One-loop matching for spin-dependent quasi-TMDs

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arXiv: 2004.14831

Internal structure of a hadron

Longitudinal PDFs

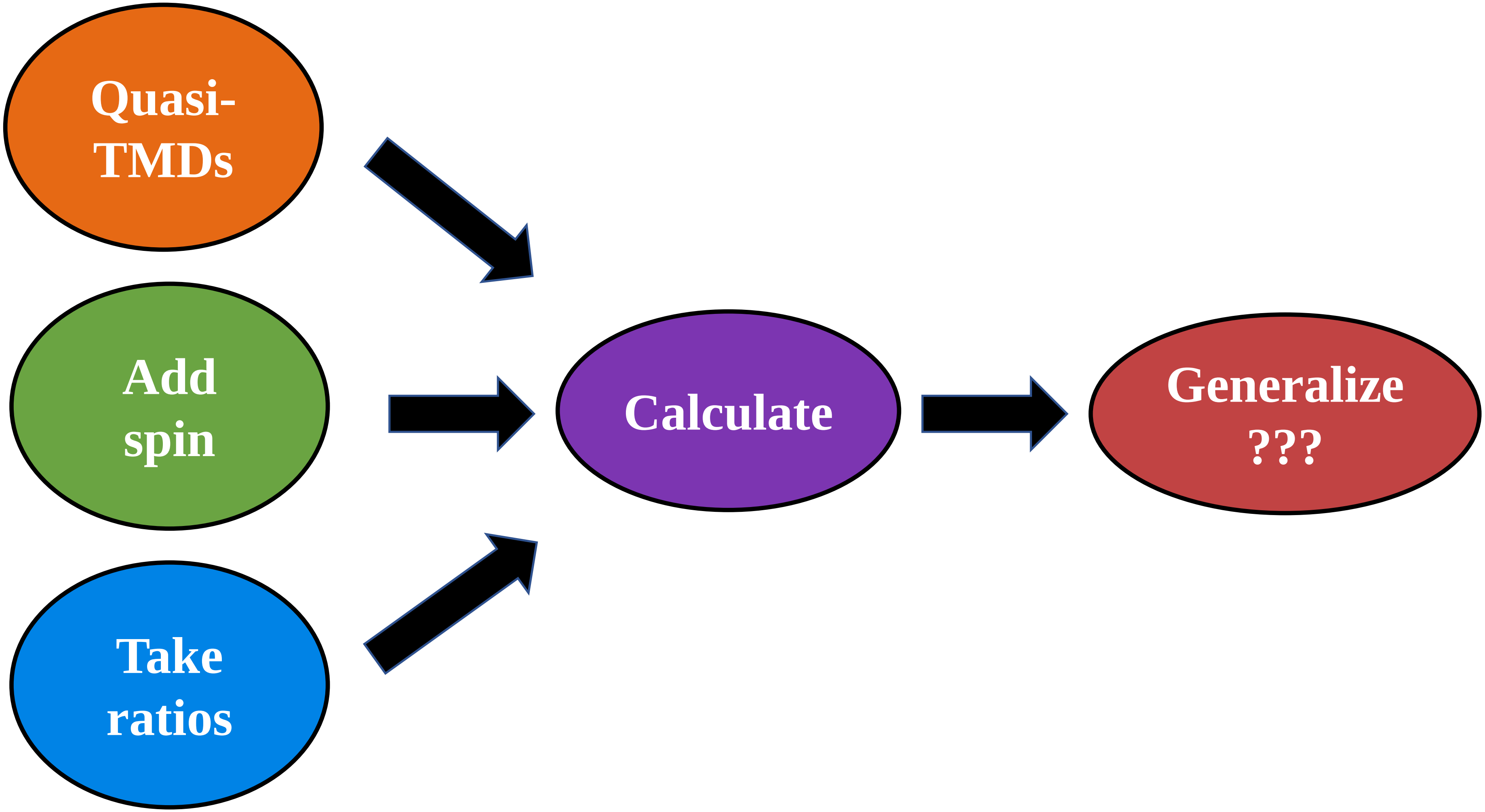
- ☑ Experiment
- ☑ Theory [see, e.g. Ji, 1305.1539]

Transverse momentum-dependent PDFs

- ☑ Experiment [HERA, COMPASS, RHIC, Fermilab…]
- ❑ First principles
  - ☑ Large p_T
  - ☑ Small p_T, unpolarized
  - ❑ Small p_T, spin dependent?

Ji et al., 1405.7640
Ji et al., 1801.05390
Ebert et al, 1811.00026
Ebert et al., 1901.03685
Ebert et al., 1910.08569
Ji et al., 1910.11415
Ji et al, 1911.03840
Vladimirov et al., 2002.07257

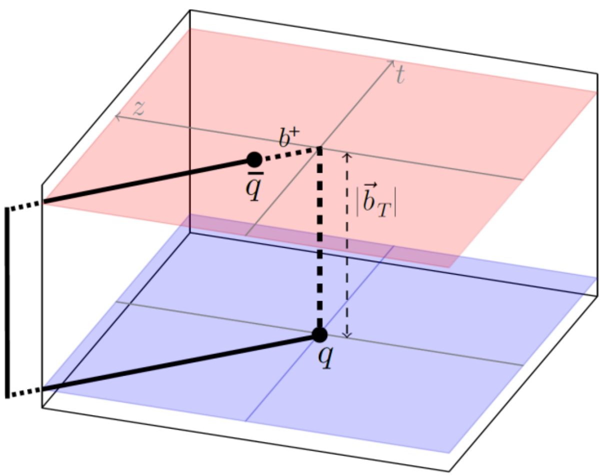
Roadmap for this talk



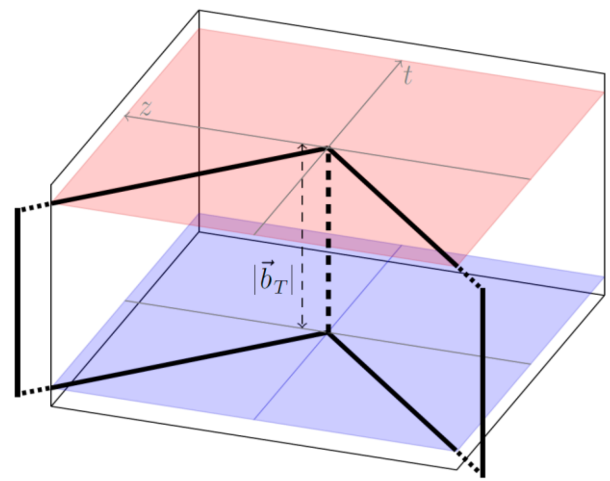
Ebert, **STS**, Stewart, Zhao, 2004.14831

TMD-PDFs

$$f^{[\Gamma]}_{q/h_S}(x, \vec{b}_T, \mu, \zeta) = \lim_{\substack{\epsilon \to 0 \\ \tau \to 0}} \underbrace{Z^q_{uv}(\mu, \zeta, \epsilon)}_{\overline{\text{MS}}} \underbrace{B^{[\Gamma]}_{q/h_S}}_{\text{Beam}}(x, \vec{b}_T, \epsilon, \tau, xP^-) \underbrace{\Delta^q_S}_{\text{Soft}}(b_T, \epsilon, \tau)$$



Beam



Soft

Lightlike Wilson lines → bad for lattice!

Solution: *quasi*-TMDs

Quasi-TMDs

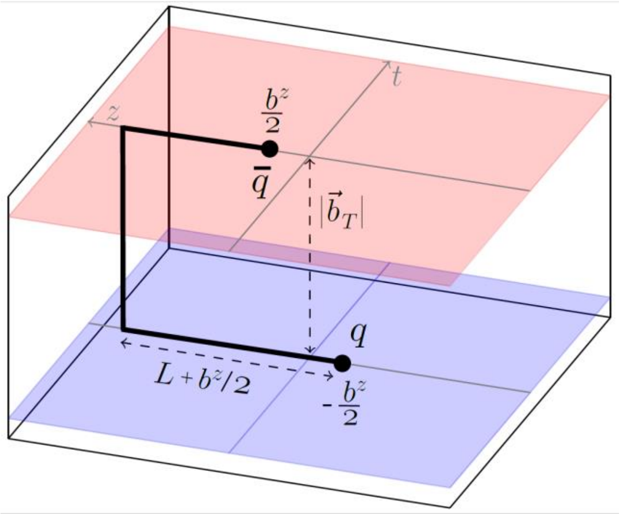
$\overline{\text{MS}}$

Lattice renormalization

$$\tilde{f}^{[\tilde{\Gamma}]}_{q/h_S}(x,\vec{b}_T,\mu,P^z) = \int \frac{db^z}{2\pi} e^{ib^z x P^z} \underline{\tilde{Z}'_q}(b^z,\mu,\tilde{\mu}) \underline{\tilde{Z}^q_{uv}}(b^z,\tilde{\mu},a)$$
$$\times \underline{\tilde{B}^{[\tilde{\Gamma}]}_{q/h_S}}(b^z,\vec{b}_T,a,L,P^z) \underline{\tilde{\Delta}^q_S}(b_T,a,L)$$

Quasi-beam

Quasi-soft



$\tilde{\Delta}^q_S$: Needed to cancel L-dependent divergences

$$\tilde{B}^{[\tilde{\Gamma}]}_q(b^z,\vec{b}_T,a,L,P^z) = N_{\tilde{\Gamma}} \langle h_S(P)|\bar{q}\tilde{W}\frac{\tilde{\Gamma}}{2}q|h_S(P)\rangle$$

Spin dependence

		Quark polarization		
		U	L	T
Hadron polarization	U	f_1 unpolarized		$h_1^\perp$ Boer-Mulders
	L		g_{1L} helicity	$h_{1L}^\perp$ worm-gear
	T	$f_{1T}^\perp$ Sivers	g_{1T} worm-gear	h_1, $h_{1T}^\perp$ transversity, pretzelosity

Choice of spin structure:

$$\Gamma \in \{\not{\bar{n}},\ \not{\bar{n}}\gamma_5,\ i\sigma^{\alpha-}\gamma_5\}$$

$$\tilde{\Gamma} \in \{\gamma^\lambda,\ \gamma^\lambda\gamma_5,\ i\sigma^{\alpha\lambda}\gamma_5\}$$

Each spin structure gives certain distributions, e.g.,

$$f_{q/h_S}^{[\not{\bar{n}}]} = f_1 - \frac{\epsilon_{\rho\sigma} b_\perp^\rho S_\perp^\sigma}{b_T} f_{1T}^\perp$$

$$\tilde{f}_{q/h_S}^{[\gamma^\lambda]} = \tilde{f}_1^\lambda - \frac{\epsilon_{\rho\sigma} b_\perp^\rho S_\perp^\sigma}{b_T} \tilde{f}_{1T}^{\lambda\perp}$$

Ebert, **STS**, Stewart, and Zhao, 2004.14831

Matching

Relationship between nonsinglet TMDs and quasi-TMDs:

$$\tilde{F}_{ns/h_S}(x, b_T, \mu, P^z) = C_{ns}^{\tilde{F}}(\mu, xP^z) F_{ns/h_S}(x, b_T, \mu, \zeta) \times \exp\left[\frac{1}{2}\mu_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] + \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

Ebert et al., 1901.03685
Vladimirov et al., 2002.07527
Ji et al., 1911.03840
Ji et al., 2004.03543

Take ratio of polarized to unpolarized TMD:

$$\frac{\tilde{F}_{ns/h_S}}{\tilde{f}_{ns}} = \frac{\tilde{B}_{ns/h_S}}{\tilde{B}_{ns}} = \frac{C_{ns}^{\tilde{F}}}{C_{ns}} \frac{F_{ns/h_S}}{f}$$

Main point of this talk!

$$C_{ns}^{\tilde{F}} = C_{ns}$$

Ebert, **STS**, Stewart, and Zhao, 2004.14831

Calculation: Tree-level

TMD:

$$f_{q/h_S}^{[\Gamma]\,(0)}(x,\vec{b}_T,\mu,\zeta) = \frac{1}{p^-}\mathrm{Tr}\left[u_s(p)\bar{u}_s\,(p)\frac{\Gamma}{2}\right]\delta(1-x)$$

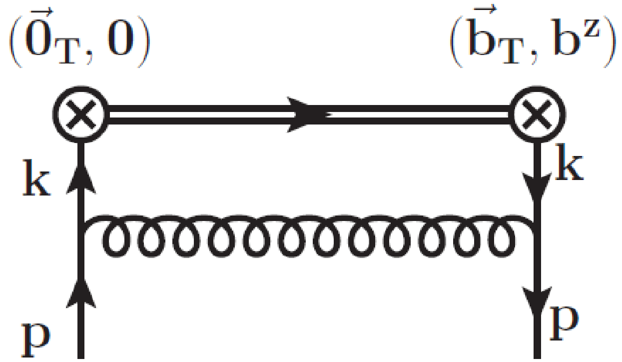
Quasi-TMD:

$$\tilde{f}_{q/h_S}^{[\tilde{\Gamma}]\,(0)}(x,\vec{b}_T,\mu,\zeta) = \frac{N_\Gamma}{p^z}\mathrm{Tr}\left[u_s(p)\bar{u}_s\,(p)\frac{\tilde{\Gamma}}{2}\right]\delta(1-x)$$

Same matching coefficient

Calculation: One-loop

Vertex topology



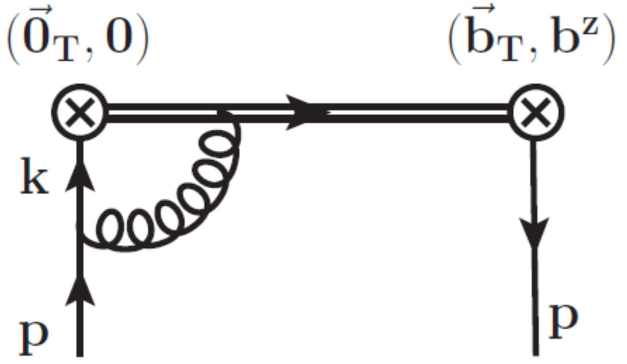
$$q^{[\tilde{\Gamma}]}(b,p) = [\bar{u}_s^a(p) u_s^b(p)] \frac{\alpha_S C_F}{4\pi} \left(\gamma^\mu \gamma_\alpha \frac{\tilde{\Gamma}}{2} \gamma_\beta \gamma_\mu \right)^{ab} \underline{I^{\alpha\beta}(p \cdot b, b^2)}$$

!!!

Same matching coefficient

Calculation: One-loop

Sail topology



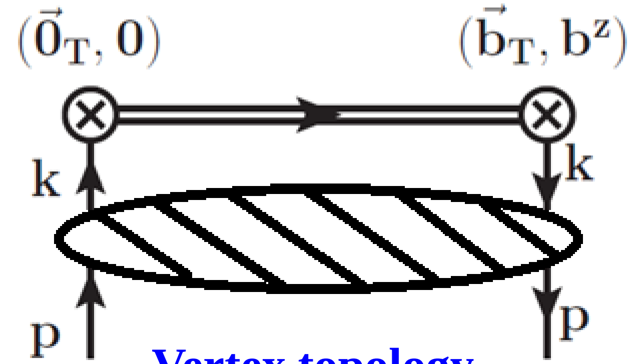
$$\tilde{q}^{\tilde{\Gamma}}(b) = -\frac{\alpha_s C_F}{2\pi} \mathrm{Tr}\left[\bar{u}_s(p)\tilde{\Gamma}\gamma^\rho\gamma^\mu u_s(p)\right] \frac{\mu_0^{2\epsilon}}{(2\pi)^{d-2}}$$
$$\int d^d k \int_0^1 ds \frac{\gamma'(s)_\mu k_\rho}{k^2(p-k)^2} e^{ip\cdot b - i(p-k)\cdot\gamma(s)}$$

Non-trivial matching, but not dependent on spin

Generalization to higher order?

One-loop order:

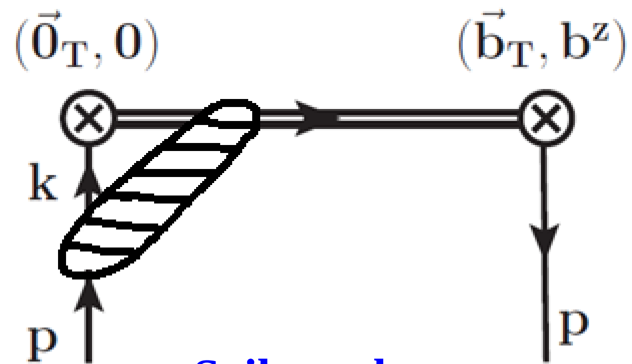
- Vertex diagrams – no matching
- Sail - at leading order, Wilson lines didn’t modify spin structure



Vertex topology

Higher order:

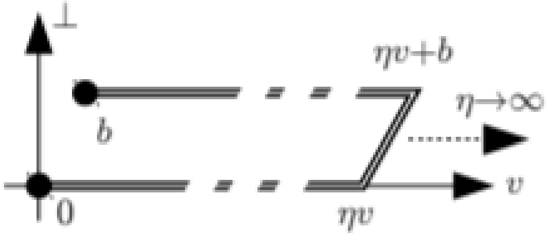
- Vertex topology: Lorentz invariant, don’t need matching
- Sail topology: nonlocal objects, local corrections. Do not expect spin structure to contribute.



Sail topology

(Dis)advantages of quasi approach

An alternate approach with a different Wilson line path



Musch et al., 1011.1213
Musch et al., 1111.4249
Engelhardt et al., 1506.07826
Yoon et al., 1601.05717
Yoon et al.,1706.03406

Our quasi-TMD result

$$\frac{\tilde{F}^{[\Gamma]}(x, b_T, \mu, P^z)}{\tilde{f}^{[\Gamma]}(x, b_T, \mu, P^z)} = \frac{F^{[\Gamma]}(x, b_T, \mu, \zeta)}{f^{[\Gamma]}(x, b_T, \mu, \zeta)}$$

Advantages of quasi-TMDs:

- Simpler Wilson line path
  → simpler matrix elements
- No rotations needed, only boost in z direction
- No need for infinite boost limit: Wilson coefficients cancel out

Disadvantages of quasi-TMDs:

- Nontrivial Wilson coefficient
- Wilson coefficient does NOT cancel when taking moments over x

Conclusion

- We extended the one-loop quasi-TMD calculation to T-even twist-2 spin structures
- The matching is the same as in the unpolarized case
- Arguments about why this will continue to be the case for higher orders
- Quasi-TMDs are advantageous for constraining *x*-dependence of spin-dependent TMDs

$$
\begin{aligned}
\frac{\tilde{f}_1(x, b_T, \mu, P^z)}{f_1(x, b_T, \mu, \zeta)} &= \frac{\tilde{g}_{1L}(x, b_T, \mu, P^z)}{g_{1L}(x, b_T, \mu, \zeta)} \\
&= \frac{\tilde{h}_1(x, b_T, \mu, P^z)}{h_1(x, b_T, \mu, \zeta)} \\
&= \frac{\tilde{h}_{1T}^{\perp}(x, b_T, \mu, P^z)}{h_{1T}^{\perp}(x, b_T, \mu, \zeta)}
\end{aligned}
$$

Thanks!

Questions?