Hybrid renormalization for quasi-light-front correlations in LaMET

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Quasi-light-front correlations

- LaMET calculation begins with the spatial correlation

  [Ji, PRL 13' & SCPMA 14']

$$\tilde{h}(z,a,P^z) = \frac{1}{2N}\langle P|O_\Gamma(z,0)|P\rangle = \frac{1}{2N}\langle P|\bar{\psi}(z)\Gamma W(z,0)\psi(0)|P\rangle$$

- The defining operator renormalizes multiplicatively

  [Ji, JHZ, Zhao, PRL 18', Ishikawa et al, PRD 17', Green et al, PRL 18']

$$O_\Gamma(z,0) = e^{\delta m|z|}Z(a)O_{\Gamma,R}(z,0)$$

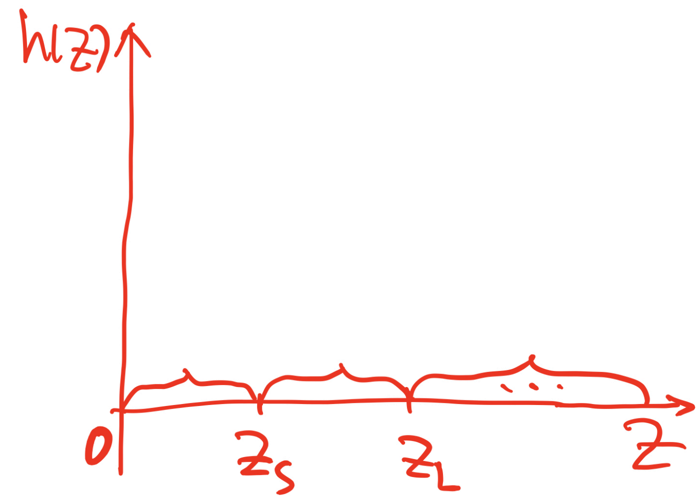
- Renormalization
  - Type 1: Determining $\delta m, Z(a)$ separately
  - Type 2: Forming a ratio of matrix elements of the same operator [Stewart, Zhao, PRD 18', Radyushkin, PRD 17'] [Braun, Vladimirov, JHZ, 19']

$$\tilde{h}_R(z,a,P^z) = \tilde{h}(z,a,P^z)/Z_X(z,a)$$

$$Z_X(z,a) = \frac{\langle p|O_\Gamma(z,0)|p\rangle|_{p^2=-\mu_R^2}}{\langle p|O_\Gamma(z,0)|p\rangle_{\text{tree}}}, \quad \tilde{h}(z,a,P^z=0), \quad |\langle 0|O_\Gamma(z,0)|0\rangle$$

Hybrid renormalization

- Separate the z-range into different parts and treat them differently



- Avoid introducing extra non-perturbative effects at large z in the renormalization stage

- At short distance ($0 \leq |z| \leq z_S$), use the ratio in type 2 renormalization
- At long distance ($z_S \leq |z| \leq z_L$), use the type 1 mass renormalization
- Beyond z_L, use physics-based extrapolation

Hybrid renormalization

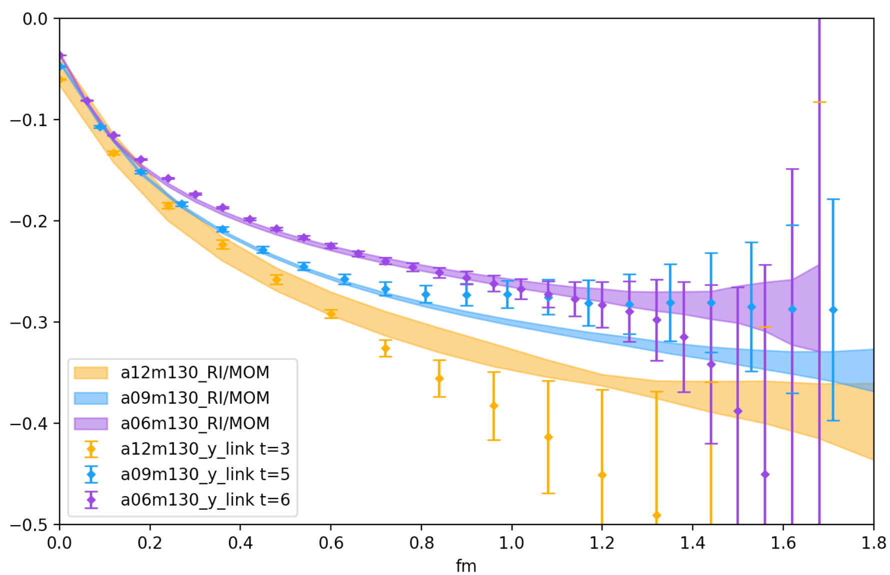
- Determination of renormalization factors

$$\delta m = \frac{m_{-1}}{a} + m_0\,, \quad Z(a)$$

- m_{-1}/a denotes power divergence, shall be independent of specific matrix elements used to extract it
- $m_0 \sim \mathcal{O}(\Lambda_{QCD})$ depends on the matrix elements

- Asymptotically, one has

$$\tilde{h}(z, a, P^z) \sim e^{-\delta m|z|}$$



Plot from Jun Hua

Hybrid renormalization

- Determination of renormalization factors

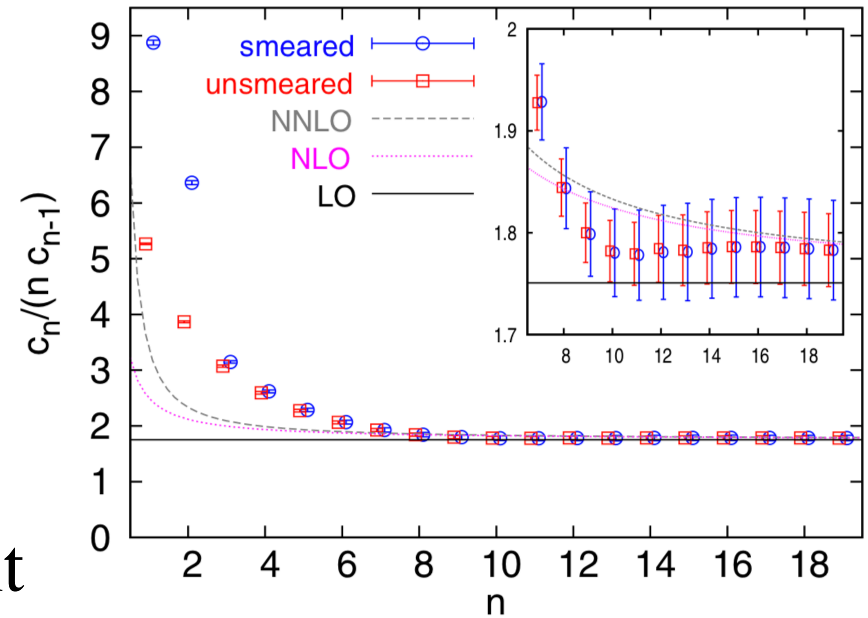
$$\delta m = \frac{m_{-1}}{a} + m_0 , \quad Z(a)$$

- In perturbation theory, m_{-1} starts from $\mathcal{O}(\alpha_s)$, can mix with m_0 due to renormalon effect

- It has been argued that

  [Bauer, Bali & Pineda, PRL 12']

$$\delta m = \frac{1}{a} \sum_{n \geq 0} c_n \alpha_s^{n+1}(1/a)$$



- m_0 is approximately constant when truncated at high enough perturbative orders

Hybrid renormalization

- Determination of renormalization factors

$$\delta m = \frac{m_{-1}}{a} + m_0 , \quad Z(a)$$

- In perturbation theory, m_{-1} starts from $\mathcal{O}(\alpha_s)$, can mix with m_0 due to renormalon effect
- m_0 can also arise from various other sources

- Ideally, it would be chosen to match non-perturbatively to the continuum MSbar scheme condition

$$\delta m_{\overline{\mathrm{MS}}} = 0$$

- In practice, an optimal way could be to vary and identify the value that yields least momentum dependence

Hybrid renormalization

- Beyond z_L , we use a physics-based extrapolated form and may begin with $x^a(1-x)^b$ for the parton distribution
- Its Fourier transform

$$h(\lambda) = \int_0^1 dx\, e^{ix\lambda} x^a (1-x)^b \sim \frac{\Gamma(1+a)}{(-i|\lambda|)^{a+1}} + e^{i\lambda} \frac{\Gamma(1+b)}{(i|\lambda|)^{b+1}}$$

at large λ

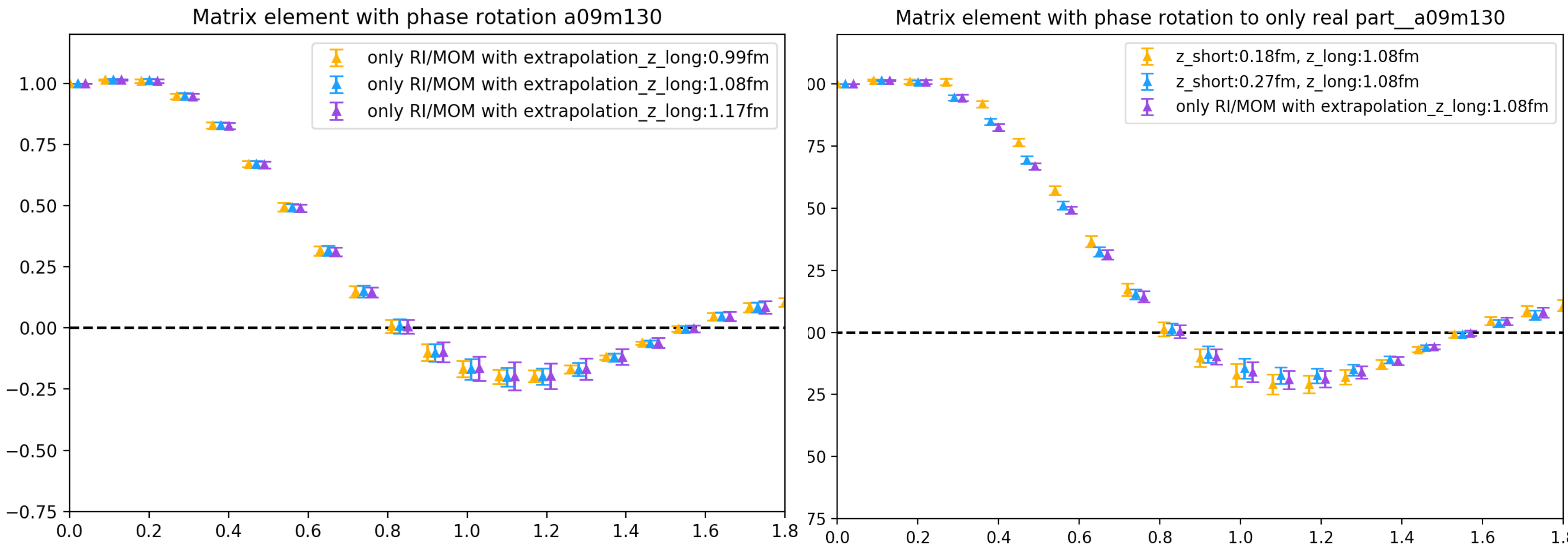
- Perturbative matching changes it to $\tilde{h}(\lambda) \sim |\lambda|^{-\alpha(a,b)+2\gamma}$, which also has a power decay behavior
- Extrapolation form can be chosen as

$$\frac{c_1}{(-i|\lambda|)^{d_1}} + e^{i\lambda} \frac{c_2}{(i|\lambda|)^{d_2}}$$

with unknown parameters being fixed from continuity requirement

Hybrid renormalization

- We need to investigate the dependence on the choice of z_S, z_L
- Example: pion distribution amplitude (see J. Hua's talk)



Plots from Jun Hua

Summary

- Hybrid renormalization offers a conceptually more viable renormalization scheme for LaMET calculations

- Further studies are required to establish robustness of this scheme
  - Optimal strategy to extract mass counterterms
  - Check independence of m_{-1} on various matrix elements used to compute it
  - Determination of m_0
  - Applications to various parton quantities