Pi/K Distribution Amplitudes with Hybrid Renormalization

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Outline

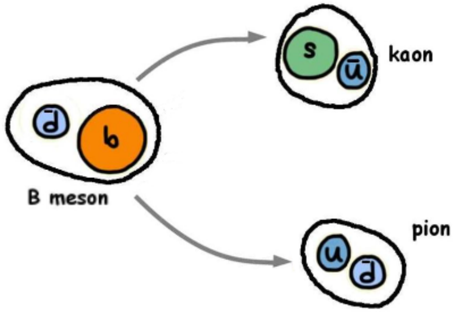
- Pi/K distribution in lattice calculation
- Hybrid renormalization on Pi/K
- Numerical results
- Summary and outlook

Pi/K distribution(DA) from lattice calculation

- Describe hard exclusive processes involving large momentum transfer
- Inputs for measuring fundamental parameters
- Recently Pi/K researches include:

  (J.H. Zhang et al arXiv:1702.00008, G.S. Bali et al arXiv: 1709.04325, J.W. Chen et al arXiv: 1712.10025……)

- Limited by lattice calculation and theoretical research, Pi/K DAs still need a more precise research specially with the Hybrid renormalization proposed by Xiangdong Ji recently on arXiv:2008.038861



Pi/K distribution(DA) from lattice calculation

- The two point matrix element are defined as:

$$C_M^{DA}(z,P_z,t)=\langle\int d^3x e^{i\vec{P}\vec{x}}\bar{\psi}(\vec{x},t)\Gamma_1 U(\vec{x},\vec{x}+z)\psi(\vec{x}+z,t)\bar{\psi}(0,0)\Gamma_2\psi(0,0)\rangle$$

with $\Gamma_1=\gamma_t\gamma_5, \Gamma_2=\gamma_5$

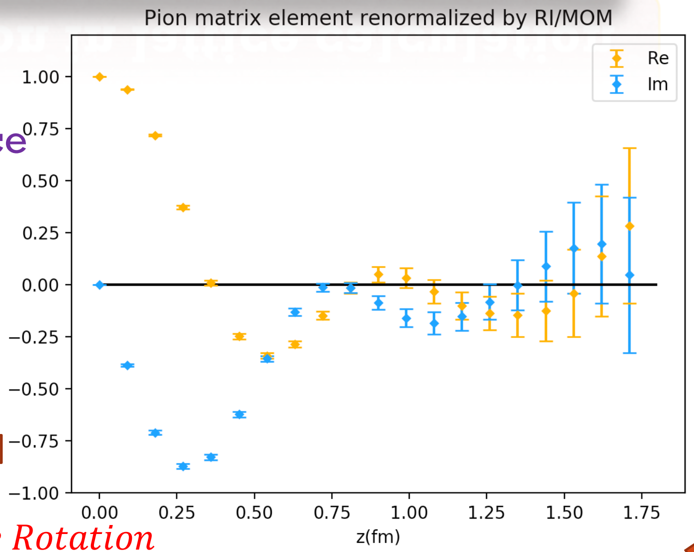
- Renormalize the UV divergence by RI/MOM or hybrid method

- Fourier transform the matrix elements in the coordinate space into quasi DA

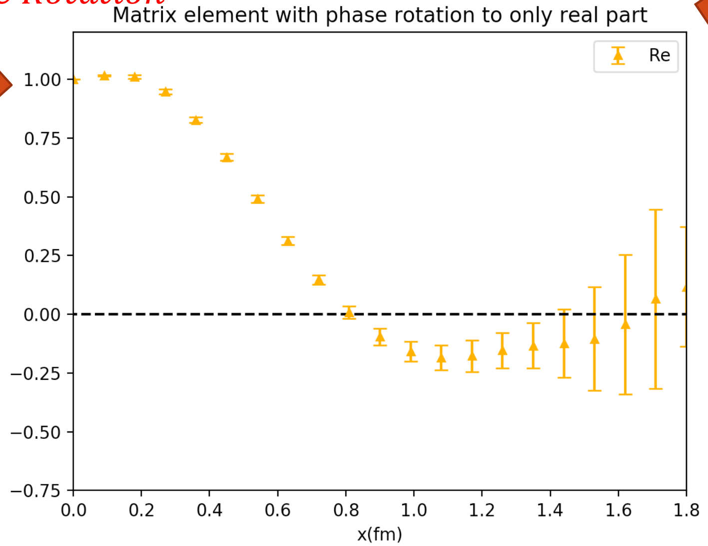
- Matching from Quasi DA to Light-cone DA (J. Xu et al PRD97.114026, Y.S. Liu et al PRD99.094036)

Pi/K distribution in lattice calculation

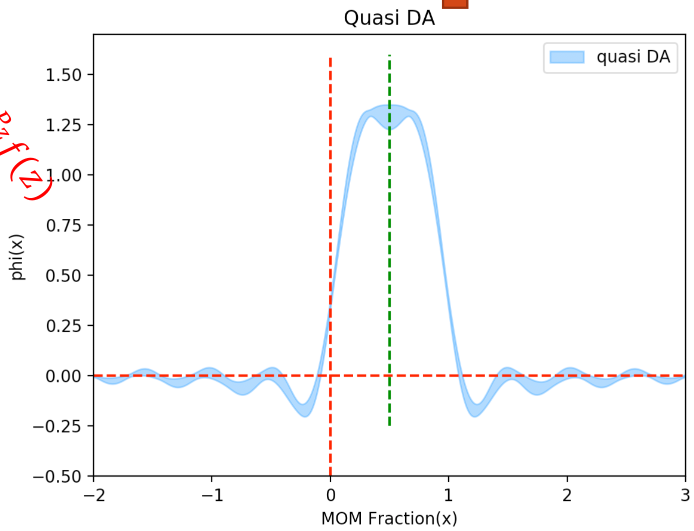
Graphical process for
Pi/K distribution on lattice


Pion matrix element renormalized by RI/MOM

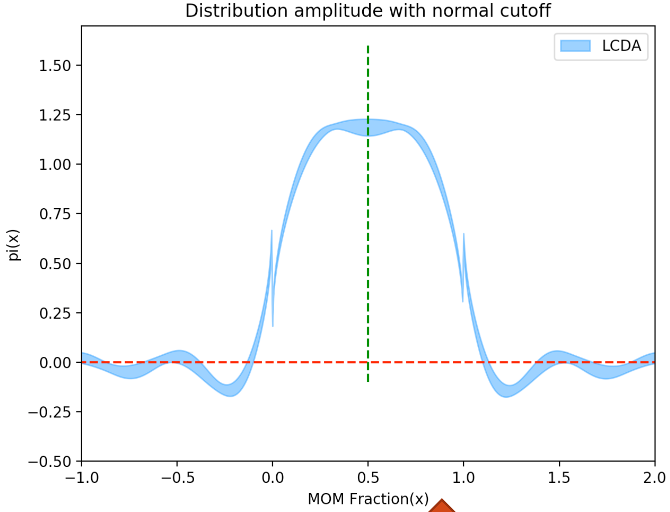
Phase Rotation


Matrix element with phase rotation to only real part

$FT: \int dz \frac{1}{2\pi} e^{ixzP_z} f(z)$

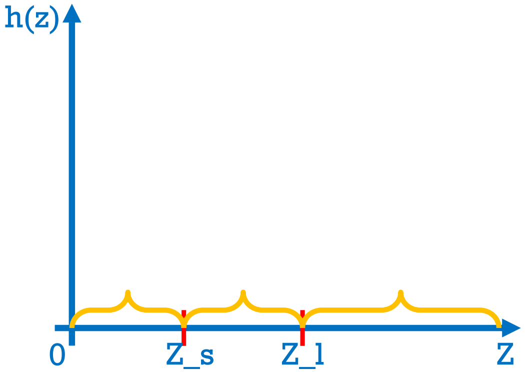

Quasi DA

Inverse Matching


Distribution amplitude with normal cutoff

Hybrid renormalization on Pion/K

- Separate the z-range into different parts and treat them differently



- At small distance ($0 \leq |z| \leq zS$), use RI/MOM renormalization
- At medium distance ($zS \leq |z| \leq zL$), use modified mass renormalization
- Beyond lattice reach, use physics-based extrapolation

---- X.D. Ji et al arXiv:2008.03886

Simulation information

Calculation platform: 2+1+1 flavors of highly improved staggered quarks (HISQ) ensembles generated by MILC collaboration，with physical pion mass 130MeV and lattice spacing 0.12fm and 0.09fm

Momentum smearing technology: maintain small statistical errors and good overlaps of hadronic wave functions with the respective ground states

Conf	N_conf	Size	clovCoeff	Mass(u/s)	Smearing Size
A09m130	300 * 6	64*96	1.04239	-0.058 -0.0174	5.33/50

Test on Hybrid renormalization

Fitting the RI/MOM factor

$$C_2 = C_0 e^{-\left(1+c2\ln\frac{z}{z_s}\right)\left\{\underset{\delta_m}{\underline{\left(\frac{m_{-1}}{a}+m_0\right)}}*z+c_1\ln(z)\right\}}$$

Fitted Chi2: 0.15

$m_{-1} = 0.1785(24)$, $c1 = 0.5048(62)$,

$m_0 = 0.1462(42)$, $c2 = 0.2213(94)$,

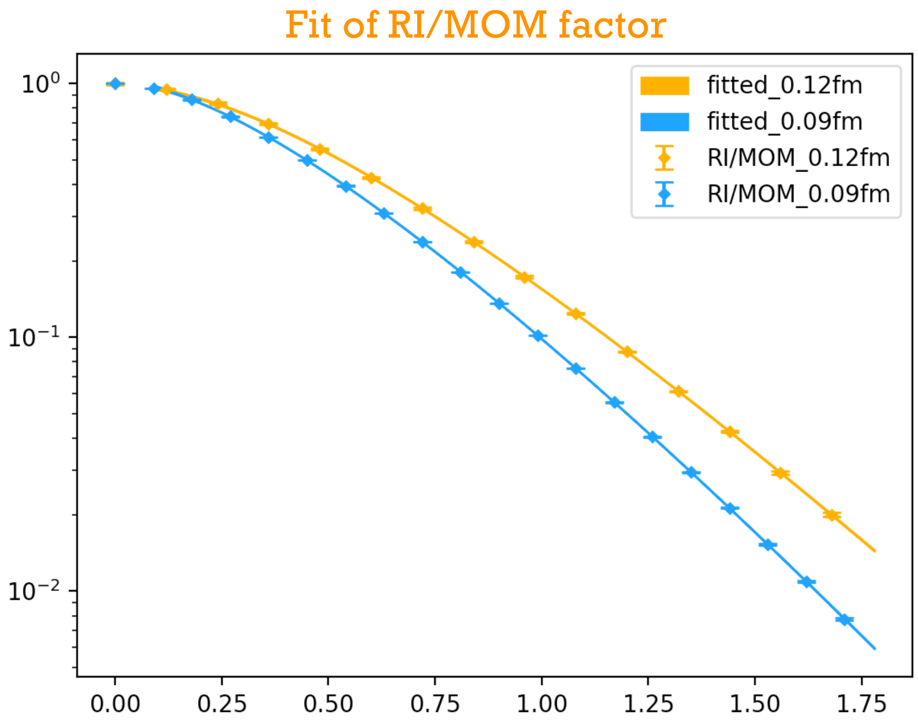
$C0_{a09} = 2.914(46)$, $C0_{a12} = 2.801(46)$

m_{-1}: The coefficient of the power divergence

m0: a. Renormalon effect
b. Pole mass
c. Finite P^z effects
d. Fitting effect.

c1,c2: ln(z) scale infrared divergence

c0: overall factor



Test on Hybrid renormalization

- We can begin with a form that incorporates the asymptotic behavior of parton distributions at $x \to 0$ or 1 $\quad x^a(1-x)^b$.

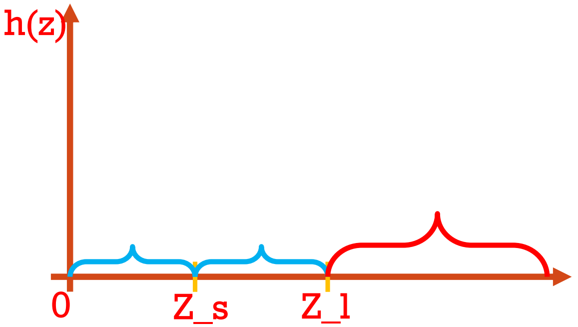
- Its FT: $q(\lambda) = \int_0^1 dx e^{ix\lambda} x^a (1-x)^b$

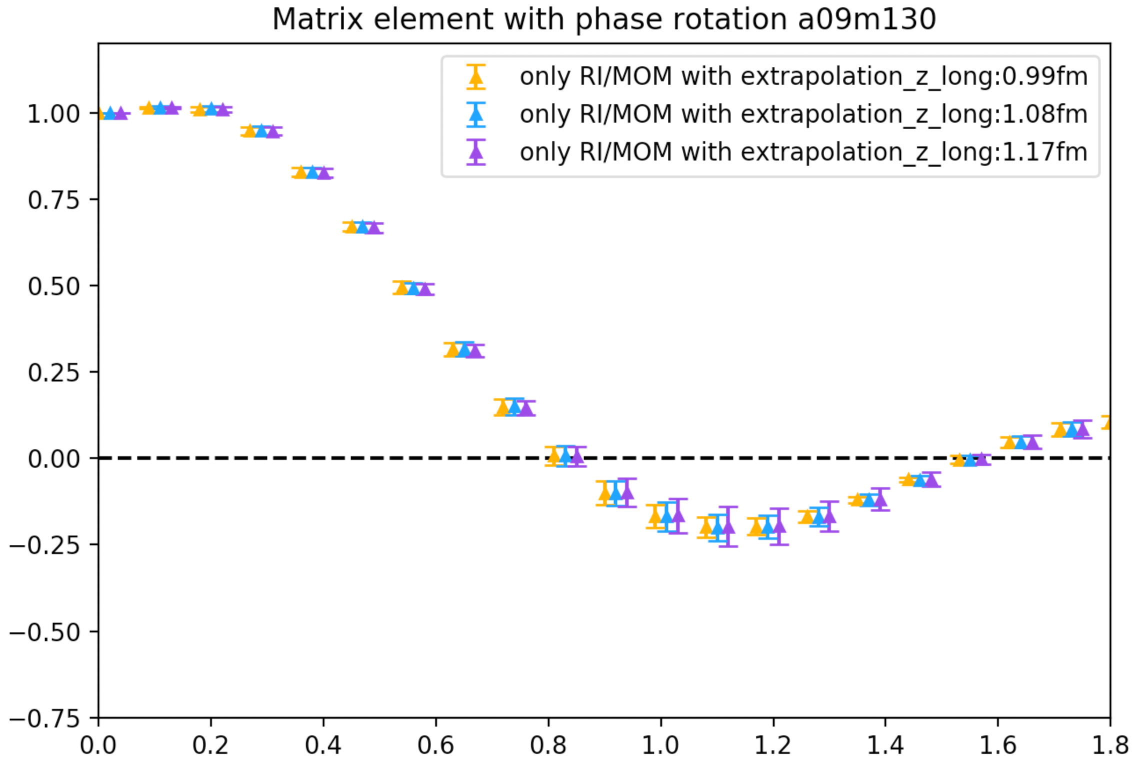
- Behaves at large $\lambda(zp_z)$ like

  (Pi is symmetric about x=1/2 , b=a):

$$q(z) = \frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^a}$$

phase rotation $\rightarrow \frac{c}{(\lambda)^a}\cos((1+a)\lambda)$



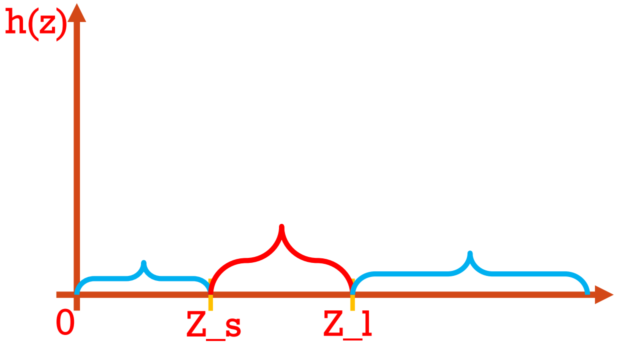


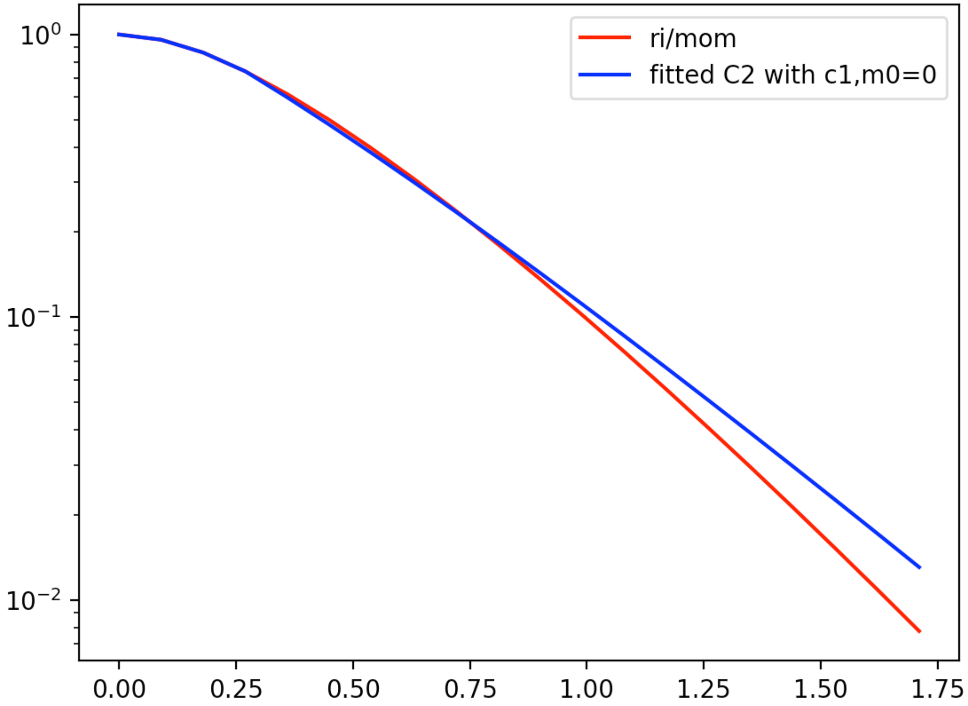
Test on Hybrid renormalization

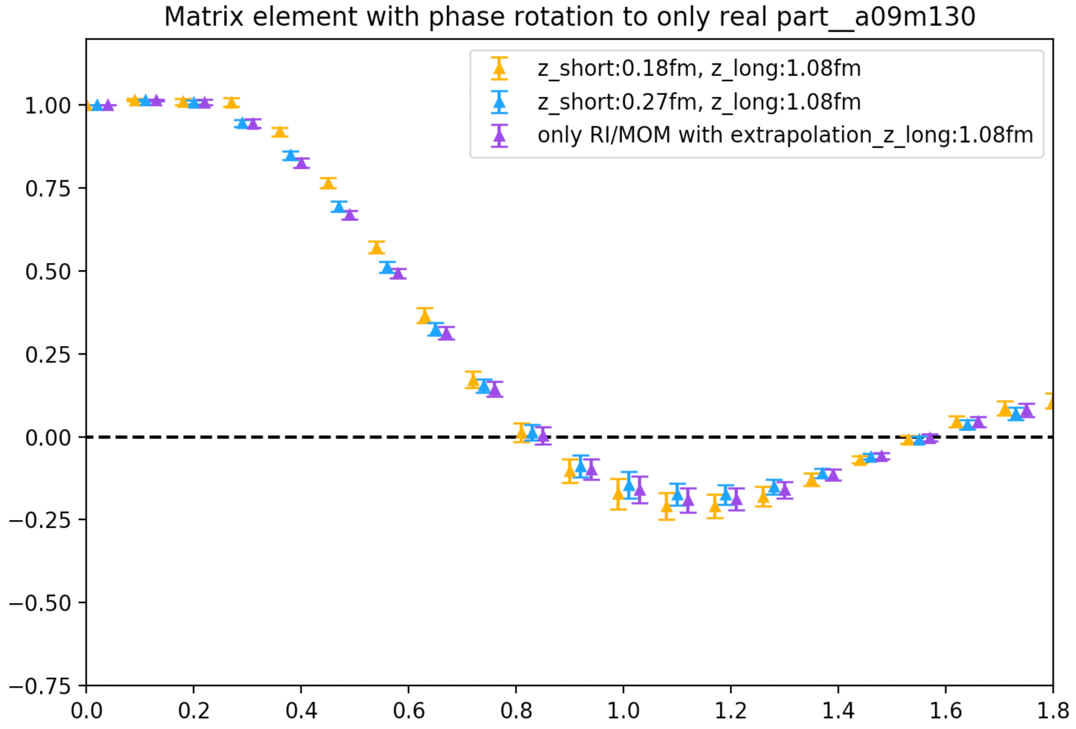
$$C_2 = C_0 e^{-\left(1+c2\ \ln\frac{z}{z_s}\right)\left\{\left(\frac{m_{-1}}{a}+\frac{1}{0.197}*m_0\right)*z+c_1\ln(\frac{z}{z_s})\right\}}$$

$m_{-1} = 0.1785(24)$, $\quad$ $\mathrm{c1} = 0.5048(62)$,
$\mathrm{m}_0 = 0.1462(42)$, $\quad$ $\mathrm{c2} = 0.2213(94)$,
$\mathrm{C0}_{\mathrm{a09}} = 2.914(46)$

Set m0=0 and c1=0







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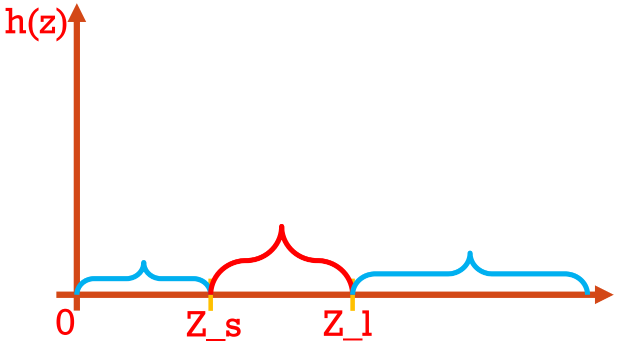
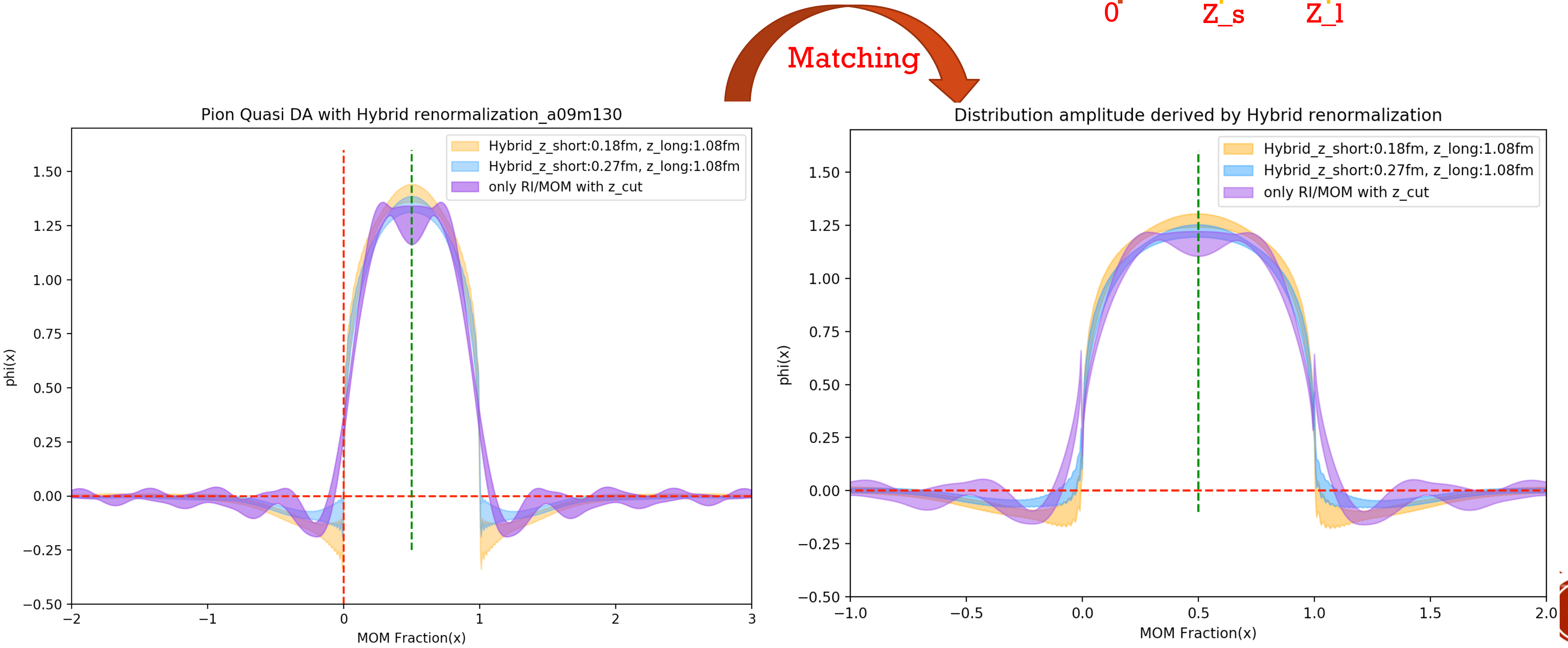




Summary

- We applied the Hybrid renormalization approach on Pi/K distribution amplitudes
- The results with Hybrid renormalization are more reasonable between x in[0,1]

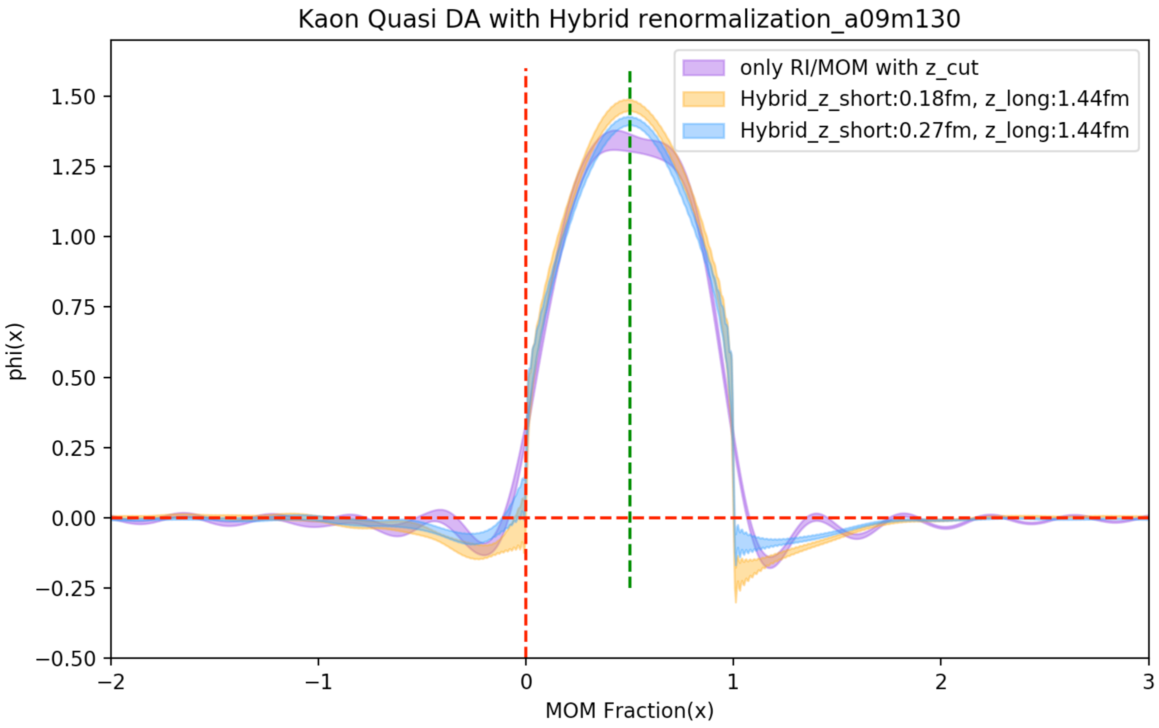
Outlook

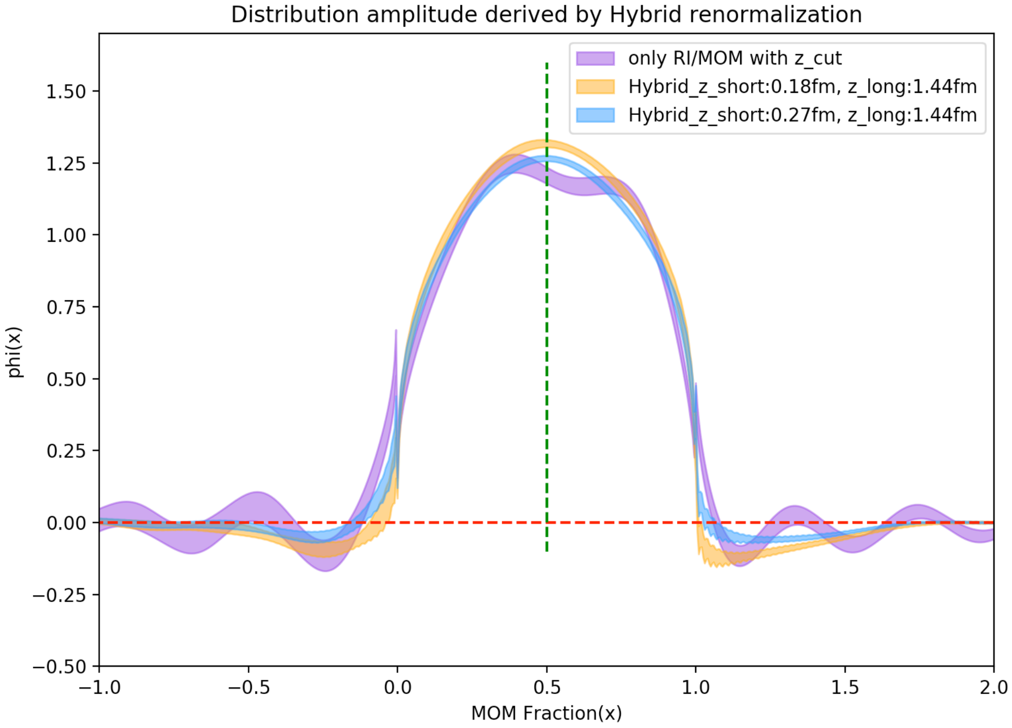
- We will continue to study Hybrid renormalization approach and apply this approach to other cases.

Thank You

Backup Slides

Kaon case


Kaon Quasi DA with Hybrid renormalization_a09m130


Distribution amplitude derived by Hybrid renormalization