Collins-Soper Kernel from lattice QCD

Phiala Shanahan, MIT

Image Credit: 2018 EIC User's Group Meeting

LaMET 2020, Sept. 11

Massachusetts Institute of Technology

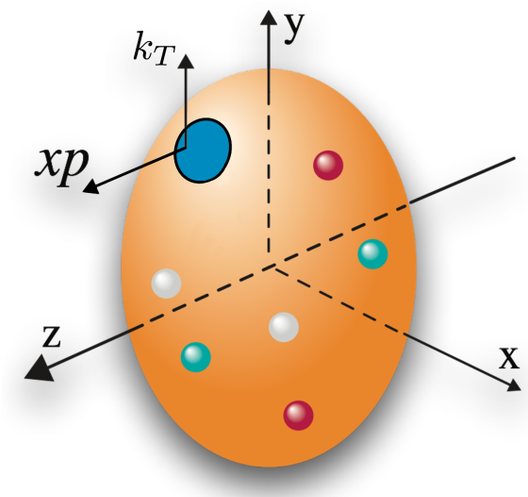
Transverse Momentum Dependent PDFs

- More detailed picture of nucleon including transverse structure

PDF
$f_{q/P}(x)$
longitudinal

→

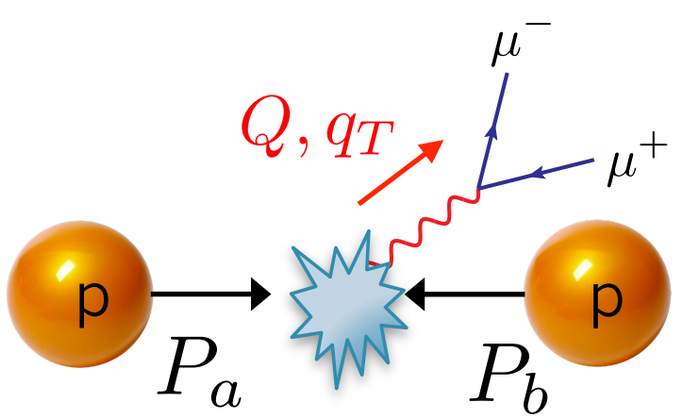
TMD
$f_{q/P}(x, k_T)$
+ transverse



- TMDPDFs accessed in e.g.,

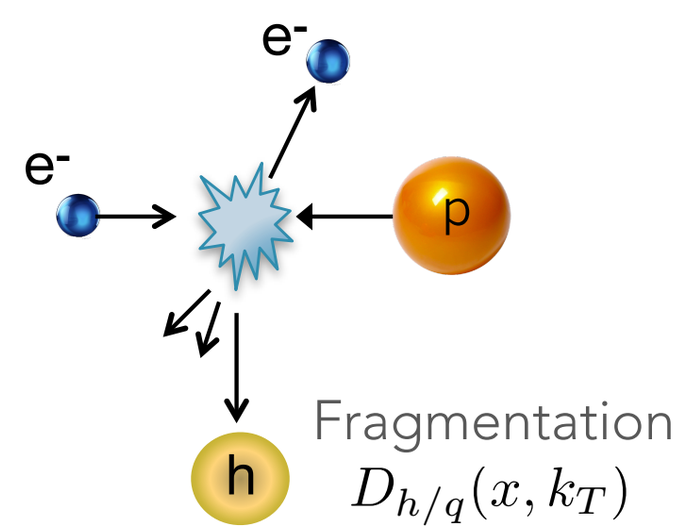
Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



Semi-inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



Figures: Iain Stewart

Transverse Momentum Dependent PDFs

- e.g., Drell-Yan, more precisely

$$\sigma\left(q_T, Q\right) \sim \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_q(x_a, \vec{b}_T, \mu, \zeta_a)\, f_q(x_b, \vec{b}_T, \mu, \zeta_b) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

Fourier transform

$$f_q(x_a, \vec{k}_T, \mu, \zeta_a)$$

Renormalisation scale

Collins-Soper parameter

$$\zeta_a = (x_a P_a^-)^2 = (2x_a P^z)^2$$

$$\zeta_a \zeta_b = Q^4$$

- Two evolution equations for TMDPDFs

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper kernel
i.e., rapidity anomalous dimension

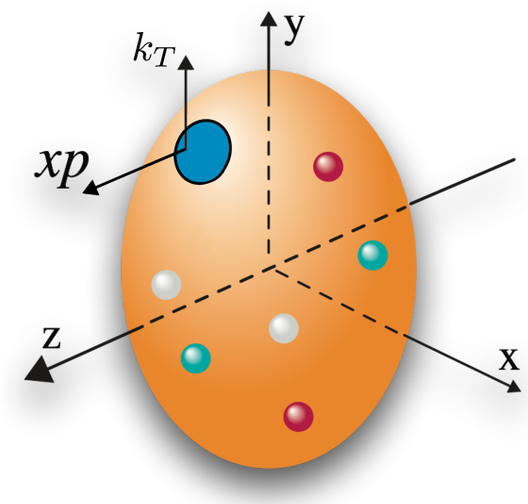
Collins-Soper evolution kernel

- More detailed picture of nucleon including transverse structure

PDF
$f_{q/P}(x)$
longitudinal

→

TMD
$f_{q/P}(x, k_T)$
+ transverse



Collins-Soper Evolution Kernel

$$\gamma_\zeta^q(\mu, b_T) = \zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta)$$

- Governs TMD evolution
- Needed to match quasi-TMD to physical TMD

- Perturbative at short distances $\mu, b_T^{-1} \gg \Lambda_{\rm QCD}$
- Non-perturbative for $b_T^{-1} \lesssim \Lambda_{\rm QCD}$

Collins-Soper evolution kernel

Estimates of size of nonperturbative contributions to CS kernel

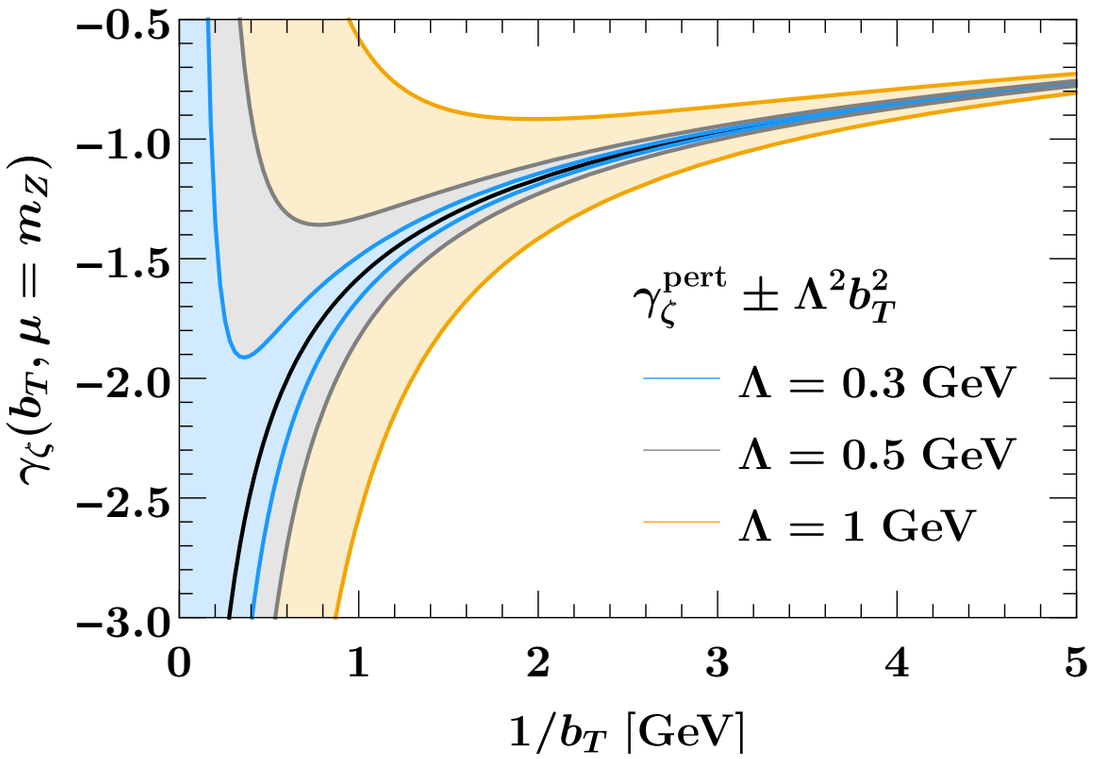


Figure: Iain Stewart

- Large uncertainties in estimates of the size of nonperturbative contributions to the CS kernel

CS kernel from phenomenological fits

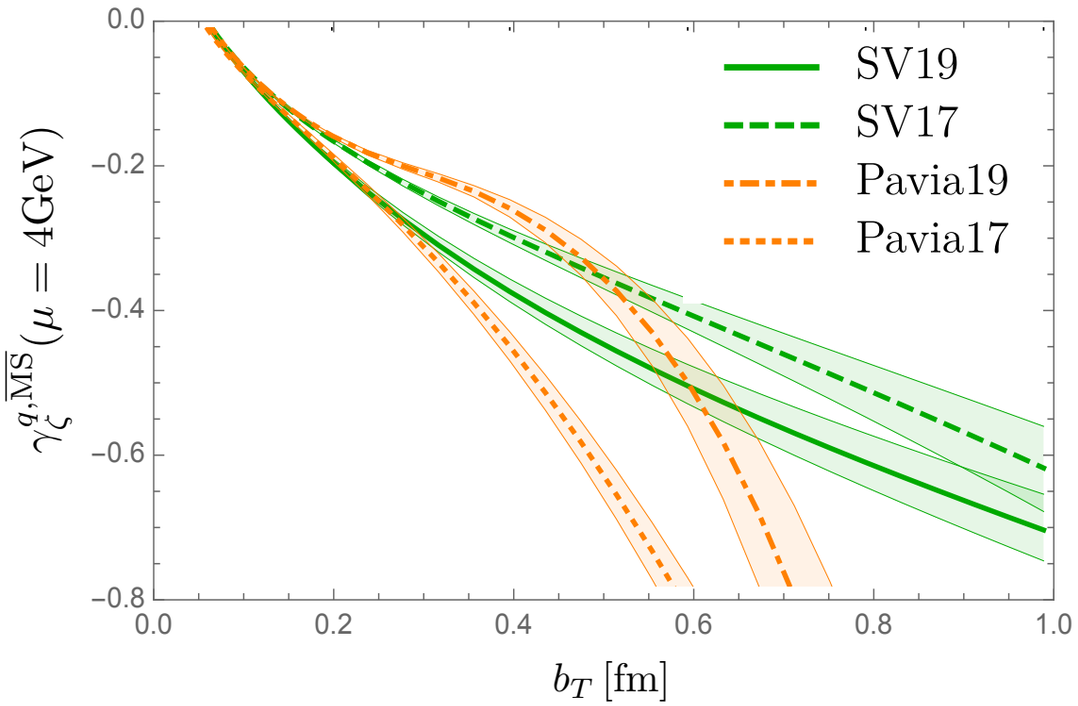


Figure adapted from Vladimirov [2003.02288]
[see also e.g., Kang, Prokudin, Sun,Yuan Phys.Rev.D 93 (2016), Collins, Rogers Phys.Rev.D 91 (2015), Sun, Yuan Phys.Rev.D 88 (2013)]

- LQCD constraints in nonperturbative region with ~10% uncertainties would contribute to model differentiation

Collins-Soper evolution kernel

- Sub-percent precision in measurements of e.g., Drell-Yan differential cross-sections at the LHC
- Lack of knowledge of the Collins-Soper kernel in the non-perturbative region limits theory predictions to much lower precision

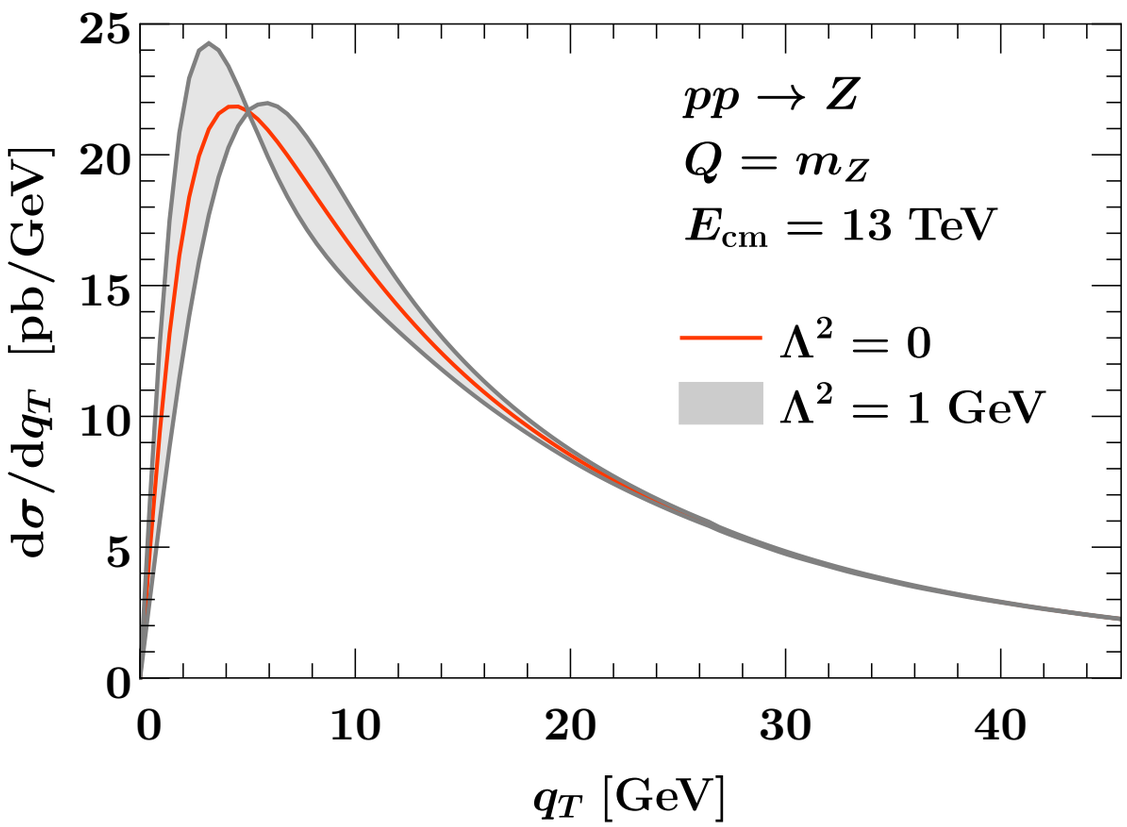
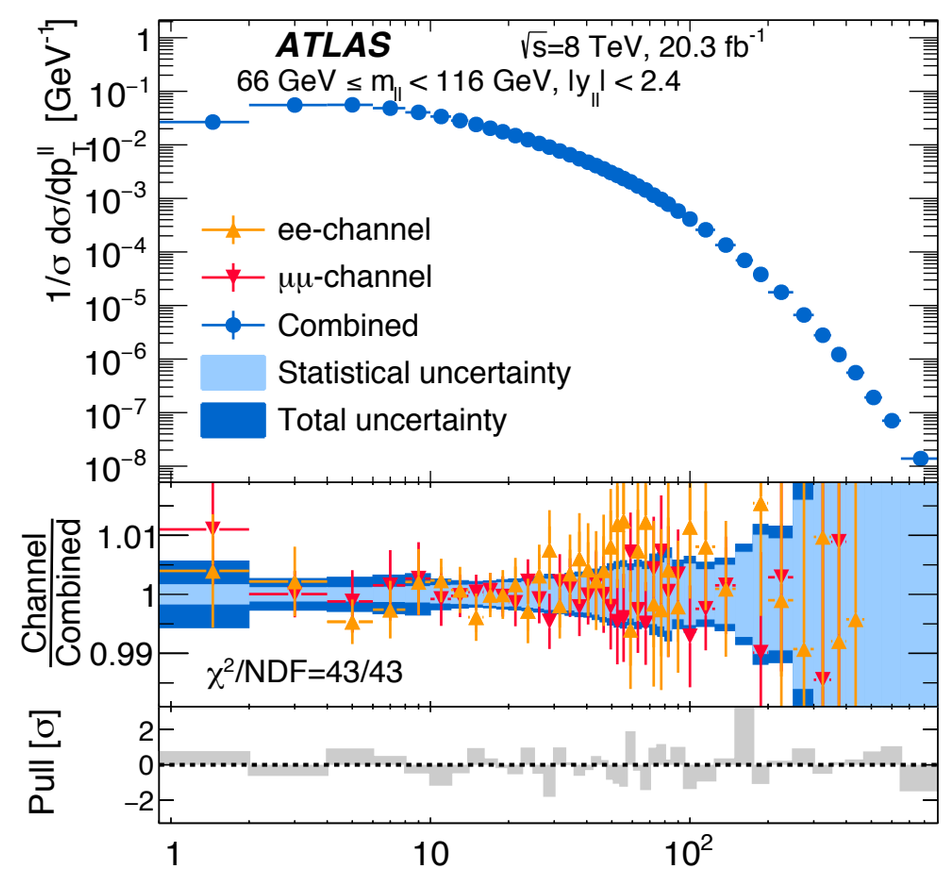


Figure: Iain Stewart



[ATLAS '15]

TMDPDFs from lattice QCD

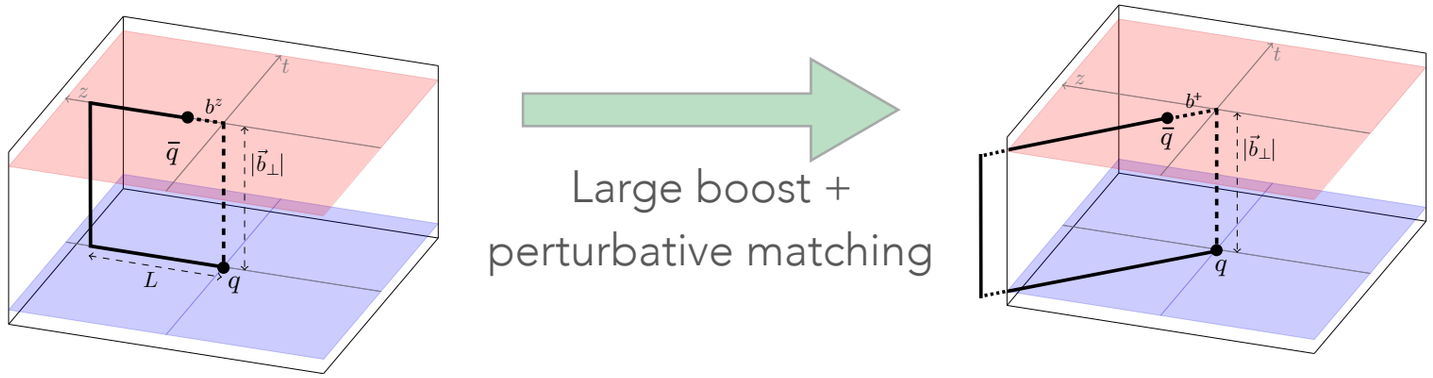
- More detailed picture of nucleon structure

PDF $f_{q/H}(x)$ longitudinal → **TMD** $f_{q/H}(x, k_T)$ + transverse

- TMDPDF defined by matrix element of non-local light-cone quark bilinear operator
  - Calculate via quasi-TMD prescription analogous to PDF case

[Ji, Sun, Xiong,Yuan '14, Ji, Jin, Yuan, Zhang, Zhao, PRD99 (2019), Ebert, Stewart, Zhao, PRD99 (2019), JHEP09 (2019), Ji, Liu, Liu, [1911.03840] (2019), Nucl.Phys.B (2020), Vladimirov, Schäfer, Phys.Rev.D 101 (2020)]

Spatial staple calculable in LQCD



$$\tilde{f}_{u\text{-}d}(x, \vec{b}_T, \mu, P^z) = C^{\mathrm{TMD}}_{u\text{-}d}(\mu, xP^z)\, g^S_q(b_T, \mu) \exp\left[\frac{1}{2}\gamma^q_\zeta(\mu, b_T) \ln \frac{(2xP^z)^2}{\zeta}\right] f_{u\text{-}d}(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMD from LQCD | perturbative matching | non-pert. soft factor | Collins-Soper evolution kernel | desired TMDPDF

Collins Soper kernel from lattice QCD

- Collins-Soper kernel calculable from ratios of quasi-TMDPDFs

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)} \frac{\int {\rm d}b^z e^{-ib^z x P_1^z} P_1^z \lim_{\substack{a\to 0\\ \eta\to\infty}} B_{\gamma^4}^{\overline{\rm MS}}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_1^z)}{\int {\rm d}b^z e^{-ib^z x P_2^z} P_2^z \lim_{\substack{a\to 0\\ \eta\to\infty}} B_{\gamma^4}^{\overline{\rm MS}}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_2^z)}\right]$$

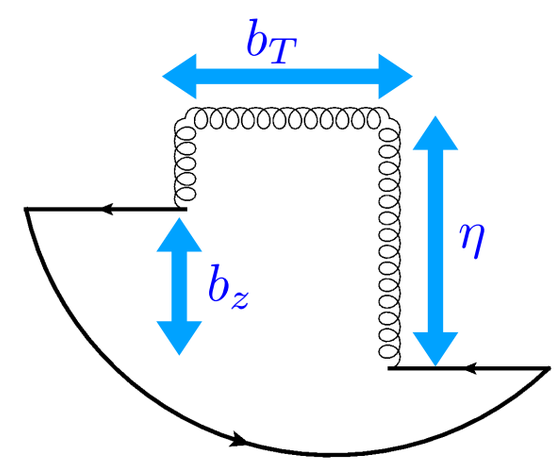
Perturbative matching

Computed at 1-loop

[Ebert, Stewart, Zhao, PRD99+JHEP09 (2019)]

Beam function

Calculated from matrix elements of operators with staple-shaped Wilson lines



- Do not need soft factor (cancels in ratio)
- Independent of hadron state, choice of momenta, choice of x, up to power corrections b_T/η, $1/(p^z b_T)$, M/p^z
- Choose γ^4 or γ^3: boost onto γ^+ and can be matched to spin-independent TMDPDF in infinite-momentum limit

Collins Soper kernel from lattice QCD

Three steps to CS kernel from lattice QCD:

1. Calculation of bare quasi-beam functions
2. Renormalisation and matching to the $\overline{\text{MS}}$ scheme
3. Fourier transform and ratio

Proof-of-principles study in quenched QCD

- Exploit state-independence: study heavy pseudoscalar meson $m \sim 1.2$ GeV

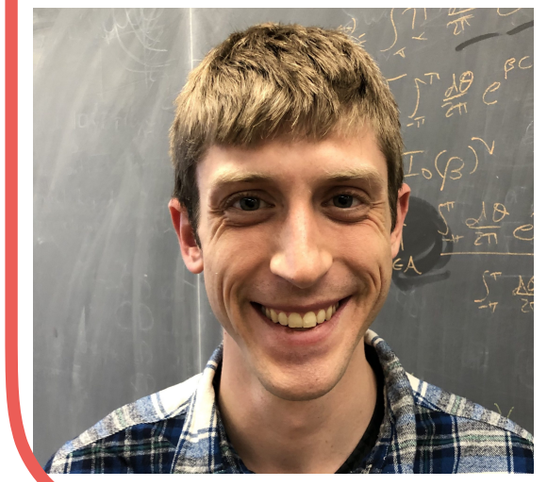
β	a [fm]	$L^3 \times T$	κ	n_{src}	n_{cfg}
6.3017	0.06	$32^3 \times 64$	0.1222	2	200

- Vary momenta to study effect of power corrections
- Study operator mixing nonperturbatively

β	a [fm]	$L^3 \times T$	κ	n_{cfg}
6.1005	0.08	$24^3 \times 48$	0.121,0.1248	30
6.3017	0.06	$32^3 \times 64$	0.1222	30
6.5977	0.04	$48^3 \times 96$	0.1233	10

Work with **Michael Wagman** and **Yong Zhao**

PRD101, 074505 (2020), PRD102, 014511 (2020)



Collins Soper kernel from lattice QCD

I Calculation of bare quasi-beam functions

- 3pt function with wall source and momentum-smeared sink, boosted hadron state

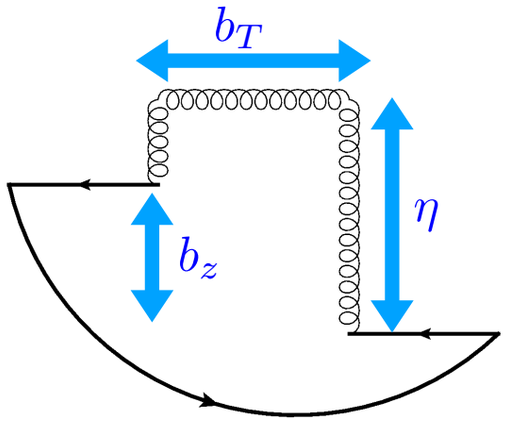
$$C^{\Gamma,i}_{3\mathrm{pt}}(t,\tau,b^\mu,a,\eta,\vec{P}=P^z\vec{e}_z)$$
$$=\sum_{\vec{x},\vec{z}} e^{i\vec{P}\cdot\vec{x}}\langle 0|\pi_{\vec{P},S}(\vec{x},t)\mathcal{O}^i_\Gamma(b^\mu,(\vec{z},\tau),\eta)\pi^\dagger_{\vec{P},W}(0)|0\rangle$$

- Ratio of 3pt and 2pt functions asymptotes to bare quasi-beam function

$$\mathcal{R}_\Gamma(t,\tau,b^\mu,a,\eta,P^z)$$
$$=\frac{C^{\Gamma,u}_{3\mathrm{pt}}(t,\tau,b^\mu,a,\eta,P^z\vec{e}_z)-C^{\Gamma,d}_{3\mathrm{pt}}(t,\tau,b^\mu,a,\eta,P^z\vec{e}_z)}{C_{2\mathrm{pt}}(t,P^z\vec{e}_z)}$$
$$\xrightarrow{t\gg\tau\gg 0} B^{\mathrm{bare}}_\Gamma(b^z,\vec{b}_T,a,\eta,P^z)+\ldots.$$

- Momentum choices $P^z\in\{1.29,1.94,2.58\}$ GeV ($\{2,3,4\}$ lattice units)

Quark bilinear operator with staple-shaped Wilson line

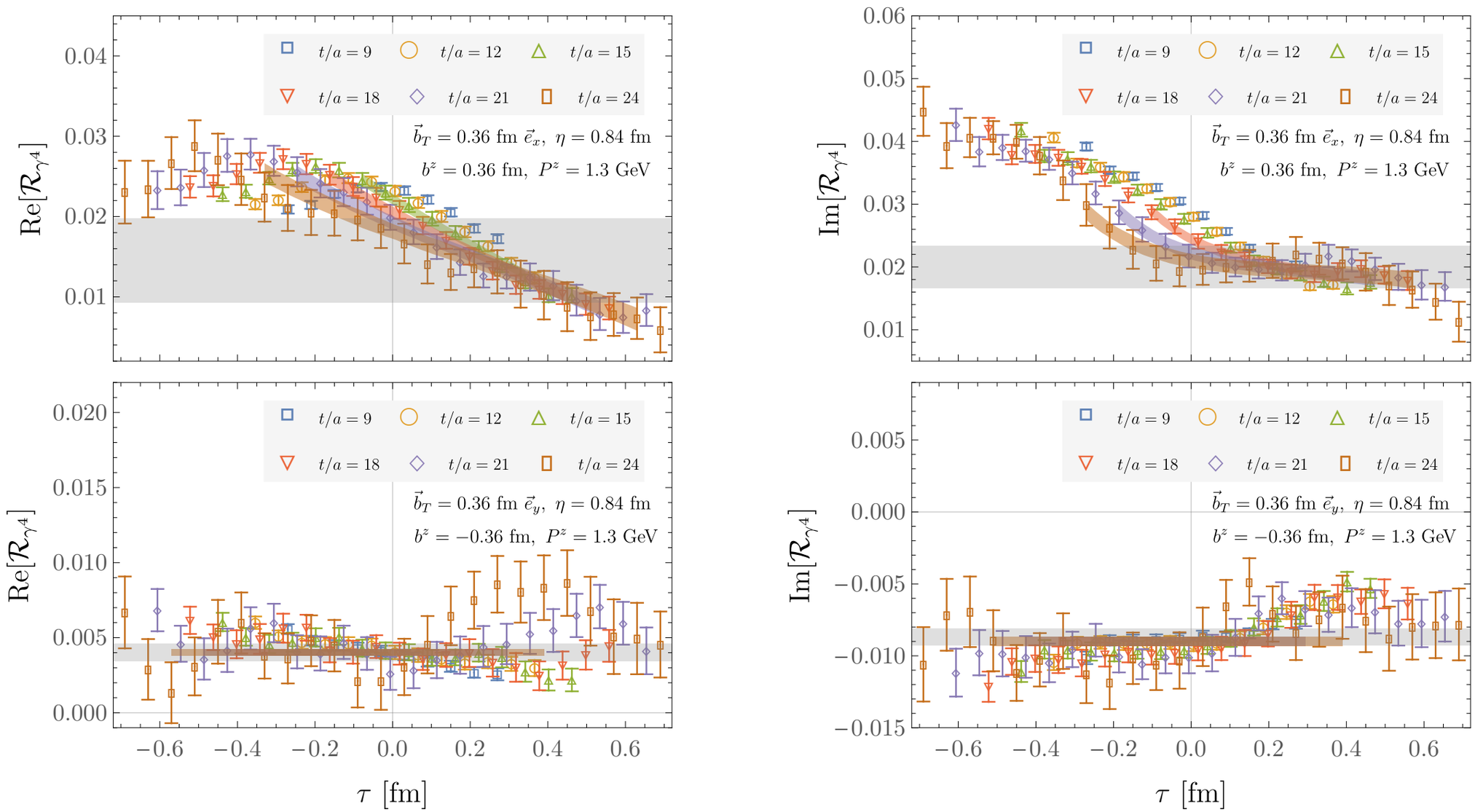


$$\mathcal{O}^i_\Gamma(b^\mu,z^\mu,\eta)=\bar{q}_i(z^\mu+b^\mu)\frac{\Gamma}{2}W_{\hat{z}}(z^\mu+b^\mu;\eta-b^z)$$
$$\times W^\dagger_T(z^\mu+\eta\hat{z};b_T)W^\dagger_{\hat{z}}(z^\mu;\eta)q_i(z^\mu)$$
$$\equiv\bar{q}_i(z^\mu+b^\mu)\frac{\Gamma}{2}\widetilde{W}(\eta;b^\mu;z^\mu)q_i(z^\mu)\,,$$

$$\eta/a\in\{10,12,14\}\iff\eta\in(0.6,0.9)\ \mathrm{fm}$$
$$-(\eta-a)\le|b_T|\le(\eta-a)$$
$$-(\eta-a)\le b^z\le(\eta-a)$$

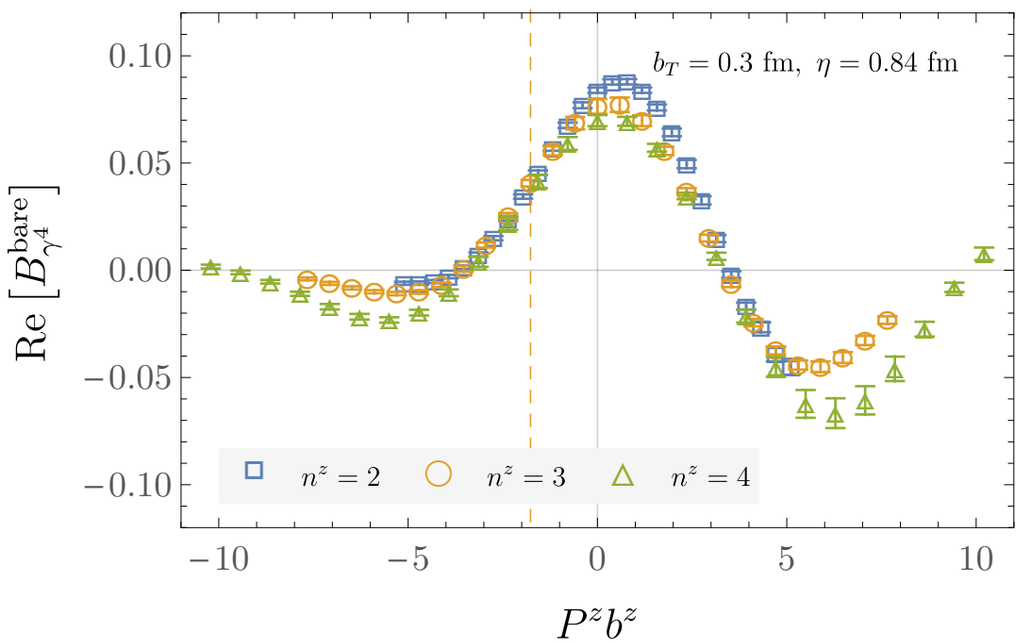
Bare quasi-beam functions

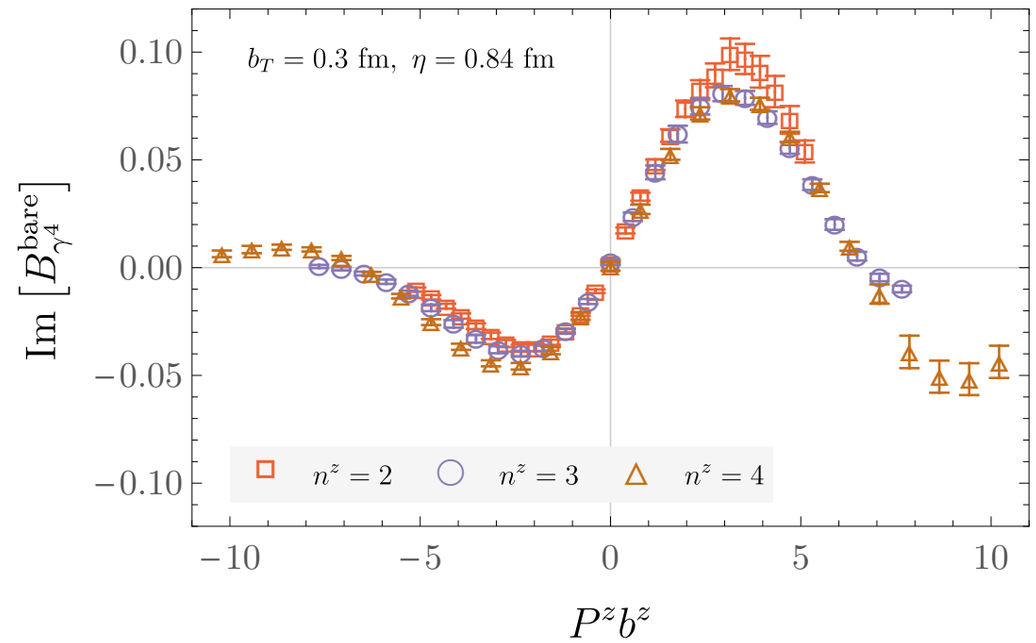
- Example fits to 3pt/2pt ratios: source/sink separations $t/a \in \{9, 12, 15, 18, 21, 24\}$
- Automated procedure: consider all time windows, Akaike information criterion to determine optimal fit form; weighted average of all fit results
- ~35,660 individual fits (different staple geometries, momenta)



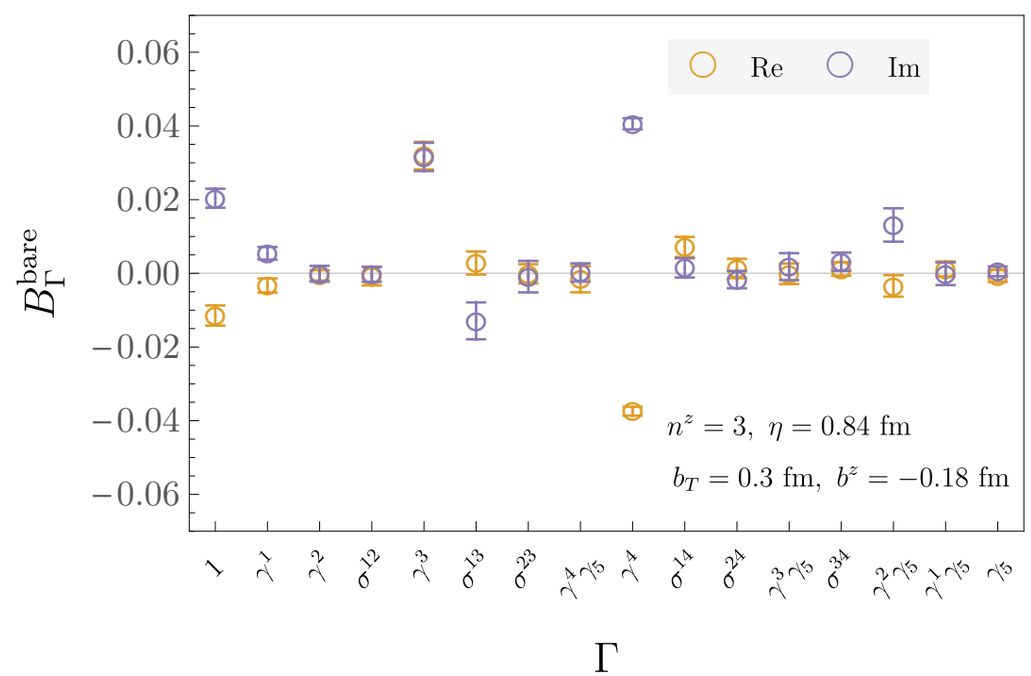
Bare quasi-beam functions

- Fits for all $P^z = n^z(2\pi/L), \eta, b_T, b^z, \Gamma, \mathrm{Re/Im}$





- Asymmetries in $P^z b^z$ from linear divergences cancelled by renormalisation
- Ultimately we are interested in contributions from mixing to $B^{\overline{\mathrm{MS}}}_{\gamma^4}(\mu, b^z, \vec{b}_T, a, \eta, P^z)$



Collins Soper kernel from lattice QCD

2 Renormalisation and matching

- Renormalise bare beam function via RI'-MOM prescription

$$B^{\overline{\rm MS}}_{\gamma^4}(\mu, b^z, \vec{b}_T, a, \eta, P^z) = Z^{\overline{\rm MS}}_{\mathcal{O}_{\gamma^4\Gamma}}(\mu, b^z, b^R_T, a, \eta)\,\tilde{R}(b_T, b^R_T, a, \eta)\,B^{\rm bare}_{\Gamma}(b^z, \vec{b}_T, a, \eta, P^z).$$

summed

$\overline{\rm MS}$ renormalisation

$$Z^{\overline{\rm MS}}_{\mathcal{O}_{\gamma^4\Gamma}}(\mu, b^z, b_T, a, \eta) = \mathcal{R}^{\overline{\rm MS}}_{\mathcal{O}_{\gamma^4\Gamma}}(\mu, p_R, b^z, \vec{b}_T, \eta) \times Z^{\rm RI'/MOM}_{\mathcal{O}_{\gamma^4\Gamma}}(p_R, b^z, \vec{b}_T, a, \eta)$$

Perturbative conversion

[Ebert, Stewart, Zhao, JHEP03 (2020), Constantinou, Panagopoulos, Spanoudes, PRD99 (2019)]

Nonperturbative renormalisation

[Shanahan, Wagman, Zhao, PRD101, 074505 (2020)]

Divergence cancellation

$$\tilde{R}(b_T, b^R_T, a, \eta) = \frac{Z^{\rm RI'/MOM}_{\mathcal{O}_{\gamma^4\gamma^4}}(p_R = \tilde{p}_R, b^z = 0, \vec{b}_T, a, \eta)}{Z^{\rm RI'/MOM}_{\mathcal{O}_{\gamma^4\gamma^4}}(p_R = \tilde{p}'_R, b^z = 0, \vec{b}^R_T, a, \eta)}$$

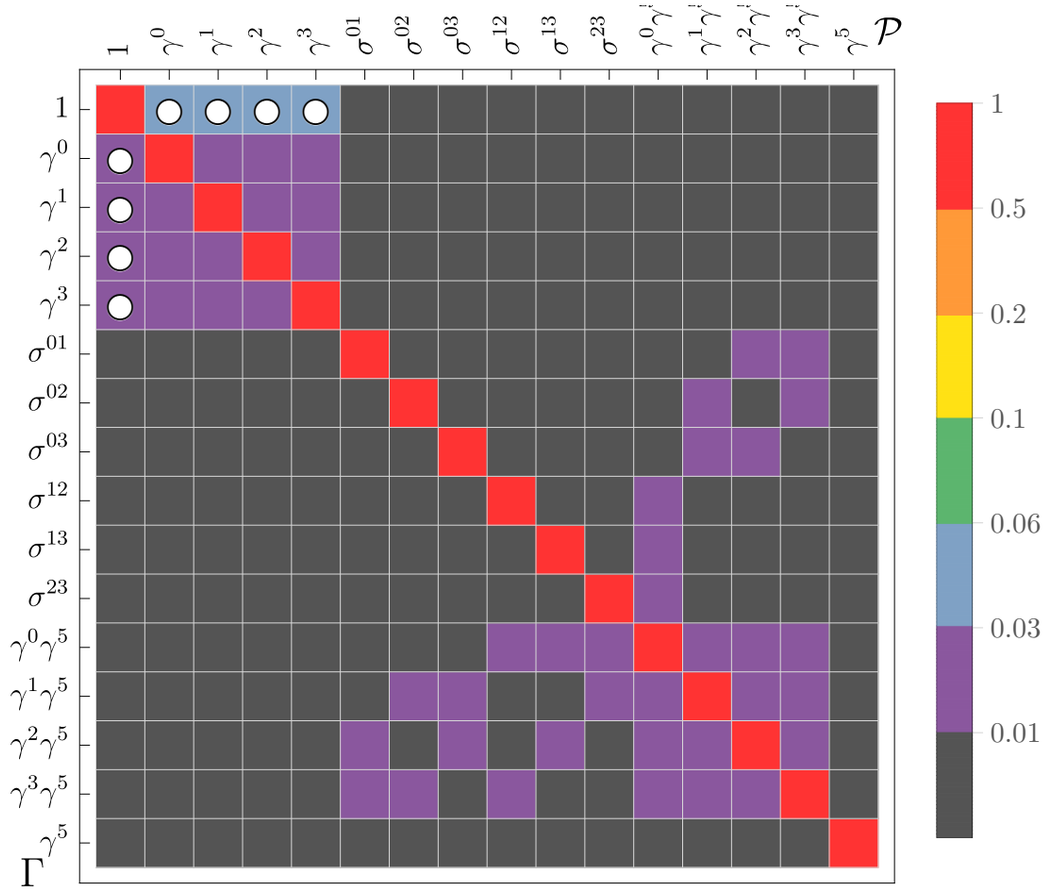
Introduced to compensate for power-law divergences in $\{\eta, b_T\}$ in both numerator and denominator of ratio determining CS kernel

RI'-MOM Renormalisation

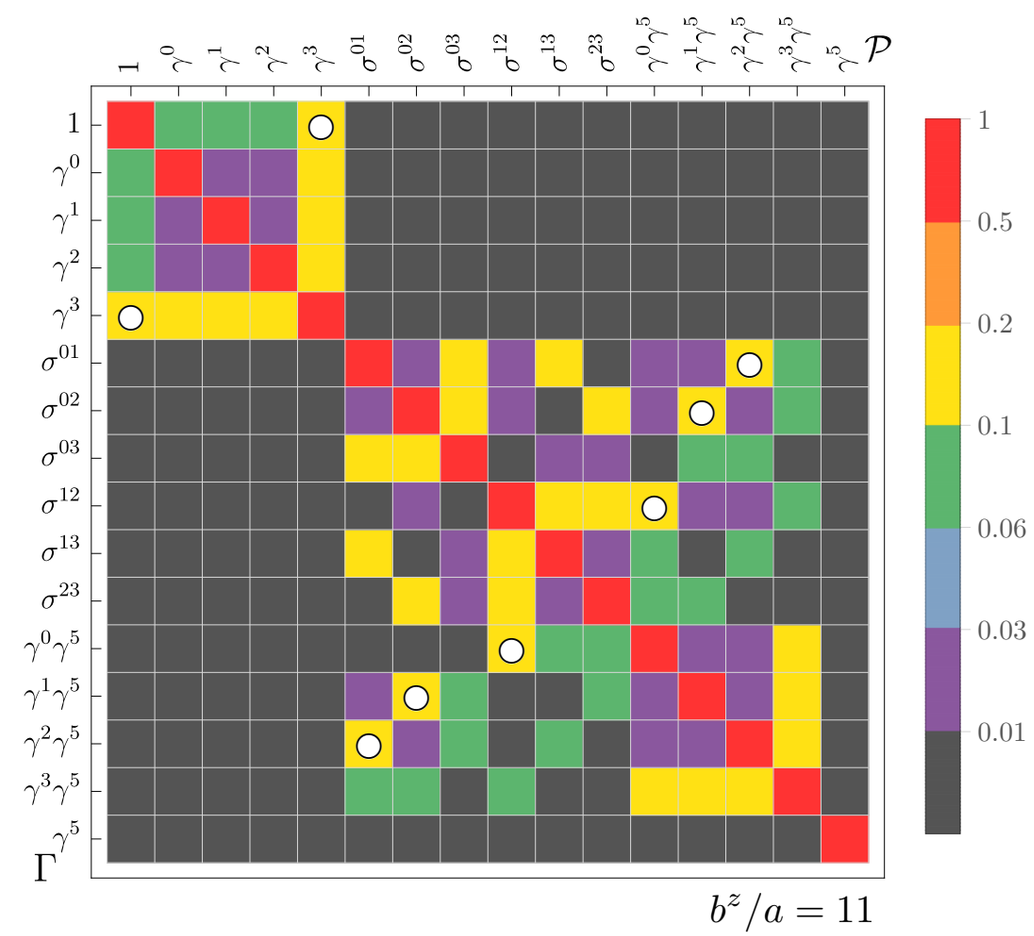
- 16x16 matrix: mixing of operators with different Dirac structures
- Additional mixing beyond that predicted by 1-loop lattice perturbation theory (dots)

$$\mathcal{M}^{\mathrm{RI'/MOM}}_{\mathcal{O}_{\Gamma\mathcal{P}}} \equiv \max_{p_R} \frac{\mathrm{Abs}[Z^{\mathrm{RI'/MOM}}_{\mathcal{O}_{\Gamma\mathcal{P}}}(p_R)]}{\frac{1}{16}\sum_i \mathrm{Abs}[Z^{\mathrm{RI'/MOM}}_{\mathcal{O}_{\Gamma_i\Gamma_i}}(p_R)]}$$

[Constantinou, Panagopoulos, PRD96 (2017), Green, Jansen, Steffens, PRL 121 (2018), Chen et al (LP3), Chin. Phys. C43 (2019), Constantinou, Panagopoulos, Spanoudes, PRDD99 (2019)]



Local operators



Straight Wilson lines

RI'-MOM Renormalisation

- 16x16 matrix: mixing of operators with different Dirac structures
- Additional mixing beyond that predicted by 1-loop lattice perturbation theory (dots)

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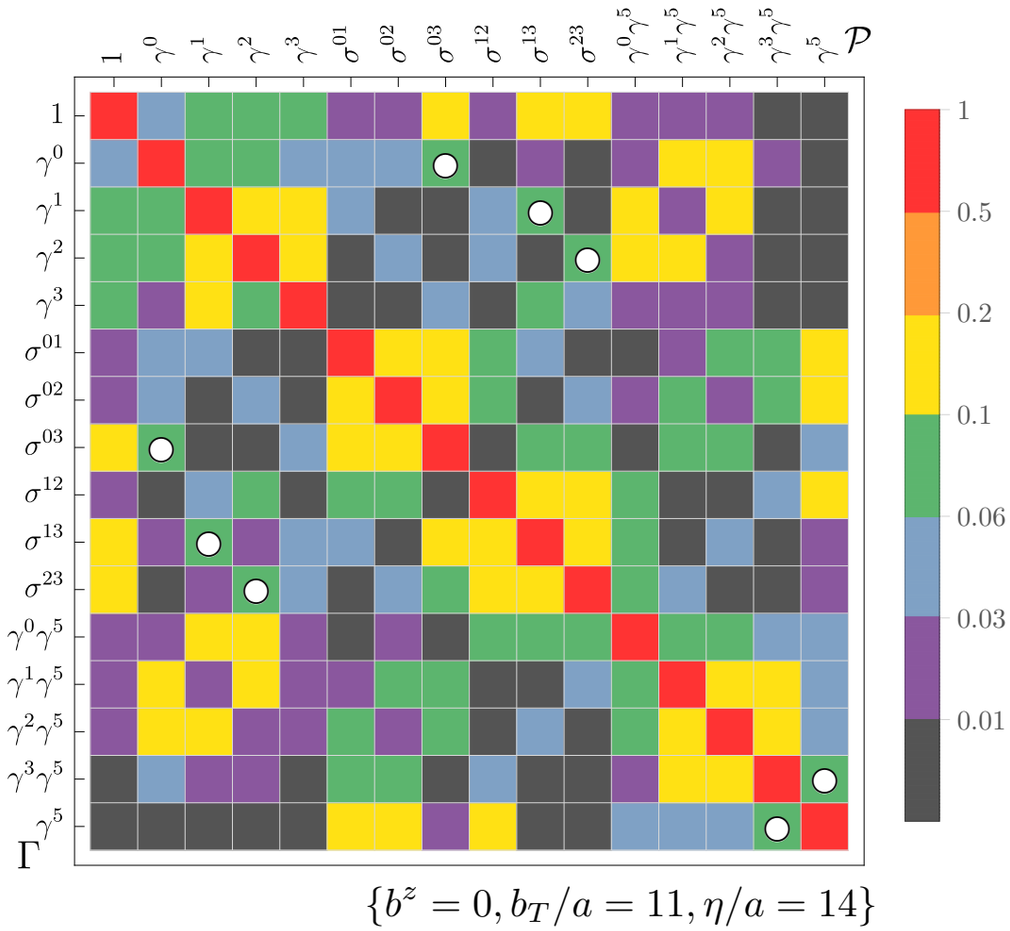
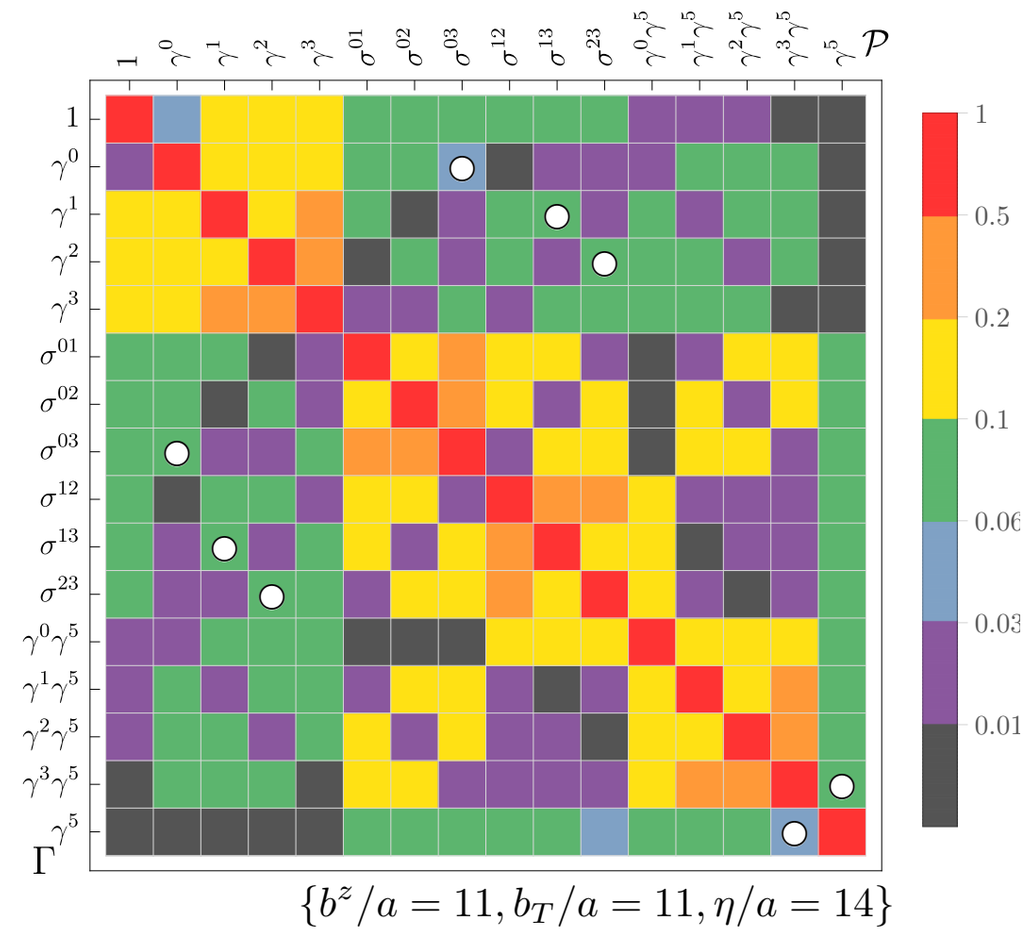


Symmetric staple Wilson lines



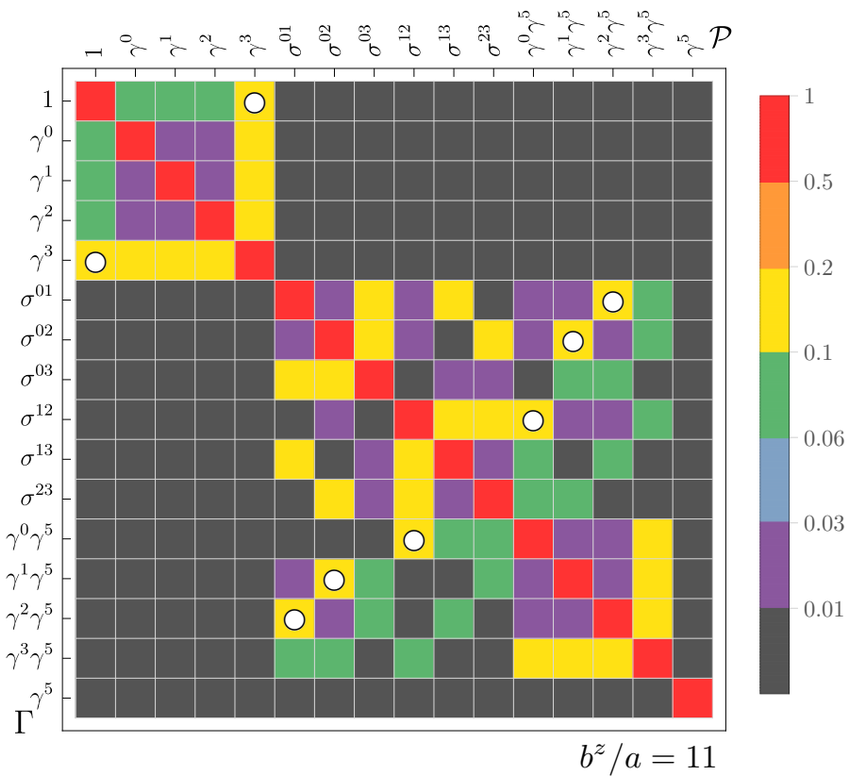
Asymmetric staple Wilson lines

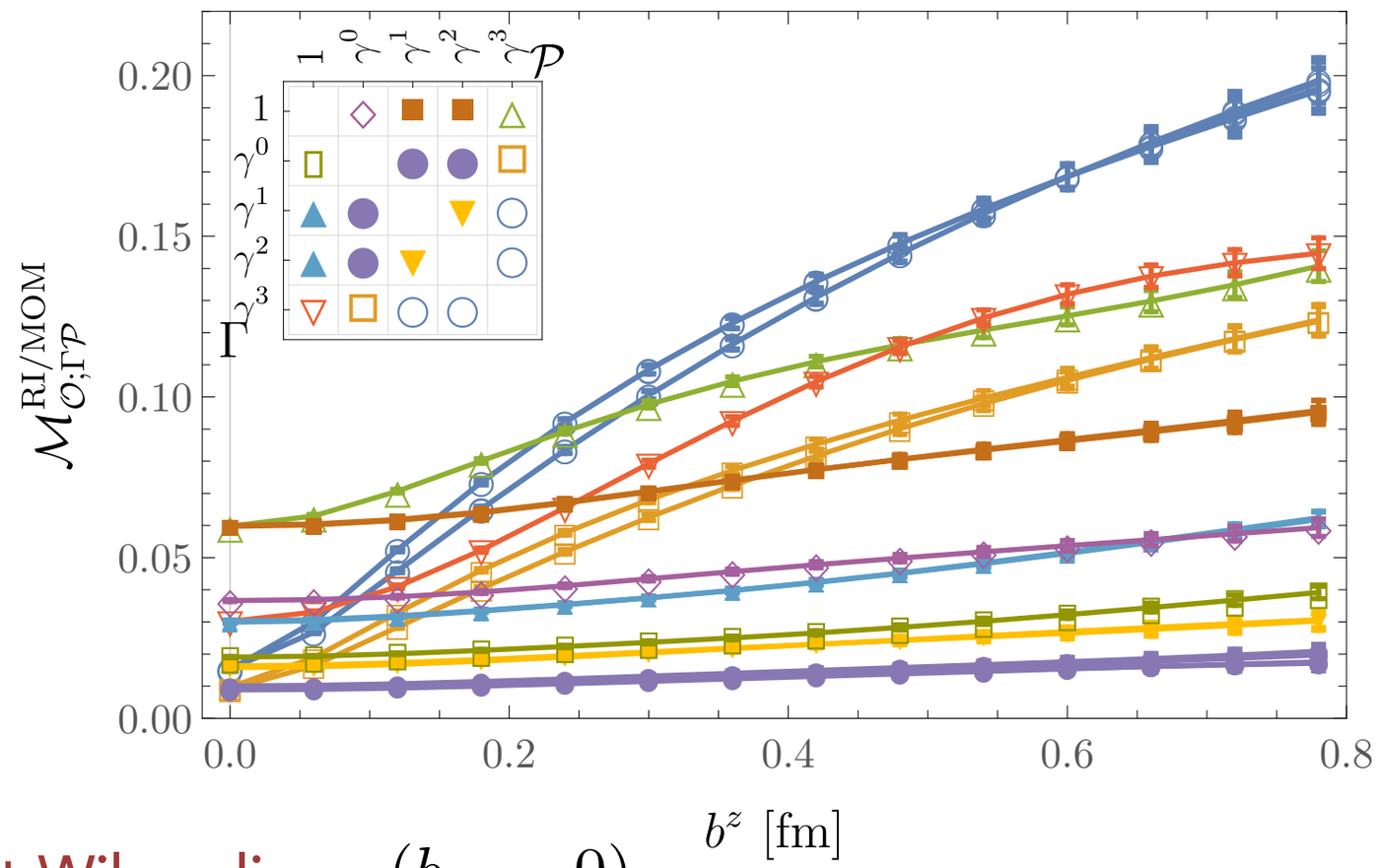
RI'-MOM Renormalisation

- 16x16 matrix: mixing of operators with different Dirac structures

$$\mathcal{M}^{\mathrm{RI'/MOM}}_{\mathcal{O}_{\Gamma\mathcal{P}}} \equiv \max_{p_R} \frac{\mathrm{Abs}[Z^{\mathrm{RI'/MOM}}_{\mathcal{O}_{\Gamma\mathcal{P}}}(p_R)]}{\frac{1}{16}\sum_i \mathrm{Abs}[Z^{\mathrm{RI'/MOM}}_{\mathcal{O}_{\Gamma_i\Gamma_i}}(p_R)]}$$

- Mixing increases with operator Wilson line extent





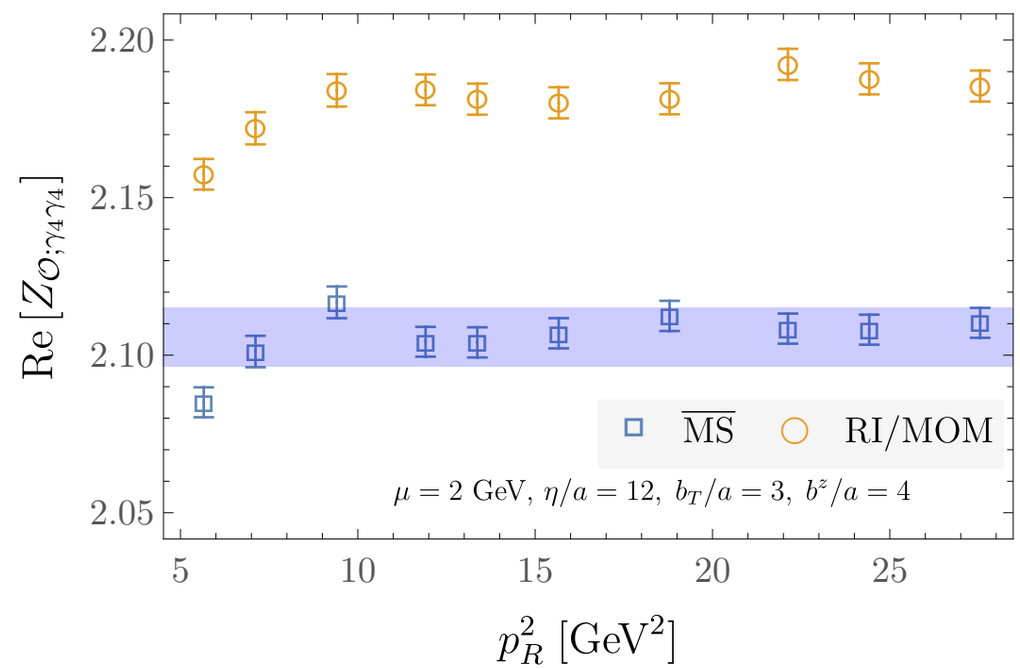
Straight Wilson lines $(b_T = 0)$

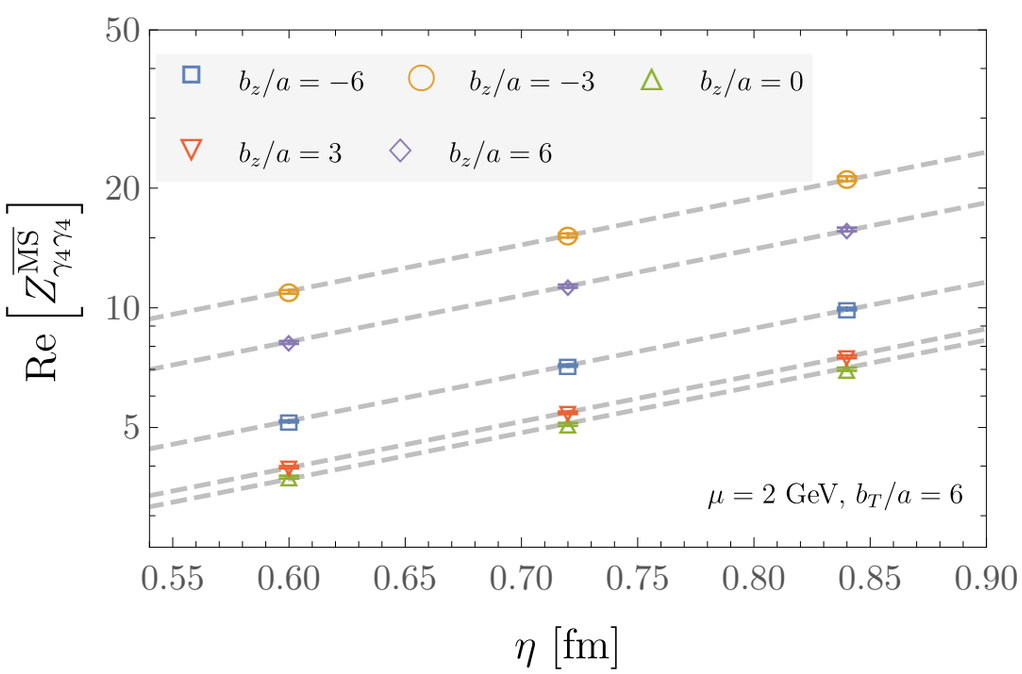
$\overline{\text{MS}}$ Renormalisation

- Match from RI'-MOM to MS using one-loop perturbative conversion factor

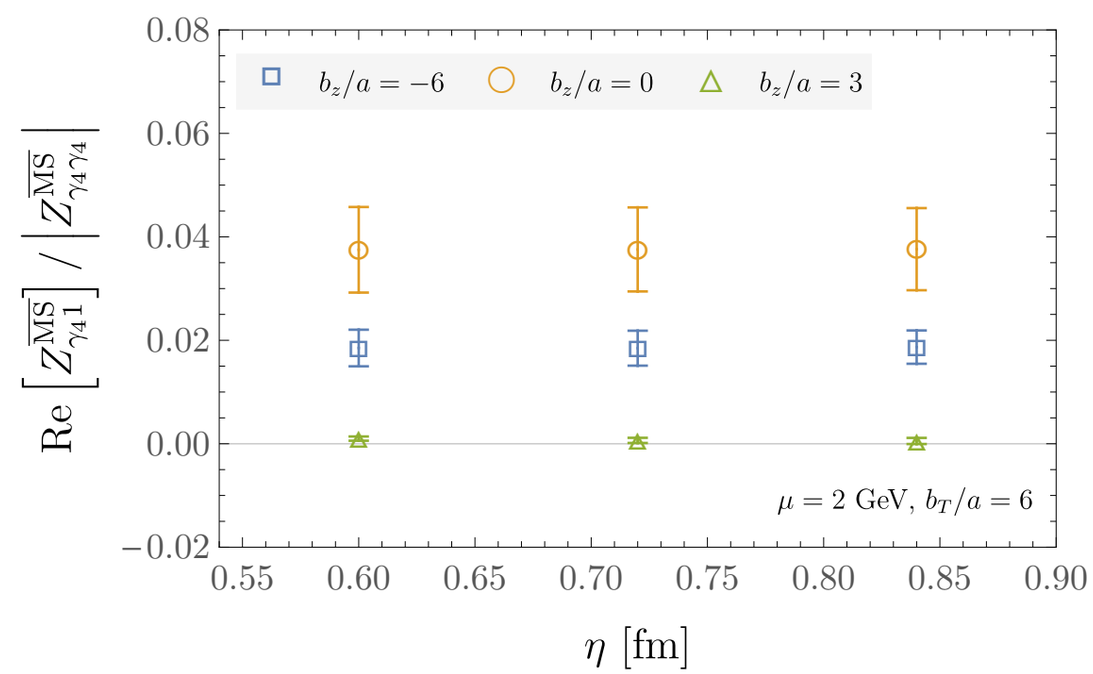
  [Ebert, Stewart, Zhao, JHEP03 (2020), Constantinou, Panagopoulos, Spanoudes, PRD99 (2019)]

$$Z^{\overline{\text{MS}}}_{\mathcal{O}_{\gamma^4\Gamma}}(\mu, b^z, b_T, a, \eta) = \mathcal{R}^{\overline{\text{MS}}}_{\mathcal{O}_{\gamma^4\Gamma}}(\mu, p_R, b^z, \vec{b}_T, \eta) \times Z^{\text{RI}'/\text{MOM}}_{\mathcal{O}_{\gamma^4\Gamma}}(p_R, b^z, \vec{b}_T, a, \eta)$$





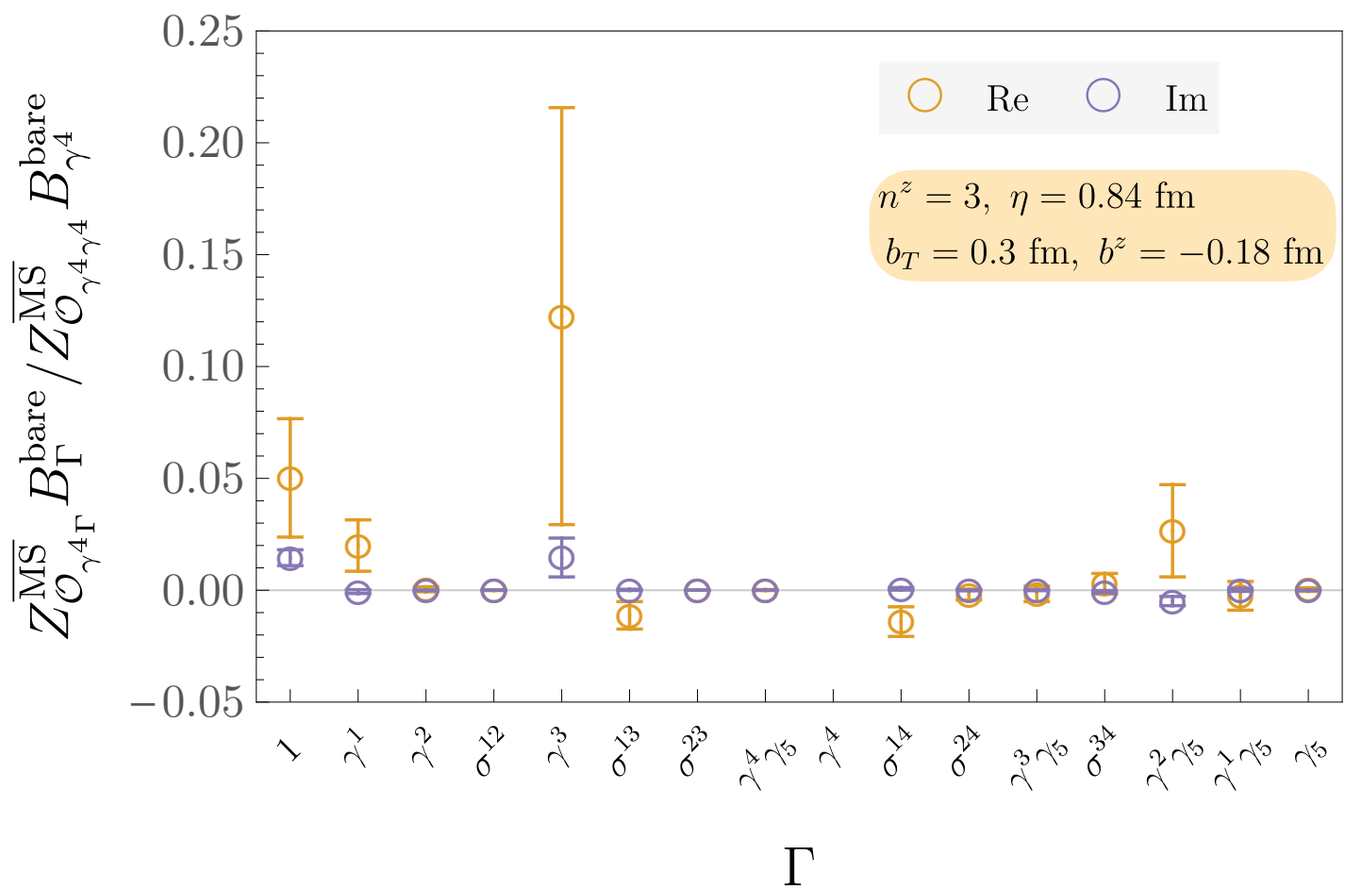
Diagonal element



Off-diagonal/diagonal ratio

Operator mixing under renormalisation

- Mixing of operators with different Dirac structures into $\mathcal{O}_{\gamma^4}$ at the level of 10-20%; significant mixing not limited to that predicted by 1-loop PT
- Large uncertainties: lower statistics (10%) used to calculate quasi-beam functions for operators $\mathcal{O}_{\Gamma\neq\gamma^4}$ for computational expediency

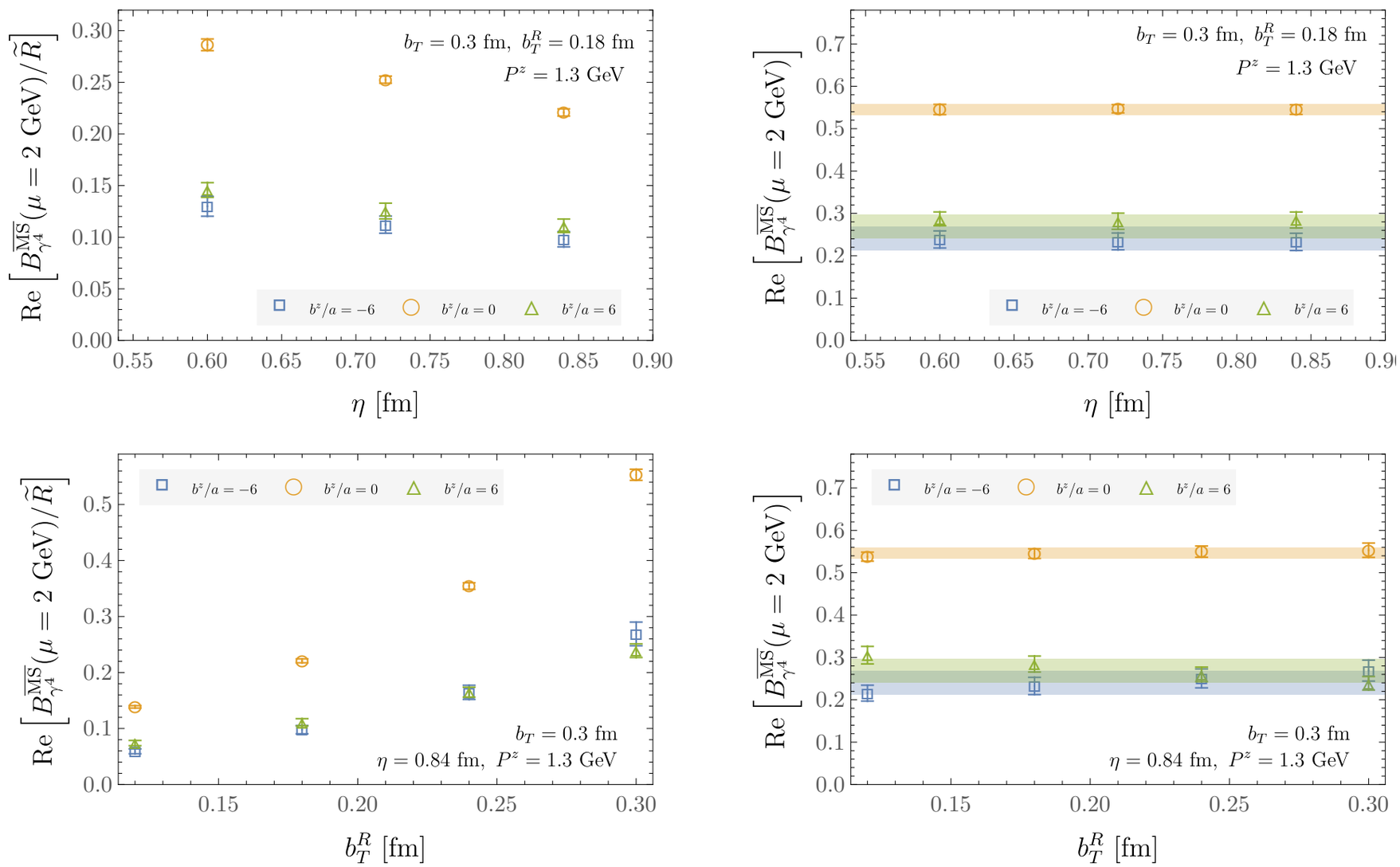


Example: just one set of parameters, similar patterns for other $\{n^z, \eta, b_T, b^z\}$

Renormalised quasi-beam functions

- Fit beam function to a constant in $\{\eta, b_T^R\}$ (i.e., constant ansatz for $\eta \to \infty$ limit)

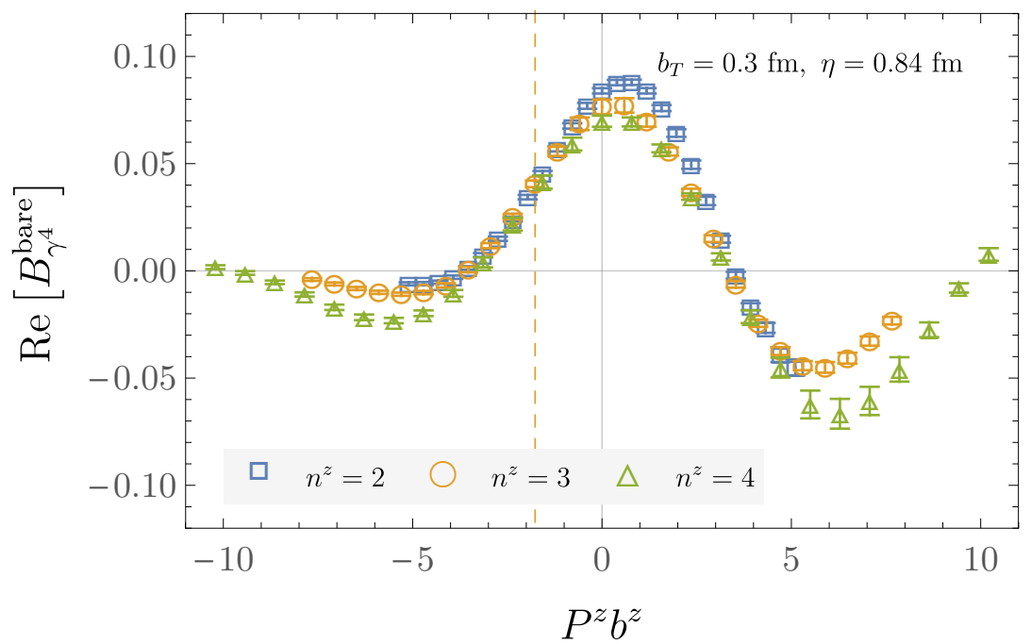
$$B_{\gamma^4}^{\overline{\rm MS}}(\mu, b^z, \vec{b}_T, a, \eta, P^z) = Z_{\mathcal{O}_{\gamma^4\Gamma}}^{\overline{\rm MS}}(\mu, b^z, b_T^R, a, \eta)\tilde{R}(b_T, b_T^R, a, \eta)B_\Gamma^{\rm bare}(b^z, \vec{b}_T, a, \eta, P^z).$$



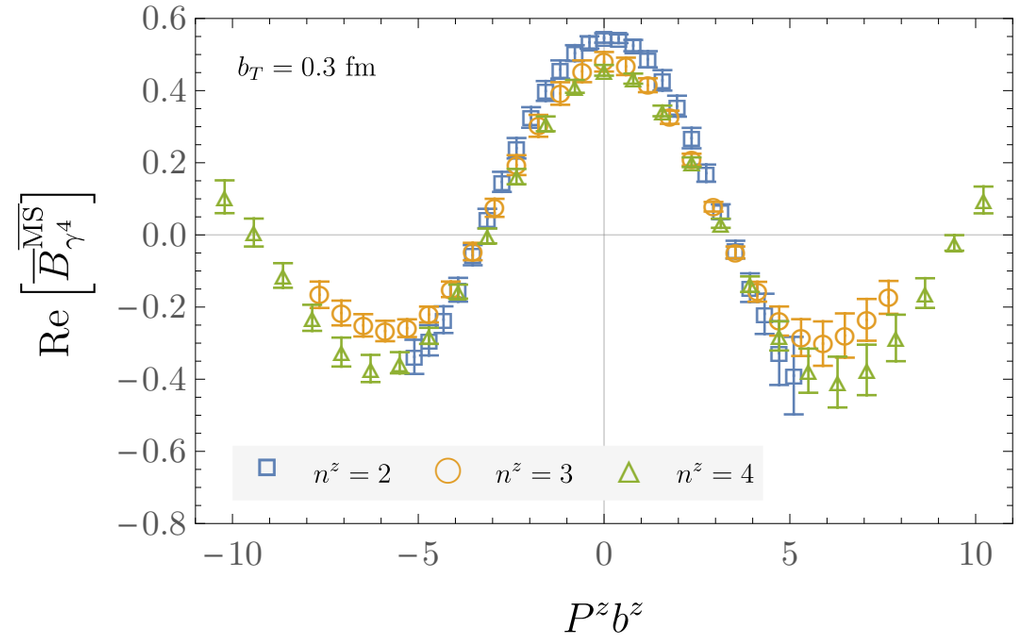
$\overline{\rm MS}$ quasi-beam function Incl. divergence cancellation factor

Renormalised quasi-beam functions

Bare



Renormalised



- To determine CS kernel, must take Fourier transform in $P^z b^z$
- Quasi-beam functions still oscillatory in data range → truncation effects in Fourier transform

$$\gamma^q_\zeta(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{C^{\mathrm{TMD}}_{\mathrm{ns}}(\mu, xP_2^z)}{C^{\mathrm{TMD}}_{\mathrm{ns}}(\mu, xP_1^z)} \frac{\int \mathrm{d}b^z e^{-ib^z x P_1^z} P_1^z \lim_{\substack{a\to 0\\ \eta\to\infty}} B^{\overline{\mathrm{MS}}}_{\gamma^4}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_1^z)}{\int \mathrm{d}b^z e^{-ib^z x P_2^z} P_2^z \lim_{\substack{a\to 0\\ \eta\to\infty}} B^{\overline{\mathrm{MS}}}_{\gamma^4}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_2^z)}\right]$$

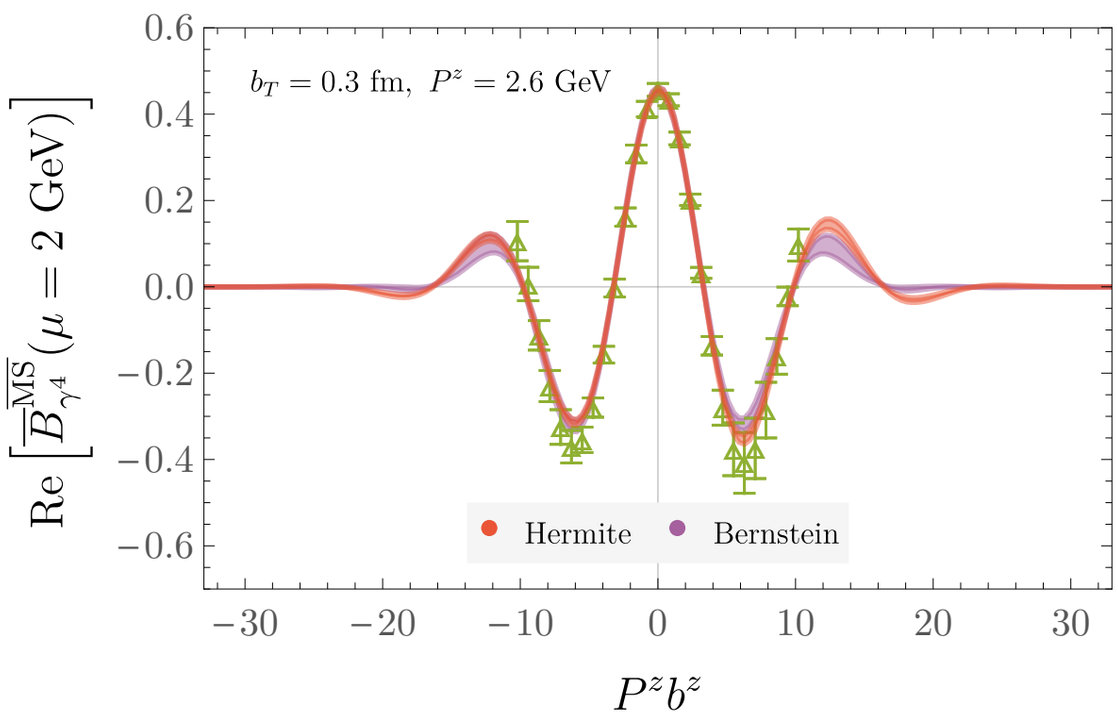
Collins Soper kernel from lattice QCD

3 Fourier transform and ratio

- Discrete FT of current data would suffer from significant truncation effects
  → choose to model the $P^z b^z$-dependence and take FT of model
- Construct models that give x-independent CS kernel γ (i.e., assuming leading-order perturbative matching)
- Models based on Hermite $x \in (\infty, \infty)$ and Bernstein polys $x \in (0, 1)$ (orthogonal polynomial bases)

$$\mathcal{F}_N^{\rm Herm}(P^z, b^z P^z; \{a_k\}, \gamma, \omega, \sigma) = \sum_{k=1}^{N} a_k \int_{-\infty}^{\infty} dx\, e^{i(b^z P^z)x} e^{-(x-\omega)^2/2\sigma} (P^z x)^{\gamma} \mathcal{H}_{k-1}(x),$$

$$\mathcal{F}_N^{\rm Bern}(P^z, b^z P^z; \{a_n^r\}, \gamma, A, B) = \sum_{r=0}^{N-1} a_r \int_0^1 dx\, e^{i(b^z P^z)x}\, x^A (1-x)^B (P^z x)^{\gamma} \mathcal{B}_{r,N-1}(x)\,,$$



Collins Soper kernel from lattice QCD

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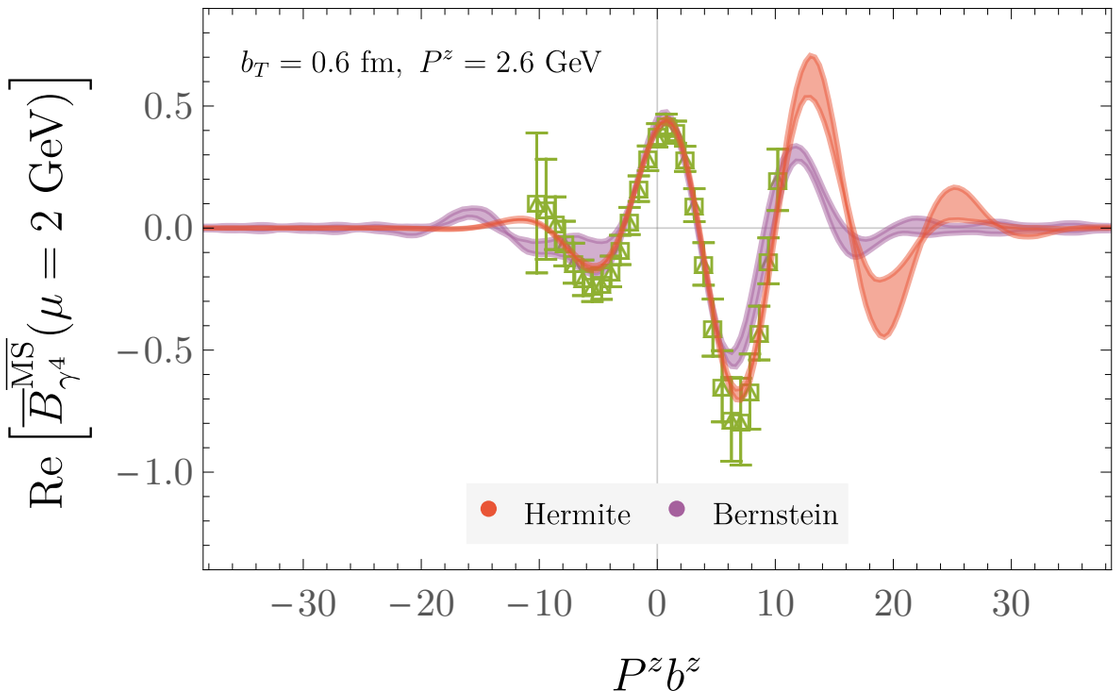
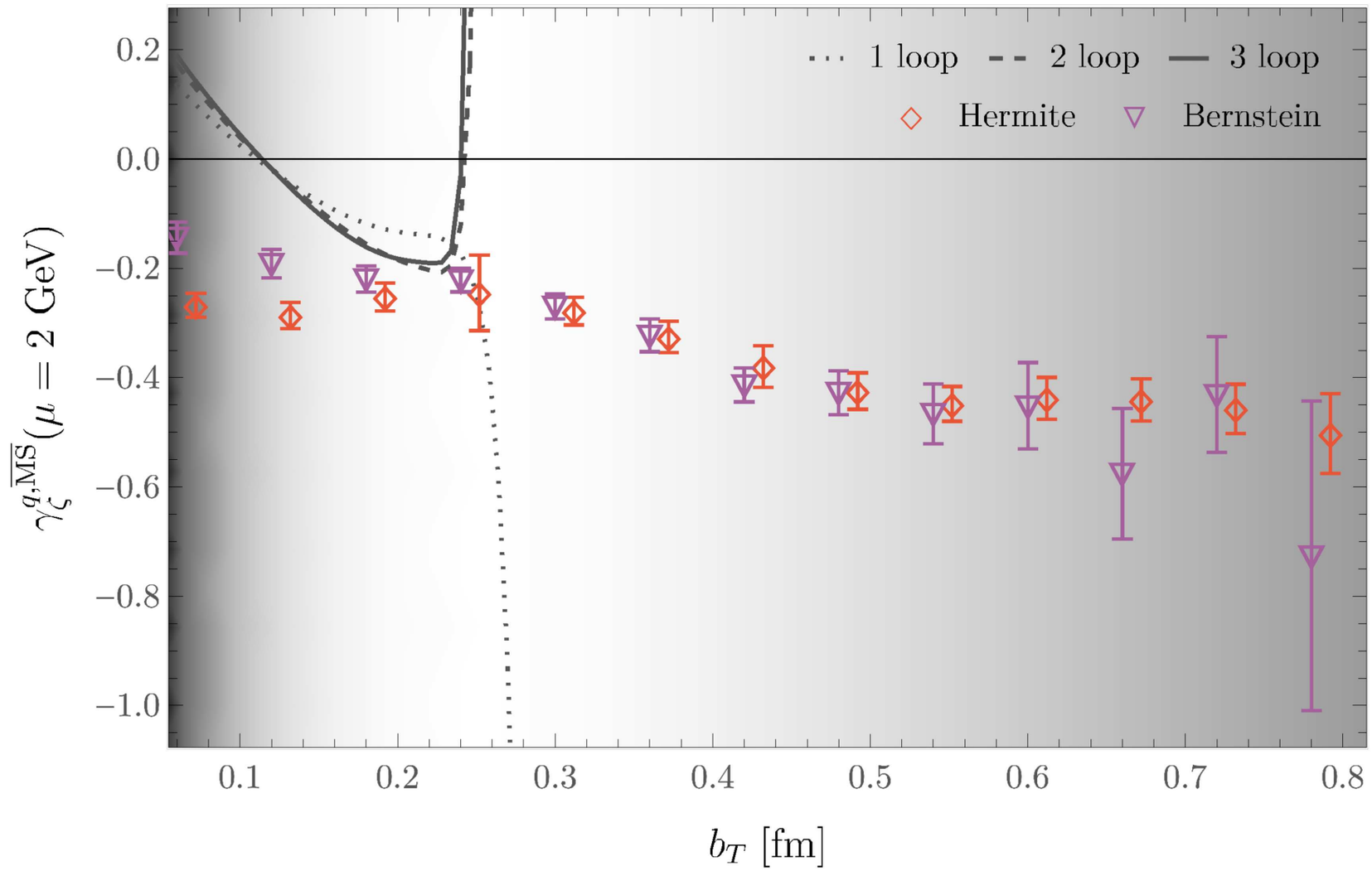
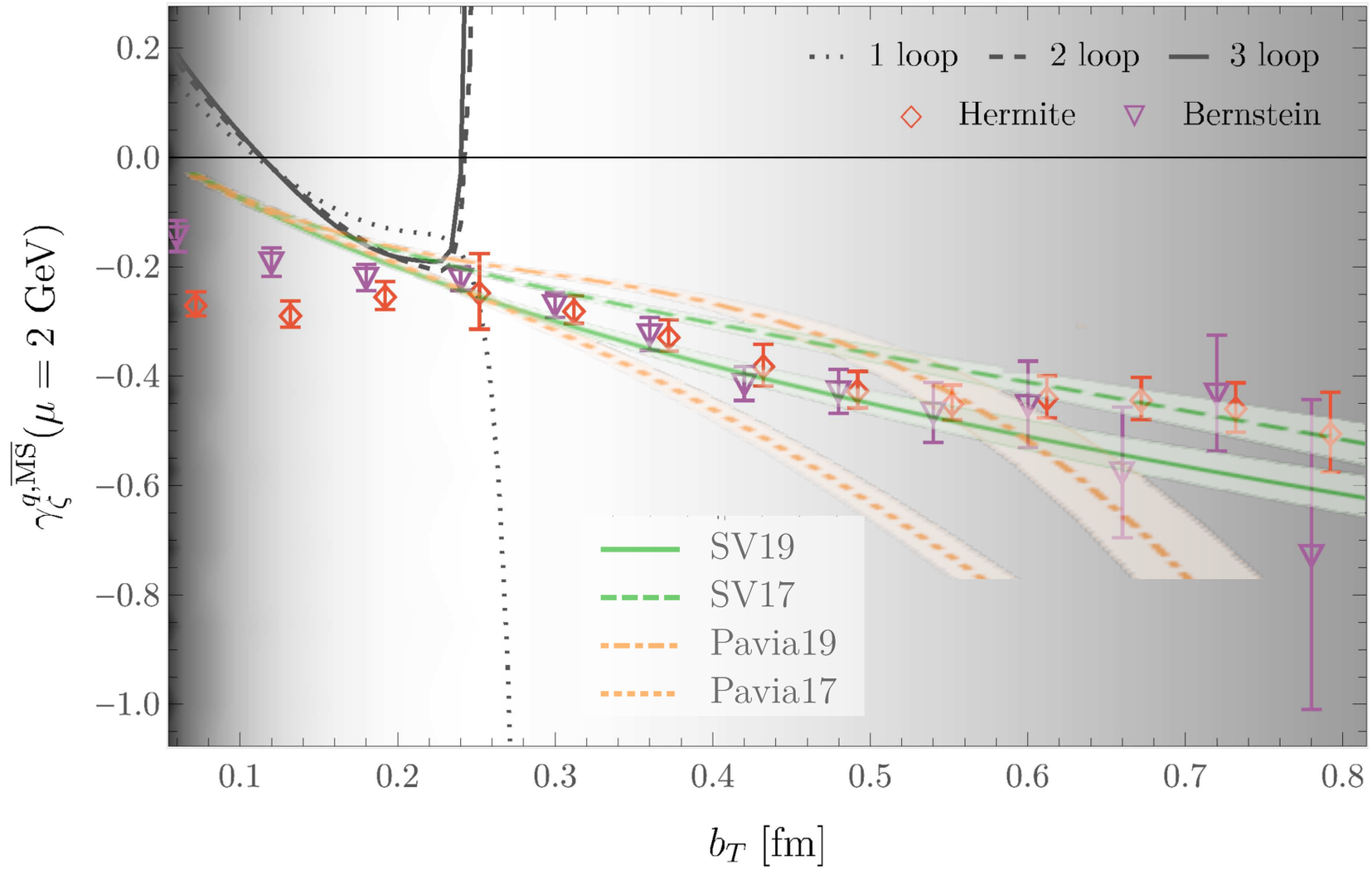


Collins Soper kernel from lattice QCD



- Background density indicates relative sensitivity to power corrections $1/(b_T P^z) + b_T/\eta$, not included in error bars
- Results using different models broadly consistent

Collins Soper kernel from lattice QCD



- Fully-controlled dynamical results at this precision would already impact phenomenology
- **CAUTION**: Photoshop overlay of pheno. models with zero-flavour lattice calc

Collins Soper kernel from lattice QCD

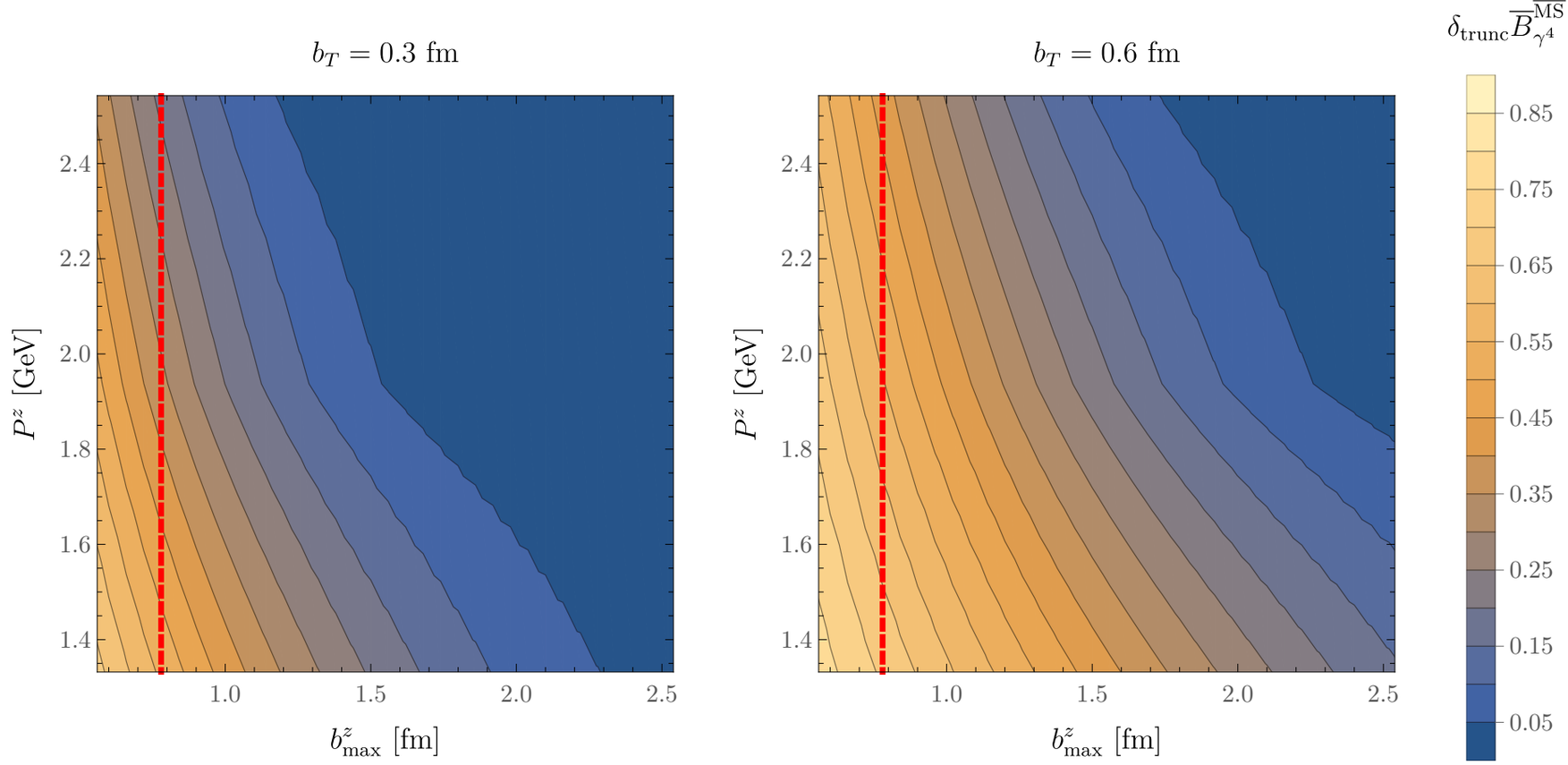
- Ultimate goal: first-principles calculation without modelling

  → must reduce truncation effects in discrete Fourier transform

  i.e., use larger lattice volumes

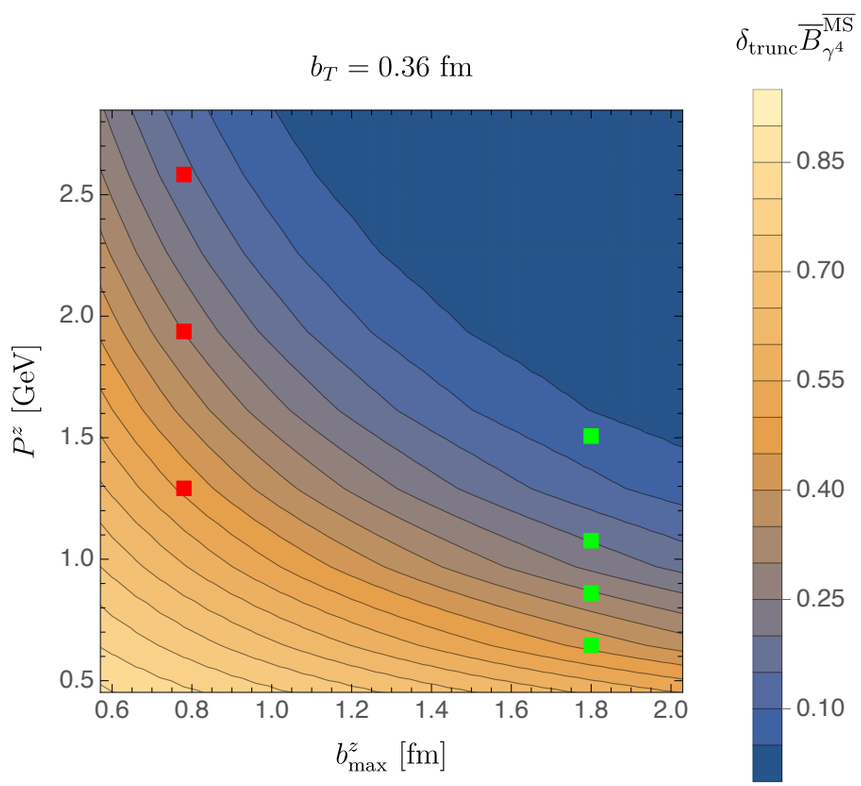
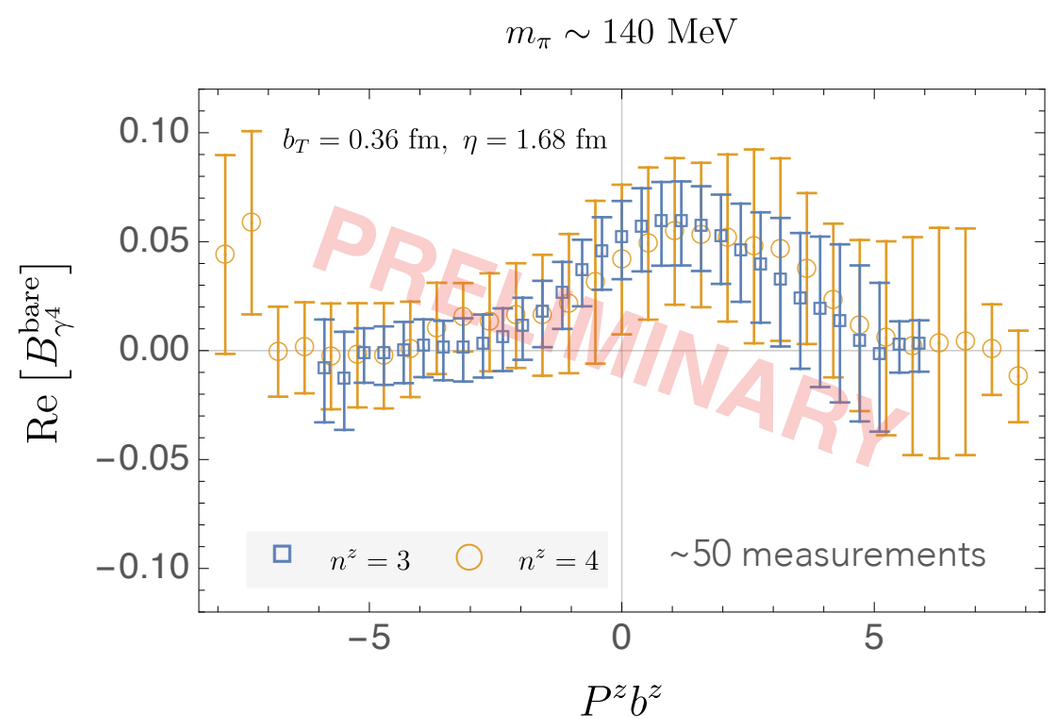
- Use models to study truncation in discrete Fourier transform and predict volume needed

$$\delta_{\text{trunc}}\overline{B}^{\text{MS}} = \left| \frac{\text{DFT}_{b^z_{\max}}\left[\mathcal{F}_N^{\text{Herm}}\right](P^z, x)}{\text{FT}_\infty\left[\mathcal{F}_N^{\text{Herm}}\right](P^z, x)} - 1 \right|$$



Need to target sufficiently large for discrete Fourier $\{b^z, P^z\}$ transform to be reliable

Collins Soper kernel from lattice QCD

Next steps: Dynamical calculation of CS kernel

- MILC ensemble $L^3 \times T = 48^3 \times 64 \quad a \sim 0.12 \text{ fm}$
- $m_\pi \sim 140 \text{ MeV} \quad m_\pi \sim 500 \text{ MeV}$
- Target calculation with ~20% truncation effects in discrete FT
- Preliminary NPR study confirms 16x16 mixing matrix is similarly important in dynamical case