Valance parton distribution of pion from lattice QCD

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Outline

- Extract PDFs from equal time correlation function
- Pion valence quark PDF from fine lattices
- Preliminary results of DWF calculations
- Summary

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- **Extract PDFs from equal time correlation function**
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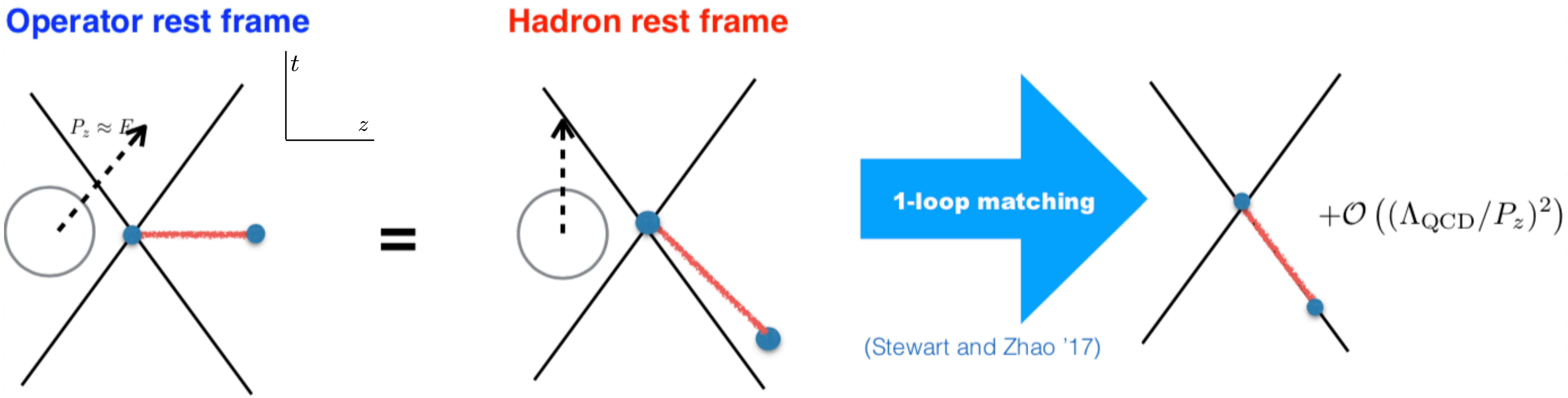
Large momentum effective theory

Equal time correlation function can be determined on lattice (X. Ji, PhysRevLett.110.262002):

$$\tilde{q}(x) \equiv \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P \,|\, \tilde{O}_\Gamma(z,\epsilon) \,|\, P \rangle, \; \tilde{O}_\Gamma(z,\epsilon) = \overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)$$

$$\Gamma = \gamma^t \text{ or } \gamma^z$$



Factorization of Quasi-PDFs

The form of the factorization formula for quark quasi-PDFs in the $\overline{\mathrm{MS}}$ scheme is (iso non-singlet case):

$$\tilde{q}(x, P_z, \mu) = q(x, \mu) + \alpha_s(\mu)(\tilde{q}^{(1)}(x, P_z, \mu) - q^{(1)}(x, \mu))$$

$$= \int \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu}{yP_z}) q(y, \mu) + \mathcal{O}(\frac{m_h^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{x^2 P_z^2})$$

X. Ji, Y. Zhao, et al, arXiv:2004.03543

The difference of qPDF and PDF is only UV, and the IR physics must be the same.

The $\frac{m_h^2}{x^2 P_z^2}$ and $\frac{\Lambda_{QCD}^2}{x^2 P_z^2}$ corresponding to the target mass correction and higher twist correction, thus the key ingredient is large P_z.

Factorization in Coordinate-Space

By OPE, the spacial non-local correlator:

$$\frac{1}{2}\langle P | \tilde{O}_{\Gamma}(z,\mu) | P \rangle = P^{\mu} h^{R}_{\gamma^{\mu}}(zP_z, \mu^2 z^2)$$

can be expanded in terms of local operators, which give the moments of PDFs (iso non-singlet case):

$$h^{R}_{\gamma^{t}}(zP_z, \mu^2 z^2) = \sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy y^n q(y,\mu) + \mathcal{O}(z^2 m_h^2, z^2 \Lambda^2_{QCD})$$

in which, $C_n(\mu^2 z^2)$ is the Wilson coefficient and $\nu = zP_z$ is so called Pseudo Ioffe time. Again the key ingredient is large P_z , then one can extract more information with large ν.

Matching to PDF

Bare
matrix elements

→

PDFs

Matching to PDF

RI-MOM scheme:

$$h^R(zP_z, p^R) = Z_{\gamma_t\gamma_t}(z, p^R, a)h^B_{\gamma^t}(zP_z, a)$$

RI-MOM renormalization

Fit

Quasi PDF

One-loop matching

$$\tilde{q}(x; P_z, P^R) = \int \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{P^R_\perp}{P^R_z}, \frac{yP_z}{P^R_z}) q(x, \mu)$$

Bare matrix elements

Phenomenology inspired PDF

$$q(x, \mu) = Ax^a(1 - x)^b(1 + \text{subleading terms})$$

Renormalization group invariant ratio

Fit

Model independent Mellin moments

$$\frac{h^B_{\gamma^t}(zP_z, a)}{h^B_{\gamma^t}(zP^0_z, a)} = \frac{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy y^n q(y, \mu)}{\sum_n C_n(\mu^2 z^2) \frac{(-izP^0_z)^n}{n!} \int_{-1}^{1} dy y^n q(y, \mu)}$$

$$\langle x^n \rangle = \int_{-1}^{1} dx x^n q(x, \mu)$$

Ratio scheme:

$$\tilde{O}_\Gamma(z, \mu) = Z_{\psi,z} e^{\delta m|z|} \tilde{O}_\Gamma(z, \epsilon)$$

- **Hadron state independent**

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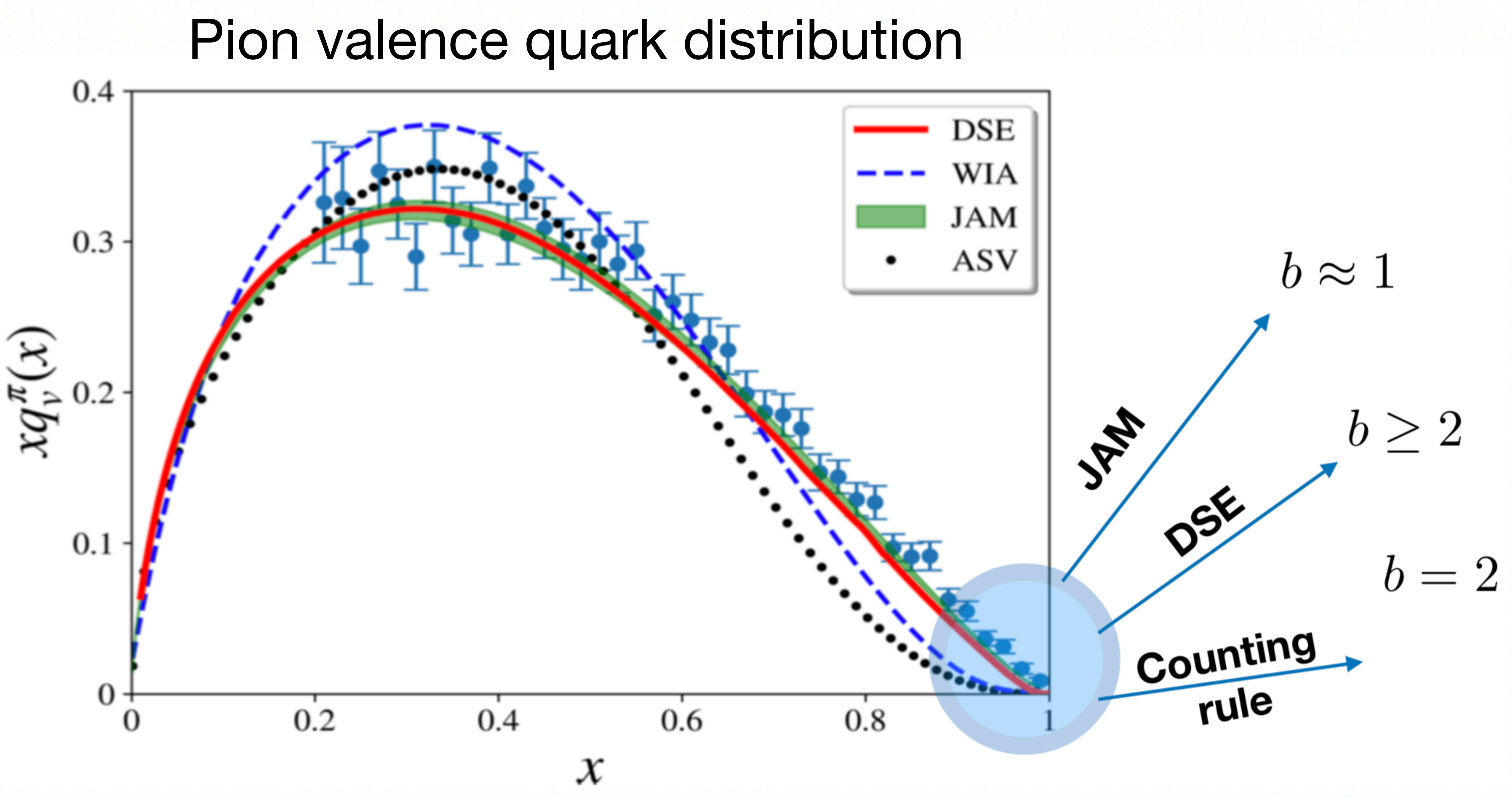
Pion valence quark PDF: Motivation



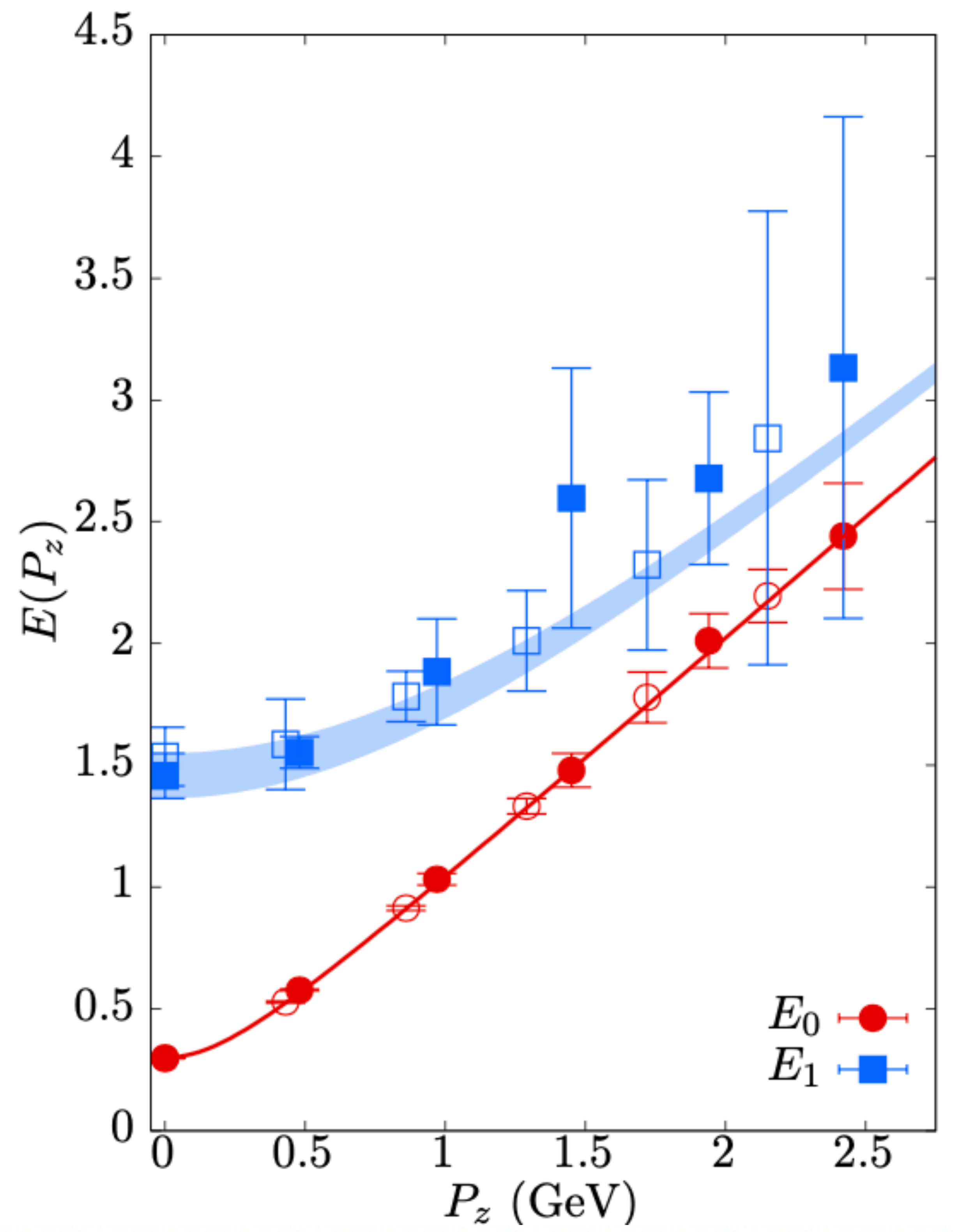
Physical reason:

Pion play a central role in the study of the strong interactions.
However, The absence of fixed pion targets has made it difficult to determine the pion's structure experimentally. One of the key physics issue is x=1 behavior:

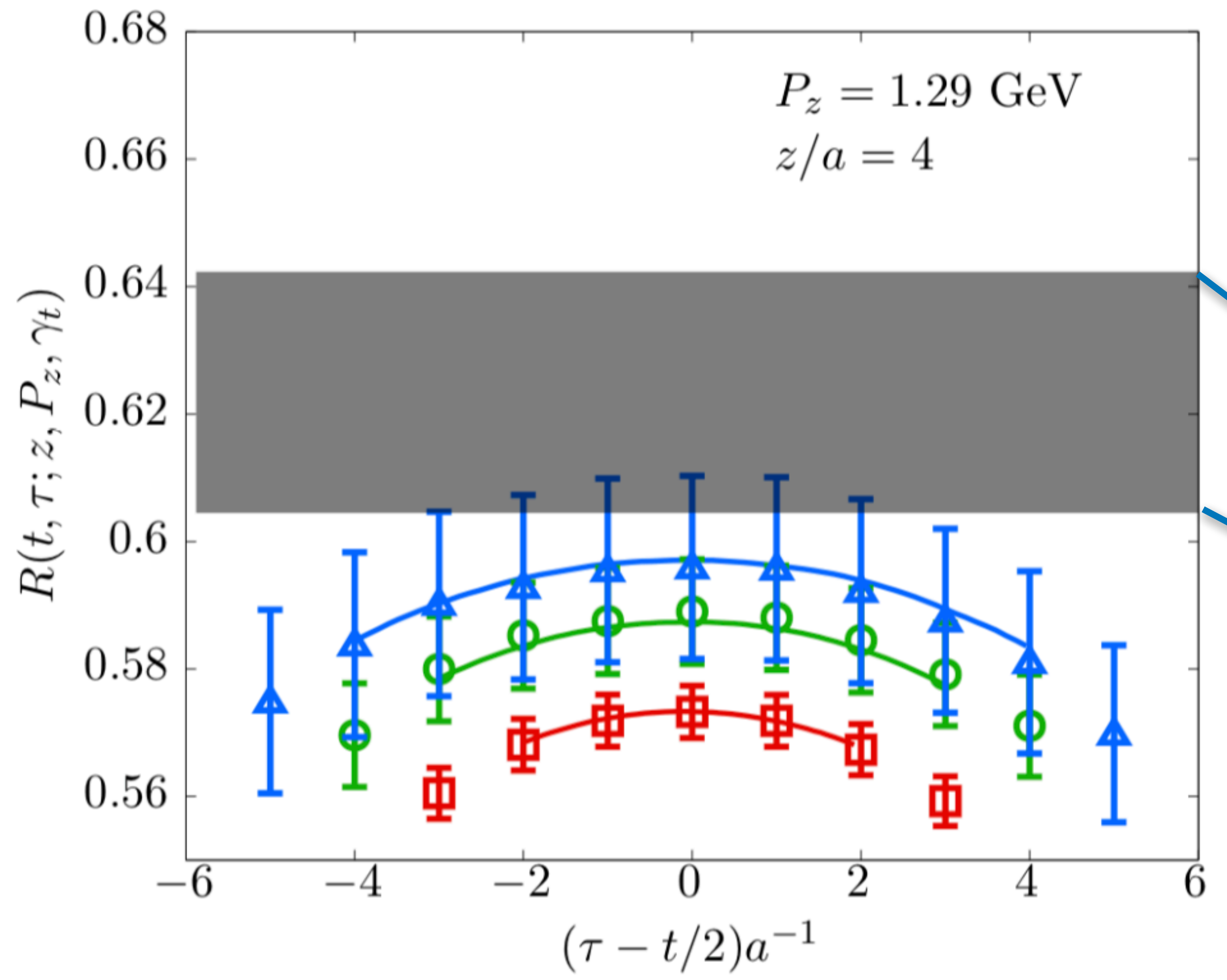
$$\lim_{x \to 1} f_v^\pi(x) \sim (1-x)^b$$

Technical reason:

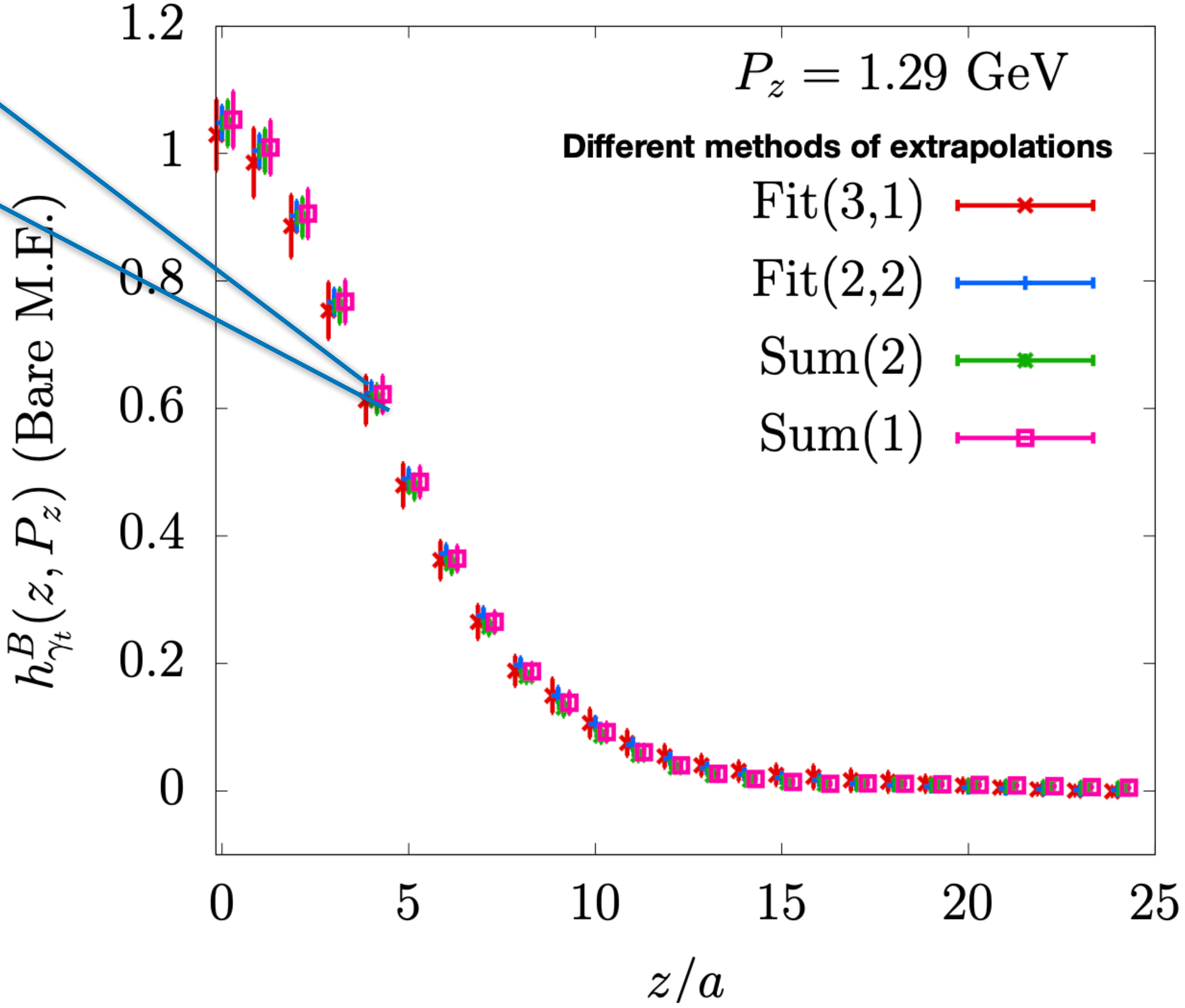
- It's easier to get close to the light-cone.
- The excited state contamination is less problematic.



Pion valence quark PDF: Bare matrix elements



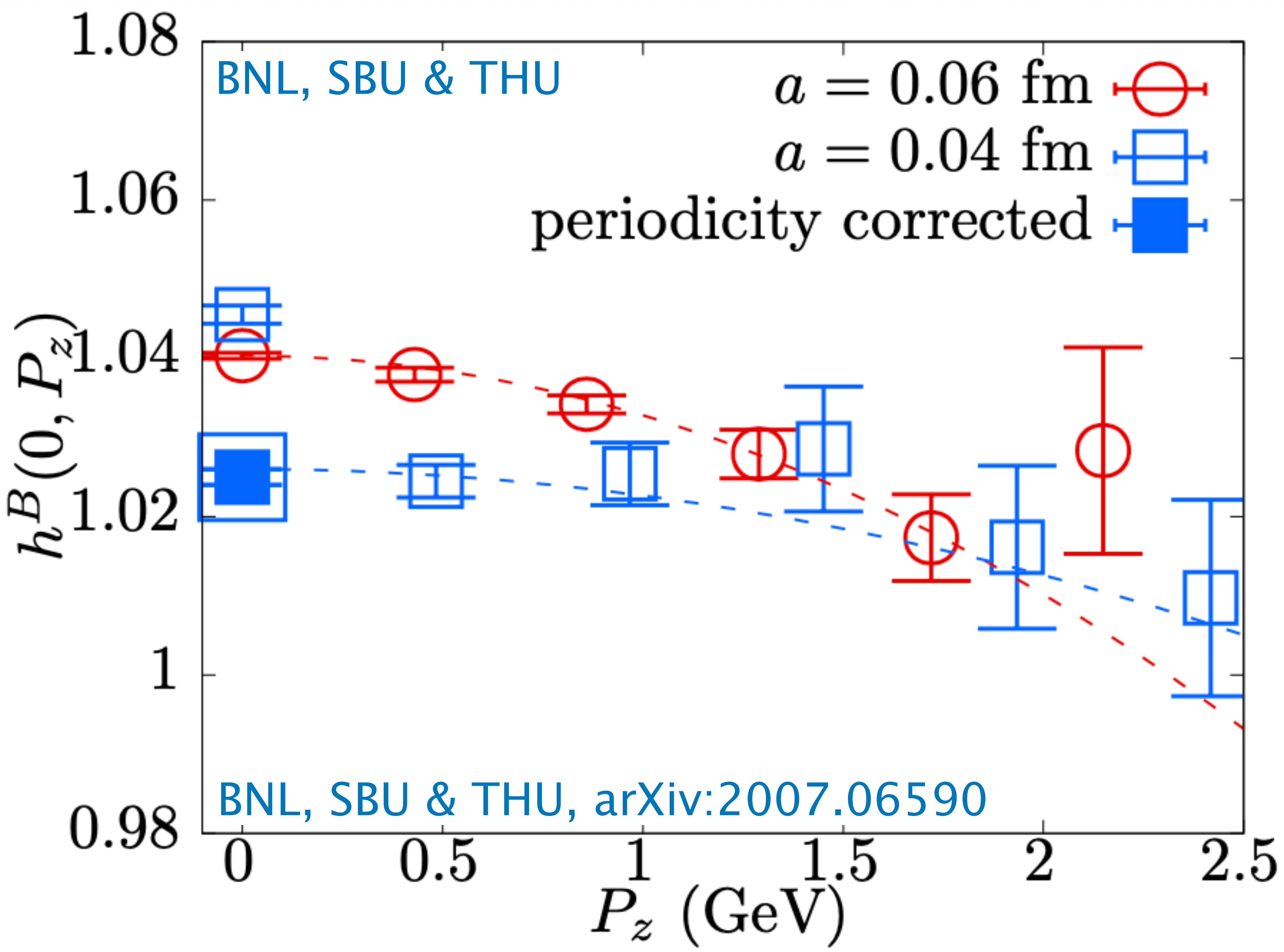
$$h^B(z, P_z) = \lim_{t\to\infty} \frac{\langle \pi(t; P_z) O_\Gamma(z; \tau) \pi^\dagger(0; P_z) \rangle}{\langle \pi(t; P_z) \pi^\dagger(0; P_z) \rangle}$$



ensemble $a, L_t \times L^3$	$m_q a$	$m_\pi L_t$	n_z	z range	#cfgs	(#ex,#sl)
$a = 0.06$ fm, 64×48^3	-0.0388	5.85	0,1	[0,15]	100	(1, 32)
			2,3,4,5	[0,8]	525	(1, 32)
				[9,15]	416	(1, 32)
				[16,24]	364	(1, 32)
$a = 0.04$ fm, 64×64^3	-0.033	3.90	0,1	[0,32]	314	(3, 96)
			2,3	[0,32]	314	(4, 128)
			4,5	[0,32]	564	(4, 128)

- HISQ sea quarks and Wilson-Clover valence quarks.

Pion valence quark PDF: Lattice artifacts



The red and blue open symbols are the estimates of the bare matrix elements $h^B(z=0,\ P_z)$, which in the continuum limit will be the total isospin of pion (π^+), which is 1.

- The estimated value for a = 0.04 fm after correcting for wrap-around effect is shown as the filled blue square.

$$h^B_w(z=0,\ P_z) \approx \frac{1+2e^{-E_\pi L_t}}{1+e^{-E_\pi L_t}}\, h^B(z=0,\ P_z)$$
$$\approx 1.024\ h^B(z=0,\ P_z)$$

- The red and the blue dashed curves are the modeled lattice spacing effects using an ansatz, with b=-3.21 in both cases.

$$h^B(z=0,\ P_z) = h^B(z=0,\ P_z=0) + b(P_z a)^2$$

Pion valence quark PDF: Lattice artifacts

We modify the ratio by adding a free parameter r:

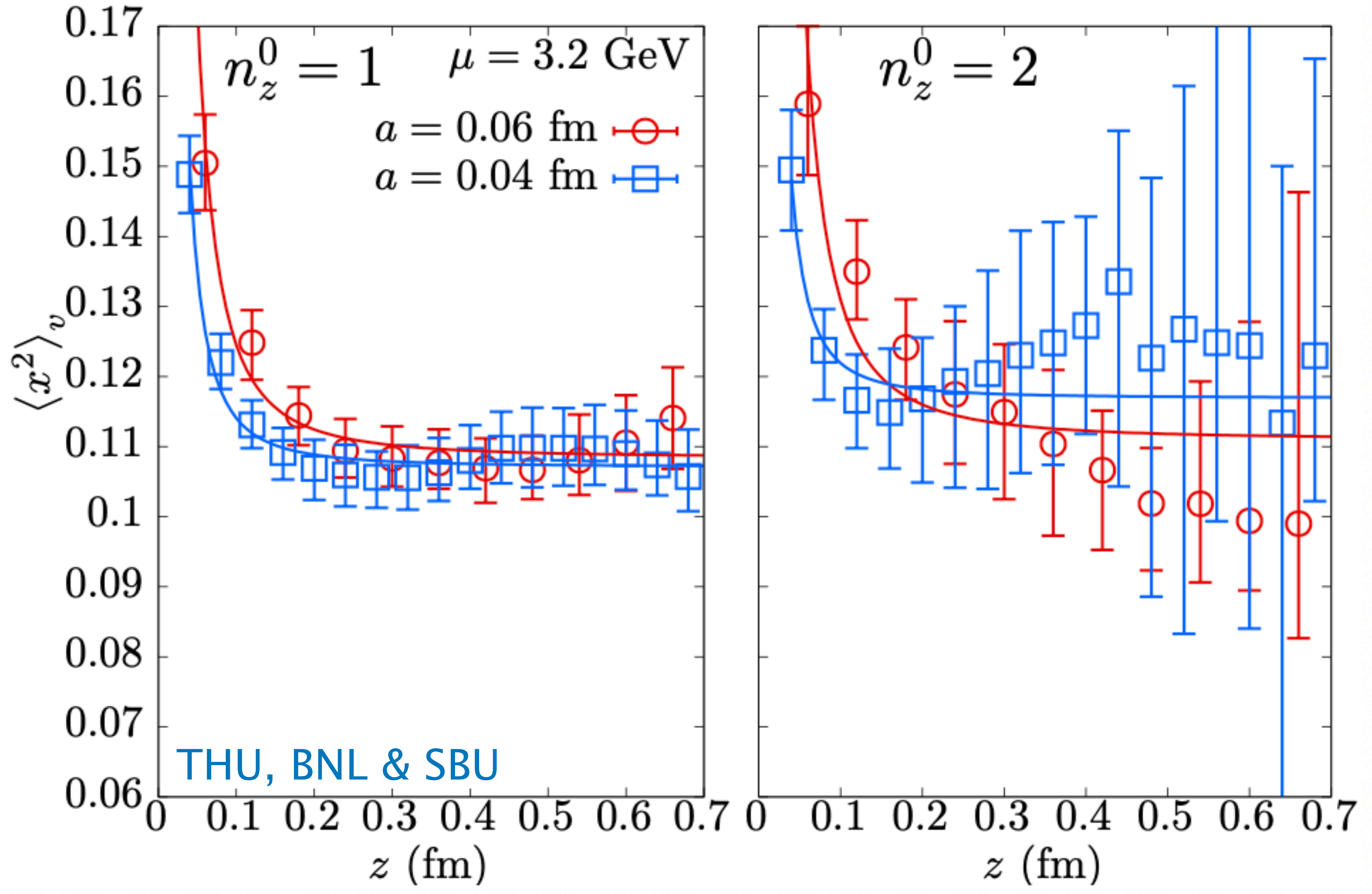
$$\mathscr{M}(z, P_z, P_z^0) = \frac{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy y^n q(y,\mu)}{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z^0)^n}{n!} \int_{-1}^{1} dy y^n q(y,\mu)}$$

↓

$$\mathscr{M}'(z, P_z, P_z^0) = \frac{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy y^n q(y,\mu) + r(aP_z)^2}{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z^0)^n}{n!} \int_{-1}^{1} dy y^n q(y,\mu) + r(aP_z^0)^2}$$

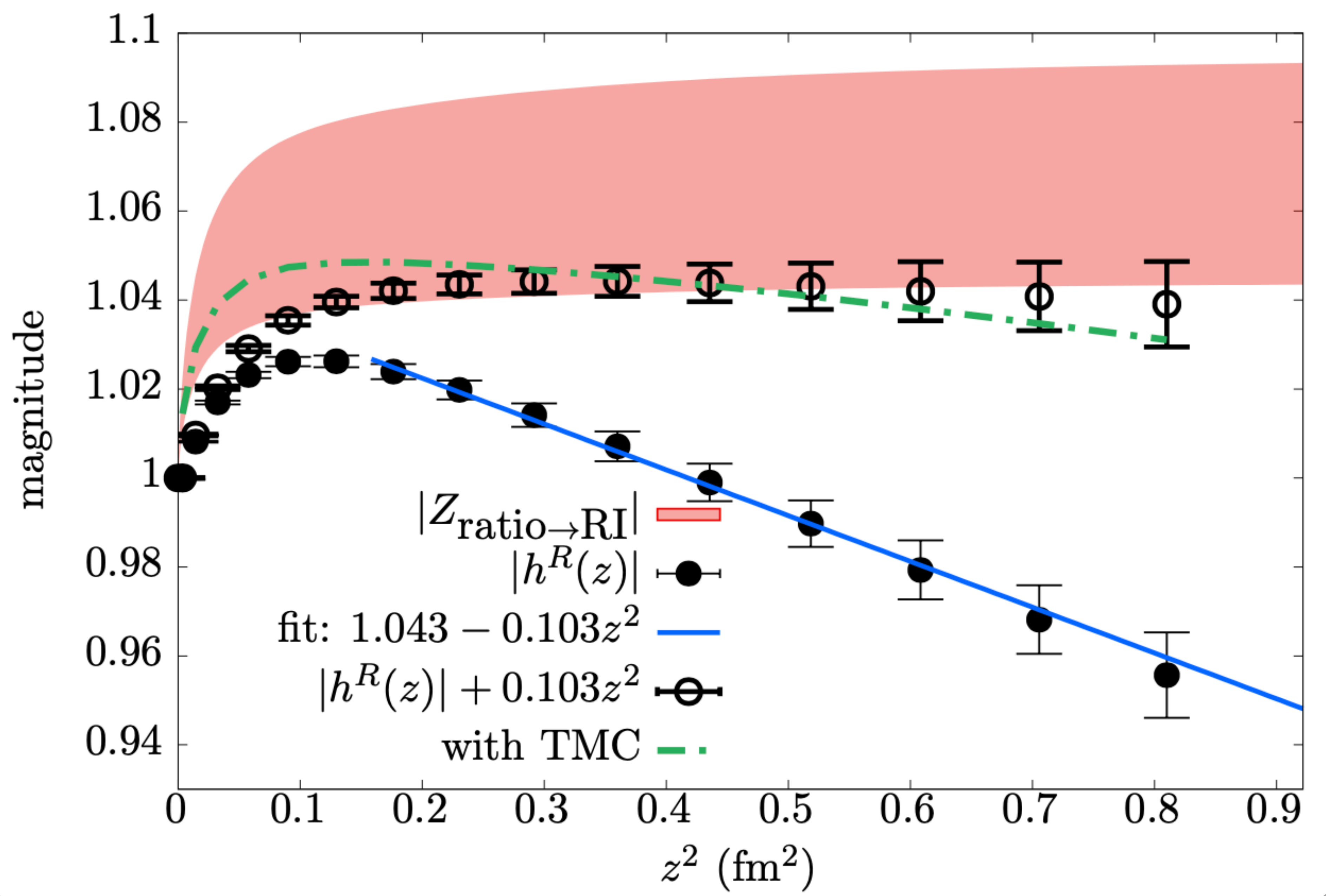
such a correction shift the 2nd moment in $(z/a)^{-2}$ manner

$$\langle x^2 \rangle \to \langle x^2 \rangle - \frac{2r}{c_2(\mu^2 z^2)} \frac{1}{(z/a)^2}$$



The z dependence of $\langle x^2 \rangle(z)$ obtained by fitting the ratio $\mathscr{M}(z, P_z, P_z^0)$ at different fixed values of z.

Pion valence quark PDF: Higher twist effect



The expectations from the 1-loop leading twist results are shown as the red bands. The lattice data after subtracting the z^2 correction term is shown using open circles.

For Pz = 0, the only non-zero twist-2 contribution is from the local current operator, in $\overline{\text{MS}}$ scheme:

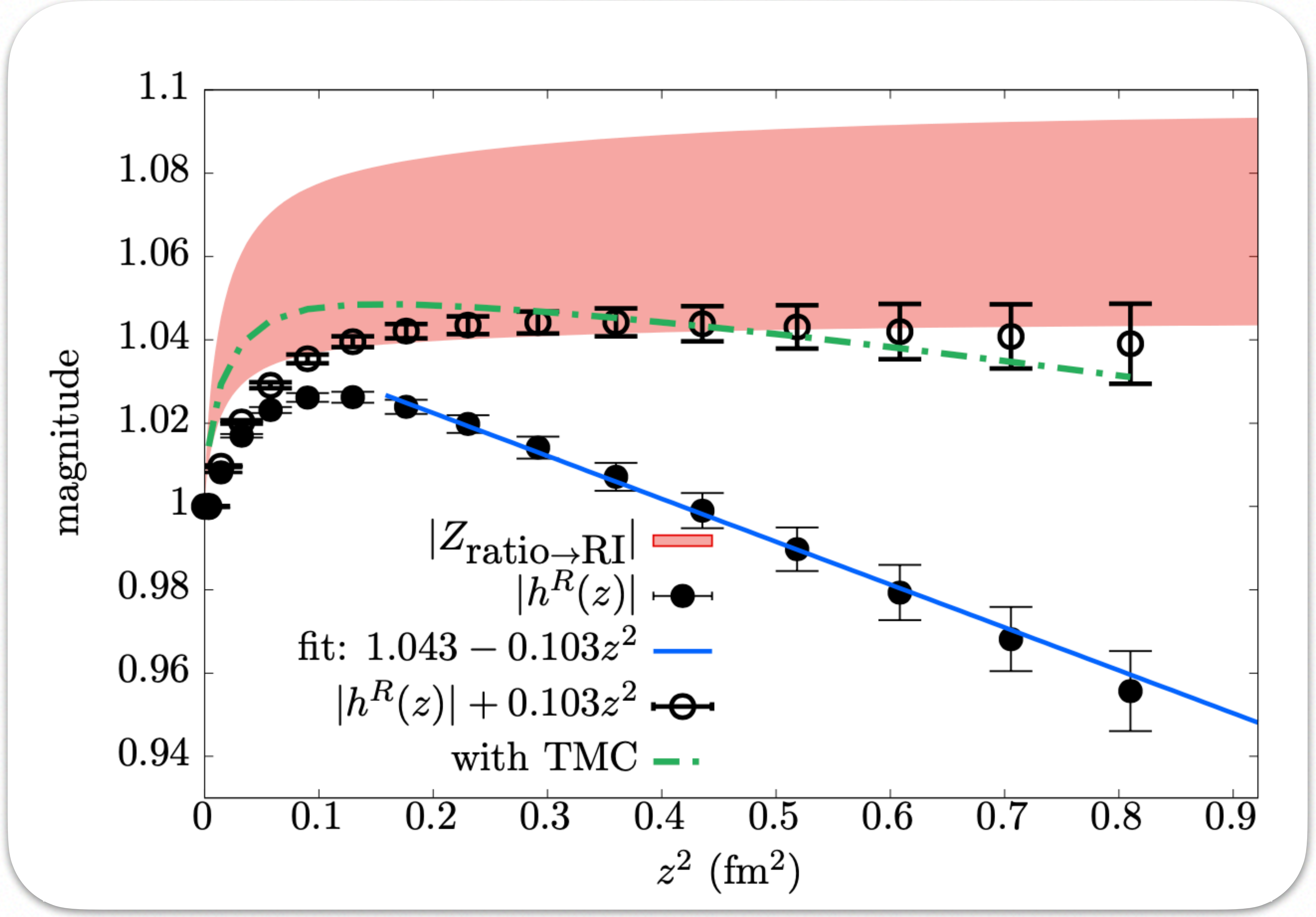
$$h^R_{\gamma^t}(z, P_z = 0, \mu^2 z^2) \;= C_0(\mu^2 z^2) + \mathcal{O}(z^2 m_h^2, z^2 \Lambda^2_{QCD})$$

Then we compare with the $P_z = 0$ lattice data renormalized in RI-MOM scheme:

$$h^R(z, P_z, p^R) = Z_{\gamma_t\gamma_t}(z, p^R, a) h^B_{\gamma^t}(z, P_z, a)$$

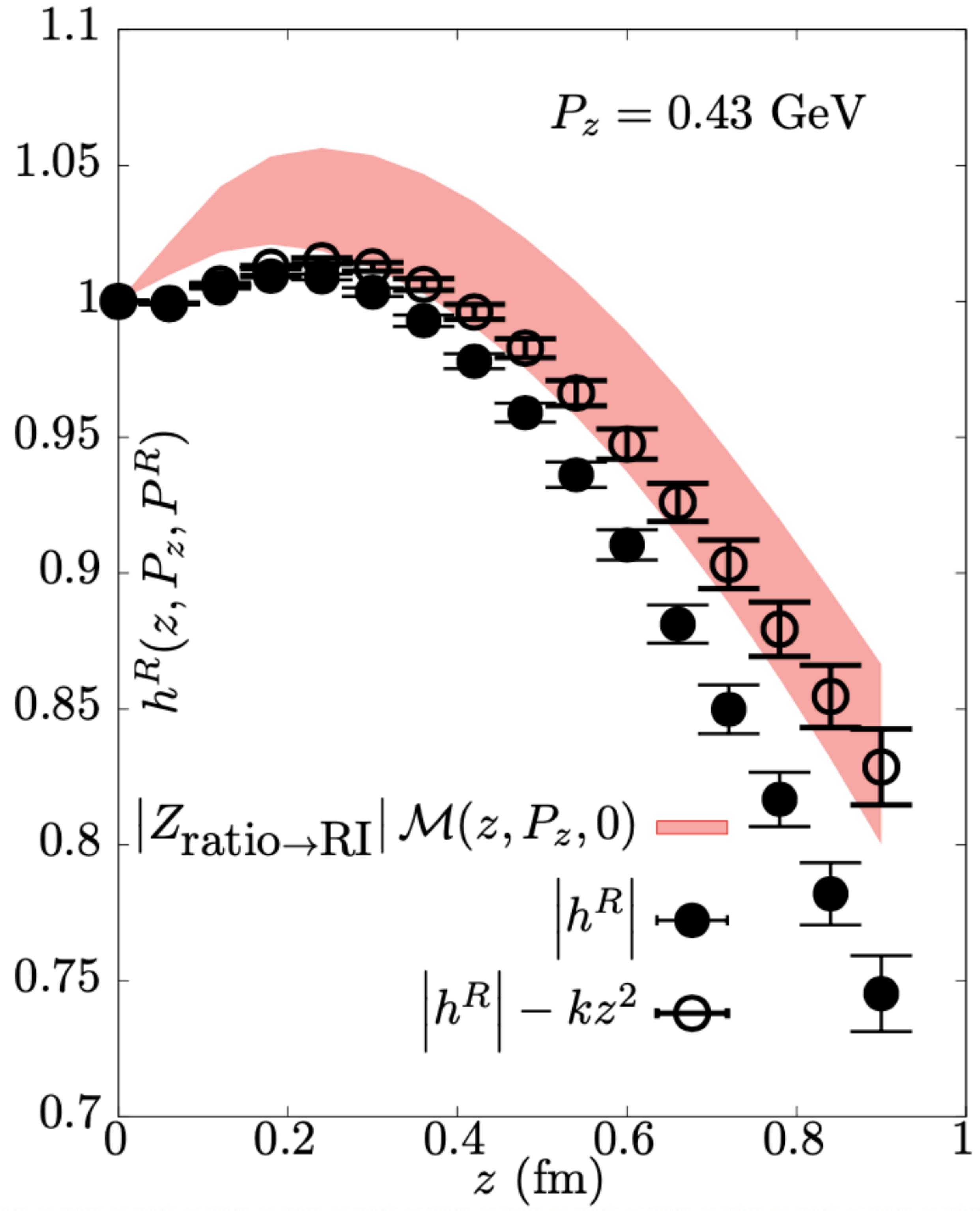
$$Z_{\overline{\text{MS}}\to\text{RI}}(z, p^R, \mu) h^R_{\gamma^t}(z, P_z = 0, \mu^2 z^2)$$
$$= h^R(z, P_z = 0, p^R) + kz^2$$

Pion valence quark PDF: Higher twist effect

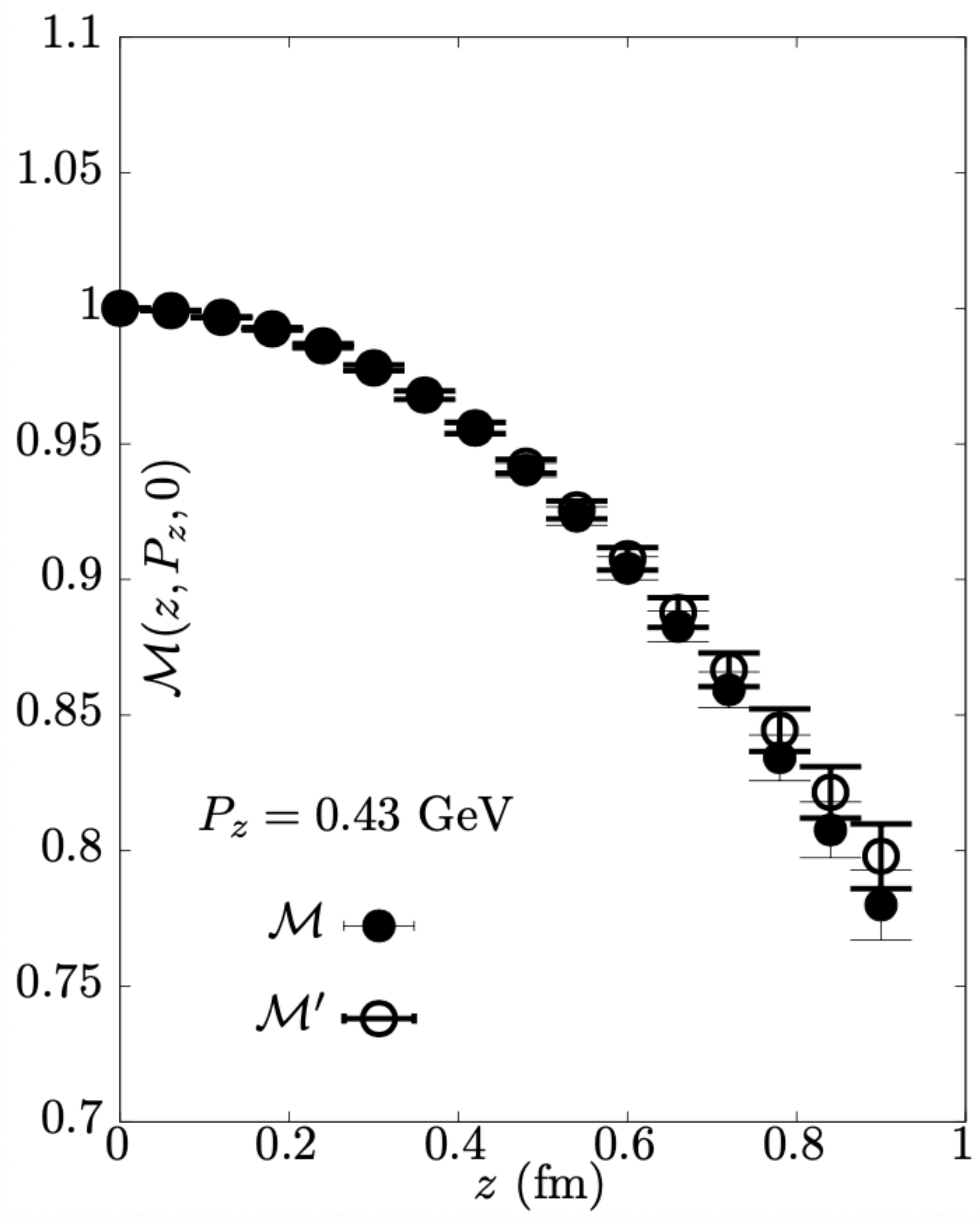


The expectations from the 1-loop leading twist results are shown as the red bands. The lattice data after subtracting the z^2 correction term is shown using open circles.

We apply this kz^2 term to non-zero momentum case, and self-consistently justified that the observed kz^2 effect in P_z = 0.43 GeV is almost the same as in P_z= 0.



Pion valence quark PDF: Higher twist effect



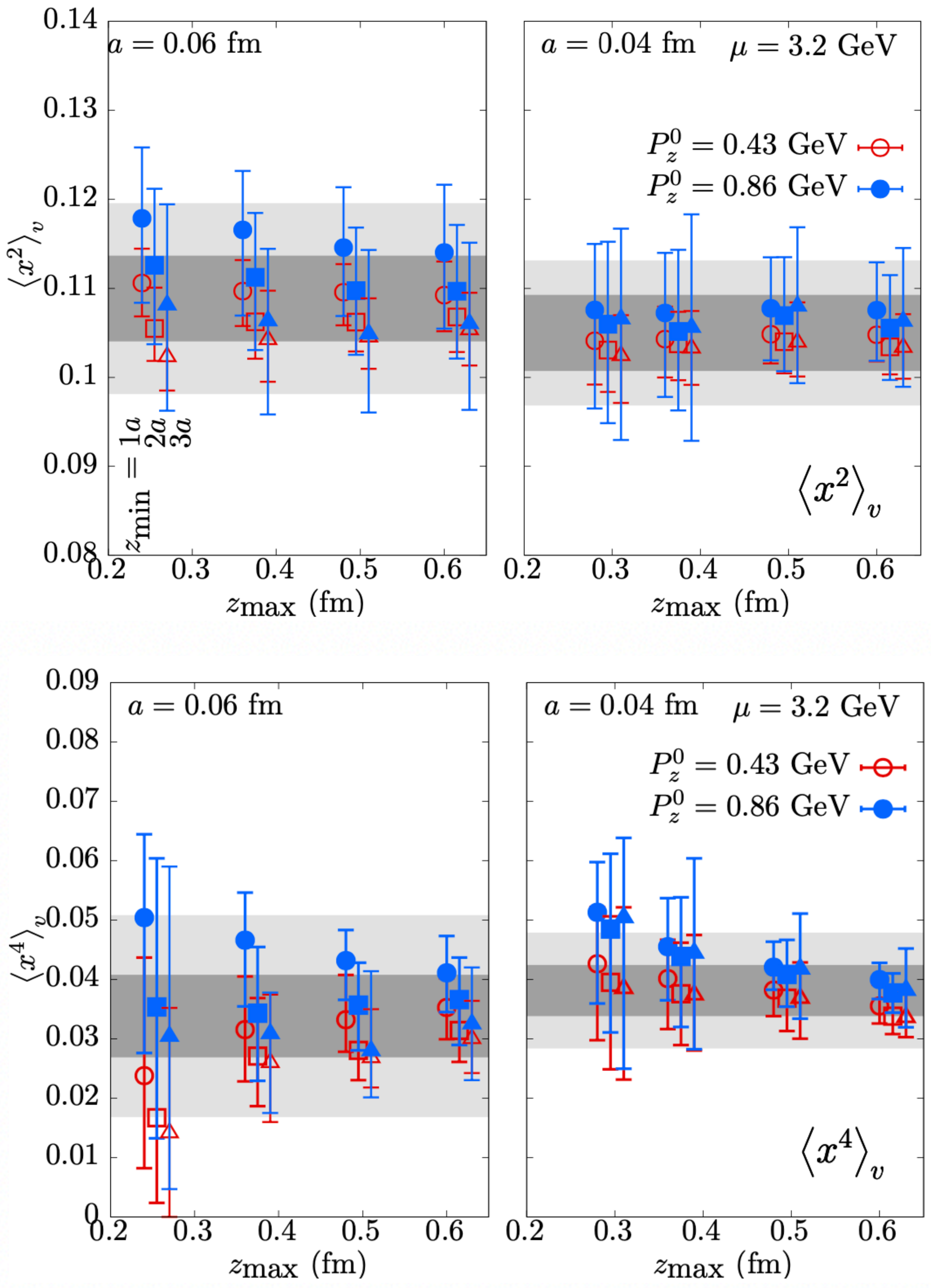
We compare the improved ratio with the standard reduced ITD, and they agree with each other within the error. This help us understand that benefit from the cancelation between numerator and denominator, the Ratio Scheme suffer less higher twist effect.

$$\mathscr{M}(z, P_z, P_z^0) = \frac{h^B_{\gamma^t}(zP_z, a)}{h^B_{\gamma^t}(zP_z^0, a)}$$

$$\mathcal{M}'(z, P_z, P_z^0 = 0) \equiv \frac{|h^R(z, P_z, P^R)| - kz^2}{|h^R(z, P_z = 0, P^R)| - kz^2}$$

Pion valence quark PDF: Moments

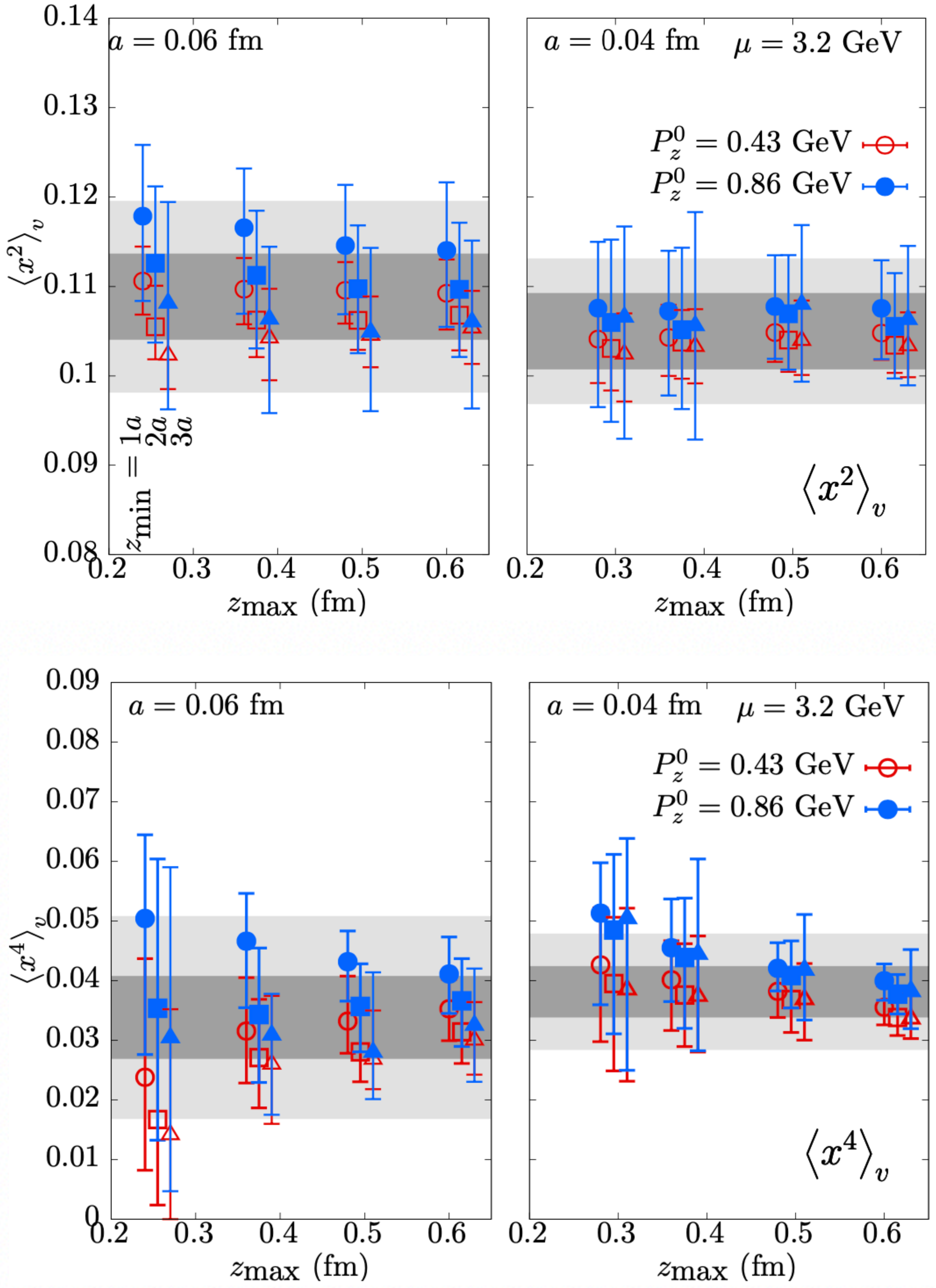
Model independent analysis of moments

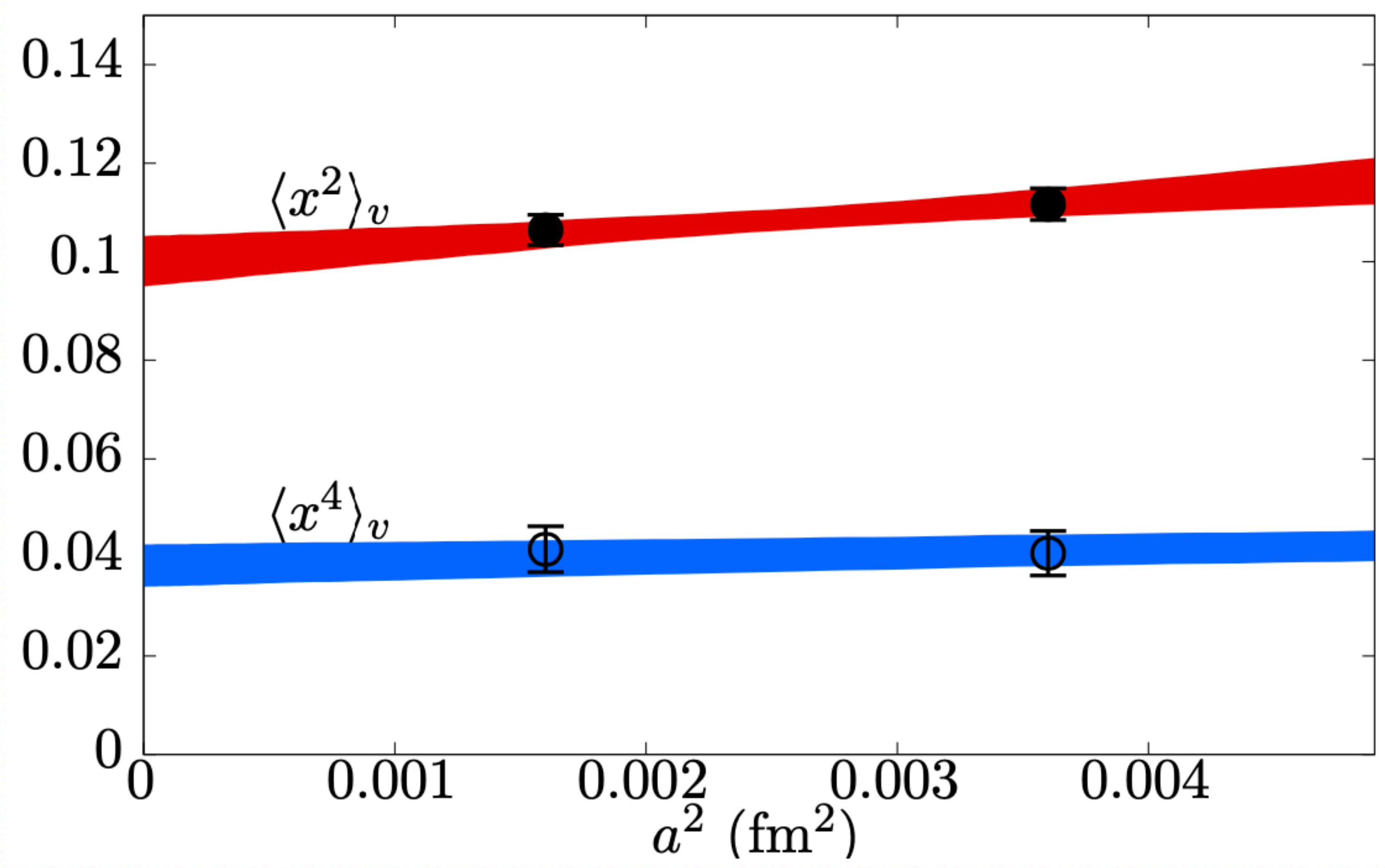


$$\mathcal{M}(z, P_z, P_z^0) = \frac{\sum_n c_n(z^2\mu^2)\langle x^n\rangle(\mu)\frac{(-iP_z z)^n}{n!} + r(aP_z)^2}{\sum_n c_n(z^2\mu^2)\langle x^n\rangle(\mu)\frac{(-iP_z^0 z)^n}{n!} + r(aP_z^0)^2}$$

Pion valence quark PDF: Moments

Model independent analysis of moments





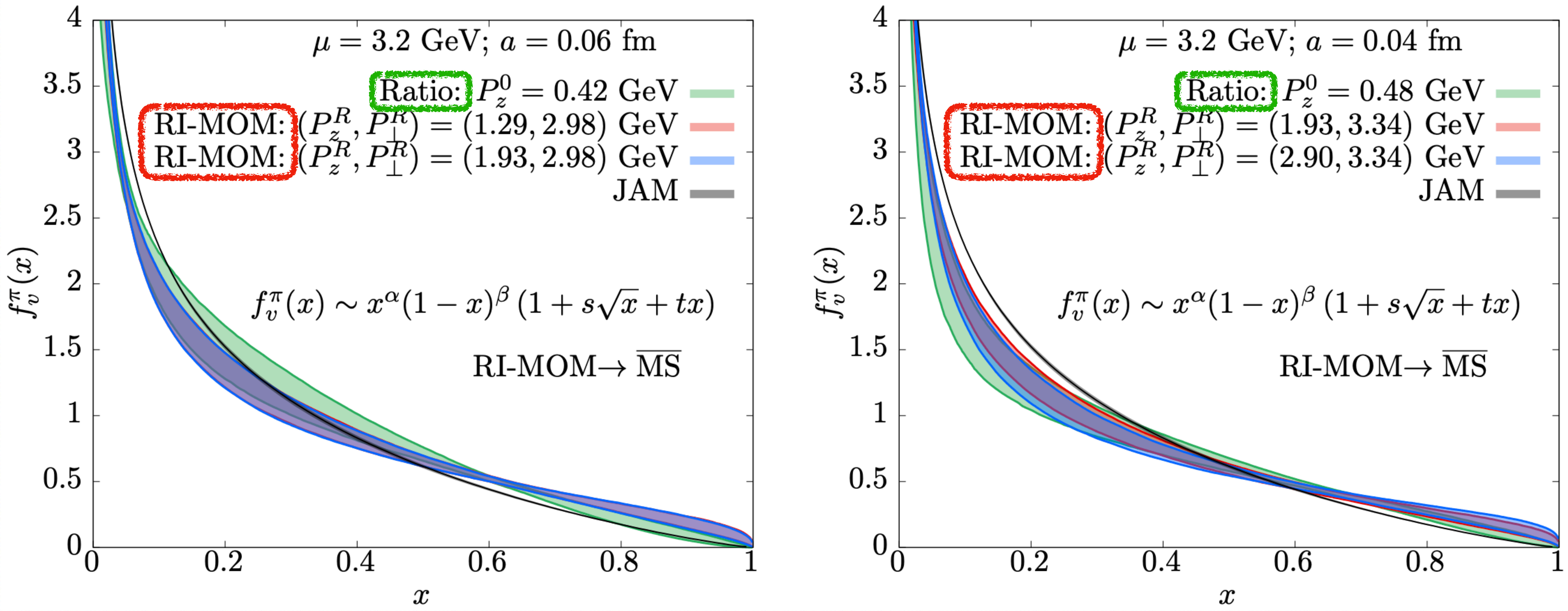
Estimate of continuum extrapolation

$$\langle x^n \rangle_v(a) = \langle x^n \rangle_v + d_n a^2$$

This work	JAM
$\langle x^2 \rangle$=0.0993(71)(54)	0.095
$\langle x^4 \rangle$=0.0356(39)(60)	0.032

Pion valence quark PDF: Model dependent fit

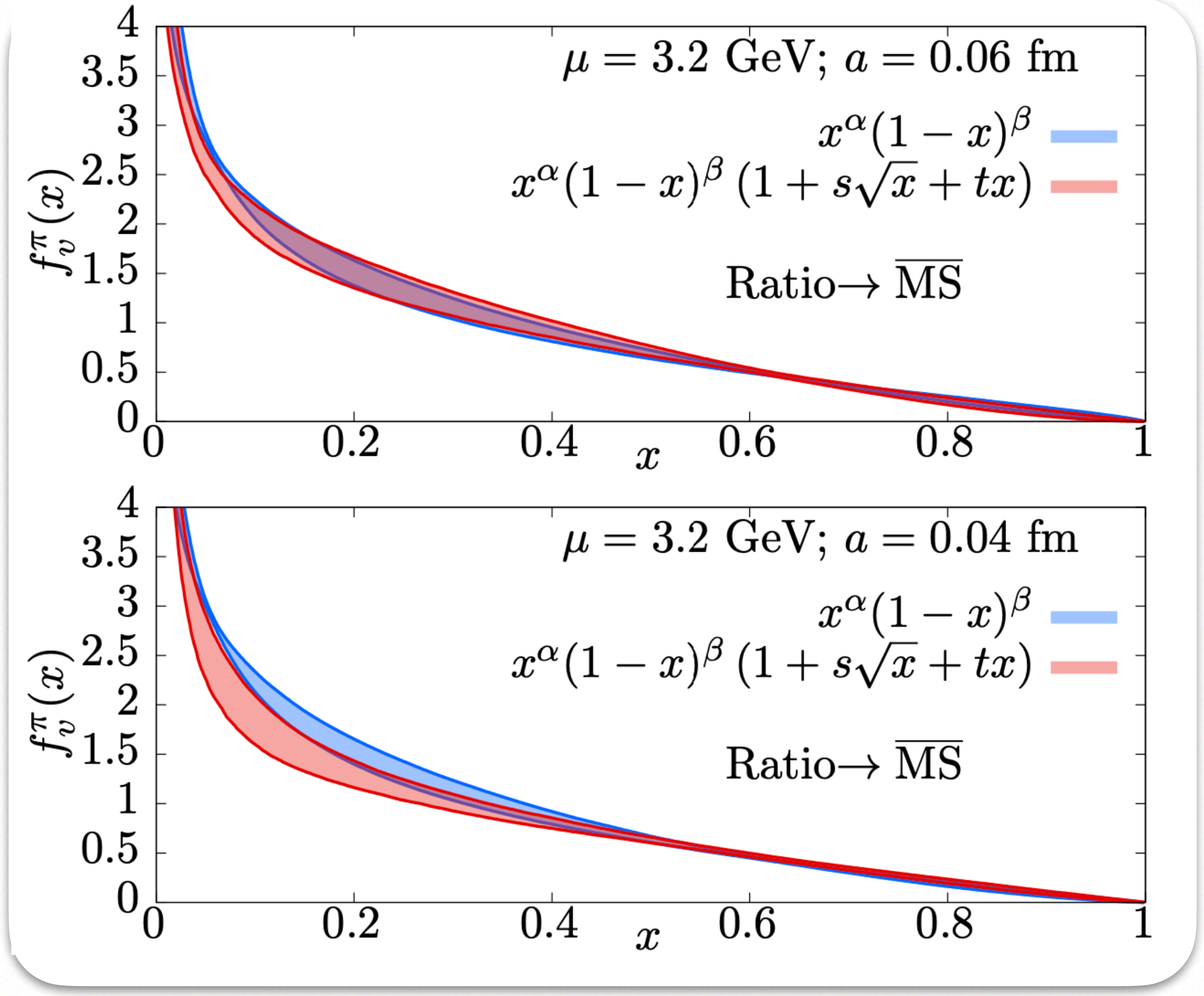
Model dependent fit by two different methods:



- The results using two different renormalization scales P^R (red and blue band) are consistent with each other as one would expect.
- The RI- MOM (red and blue band) results agree overall with the one from ratio scheme (Green band).

Pion valence quark PDF: Model dependent fit

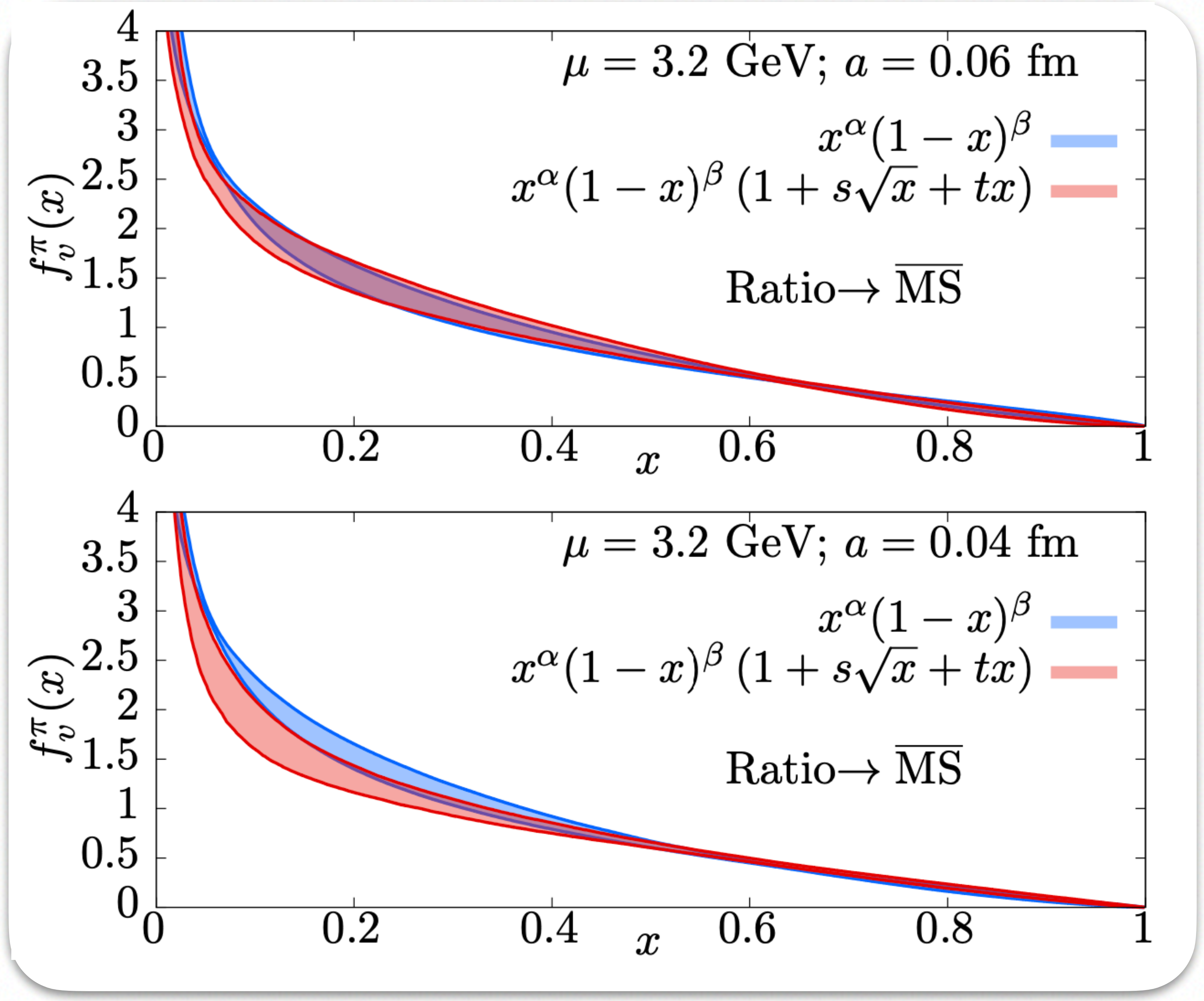
Model dependent fit



Two model ansatz are used, and consistent within the error, thus the model dependence is little with current statistics.

Pion valence quark PDF: Model dependent fit

Model dependent fit



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The large x behavior:

$$\lim_{x\to 1} f_v^\pi \sim (1-x)^\beta$$

$a \to 0$

$$\alpha = \begin{cases} -0.37^{+(16)(13)}_{-(11)(13)}, & a = 0.06 \text{ fm} \\ -0.55^{+(11)(09)}_{-(08)(09)}, & a = 0.04 \text{ fm} \end{cases} \quad -0.61(16)(08)$$

$$\beta = \begin{cases} 1.05^{+(42)(30)}_{-(42)(33)}, & a = 0.06 \text{ fm} \\ 0.76^{+(22)(24)}_{-(20)(24)}, & a = 0.04 \text{ fm} \end{cases} \quad 0.77(26)(30)$$

The downside of the model dependent analysis is the question of whether by using a sufficiently general functional form $f_v^\pi(x)$, it is possible to find $\beta \approx 2$. For example, if we performed the analysis with $\beta = 2$ fixed, the χ^2/dof is between 1.5 and 2 while the global minimum between 0.5 and 1 when β was allowed as a free parameter.

Pion valence quark PDF: Large x behavior

Model independent estimate of β

The moments $\langle x^n \rangle$ approach zero in the large-n limit in a manner dependent on β as:

$$\langle x^n \rangle \propto n^{-\beta-1}(1 + \mathcal{O}(1/n))$$

then one can determine β in a model independent way:

$$\beta + 1 = -\frac{d\log(\langle \mathrm{x}^{\mathrm{n}} \rangle)}{d\log(\mathrm{n})} + \mathcal{O}(1/n)$$

A discretised form of the above expression that is suitable for a practical implementation:

$$\beta_{\mathrm{eff}}(n) \equiv -1 + \frac{\langle x^{n-2} \rangle - \langle x^{n+2} \rangle}{\langle x^n \rangle}\frac{n}{4}$$

Pion valence quark PDF: Large x behavior

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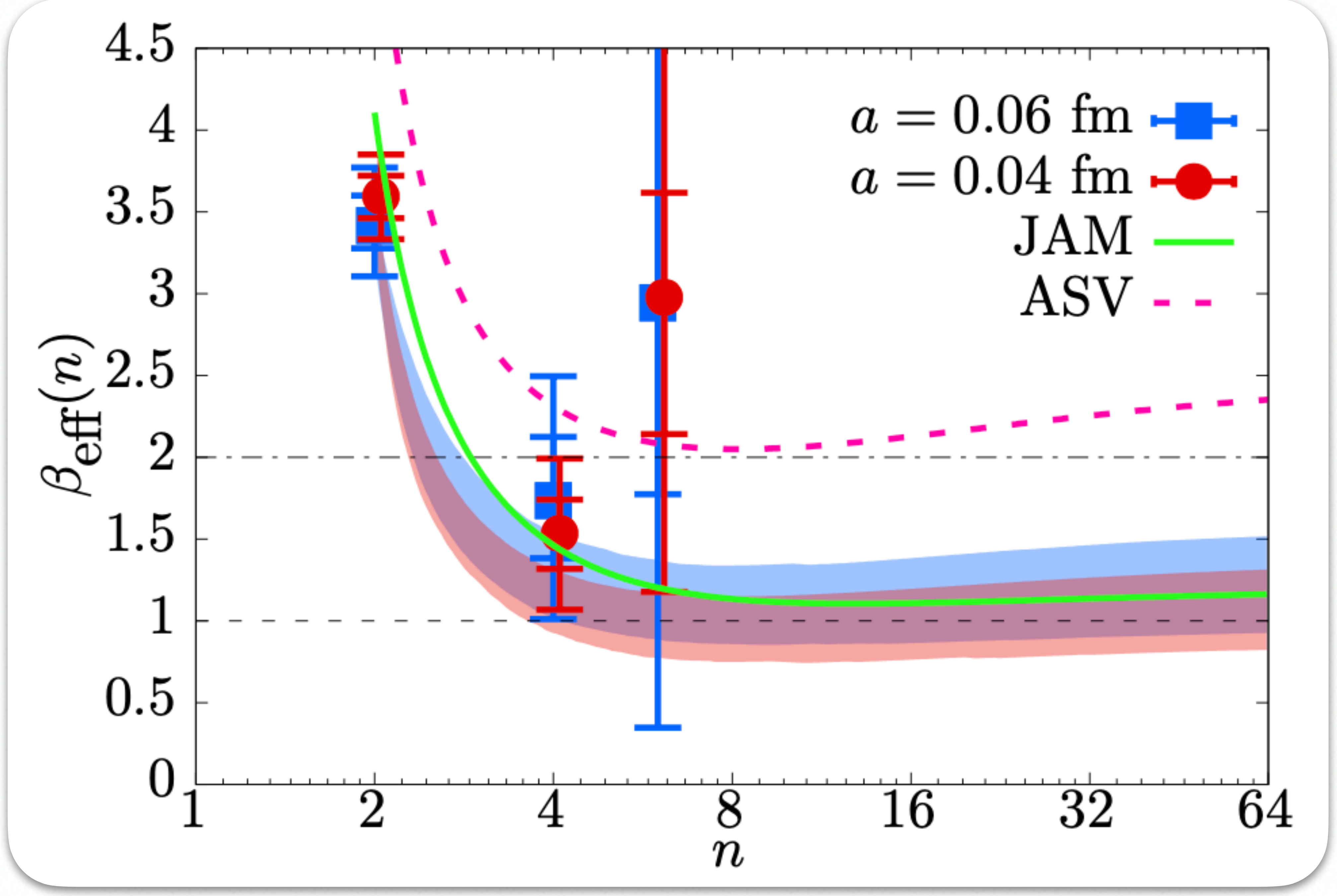
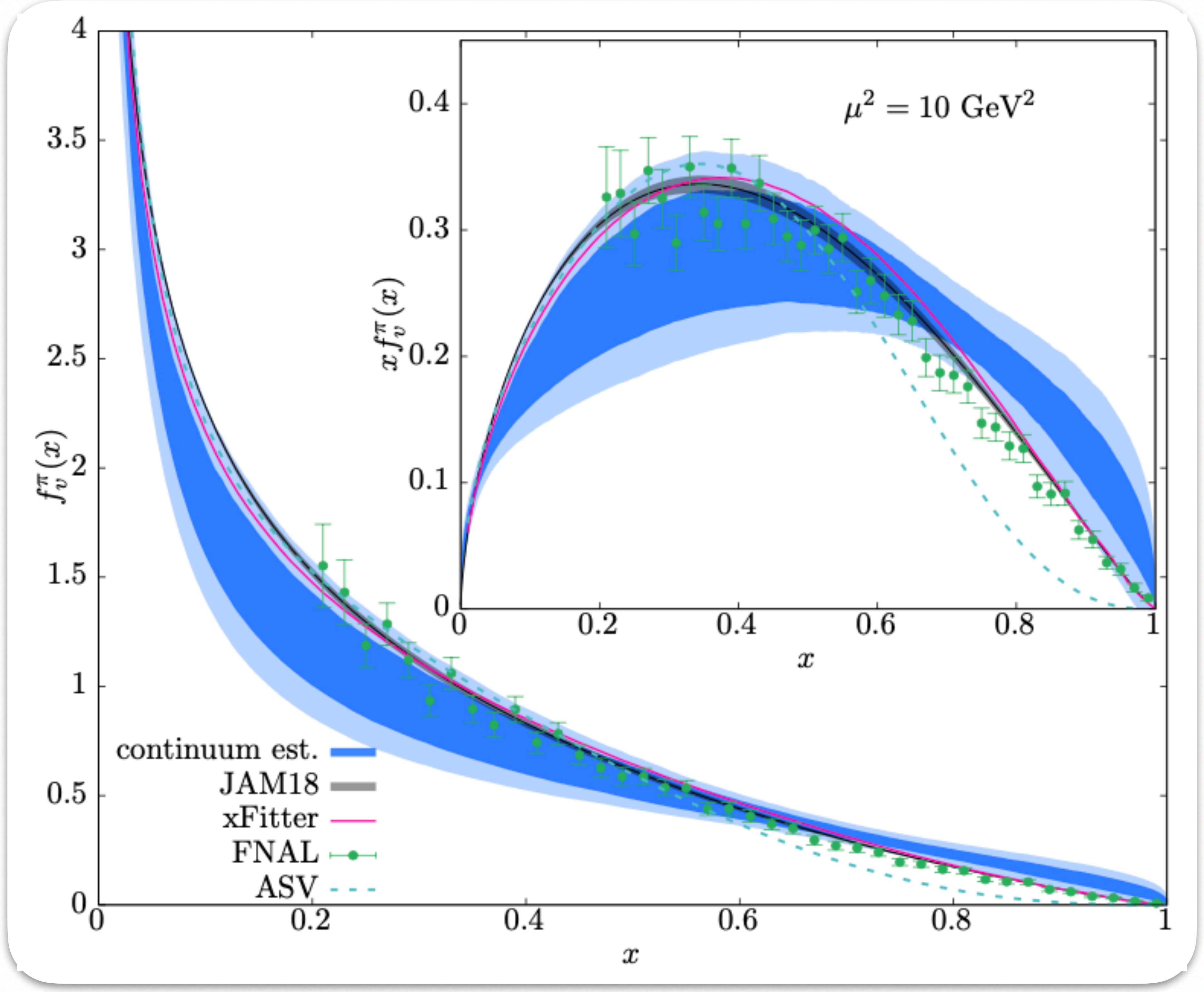


The blue and red band are from our model dependent PDF of two different lattice, and the green line and dashed line are corresponding to JAM and ASV.

$$\beta_{\text{eff}}(n=4) = \begin{cases} 1.73^{+(39)(37)}_{-(35)(37)}\,, & a = 0.06 \text{ fm} \\ 1.53^{+(21)(25)}_{-(21)(25)}\,, & a = 0.04 \text{ fm}. \end{cases}$$

Pion valence quark PDF: Summary



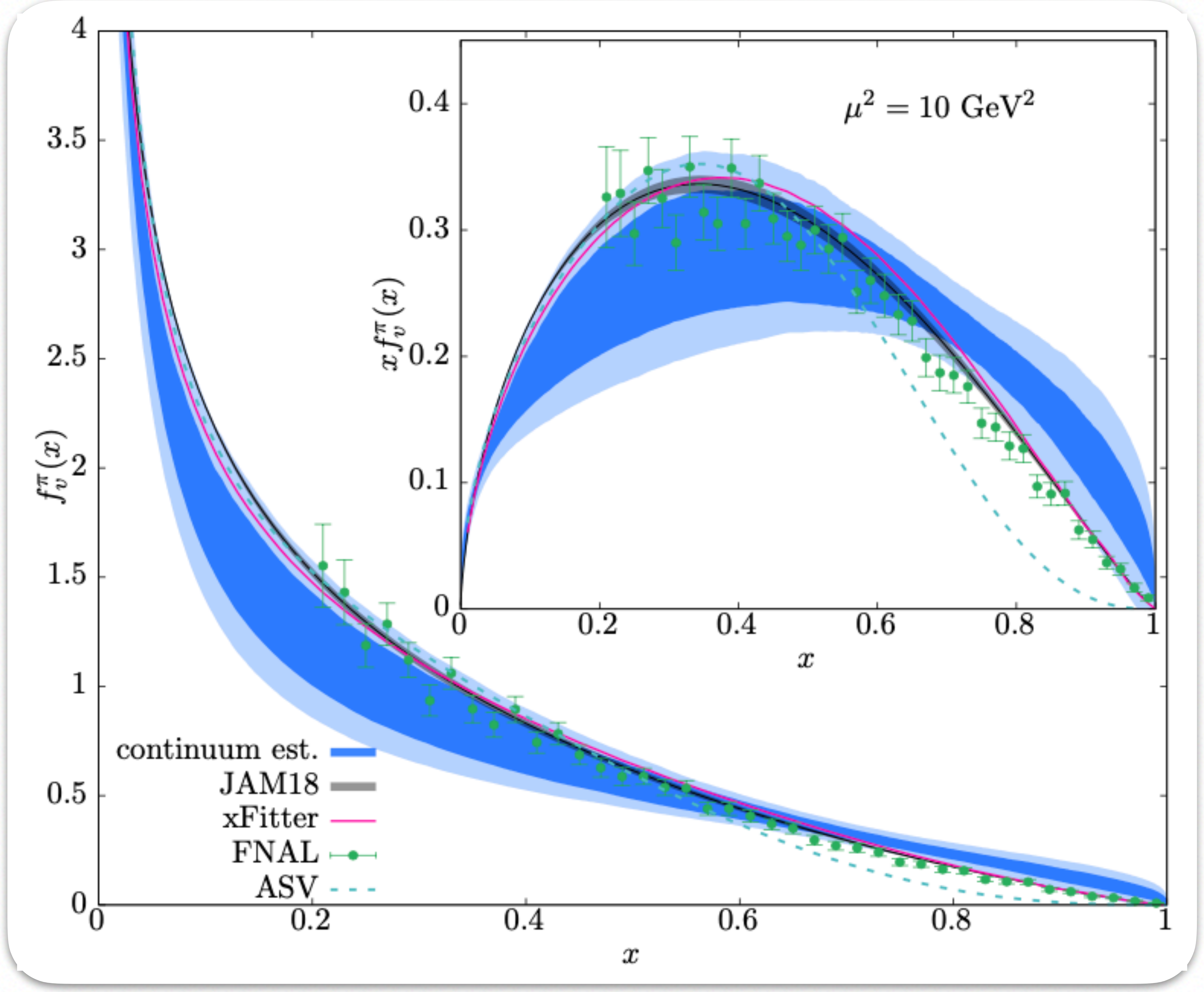
BNL, SBU & THU, arXiv:2007.06590

We compare our estimate for the PDF in the continuum limit for 300MeV pion with other global fit analysis. One can find an overall agreement of our determinations with the phenomenological results; with better agreement with JAM, xFitter and the initial E-0615 estimates, than with the ASV result.

Possible improvement:

- More statistics to extract higher moments.
- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.
- …

Pion valence quark PDF: Summary



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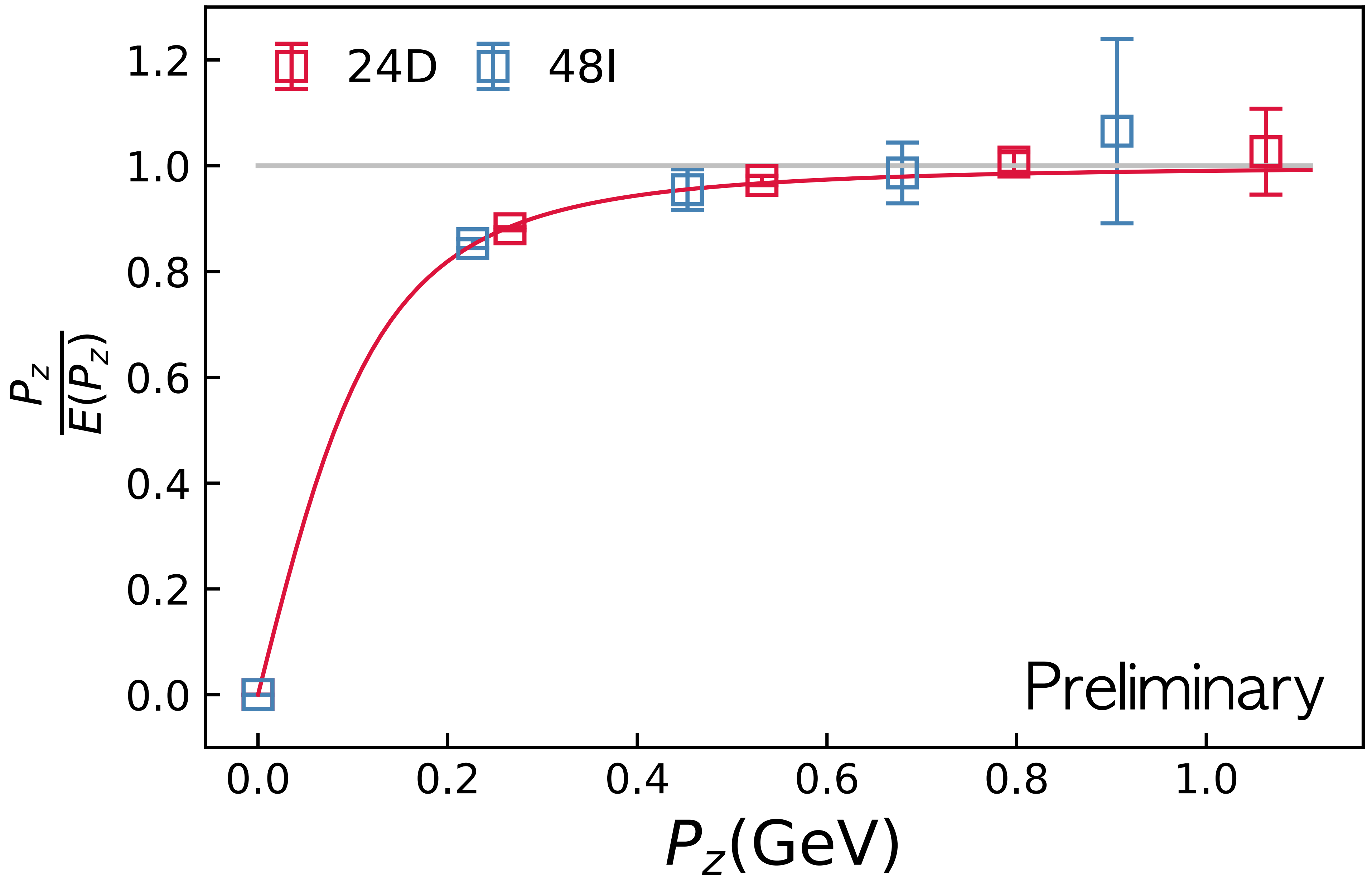
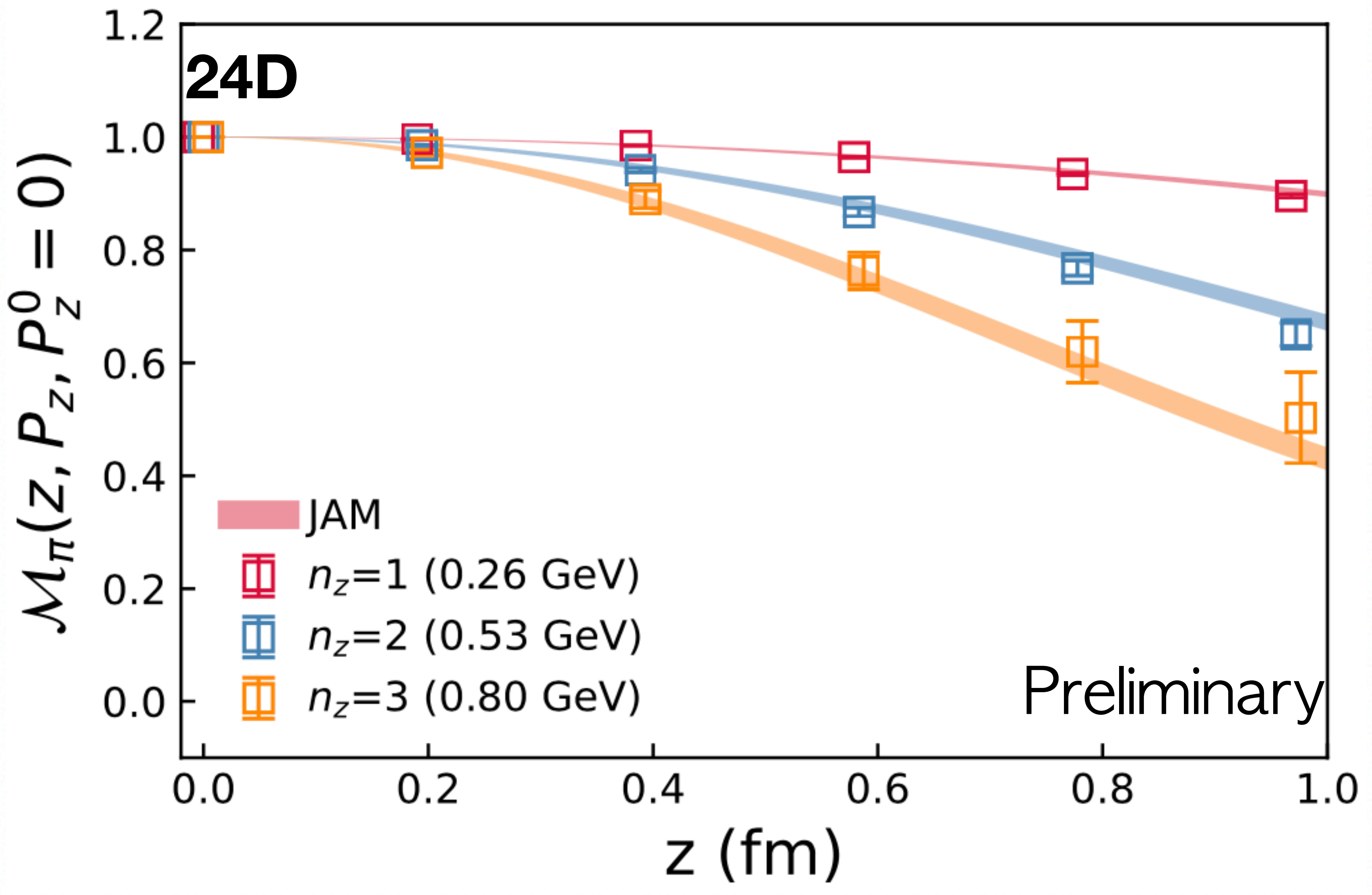
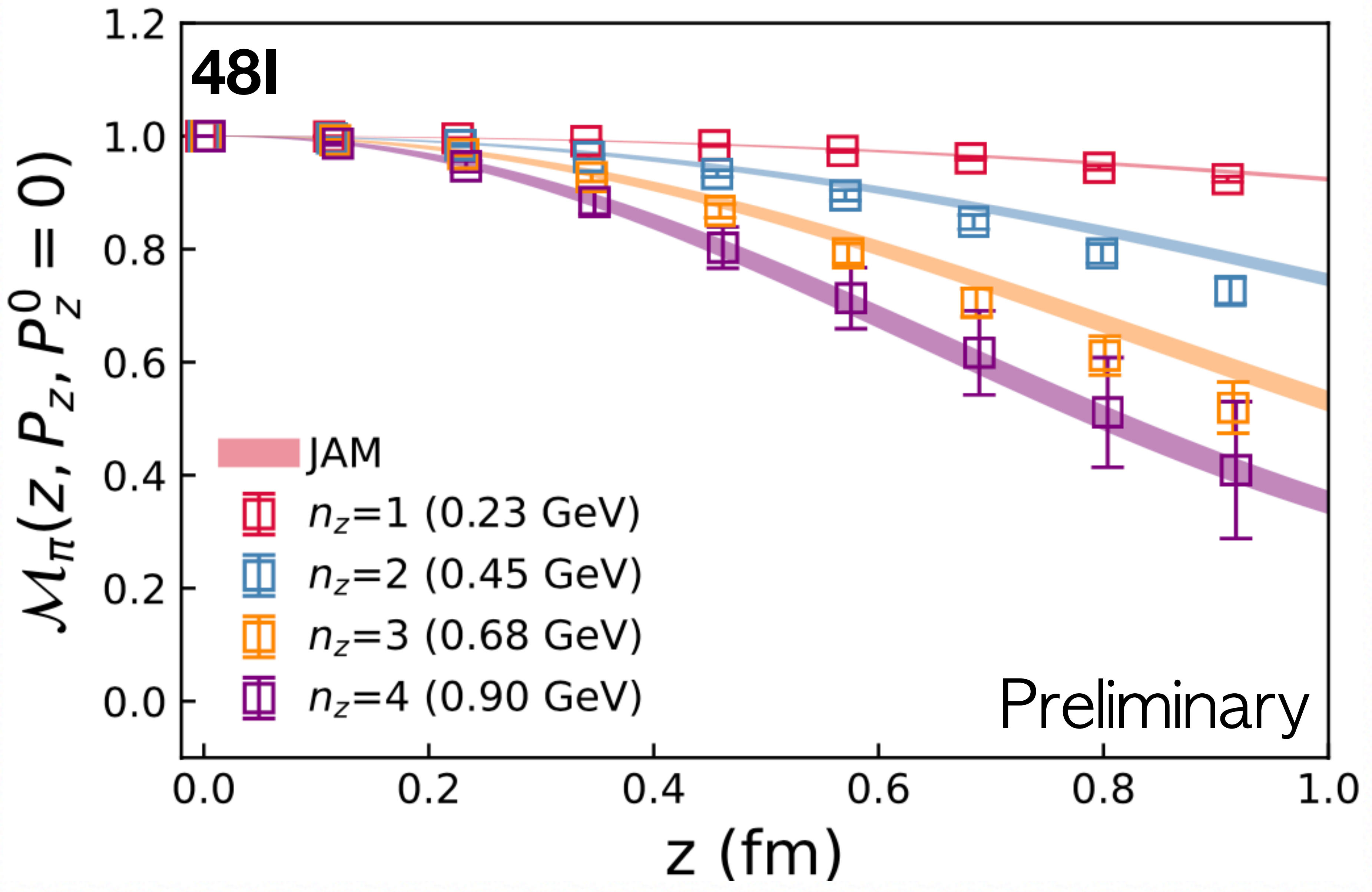
Preliminary results from DWF



Lattce	a(fm)	P_z	t_s	z	#cfgs	M_π(GeV)	M_N(GeV)
24D ($24^3 \times 64$)	0.1944	[0,4]	4,6,8	[-12,12]	101(×64)	0.1420(4)	0.961(11)
48I ($48^3 \times 96$)	0.1141	[0,4]	4,6,8	[-24,24]	22(×48)	0.138(1)	0.994(55)

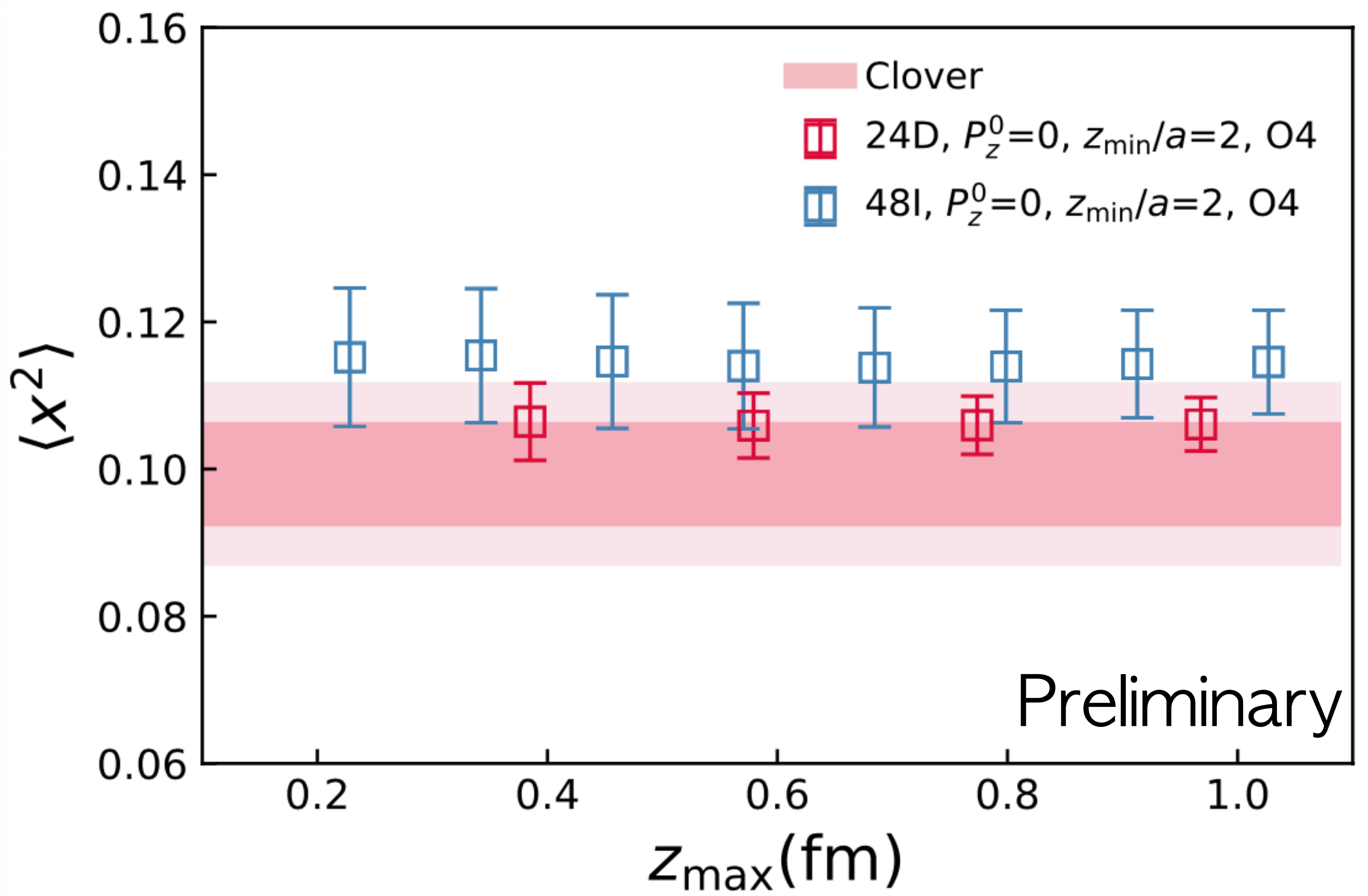
Preliminary results from DWF: rITD

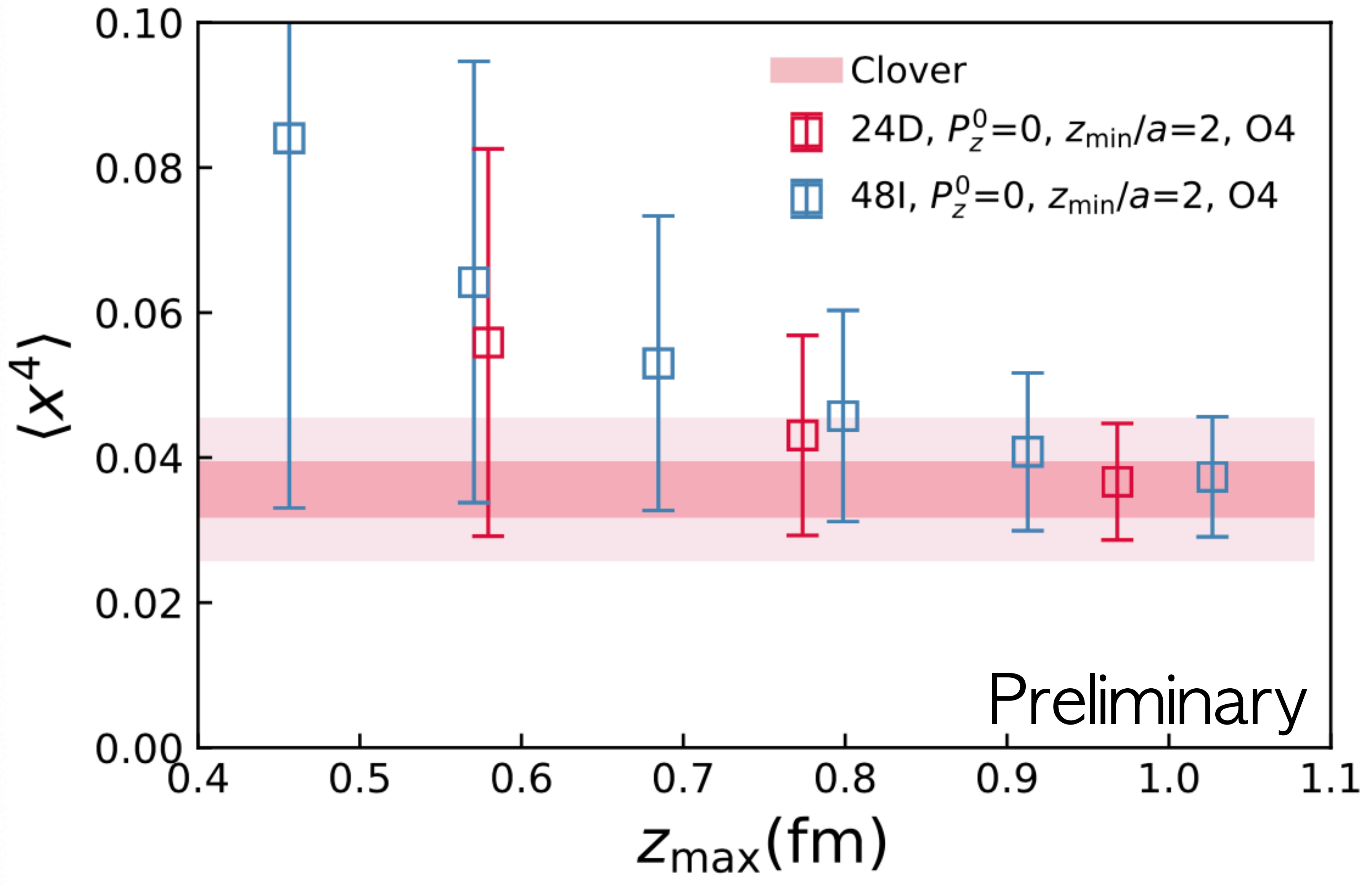




$$\frac{h^B_{\gamma^t}(z, P_z, a)}{h^B_{\gamma^t}(z, P_z^0 = 0, a)} = \sum_n \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy y^n q(y, \mu)$$

Preliminary results from DWF: Moments





$$\frac{h^B_{\gamma^t}(z,P_z,a)}{h^B_{\gamma^t}(z,P_z^0=0,a)} = \sum_n \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy y^n q(y,\mu)$$

Since the momentum we have is relative small, we truncate the function up to 4th order, then fit to extract 2nd and 4th moments.

The bands are the best estimate from our fine Clover+HISQ lattices. One can observe good agreement between this DWF calculation and our previous Clover+HISQ results.

Summary

- We studied pion valence quark PDF in the frame of LaMET using both RI-MOM and ratio-based schemes.
- The higher twist effect and lattice artifacts are investigated empirically.
- We reconstruct the x-dependent pion valence PDF, as well as infer the first few moments.
- Preliminary results from DWF show good agreement with Clover+HISQ.