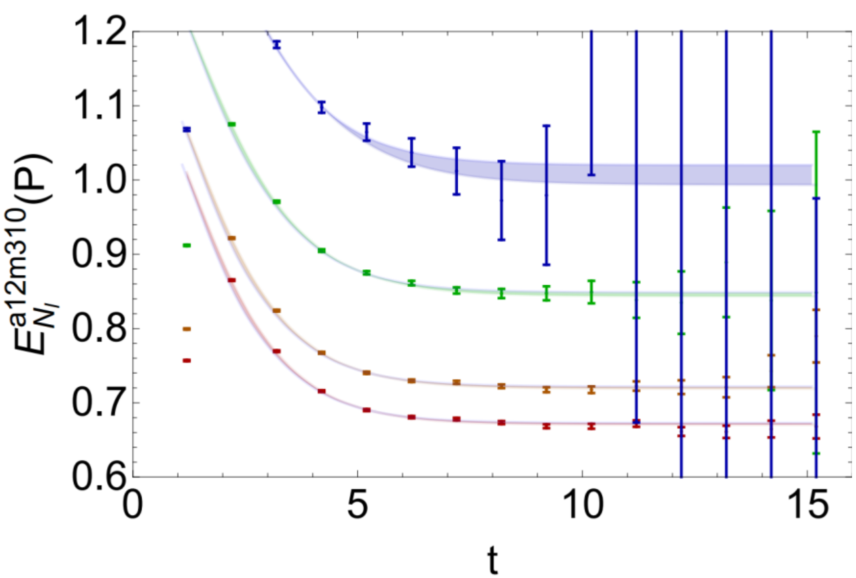
The lattice calculation are carried out with Clover valence quarks on the MILC $N_f = 2+1+1$ HISQ fermion gauge configurations with $L^3 \times T = 24^3 \times 64$, $\mathrm{a} = 0.1207(11)$ fm, $m_\pi^{sea} = 306.9(5)$ MeV, $N_{cfg} = 898$. The total measurements equal to 57472.

To reach higher-momentum states, we use **Gaussian momentum smearing** [1602.05525] for the quark field $\psi(x) + \alpha \sum_j U_j(x) e^{ik\hat{e}_j} \psi(x + \hat{e}_j)$.

- Momentum smearing parameter k=2.9



- PNDME data without momentum smearing



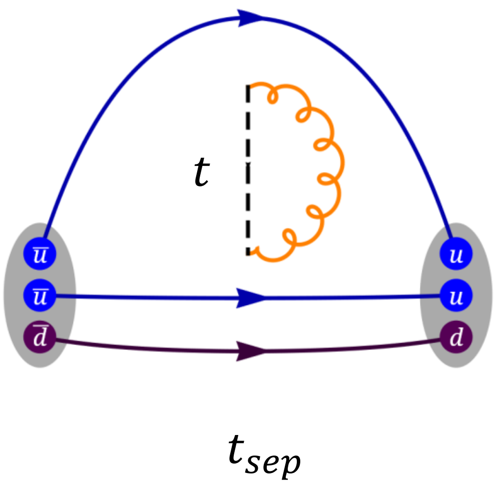
Gluon operator and ratio method

In the paper [arXiv:1910.13963], the unpolarized multiplicatively renormalizable gluon operators are defined,

$$O_g(z) = \sum_{i\neq z,t} O\left(F^{ti}, F^{ti}; z\right) - \sum_{i\neq z,t} O\left(F^{ij}, F^{ij}; z\right),$$

where the operator $O(F^{\mu\nu}, F^{\rho\sigma}; z) = F^{\mu\nu}(z)U(z,0)F^{\rho\sigma}(0)$.

We use the ratio of 3-point correlator and 2-point correlator to extract the ground state matrix element. The ratio is defined by,



$$R\left(z, P_z; t_{sep}, t\right) \equiv \frac{C_{3pt}\left(z, P_z; t_{sep}, t\right)}{C_{2pt}\left(z, P_z; t_{sep}, t\right)} = \frac{\langle 0|\Gamma^e \int d^3y e^{-iyP} \chi\left(\vec{y}, t_{sep}\right) O(z,t)\, \chi\left(\vec{0}, 0\right)|0\rangle}{\langle 0|\Gamma^e \int d^3y e^{-iyP} \chi\left(\vec{y}, t_{sep}\right)\, \chi\left(\vec{0}, 0\right)|0\rangle}$$

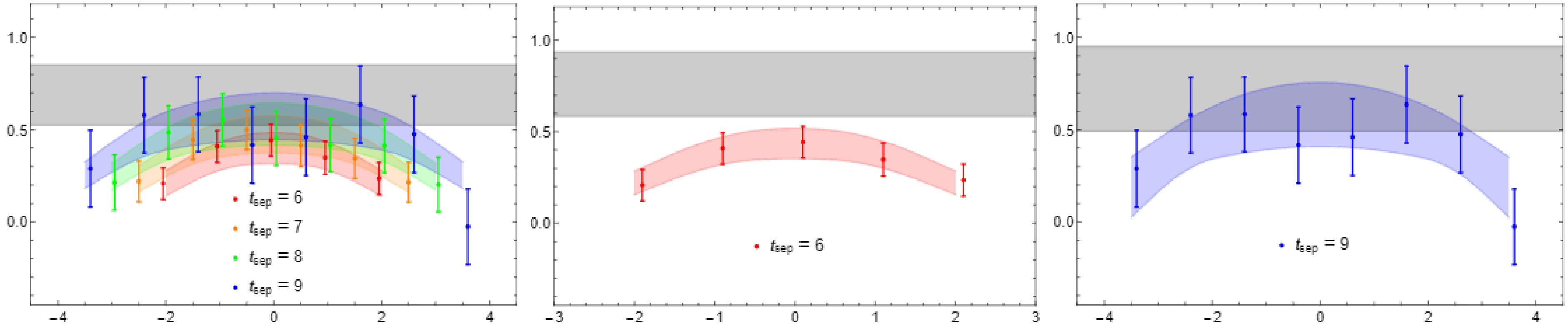
Ground state matrix element

Following the work [arXiv:1306.5435v3], the correlators C_{3pt} and C_{2pt} can be decomposed as,

$$C_{3pt}(z, P_z; t_{sep}, t) = |A_0|^2 \langle 0|O|0\rangle e^{-E_0 t_{sep}} + A_1 A_0^* \langle 1|O|0\rangle e^{-E_1(t_{sep}-t)} e^{-E_0 t} + A_0 A_1^* \langle 0|O|1\rangle e^{-E_0(t_{sep}-t)} e^{-E_1 t} + \cdots$$

$$C_{2pt}(z, P_z; t_{sep}) = |A_0|^2 e^{-E_0 t} + |A_1|^2 e^{-E_1 t} + \cdots$$

where the ground state matrix element is $\langle 0|O_g|0\rangle$.



The ground state matrix element extracted from the fit (grey band) from different fit choices are consistent within one sigma error.

Pseudo-PDF

The reduced Ioffe-time distribution (ITDs) definition arXiv:1706.05373,

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)/M(\nu, 0)}{M(0, z^2)/M(0,0)}$$

where $M(\nu, z^2) = \langle 0 | O_g(z) | 0 \rangle$, Ioffe-time $\nu = zP_z$.

The gluon pseudo-PDF matching condition,

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x, \mu^2)}{\langle x_g \rangle_{\mu^2}} \left(R_1(x\nu, z^2\mu^2) + R_2(x\nu)\right)$$

where $R_1(x\nu, z^2\mu^2)$ is the term related to evolution, $R_2(x\nu)$ is the term related to scheme conversion. The specific definition can be found in **arXiv:2007.16113**.

The evolved ITD definition,

$$G(\nu, z^2, \mu, M_\pi) = \mathcal{M}(\nu, z^2, M_\pi) + \int_0^1 du\, R_1(u, z^2\mu^2)\, \mathcal{M}(u\nu, z^2, M_\pi)$$

Extrapolation in the pion mass

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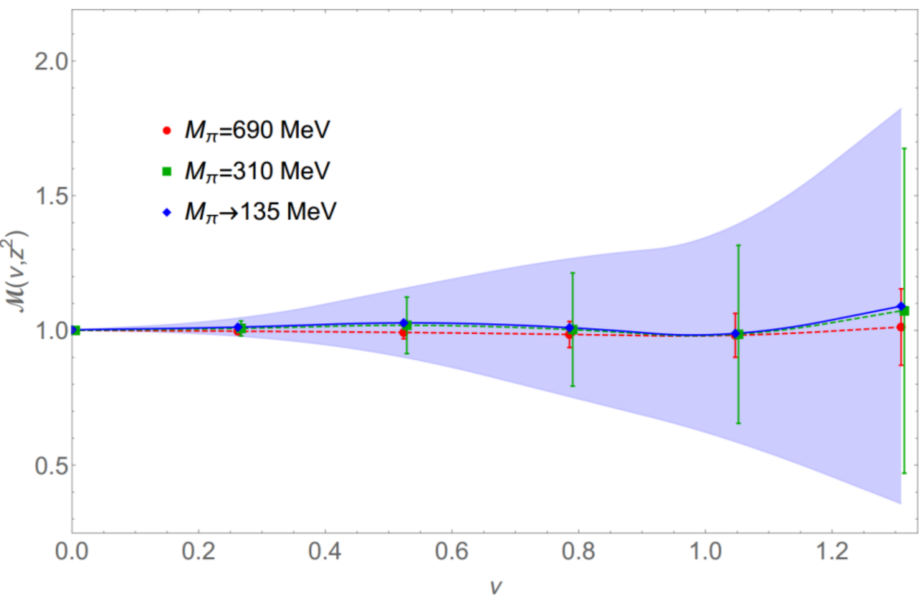
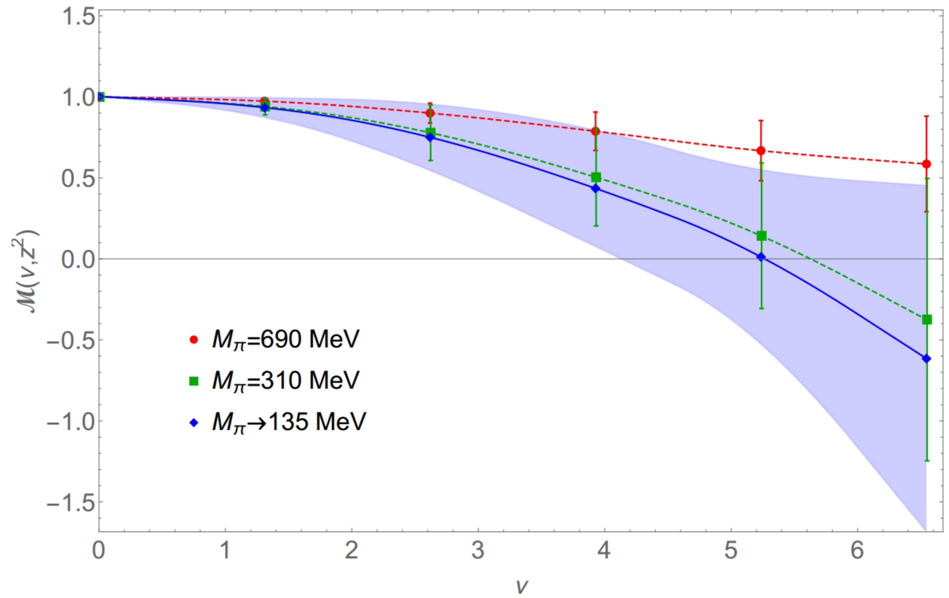
Nucleon momentum $P_z = \frac{2\pi}{L}$



Nucleon momentum $P_z = 5 * \frac{2\pi}{L}$



$$\mathcal{M}(\nu, z^2, M_\pi) = \mathcal{M}(\nu, z^2, M_\pi^{phys}) + K(\nu, z^2)(M_\pi^2 - \left(M_\pi^{phys}\right)^2)$$

The blue band show the uncertainties of the results after extrapolation to the physical pion mass.

Reduce ITDs and evolved ITDs

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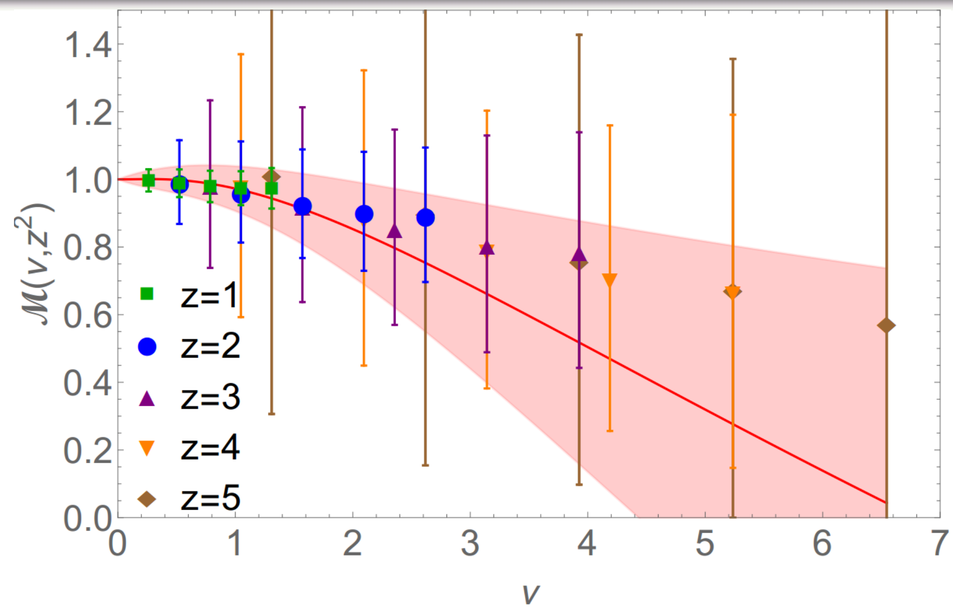
The reduced ITDs and their reconstructed fitted band from “z-expansion”,

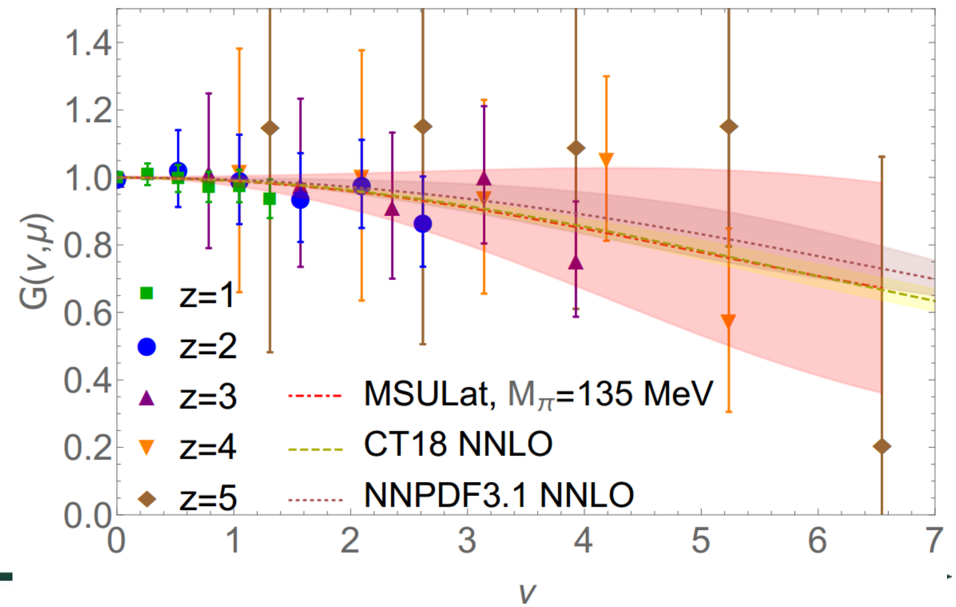
$$M(\nu, z^2, M_\pi) = \Sigma_{k=0}^{\text{kmax}} \lambda_k \tau^k,$$

$$\tau = \frac{\sqrt{\nu_{cut} + \nu} - \sqrt{\nu_{cut}}}{\sqrt{\nu_{cut} + \nu} + \sqrt{\nu_{cut}}}$$



The evolved ITDs and their reconstructed fitted band from gluon PDF functional form fit,

$$f_g(x, \mu) = \frac{x g(x, \mu)}{\langle x_g \rangle_{\mu^2}} = \frac{x^A (1-x)^C}{B(A+1, C+1)}$$



Comparison with the global fit PDFs

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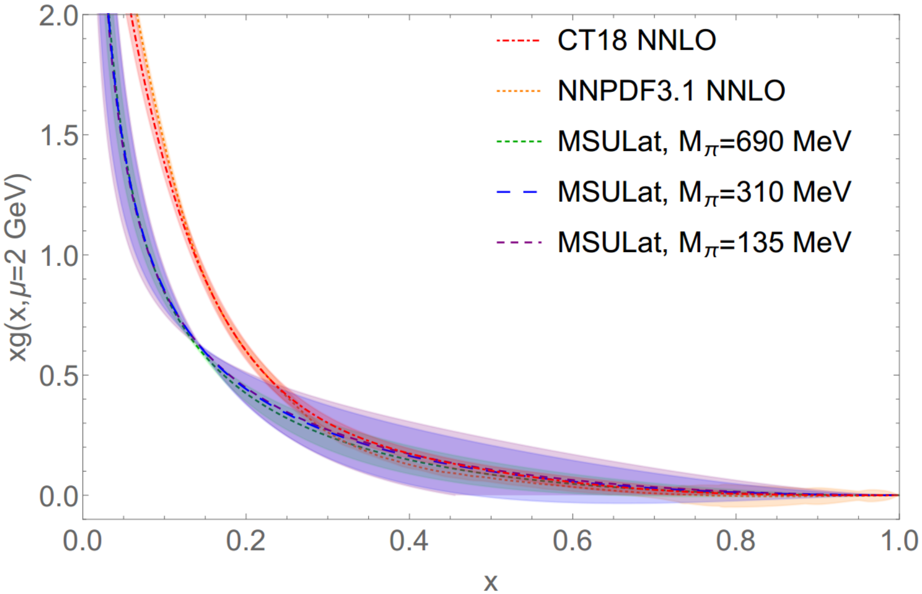
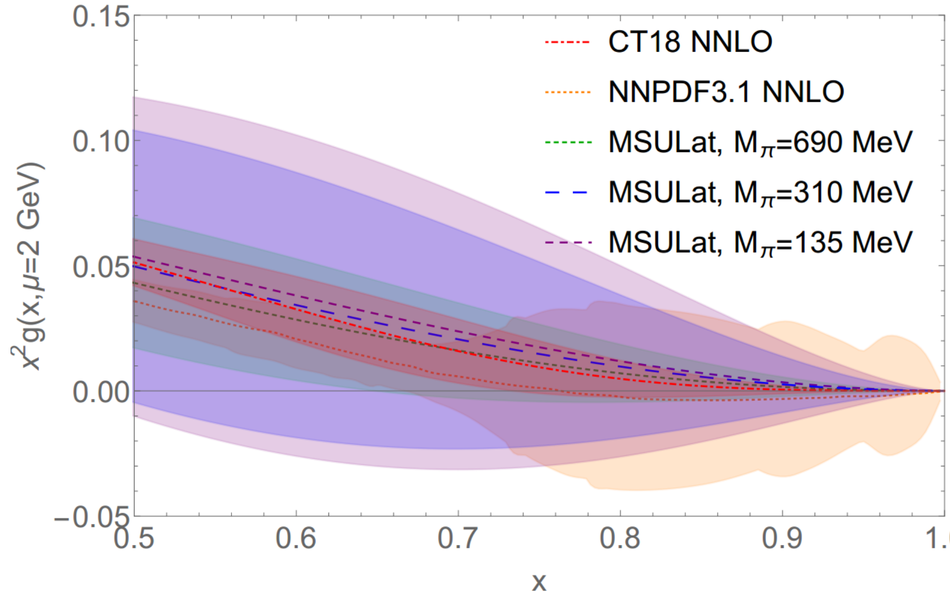
The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\mathrm{MS}}$ scheme.





Conclusion and outlook

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- We extract the gluon PDF as a function of Bjorken-x for the first time.There are systematics yet to be studied in this work.
- Future work is planned to study additional ensembles at different **lattice spacings** so that we can include the lattice-discretization systematics.
- Lighter **quark masses** should be used to control the chiral extrapolation to obtain more reliable results at physical pion mass.

Thank you!

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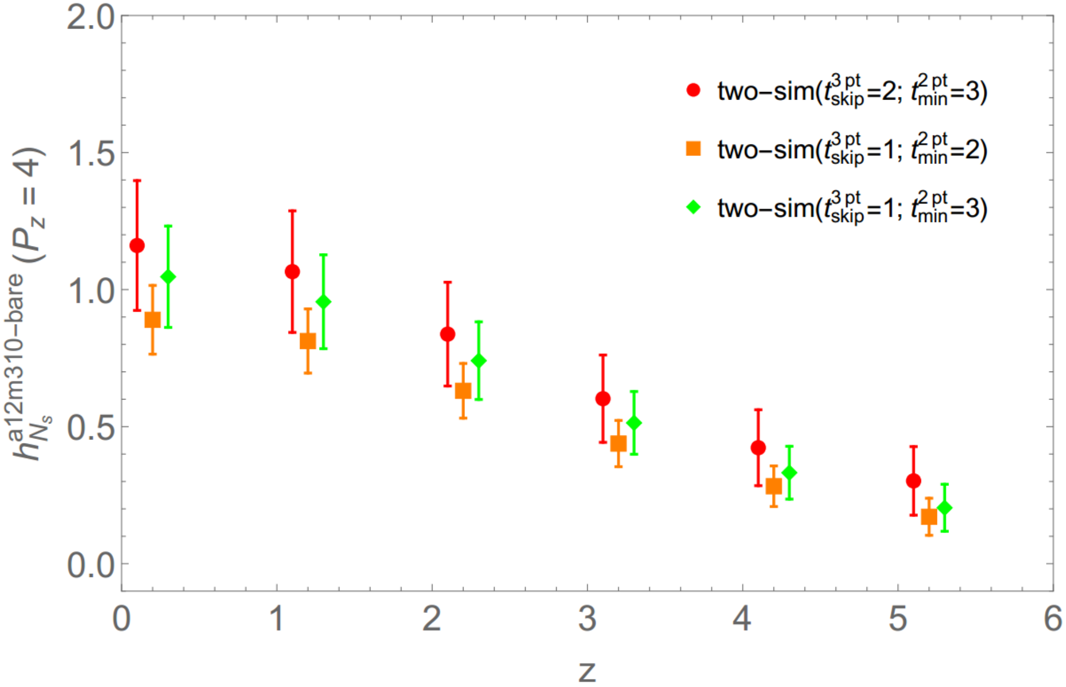
The fit range for 2pt and 3pt correlator are checked for the bare matrix elements extracted from the fit.



M_π = 690 MeV nucleon

Nucleon momentum $P_z = 4 * \frac{2\pi}{L}$

We choose $t_{skip}^{3pt} = 1, t_{min}^{2pt} = 3$ in our following fits considering the error of the fit results and χ^2/dof of the fits.

Reduce ITDs and evolved ITDs