Nucleon and Pion Ioffe time pseudo-Distributions

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As part of the

HadStruc Collaboration

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W. Morris, A. Radyushkin (Old Dominion U / JLab)
A. Rothkopf (Stavanger U)
S. Zafeiropoulos (CPT Marseille)

Musch-Hägler-Negele-Schäfer Amplitudes

B. Musch et al (2010) 1011.1213

$$\mathcal{M}(\nu, z^2) = \tilde{A}_2(l^2, l \cdot p)$$

- A general matrix element of interest
  - Analogy to the PDF's matrix element definition

$$M^\alpha(p,z) = \langle p|\bar{\psi}(z)\gamma^\alpha W(z;0)\psi(0)|p\rangle$$

- Lorentz decomposition
  - Physicists love to use of symmetries
  - Choice of p, z, and α can remove higher twist term

$$\nu = p \cdot z$$

V. Braun, et. al 9410318 (1994)

$$M^\alpha(p,z) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

A.Radyushkin (2017) 1705.01488

- The first term contains information from all twists, now called pseudo-ITD
- They calculated moments of the straight-link or primordial TMD
- The pseudo-ITD can be factorized to obtain PDF with perturbative calculable kernels

LaMET vs SDF

- In a method analogous to the space-like separated currents for the pion DA, the pseudo-ITD can be factorized into the PDF using Large Momentum Effective Theory or Short Distance Factorization
- What is the difference between LaMET and SDF
  - LaMET: factorization and expansion with respect to large momentum scale X. Ji (2013) 1305.1539
  - SDF: factorization and expansion with respect to short distance scale
- What is not the difference between LaMET and SDF
  - Renormalization choice
    - RI-Mom scheme
    - RGI ratio scheme
  - Space in which factorization occurs
    - Factorize in position space, ν, to get ITD
    - Factorize in momentum space, y, to get PDF

V. Braun and D. Müller (2007) 0709.1348
A.Radyushkin (2017) 1705.01488
Y. Q. Ma and J. W. Qiu (2017) 1709.03018

Momentum space and the Sort-of PDFs

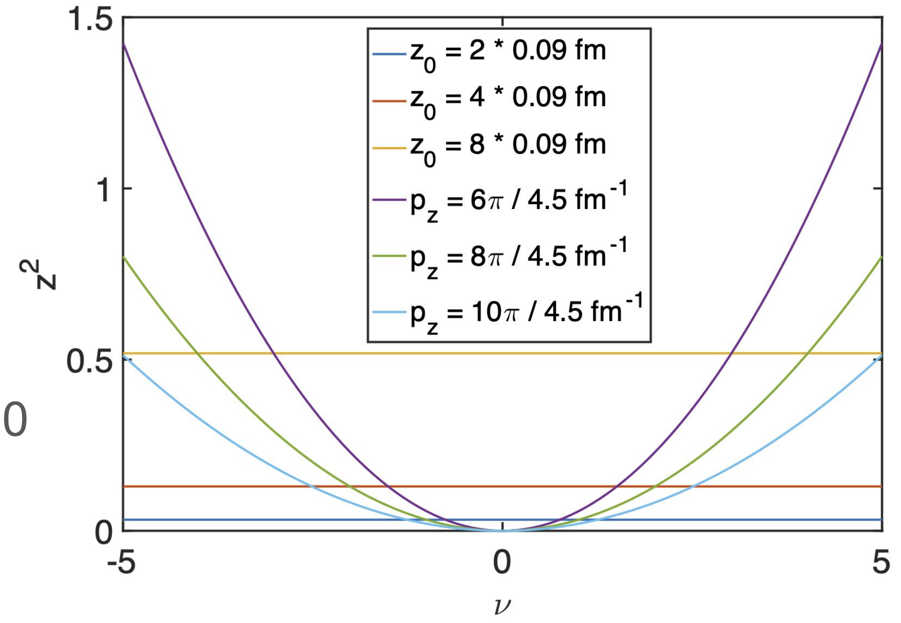
- Both are integrals of pseudo ITD
  - **Pseudo PDF** has fixed invariant scale dependence

$$P(x, z_0^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{i\nu x} \mathcal{M}(\nu, z_0^2)$$

  - **Quasi PDF** mixed invariant scales, fixed momentum

$$\tilde{q}(x, p_z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{i\nu x} \mathcal{M}(\nu, \frac{\nu^2}{p_z^2})$$

- To factorize the divergent limit of $z^2 \longrightarrow 0$
  - Pseudo-PDFs use SDF
  - Quasi-PDFs use LaMET



Position space and The Operator Product Expansion

- The OPE can be used to demonstrate the factorization of this matrix element
  - The SDF version is given by short distance expansion

Y. Q. Ma, J. W. Qiu (2017) 1709.03018
T. Izubuchi et al (2018) 1801.03917

$$\mathcal{M}(\nu, z^2) = \sum_n c_n(\mu^2 z^2) a_n(\mu^2) \frac{(i\nu)^n}{n!} + O(z^2\Lambda^2_{\mathrm{QCD}})$$

$$\mathcal{M}(\nu, z^2) = \int_{-1}^{1} du\, C(u, \mu^2 z^2) Q(u\nu, \mu^2) + O(z^2\Lambda^2_{\mathrm{QCD}})$$

- A few works have begun to study LaMET in position space
  - Pion valence PDF from position space matrix element with LaMET
  - Pheno strange and charm PDFs matched to position space with LaMET to compare to matrix elements
  - Pheno u-d PDFs matched to position space with LaMET to compare to matrix elements

T. Izubuchi et al (2019) 1905.06349
R. Zhang et al (2020) 2005.01124
Z. Fan et al (2020) 2005.12015

$$\mathcal{M}(\nu, \frac{\nu^2}{p_z^2}) = \int_{-1}^{1} du\, \tilde{C}(u, \frac{\nu^2\mu^2}{p_z^2}) Q(u\nu, \mu^2) + O(\frac{\Lambda^2_{\mathrm{QCD}}}{p_z^2})$$

The role of separation and momentum

- In LaMET and SDF, separation and momentum switch their roles

p_z^2 z^2	z p_z ν
• Scale used for factorization • Scaled used in power correction expansions • Important to keep far from Λ^2_{QCD}	• Important parameter to vary • Can obtain any possible value • Having more of them makes inverse problem easier • Want as wide a range as possible

A. Radyushkin (2017) 1710.08813
J.-H. Zhang et al (2018) 1801.03023
T. Izubuchi et al (2018) 1801.03917

Perturbative matching

Position space SDF

$$\mathcal{M}(\nu, z^2) = \int_{-1}^{1} du\, C(u, \mu^2 z^2) Q(u\nu, \mu^2)$$

Y. Q. Ma and J. W. Qiu (2017) 1709.03018

Momentum space SDF

$$P(y, z^2) = \int_{-1}^{1} \frac{dx}{|x|} K(\frac{y}{x}, \mu^2 z^2) q(x, \mu^2)$$

Position space LaMET

$$\mathcal{M}(\nu, \frac{\nu^2}{p_z^2}) = \int_{-1}^{1} du\, \tilde{C}(u, \frac{\nu^2 \mu^2}{p_z^2}) Q(u\nu, \mu^2)$$

Y. Q. Ma and J. W. Qiu (2014) 1404.6860

Momentum space LaMET

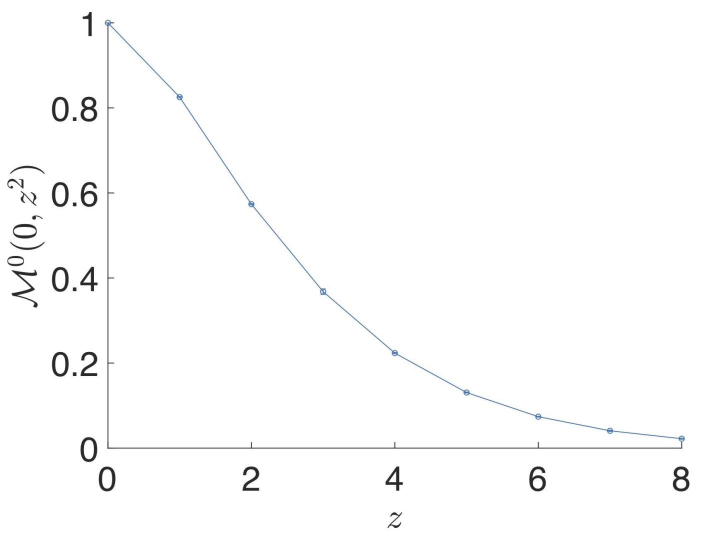
$$\tilde{q}(y, p_z^2) = \int_{-1}^{1} \frac{dx}{|x|} \tilde{K}(\frac{y}{x}, \frac{\mu^2}{(xp_z)^2}) q(x, \mu^2)$$

The Reduced distribution and normalization

A.Radyushkin (2017) 1705.01488
T. Izubuchi (2020) 2007.06590

- The pseudo-ITD is subject to many systematic errors
  - Lattice spacing, higher twist, incorrect pion mass, finite volume
- A ratio can remove renormalization constants and the low Ioffe time systematic errors
  - In style of ratios from older Lattice calculations of g_V/g_A
  - Avoids additional gauge fixed RI-Mom calculations
  - Is a renormalization group invariant quantity, guaranteeing finite continuum limit

$$\mathfrak{M}(\nu, z^2) = \frac{M^0(p,z)/M^0(p,0)}{M^0(0,z)/M^0(0,0)}$$



- New ratio method with non-zero momentum could remove different HT errors

Inverse Solutions for Lattice PDFs

- Parametric JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos (2019) 1901.05408
  - Fit a phenomenologically motivated function
    - Method used by most pheno extractions
    - Potentially significant, but controllable model dependence
  - Fit to a neural network S. Forte, L. Garrido, J. Latorre, A. Piccione (2002) 0204232
    - Machine learning is hip
    - Expensive tuning procedure
- Non-Parametric
  - Backus-Gilbert J. Liang, K-F. Liu, Y-B. Yang (2017) 1710.11145
    - No model dependence, one tunable parameter
  - Bayesian Reconstruction Y. Burnier and A. Rothkopf (2013) 1307.6106
    - Very general, Bayesian statistics has systematics included in meaningful way
  - Bayes-Gauss-Fourier transform C. Alexandrou, G. Iannelli, K. Jansen, F. Manigrasso (2020) 2007.13800
- Discrete Fourier Transform
  - The DFT adds additional information that all data outside range is 0
  - For any available lattice data this bad information creates statistically significant oscillations

Why would I want small and large momentum?

- Many momenta allows for determination of PDF moments
  - To determine previously incalculable moments, precise data along a range of momenta is required
- Many momenta and separations allow for unique Ioffe time points unavailable to a single momentum
  - Having as many Ioffe time points as possible is important for the inverse problem
- At a common Ioffe time, the low momentum points can be significantly more precise while giving the same information
- Multiple points at the same Ioffe time allows for checks of systematic errors
  - Is NLO matching sufficient or do we need NNLO
  - Deviations aside from DGLAP at small separation may be discretization errors
  - Deviations aside from DGLAP at large separation may be power corrections

Numerical Study

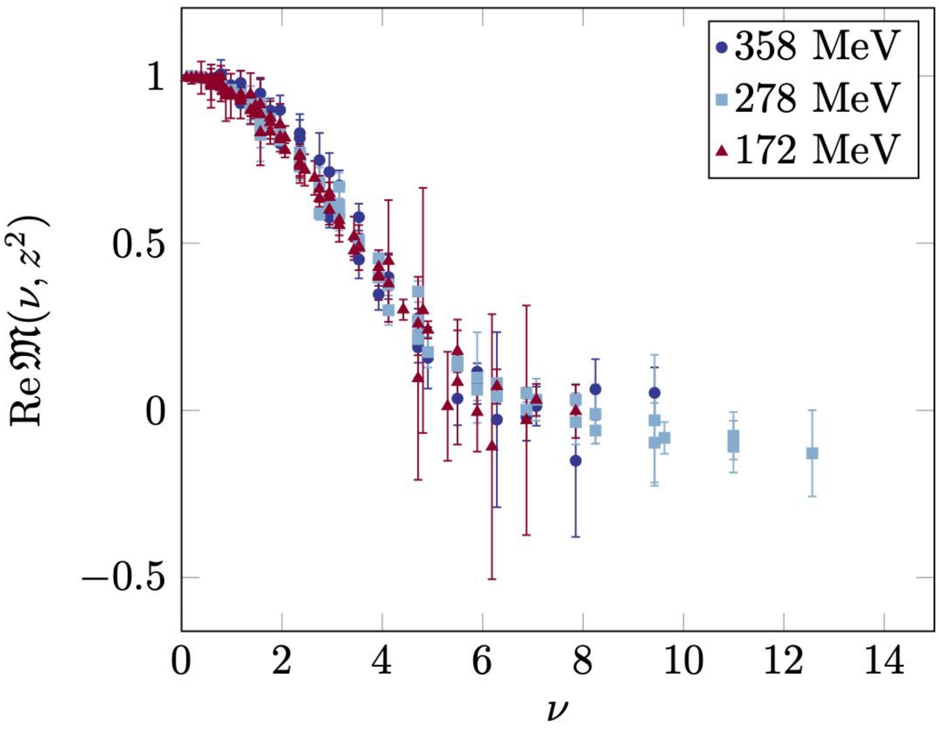
Dynamical Tree level tadpole Symanzik improved gauge action

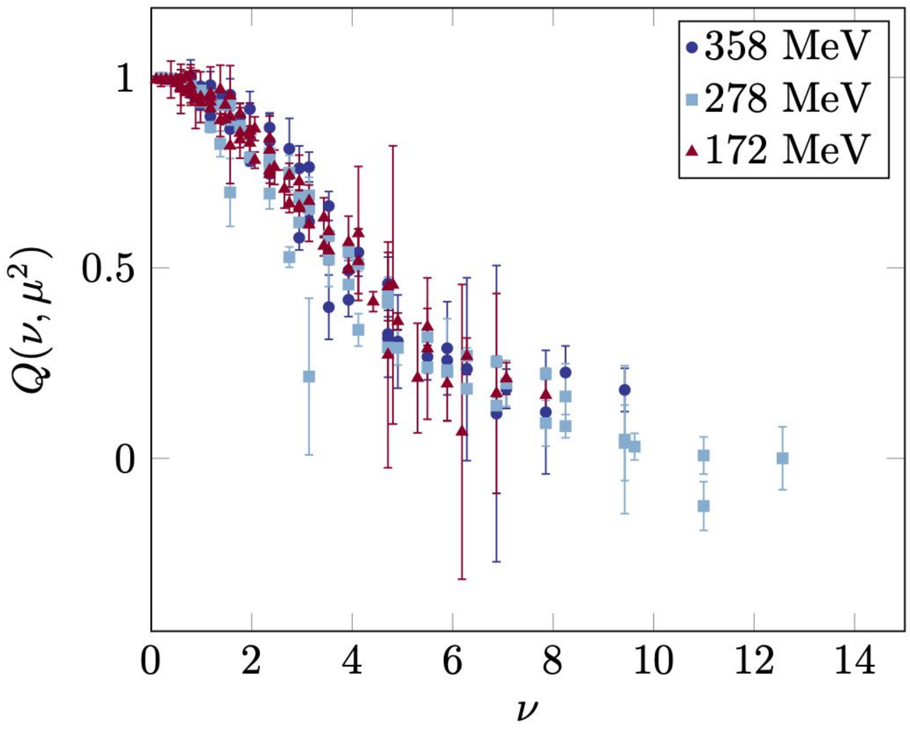
- a127m415 : $\beta = 6.1 \quad m_\pi = 415\,\mathrm{MeV} \quad 24^3 \times 64 \quad a = 0.127\,\mathrm{fm}$
- a127m415L : $\beta = 6.1 \quad m_\pi = 415\,\mathrm{MeV} \quad 32^3 \times 96 \quad a = 0.127\,\mathrm{fm}$
- a094m360 : $\beta = 6.3 \quad m_\pi = 358\,\mathrm{MeV} \quad 32^3 \times 64 \quad a = 0.094\,\mathrm{fm}$
- a094m280 : $\beta = 6.3 \quad m_\pi = 278\,\mathrm{MeV} \quad 32^3 \times 64 \quad a = 0.094\,\mathrm{fm}$
- a091m170 : $\beta = 6.3 \quad m_\pi = 172\,\mathrm{MeV} \quad 64^3 \times 128 \quad a = 0.091\,\mathrm{fm}$

Nucleon matrix elements

B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards, S. Zafeiropoulos (2020) 2004.01687 soon to be PRL

a094m360 a094m280 a091m170



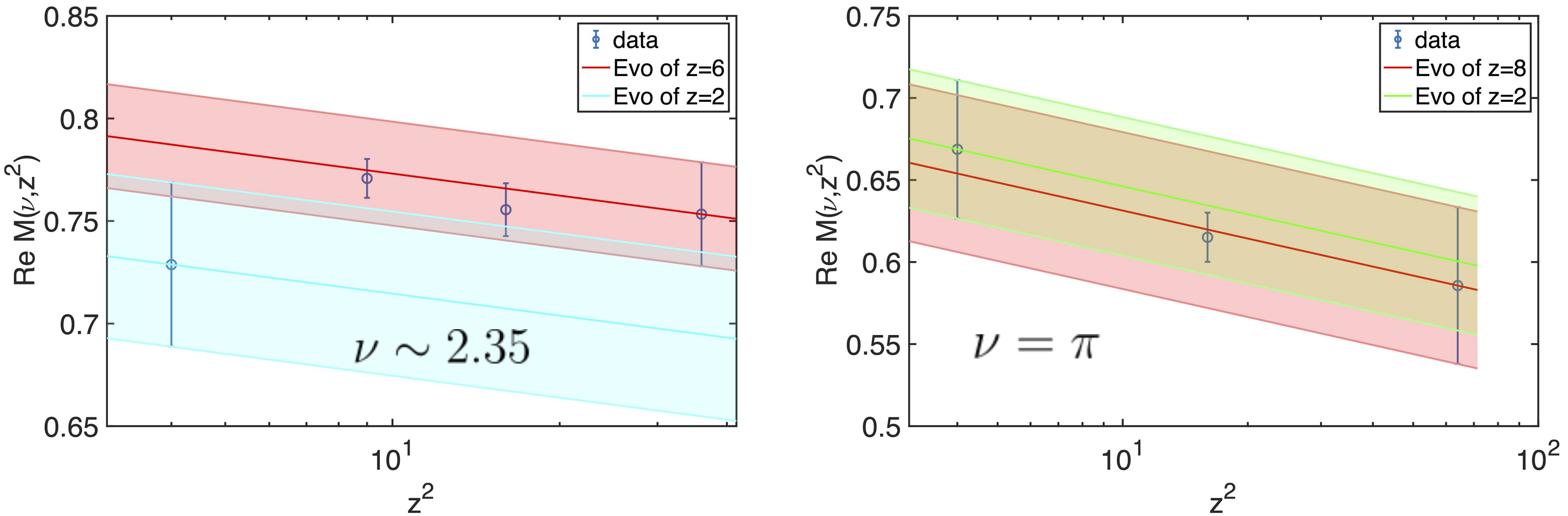


B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards,
S. Zafeiropoulos (2020) 2004.01687 soon to be PRL

A peek at fixed Ioffe time

a094m280

- No evidence of strong power corrections up to highest separation $z \sim 0.75\text{fm}$
- Possible evidence of discretization effects for shortest separation



B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards, S. Zafeiropoulos (2020) 2004.01687 soon to be PRL

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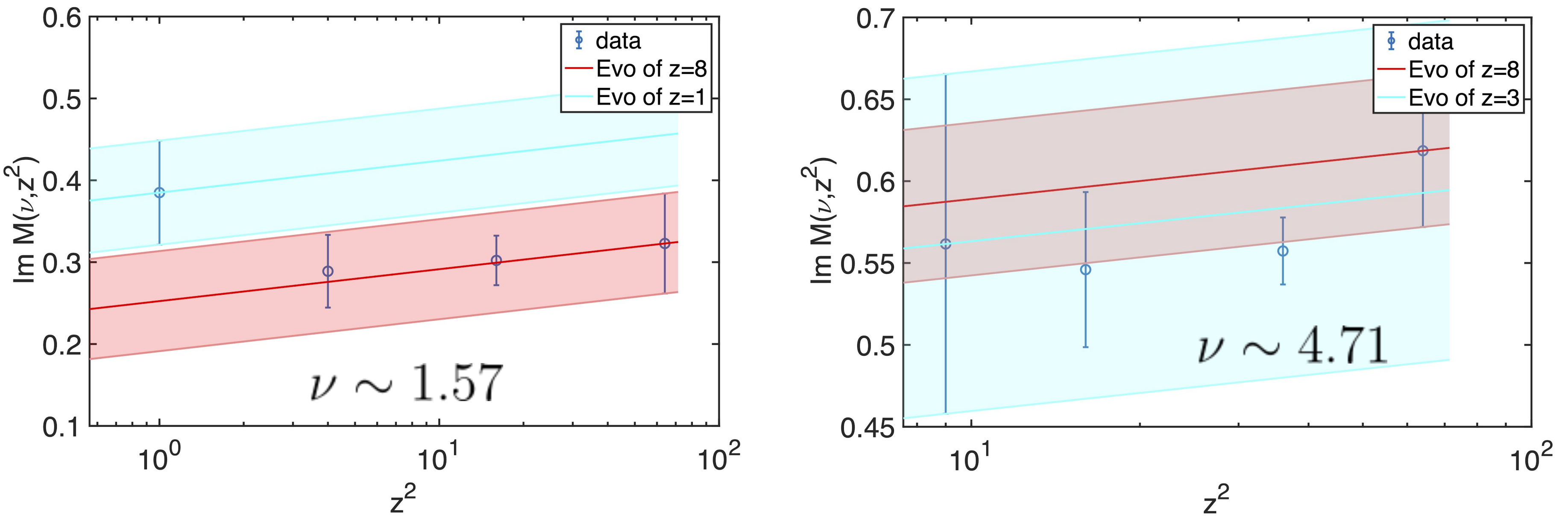
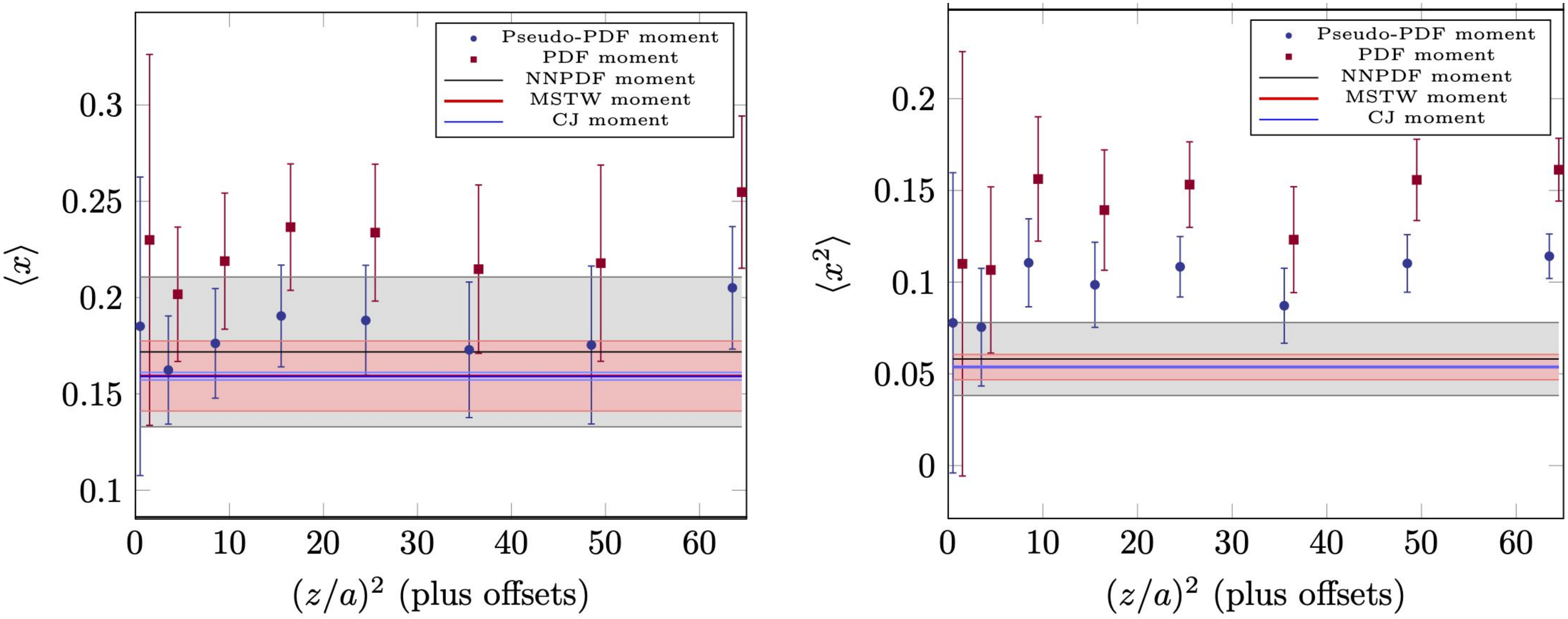


Nucleon PDF moments

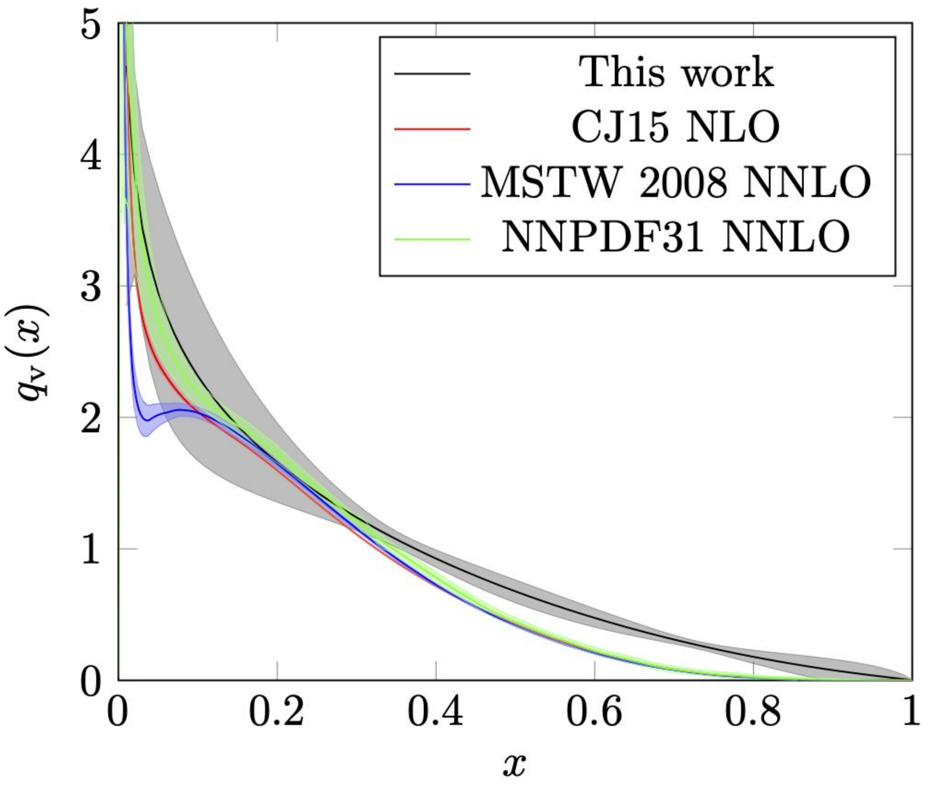
B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards, S. Zafeiropoulos (2020) 2004.01687

a091m170



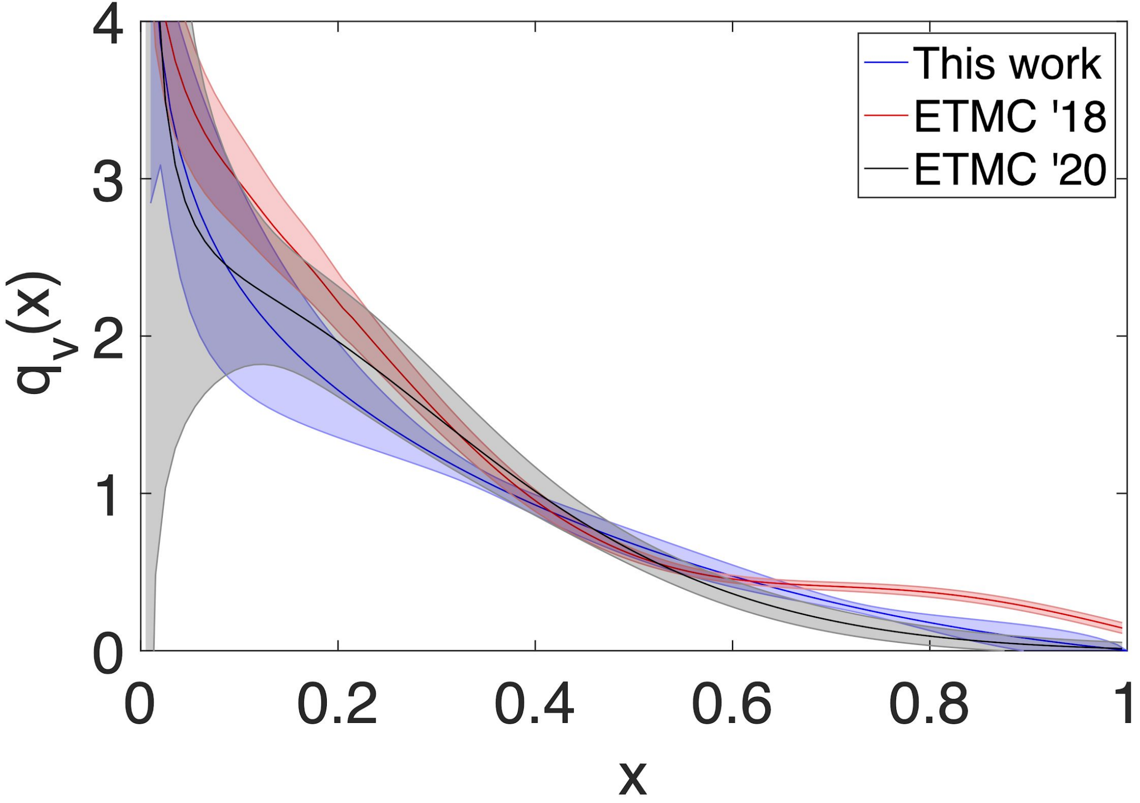
Comparison with Pheno

B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards, S. Zafeiropoulos (2020) 2004.01687 soon to be PRL



- Qualitative agreement with pheno
- Many systematics still unstudied to explain discrepancy
- Higher 2nd moment consistent with overestimate at large x

Comparison with ETMC



- Good agreement amongst groups
- Similar lattice spacings and volumes

B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards, S. Zafeiropoulos (2020) 2004.01687 soon to be PRL

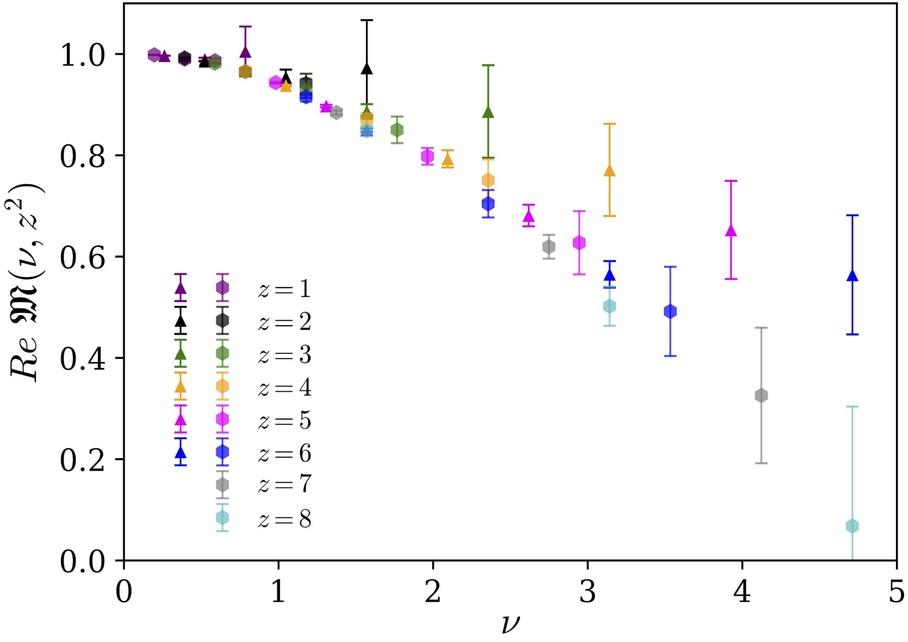
C. Alexandrou et. al. (2018) 1803.02685

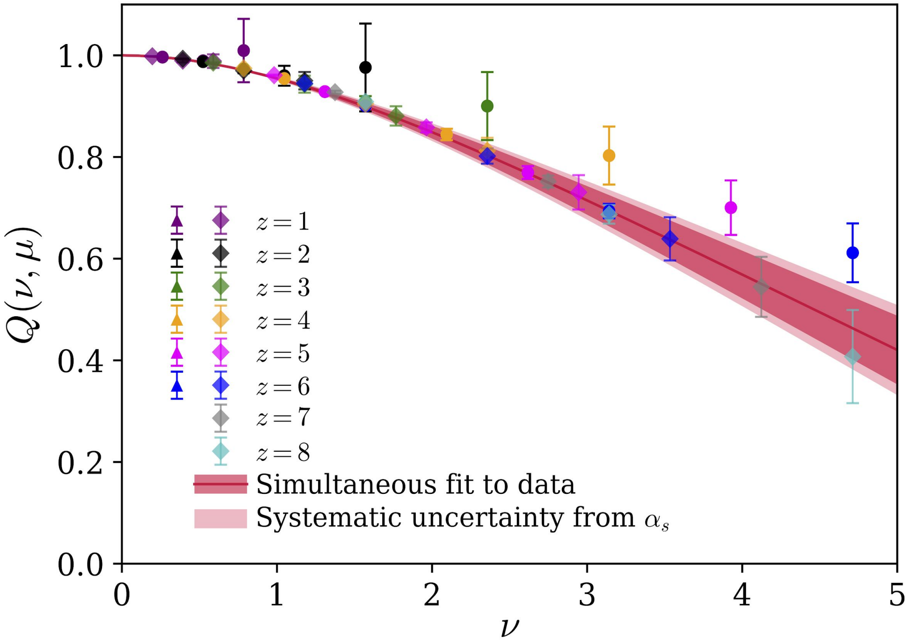
M. Bhat et. al. (2020) 2005.02102

Pion matrix element

B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards,
R. Sufian, S. Zafeiropoulos (2019) 1909.08517

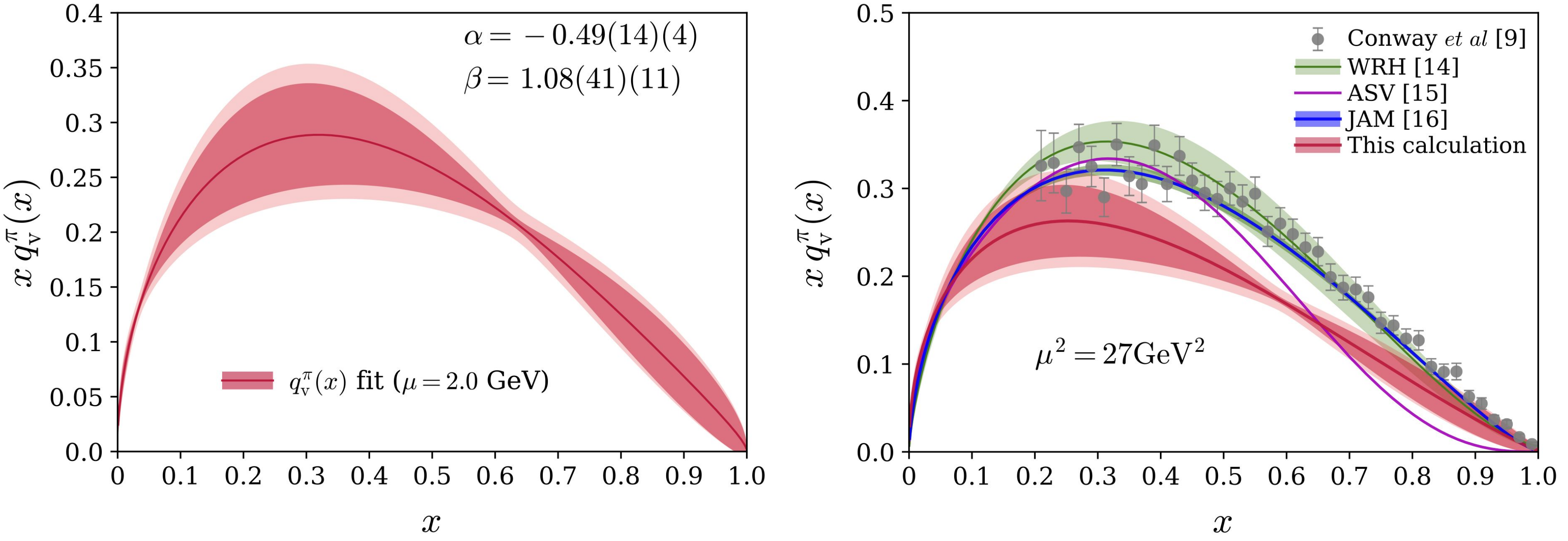
a127m415 a127m415L



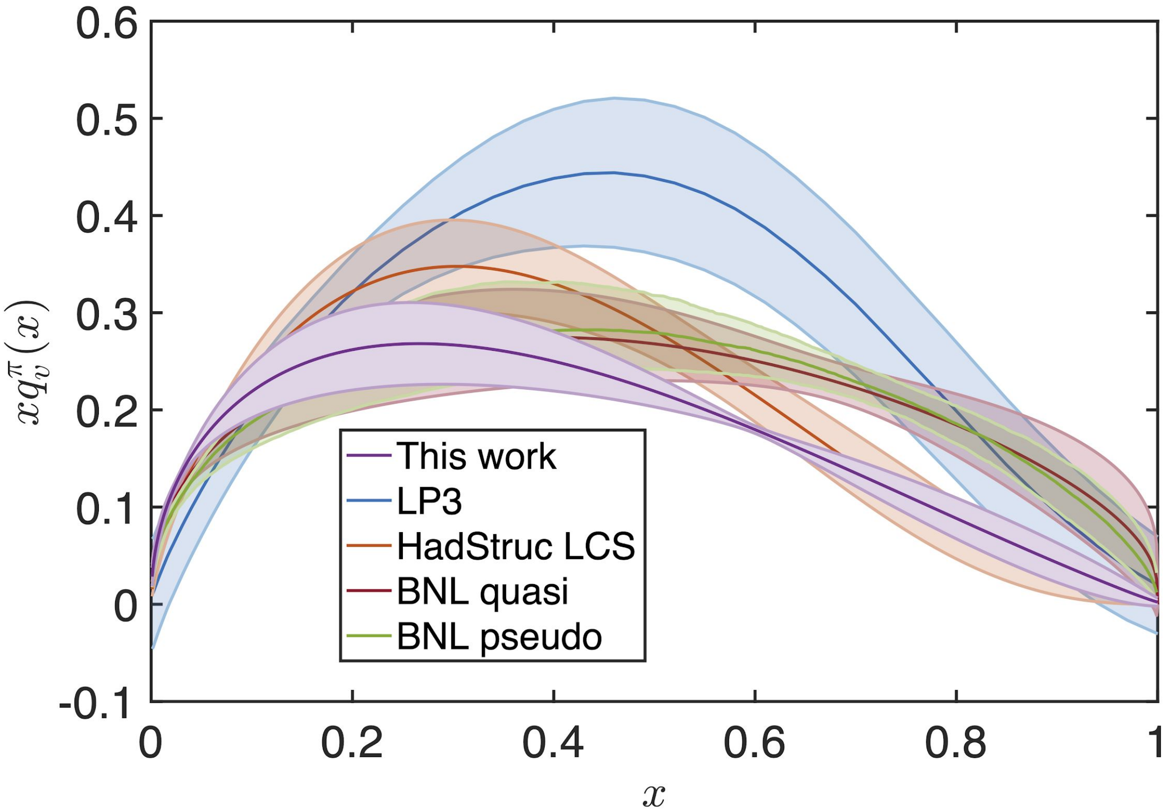


B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards, R. Sufian, S. Zafeiropoulos (2019) 1909.08517

Pion valence quark PDF



Comparison with other groups



- Range in Ioffe time is very limited ~5-6
- Matrix element at max Ioffe time far from 0
- Our study of inverse methods showed importance of longer Ioffe times extents for precise reconstructions

B. Joó, JK, K. Orginos, A. Radyushkin, D. Richards, R. Sufian, S. Zafeiropoulos (2019) 1909.08517

J.H. Zhang et. al. (2018) 1804.01483

Sufian et. al. (2020) 2001.04960

T. Izubuchi et. al. (2019) 1905.06349

X. Gao et. al. (2020) 2007.06590

Summary and Outlook

- Lattice QCD is a powerful tool for understanding parton structure
  - SDF and LaMET provide a clear pathway for considering many distributions
- SDF allows for use of data not allowed in LaMET calculation
  - Particularly, the precise small momentum data provide important constraints
  - Many momenta allow for determination of many PDF moments as well as checks for systematic error
- Nucleon PDF results with similar systematic seem to be reaching agreement
- Pion PDF results require more work to extend Ioffe time range for better determination, but results are promising
- Further studies required to produce continuum limit and more physical pion mass results
- Future applications of non-parametric inversions could remove potential biases