UV Divergence of the Quasi-PDF Operator under the Lattice Regularization

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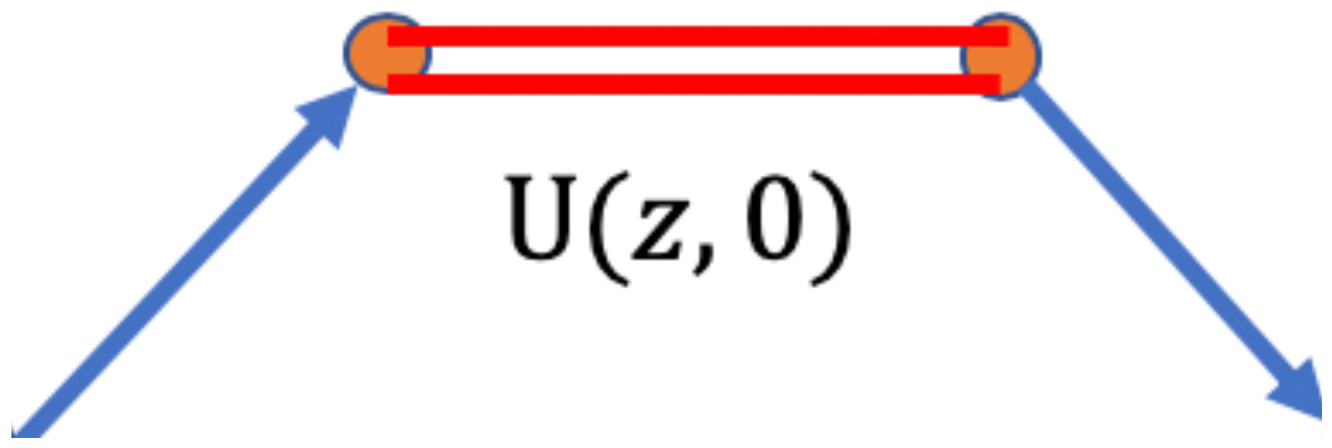
Lattice Parton Collaboration

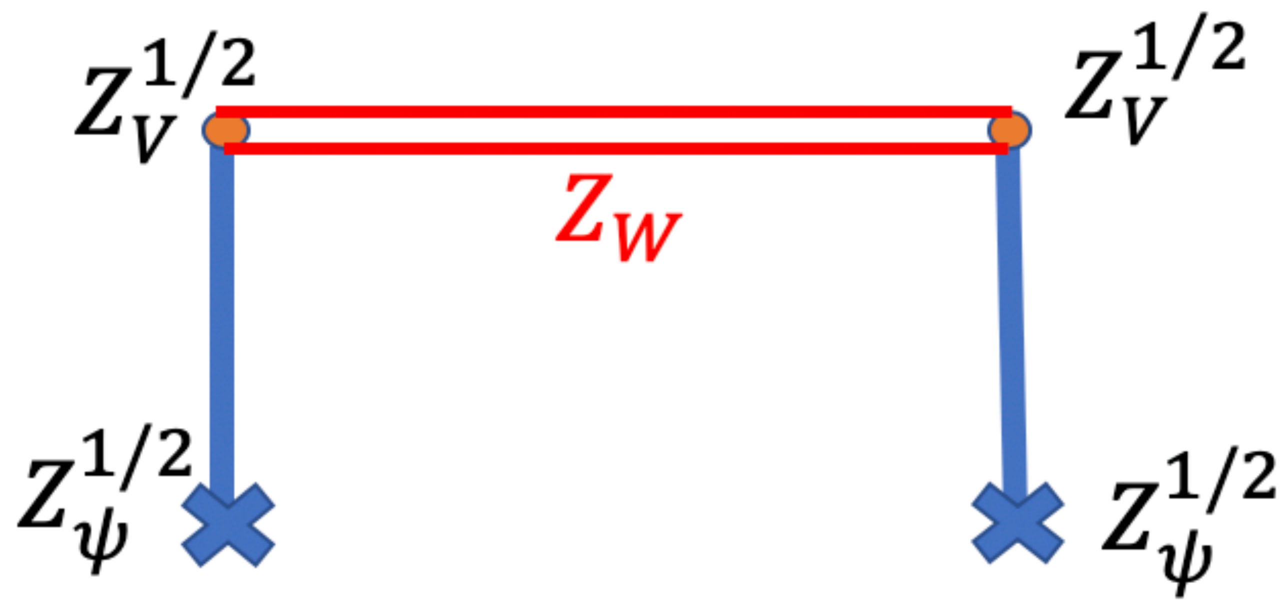
Quasi-PDF operator

Several renormalization procedures

- Naive counterterm $Z = Ce^{\frac{\delta m(a)}{a}z}$
  - $\delta m(a)$ from $\langle O_{\gamma_z}(z)\rangle$,
  - $\delta m(a)$ from $\langle O_{\gamma_t}(z,t)O^{\dagger}_{\gamma_t}(z,0)\rangle$,
  - $\delta m(a)$ from Wilson loop,
  - $\delta m(a)$ from the regularization independent momentum subtraction (RI/MOM) scheme.
- Hybrid method based on RI/MOM



$O_\Gamma(z) = \bar{\psi}(z)\Gamma U(z,0)\psi(0)$

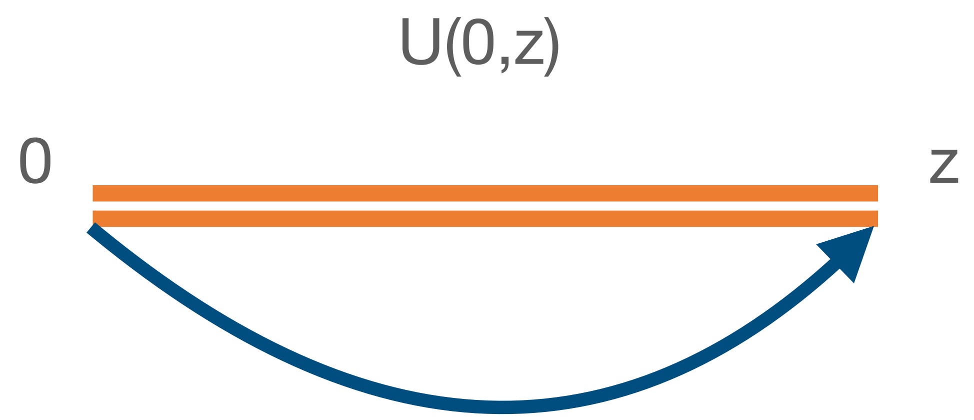


MILC configurations at 4 lattice spacings

β	L	L_t	C_{SW}	m_q	a_{phy} (fm)
3.60	24	64	1.05088	-0.0695	0.120
3.78	32	96	1.04239	-0.05138	0.090
4.03	48	144	1.03493	-0.0398	0.060
4.20	64	192	1.03144	-0.0365	0.045

Accurate lattice spacing determination is in progress.

- $\delta m(a)$ from $\langle O_{\gamma_z}(z)\rangle$

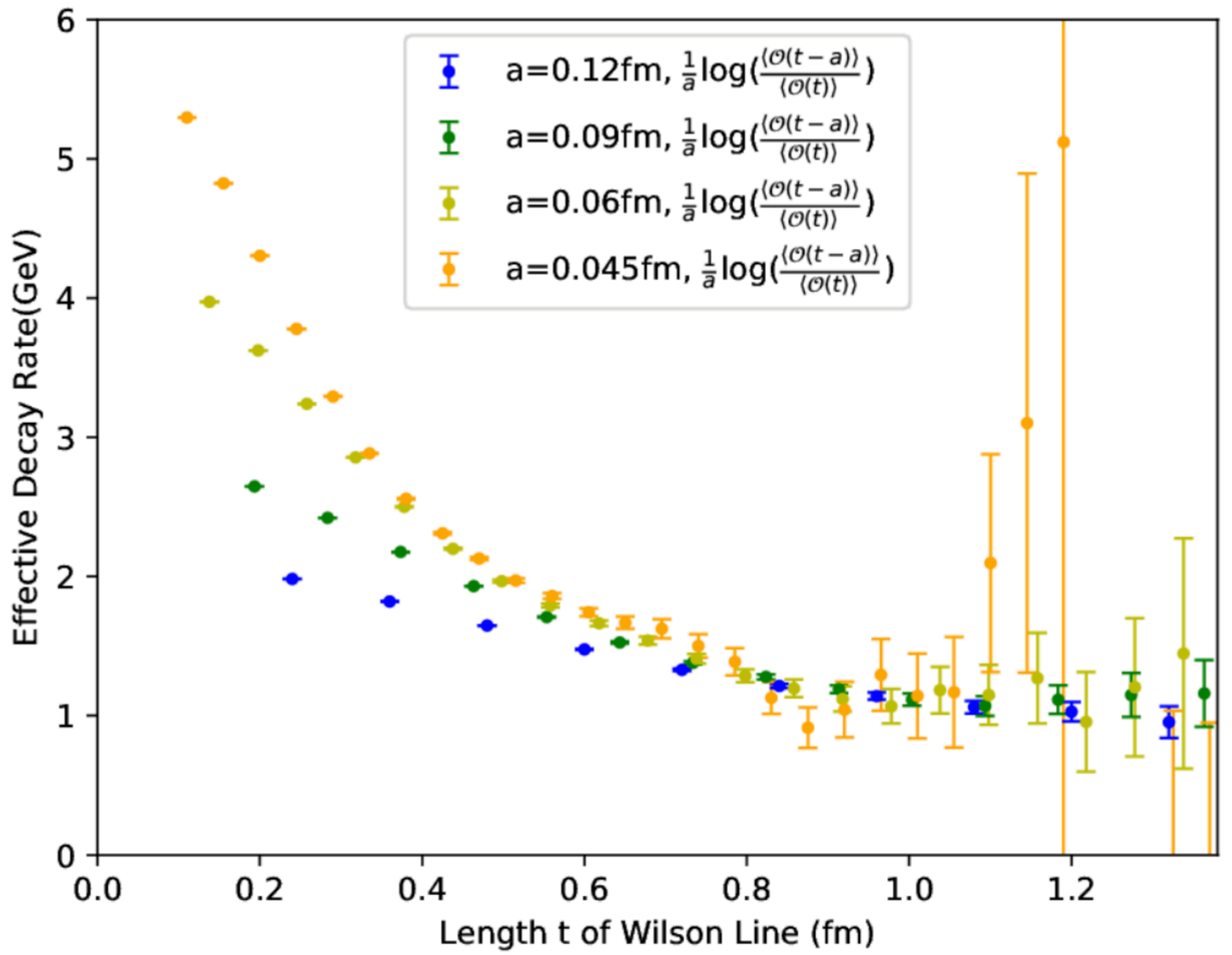


- Vacuum expectation value of $O_{\gamma_z}(z)$

- $\frac{1}{a}\log(\frac{\langle O_{\gamma_z}(z-a)\rangle}{\langle O_{\gamma_z}(z)\rangle})| = \frac{\delta m^{VEV}}{a} + f(z) + \mathcal{O}(a);$

- Above ratio seems to converge in the continuum limit, and then $\delta m^{VEV} \sim 0;$

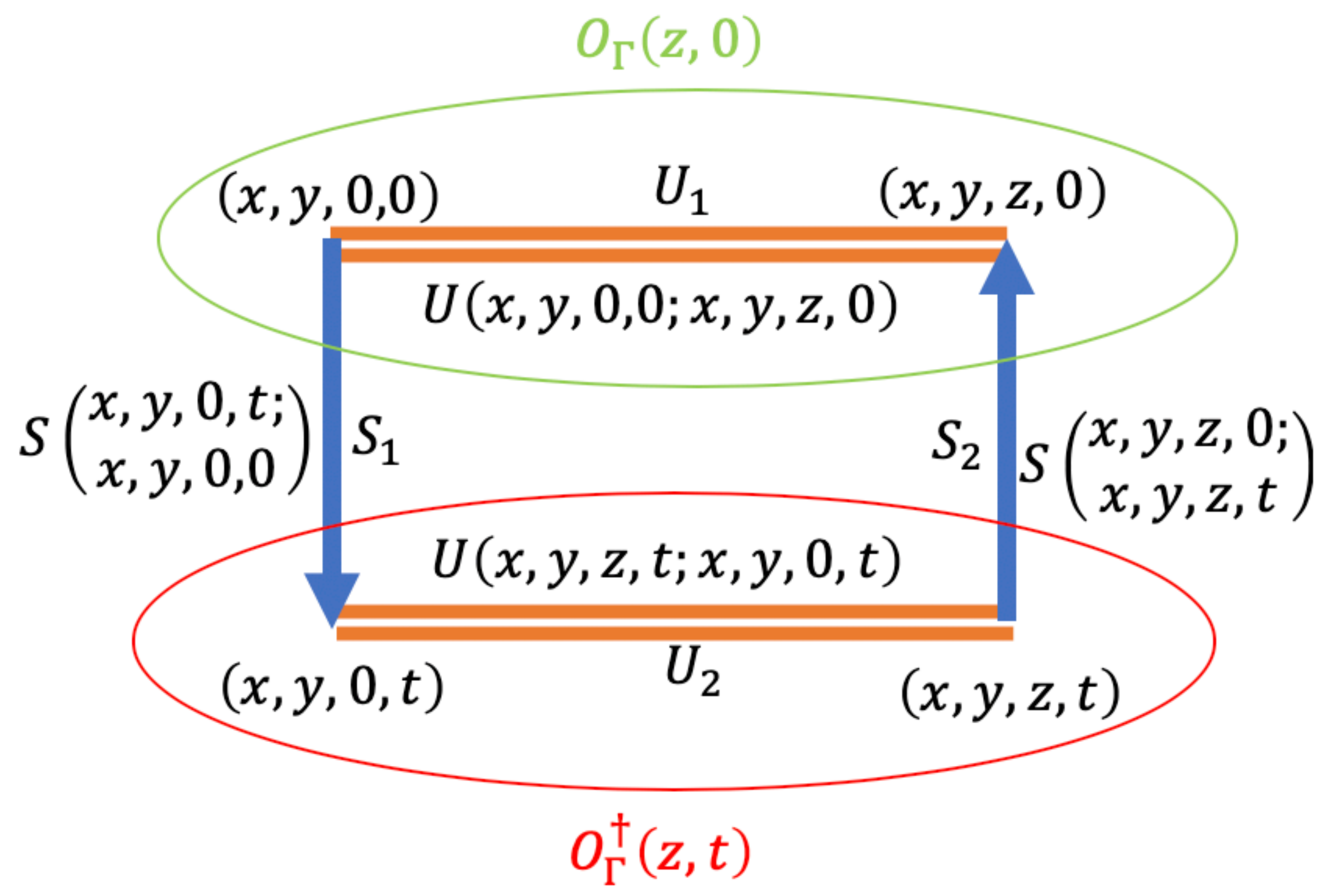
- It is understandable since the possible paths in the quark propagator increase exponentially with 1/a, and then they would cancel the linear divergence.

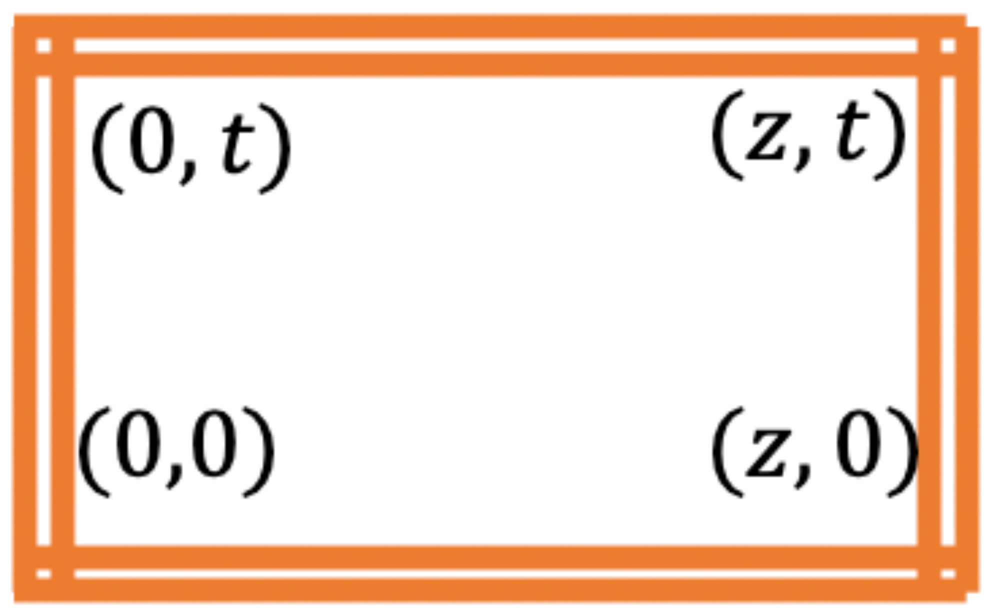


- $\delta m(a)$ from $\langle O_{\gamma_t}(z,t)O^{\dagger}_{\gamma_t}(z,0)\rangle$ and Wilson Loop

Definitions

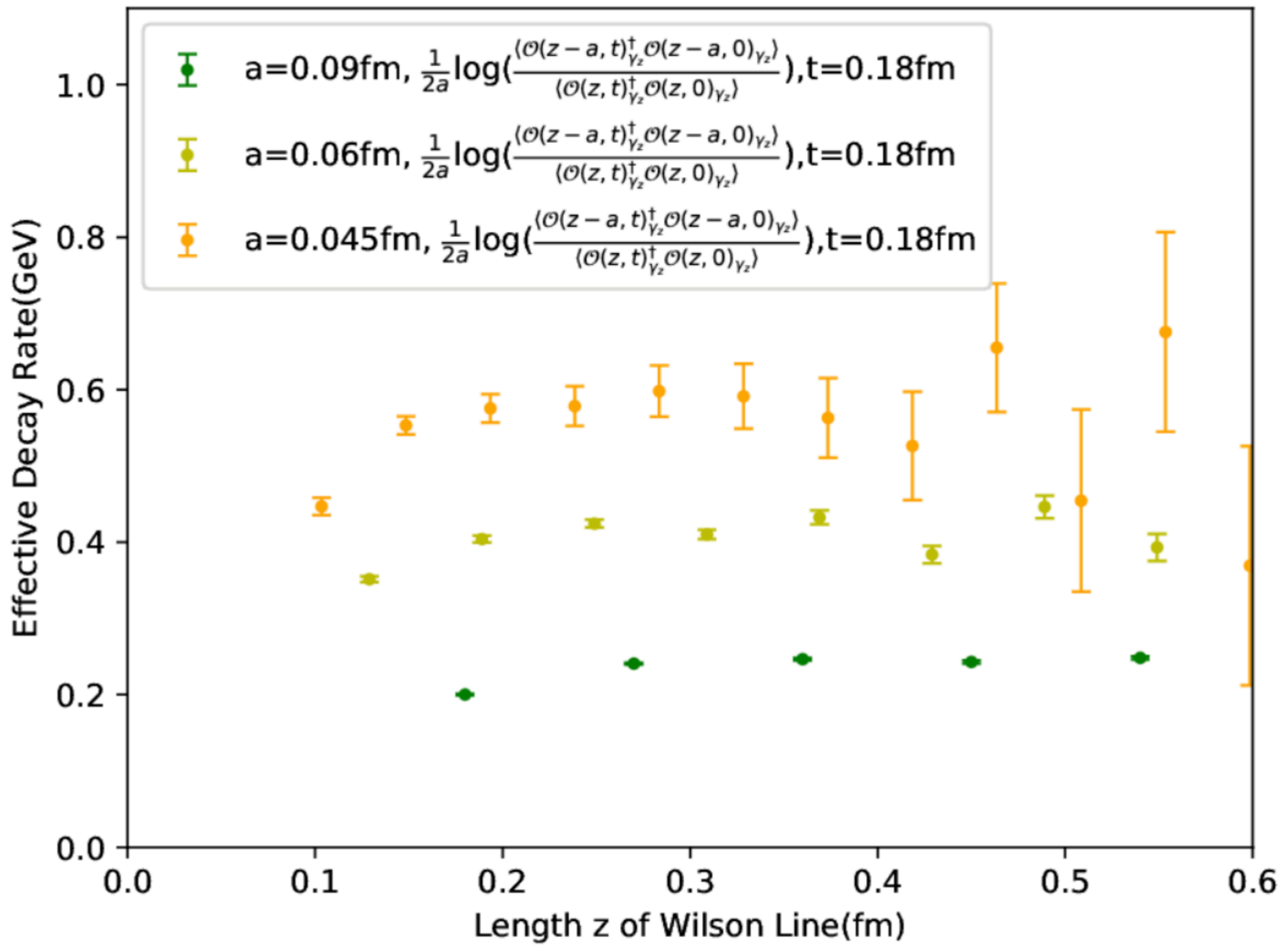
- $M(t,z) \equiv \langle O_{\gamma_t}(z)O^{\dagger}_{\gamma_t}(z)\rangle$
  - $\frac{1}{a}\log(\frac{M(t,z-a)}{M(t,z)})|_{t=0.18\text{fm}} = 2(\frac{\delta m^{VEV2}}{a} + m_0(z))$
- **Wilson loop**
  $U(t,z) = \langle U(0,t;z,t)U(z,t;z,0)U(z,0;0,0)U(0,0;0,t)\rangle$
  - $\frac{1}{2}\log(\frac{U(t-a,z)}{U(t,z)})|_{t\to\infty} = \frac{c_0}{z} + 2(\frac{\delta m^{Loop}}{a} + m_0) + \sigma z$
- Wilson loop uses infinite heavy quark line instead of the light quark propagator.

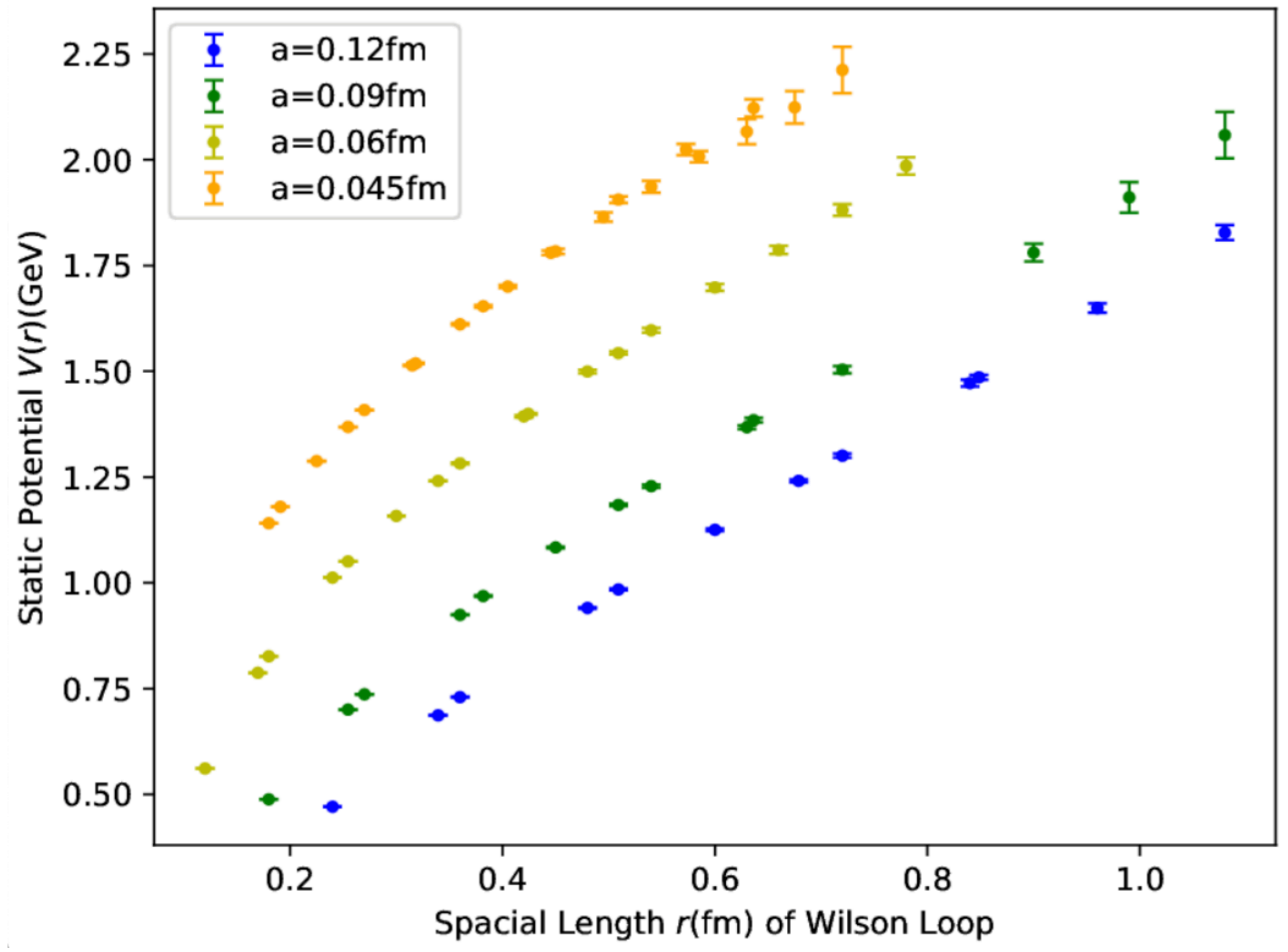




- $\delta m(a)$ from $\langle O_{\gamma_t}(z,t) O^\dagger_{\gamma_t}(z,0) \rangle$ and Wilson Loop

Results





$$\tilde{V}(z) = 2(\frac{\delta m^{VEV2}}{a} + m_0(z))$$

$$\delta m^{VEV2} \sim 0.15(2)$$

The values in these two cases are consistent with each other.

$$V(z) = \frac{c_0}{z} + 2(\frac{\delta m^{Loop}}{a} + m_0) + \sigma z$$

$$\delta m^{Loop} \sim 0.15(1)$$

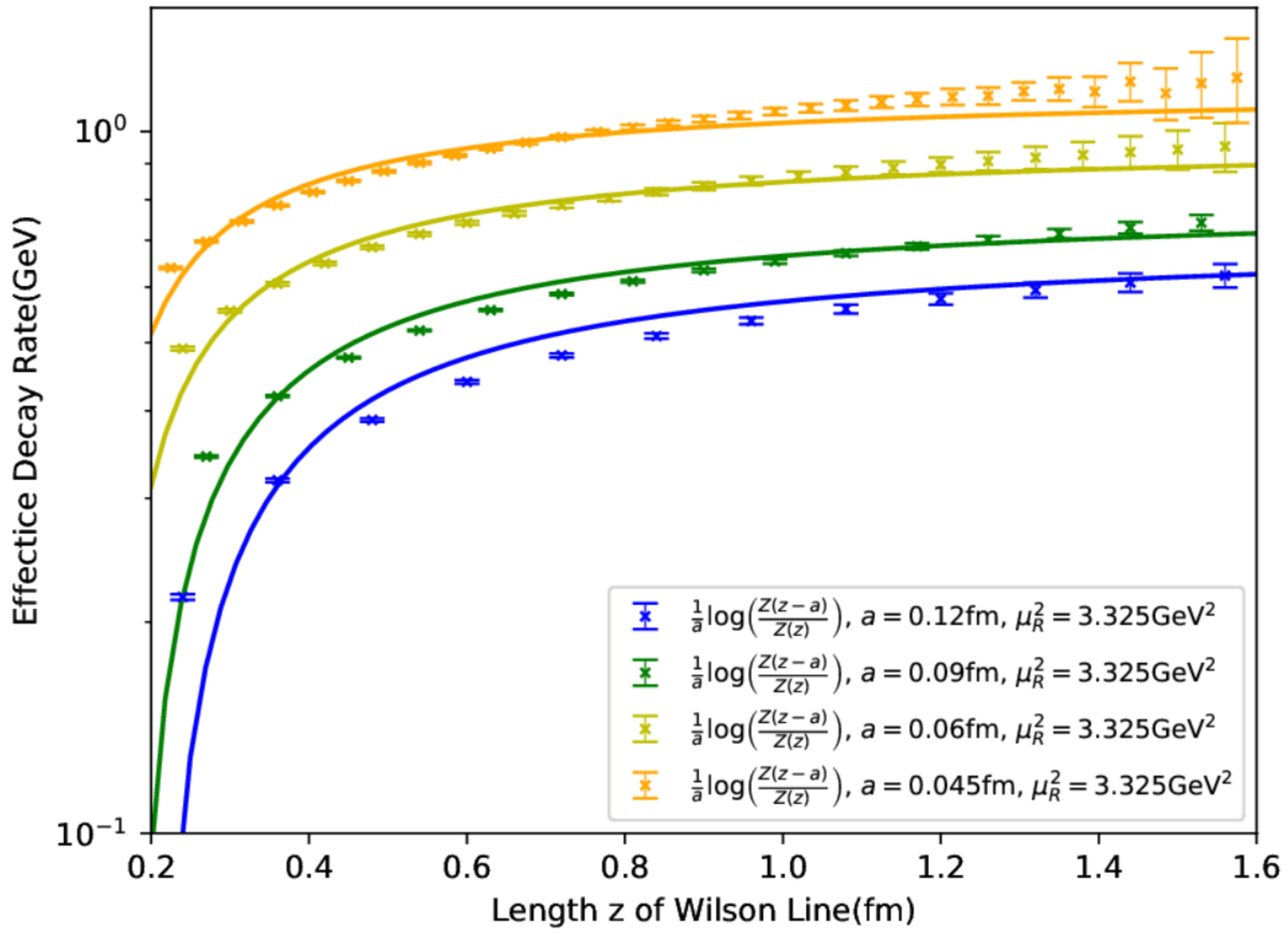
- $\delta m(a)$ from RI/MOM

Simple fit

- ○ $Z(z, a, \mu_R) = \frac{\langle p | O_{\gamma_t}(z) | p \rangle}{\langle p | O_{\gamma_t}(z) | p \rangle_{tree}} |_{p^2=-\mu_R^2, p_z=0}$

- $\frac{1}{a}\log(\frac{Z(z,a)}{Z(z-a,a)}) = \frac{\delta m^{RI/MOM}}{a} + f(z);$

- Fit $Z(z,a)$ with the form
  $Z(z,a) = C_0(a)e^{-\frac{\alpha_s(a)}{\alpha_s(a=0.12\text{fm})}\left\{\left(\frac{\delta m^{RI}}{a}+m_0^{RI}\right)z+c_1\log(\frac{z}{z_0})\right\}}$
  in the range $z \in [0.3, 2]\text{fm}$ and
  $a \in [0.04, 0.12]\text{fm}$ gives $\chi^2 \sim 130$ and
  $\delta m \sim 0.23(1)$.

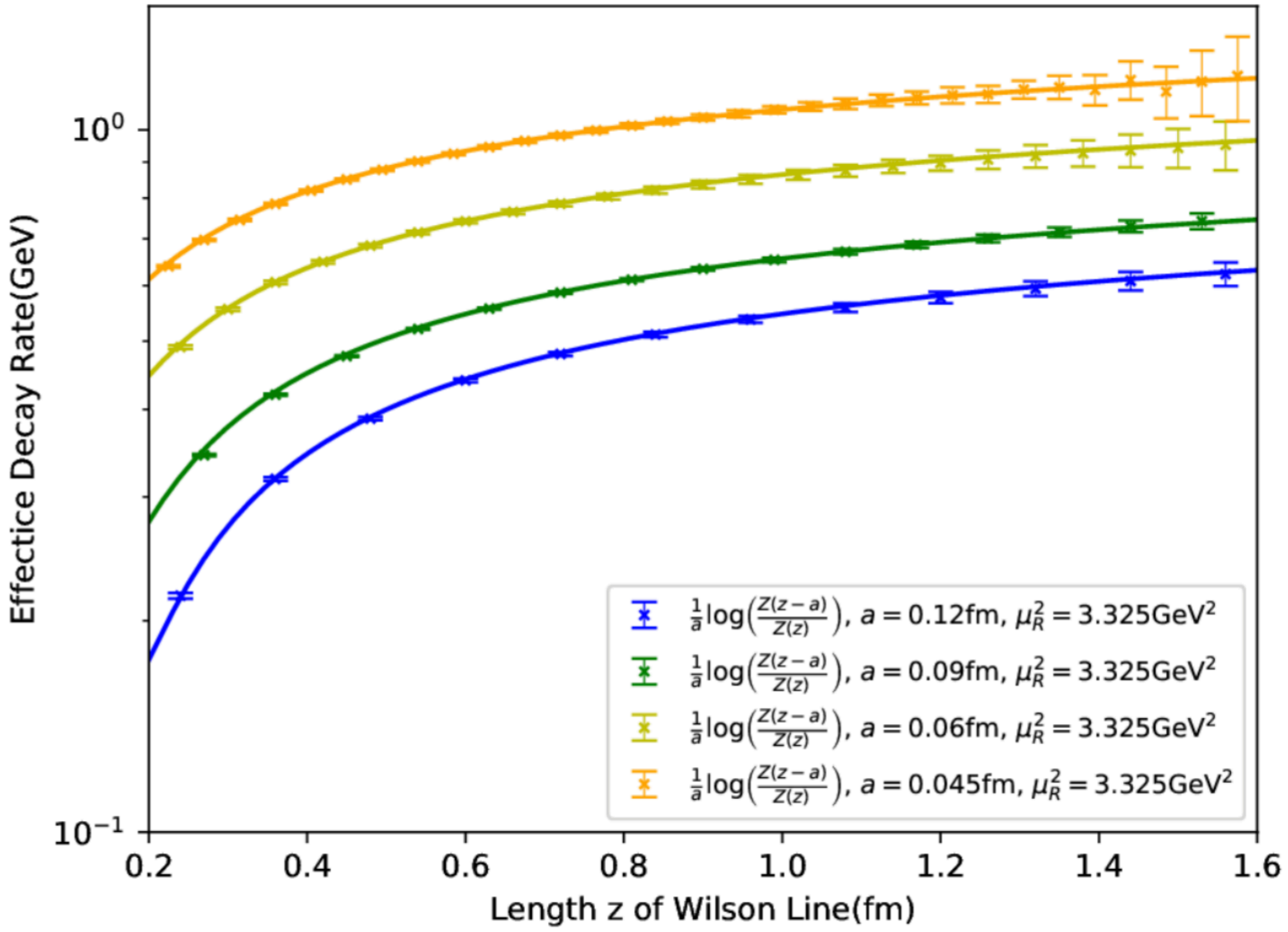


- $\delta m(a)$ from RI/MOM

Modified fit

$$Z(z,a) = C_0(a)e^{-\frac{\alpha_s(a)}{\alpha_s(a=0.12\text{fm})}(1+c_2\log(\frac{z}{z_0}))\left\{\left(\frac{\delta m^{RI}}{a}+m_0^{RI}\right)z+c_1\log(\frac{z}{z_0})\right\}}$$

- The $1+c_2\log(\frac{z}{z_0})$ factor would come from the resummation on the 1/z scale dependence, and make the linear divergence to be stronger at larger z.
- Fit $Z(z,a)$ with above form in the range $z\in[0.3,2]\text{fm}$ and $a\in[0.04,0.12]\text{fm}$ gives $\chi^2\sim 1$ and $\delta m\sim 0.16(1)$ with $z_0=0.3\text{fm}$.



- Hybrid method based on RI/MOM

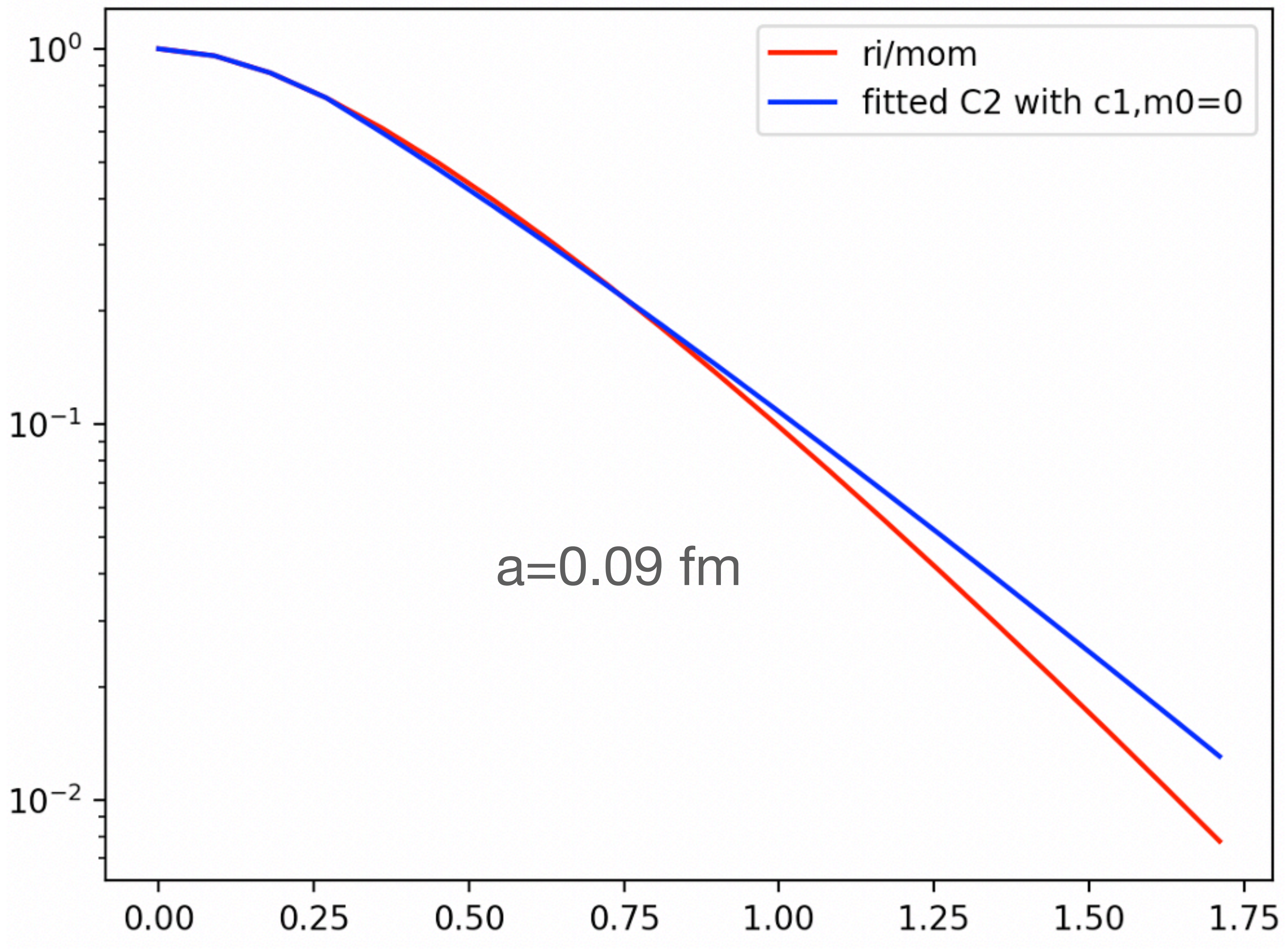
- Separate the z-range into different parts and treat them differently

- At small distance $0 \leq z \leq z_0$ ($z_0 \sim 0.3\text{fm}$), use RI/MOM renormalization

$$Z(z < z_0, a) = Z^{RI}(z, a)$$

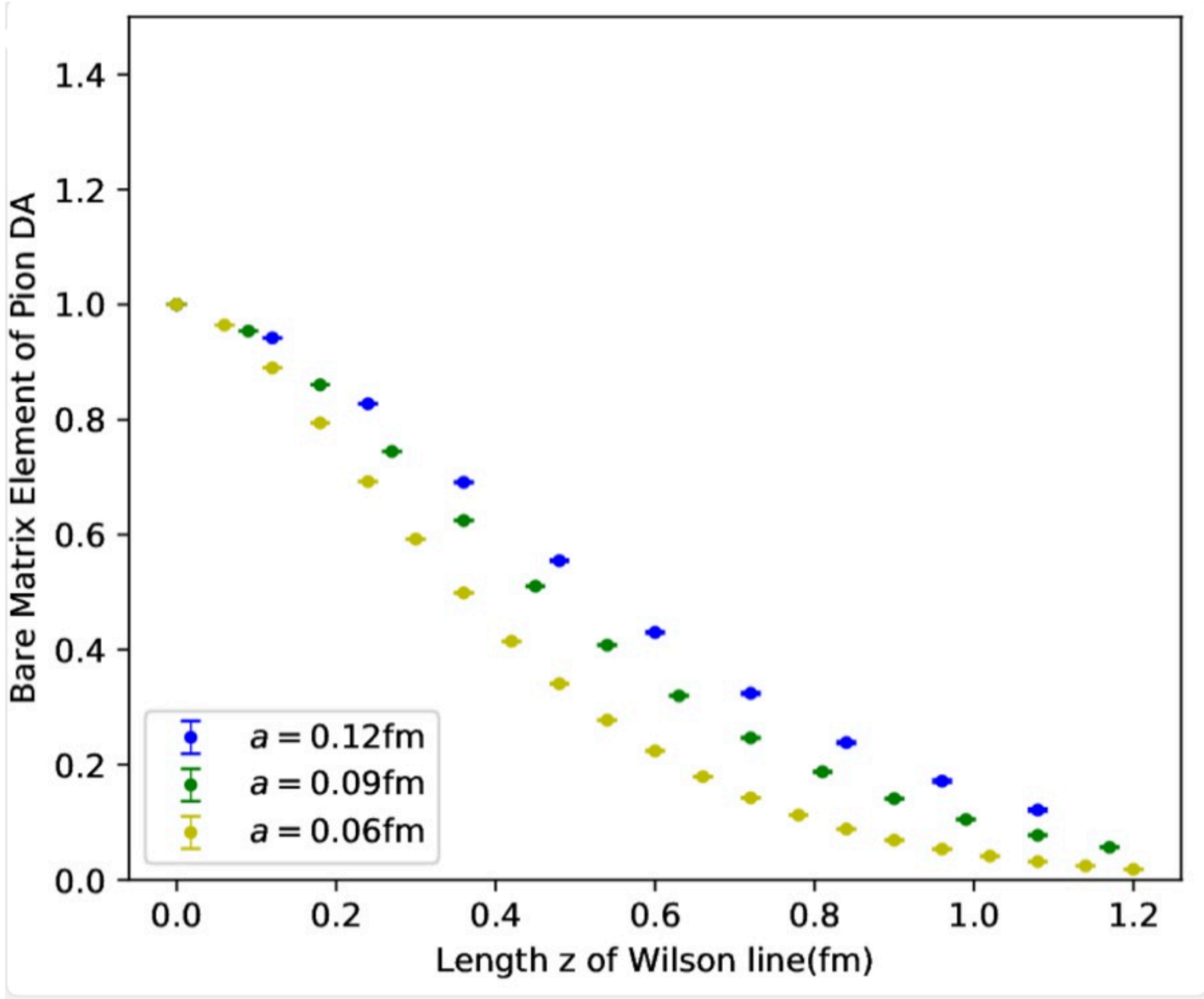
- At larger distance $z_0 \leq z$, use modified mass renormalization

$$Z(z > z_0, a) = Z(z_0, a)e^{-\frac{\alpha_s(a)}{\alpha_s(a=0.12\text{fm})}(1+c_2\log(\frac{z}{z_0}))\frac{\delta m^{RI}}{a}(z-z_0)}$$

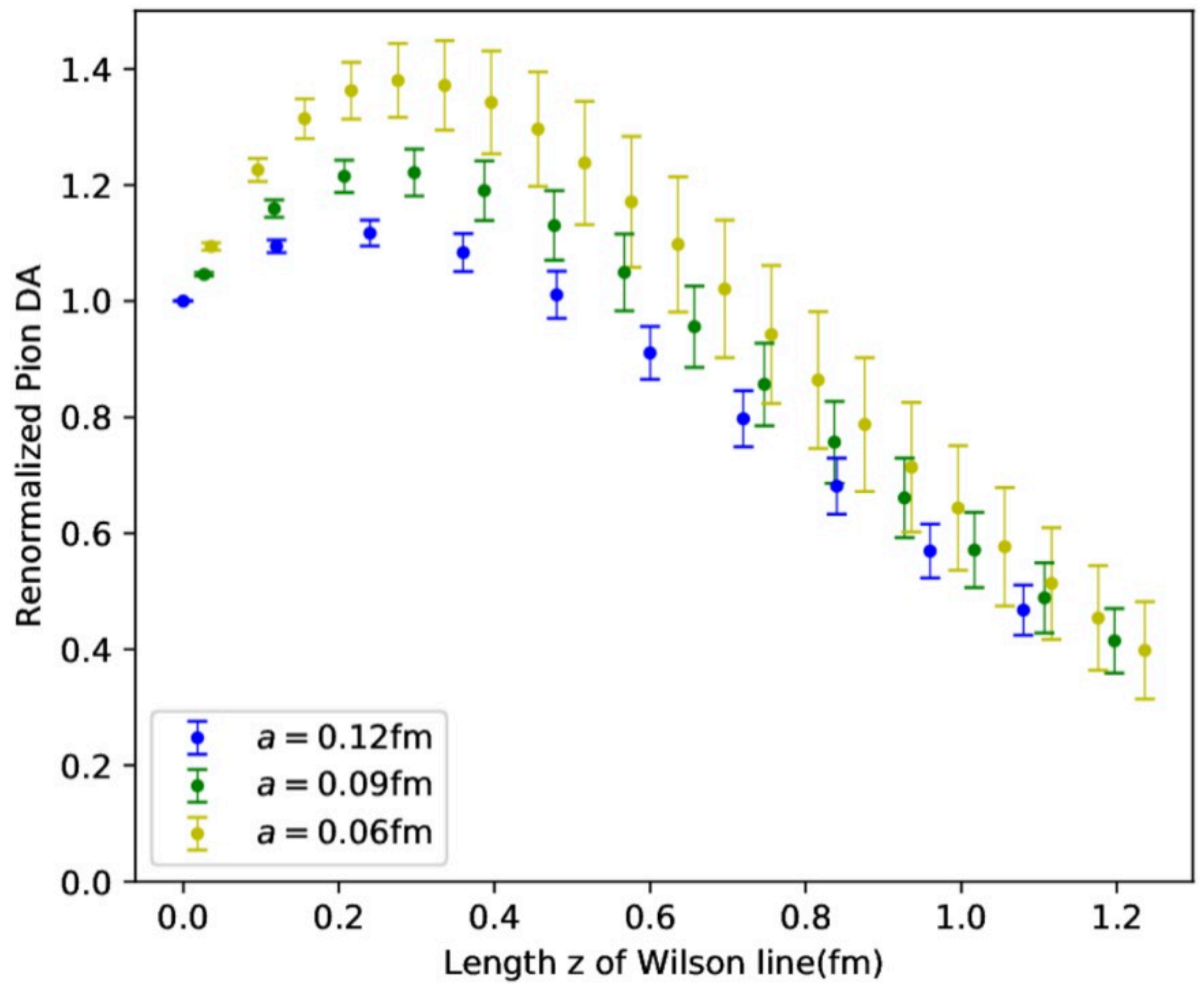


X.D. Ji et al arXiv: 2008.03886

- Renormalized pion distribution amplitude (DA) matrix elements in the rest frame



Bare DA with $\delta m = 0$

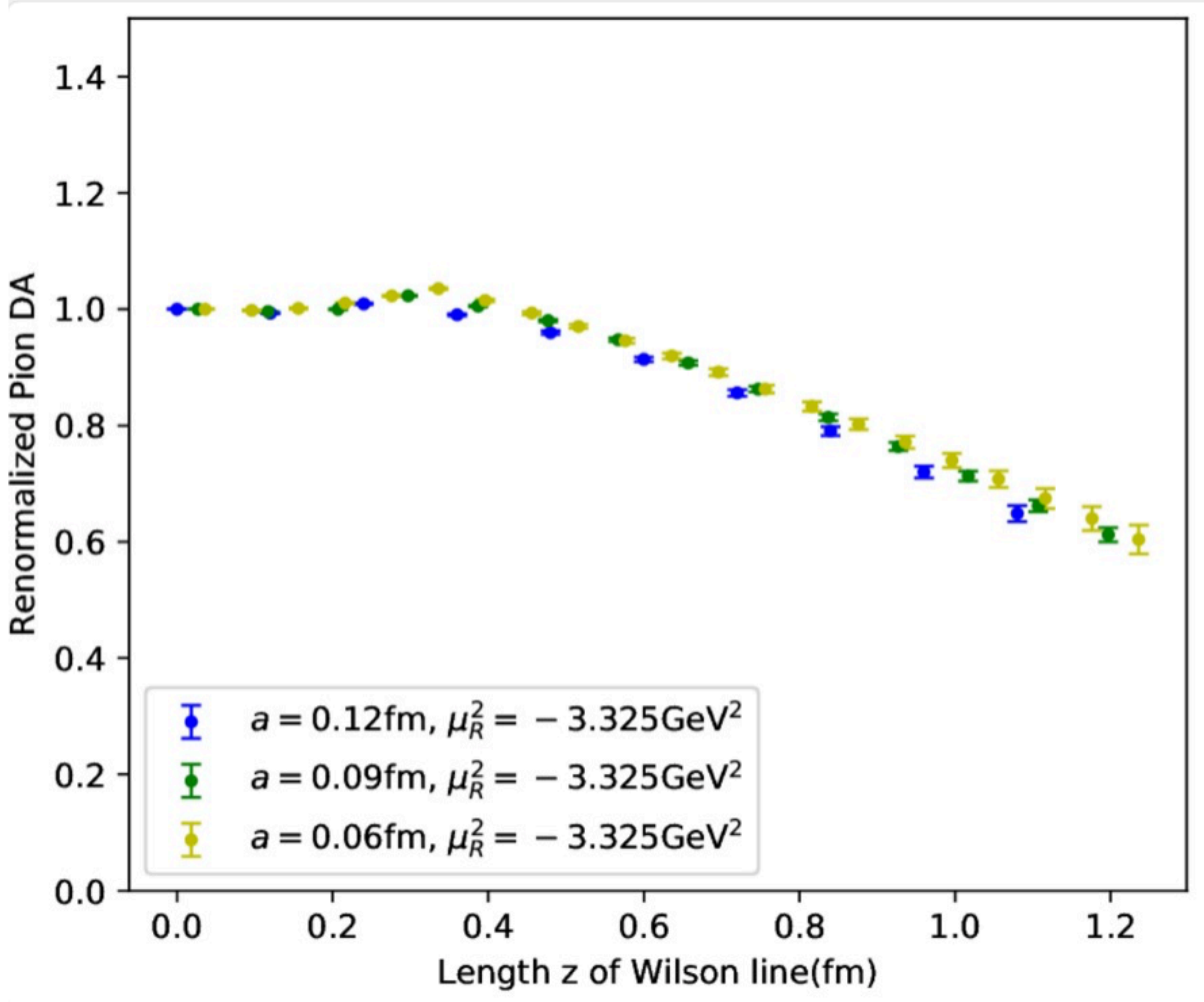


Renormalized DA with $\delta m = 0.15(2)$

$Z \propto e^{\frac{\alpha_s(a)}{\alpha_s(a=0.12\text{fm})}\frac{\delta m^{Loop}}{a}z}$

- Pion DA matrix element with $P_z = 0$;

- Only the hybrid scheme with modified mass renormalization can remove the linear divergence in the bare pion DA matrix element.



Renormalized DA with hybrid scheme

$Z \propto e^{\frac{\alpha_s(a)}{\alpha_s(a=0.12\text{fm})}(1+c_2\log(\frac{z}{z_0}))\frac{\delta m^{RI}}{a}(z-z_0)}$

Summary

- We calculated the lattice spacing dependence of kinds of matrix elements with Wilson links.
- A $\log(z)$ dependence of the linear divergence term exists in both the RI/MOM quark matrix element and hadron ones, but not $\langle O_{\gamma_z}(z)\rangle$, $\langle O_{\gamma_t}(z,t)O^\dagger_{\gamma_t}(z,0)\rangle$ or Wilson loop.
- Hybrid renormalization scheme with the modified mass renormalization, $Z(z>z_0,a)=Z(z_0,a)e^{-\frac{\alpha_s(a)}{\alpha_s(a=0.12\mathrm{fm})}(1+c_2\log(\frac{z}{z_0}))\frac{\delta m^{RI}}{a}(z-z_0)}$, can be a proper choice to remove the linear divergence in the bare hadron matrix element of $O_{\gamma_z}(z)$.