Hadron Structure from Current-Current Correlation

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with

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(HadStruc Collaboration)

WILLIAM & MARY

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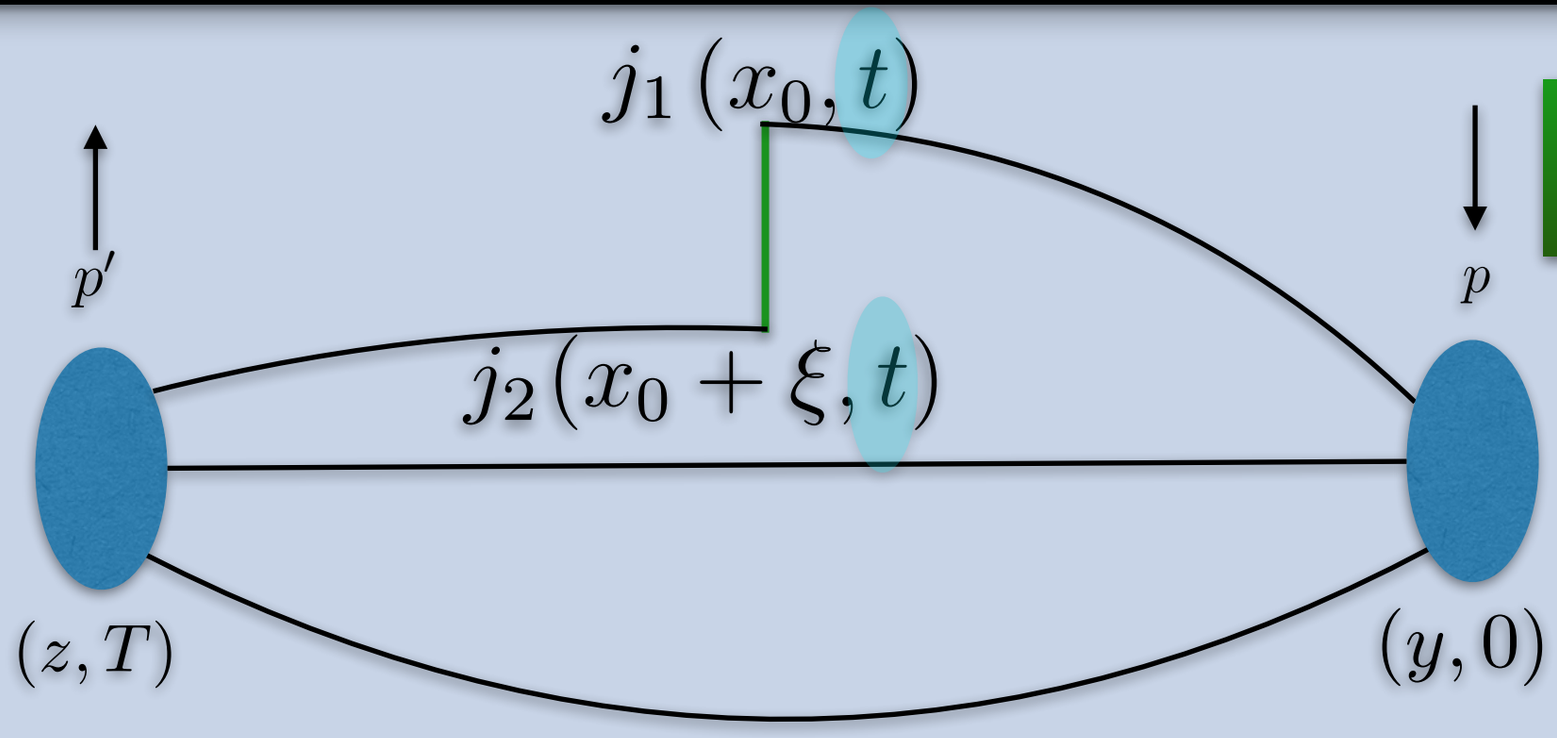
Outline

- Why current-current correlation in coordinate space
- Convergence of NLO matching in coordinate space
- Some discussions about PDF extraction from lattice
- Results in brief

Lattice cross section (LCS)



Y. Ma & J. Qiu
2014, 2018

- Spatially separated currents j_1 & j_2
- **LCS** has well defined continuum limit $(a \to 0)$
- Shares the same perturbative collinear divergences with PDFs
- Factorization holds for any finite $\omega \equiv p \cdot \xi$ and $p^2 \xi^2$ if ξ is short distance

LCS: physics interpretation

- Matrix element $\sigma_n(\omega, \xi^2)$

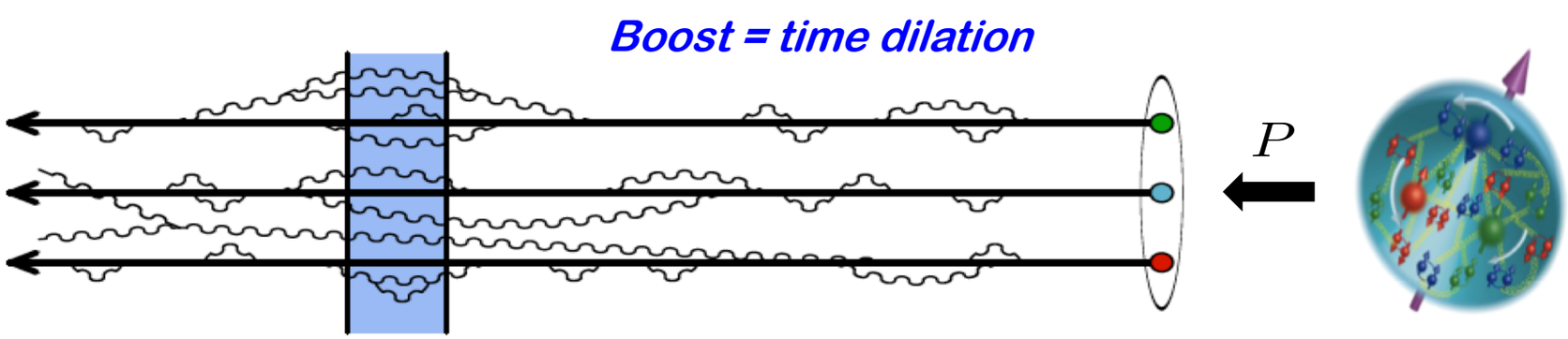
  Ioffe time $\omega \equiv P \cdot \xi$

- ξ : spatial separation between two currents

$$\xi^2 \to \frac{1}{Q^2}$$ Hard Probe

Required for pQCD factorization

❑ **Hard probes to "catch" the quantum fluctuation:**

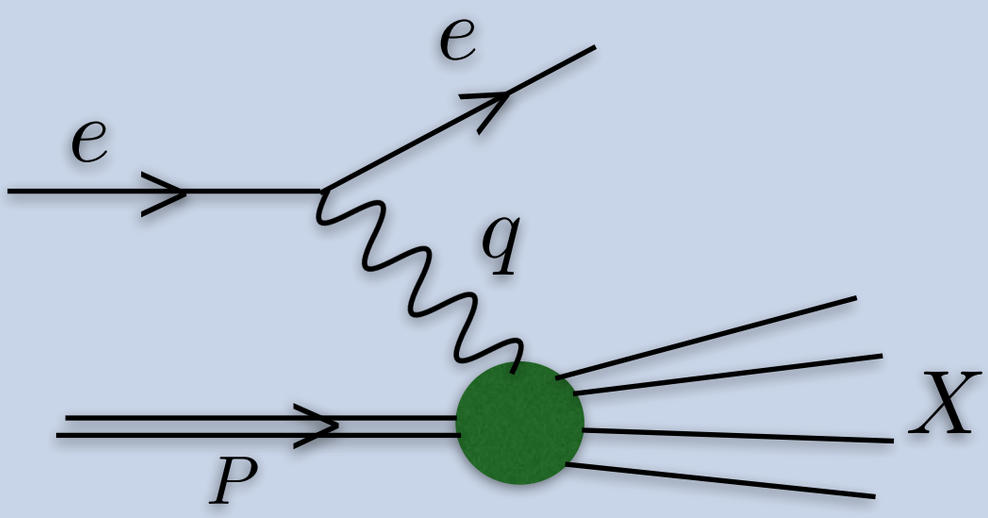


Hard probe (t ~ 1/Q < fm) ➡ Probability to "catch" the parton!

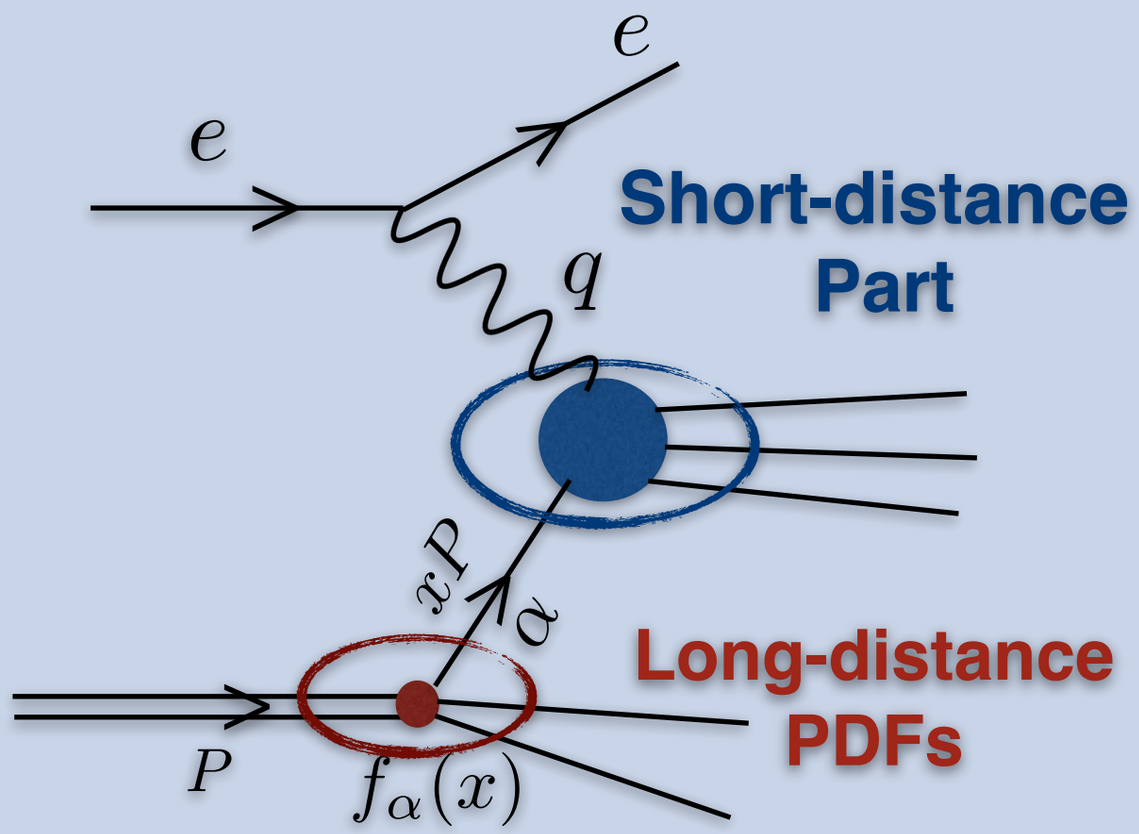
Jianwei Qiu
Snowmass 2021

Factorization in QCD & LCS

Cross section = **Short-distance physics (pQCD)** $\otimes$ **Long-distance PDFs** + Corrections



Scattering Process



Parton Picture

LQCD matrix elements -> continuum, physical limits->
-> continuum pQCD factorization -> extract PDFs

Next-to-leading order perturbative kernel

- Antisymmetric combination of V-A matrix elements for valence PDF

$$\gamma_1\,,\ \gamma_2\gamma_5 \qquad \text{and} \qquad \gamma_2\,,\ \gamma_1\gamma_5$$

- $$K(\omega,\xi^2,\mu^2) = \frac{1}{\pi^2\omega}[K^{(0)}(\omega) + \frac{\alpha_s C_{\rm F}}{2\pi}$$
$$\{K^{(1,0)}(\omega) + K^{(1,1)}(\omega)\ln(-\xi^2\mu^2 e^{2\gamma_E}/4)\}]$$

$$K^{(0)}(\omega) = \omega\cos\omega$$

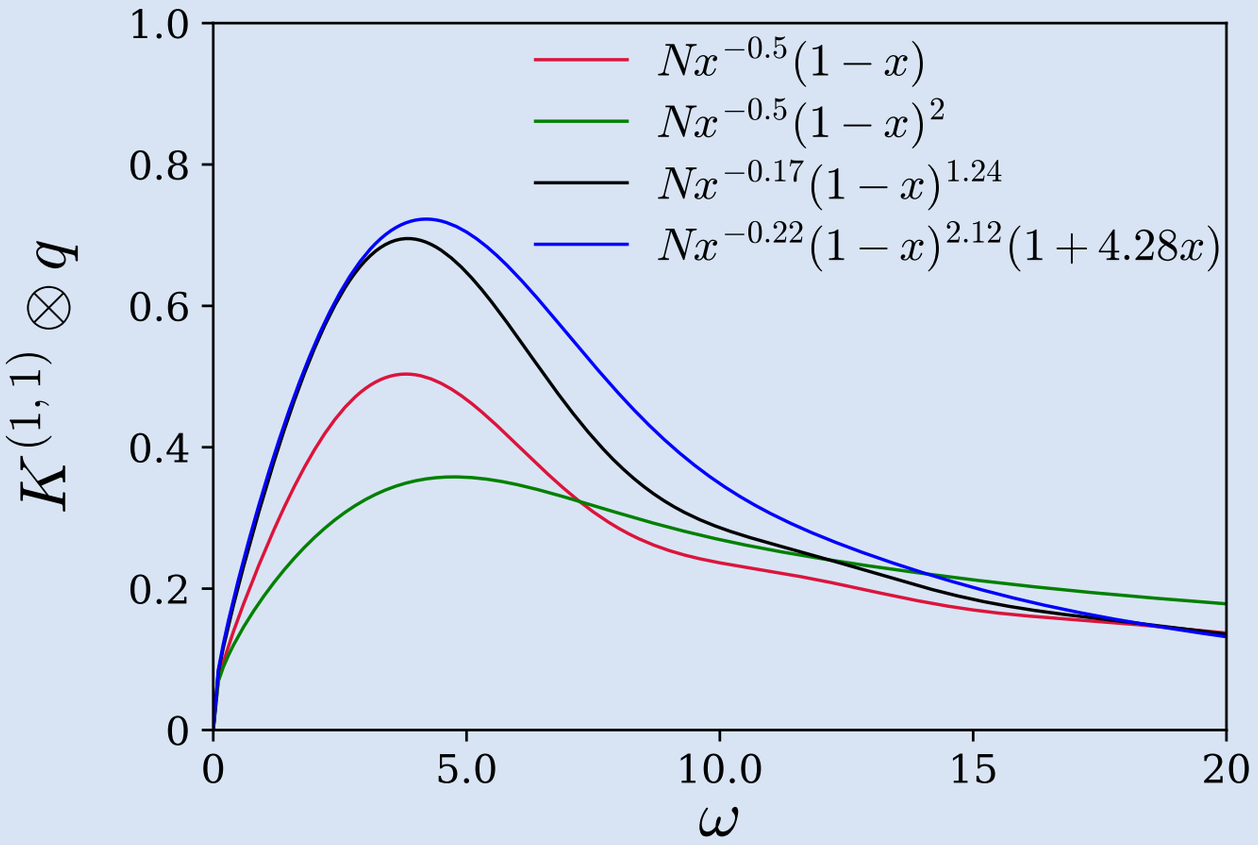
e.g. $$K^{(1,1)}(\omega) = -\omega\int_0^1 dy\cos(y\omega)\left(\frac{1+y^2}{1-y}\right)_+$$

Independent of hadrons

RSS, Karpie, Egerer, Edwards, Joo, Ma, Orginos, Qiu, Richards (2019 & 2020)

Why LCS matching in coordinate space

- LCS: Convolutions rise to a peak at $\omega \sim 4 - 5$



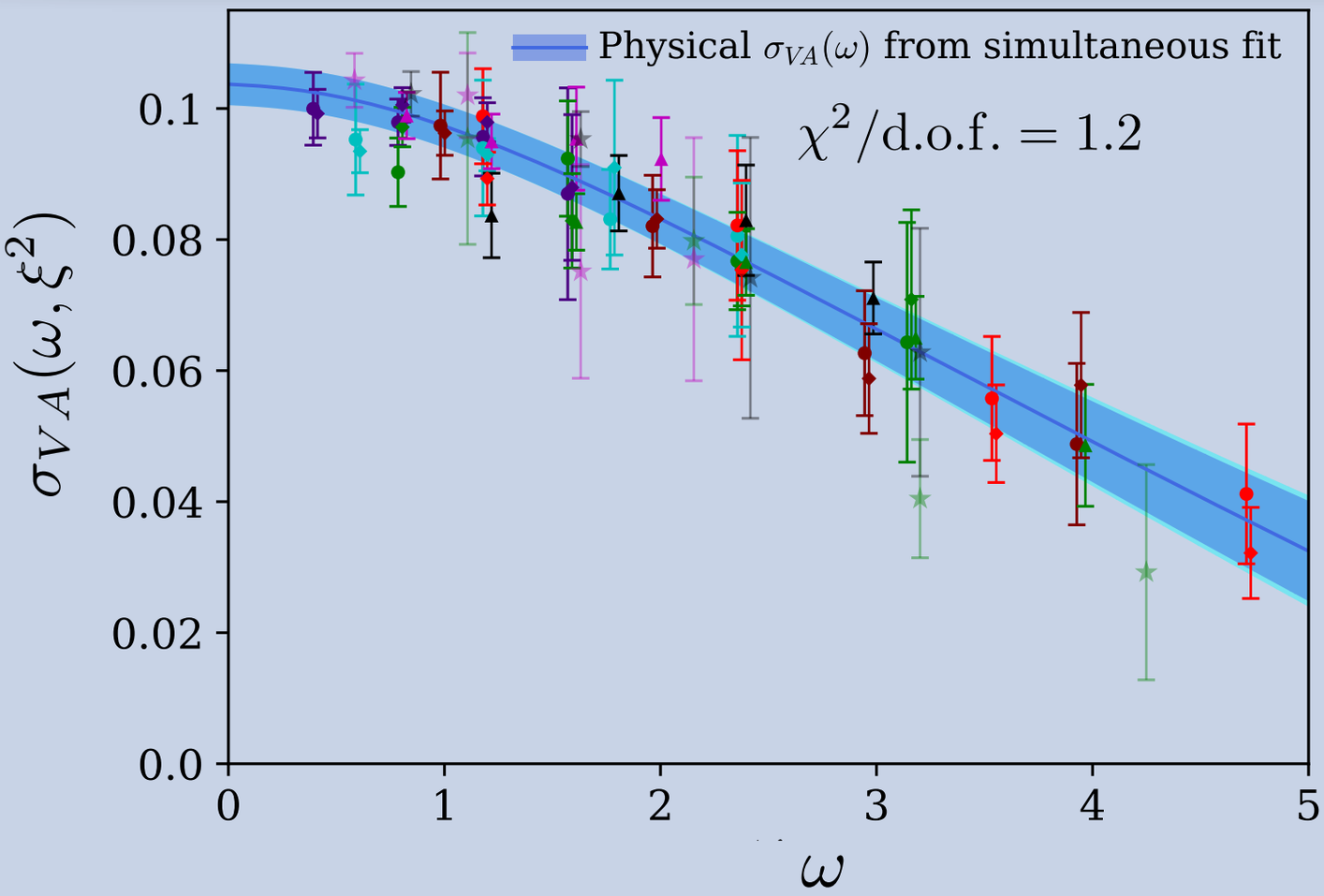
- To be multiplication by α_s

$\mathcal{O}(\alpha_s)$ effect not $\mathcal{O}(1)$

Compare with
Joo, Karpie, Orginos, Radyushkin, Richards, Zafeiropoulos (2019)

- LCS: Very stable & convergent 1-loop matching unlike NLO matching in momentum space

Lattice QCD matrix elements



a (fm)	m_π (MeV)	$L^3 \times N_t$
0.127(2)	413(4)	$24^3 \times 64$
0.127(2)	413(5)	$32^3 \times 96$
0.094(1)	358(3)	$32^3 \times 64$
0.094(1)	278(4)	$32^3 \times 64$

First attempt to go to physical limits in a pion PDF calculation

- $$\sigma_{\mathrm{VA}}(\omega, \xi^2) = \sum_{k=0}^{k_{\max}=4} \lambda_k \tau^k + b_1(m_\pi - m_{\pi,\mathrm{physical}}) + b_2 a + b_3 \xi^2 + b_4 a^2 p^2 + b_5 e^{-m_\pi (L-\xi)}$$

$$\tau = \frac{\sqrt{\omega_{\mathrm{cut}} + \omega} - \sqrt{\omega_{\mathrm{cut}}}}{\sqrt{\omega_{\mathrm{cut}} + \omega} + \sqrt{\omega_{\mathrm{cut}}}}$$

- Similar in pion PDF calculation by Joo, et al 2019
  Also adopted in gluon PDF calculation Fan, Zhang, Lin 2020

(pseudo-PDFs, Radyushkin 2017)

PDF extraction from Lattice QCD matrix elements

$$\sigma_{V\,A}(\omega) = \int_0^1 dx\, K^{\mathrm{LO+NLO}}(x,\omega)\, q_{\mathrm{v}}^{\pi}(x)$$

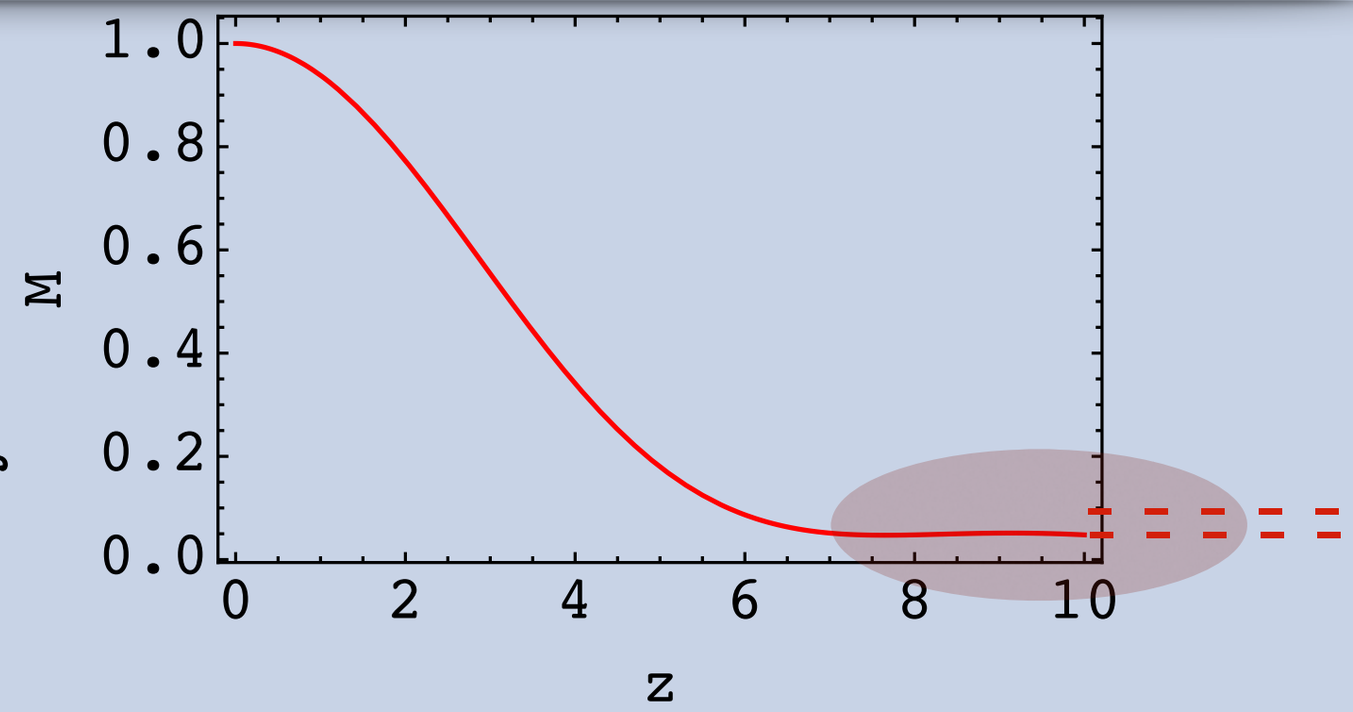
- Extraction of full x-dependence from limited data is a BIG challenge

- An important part of CNF activity!

- Global fits of PDFs

PDF extraction from Lattice QCD matrix elements

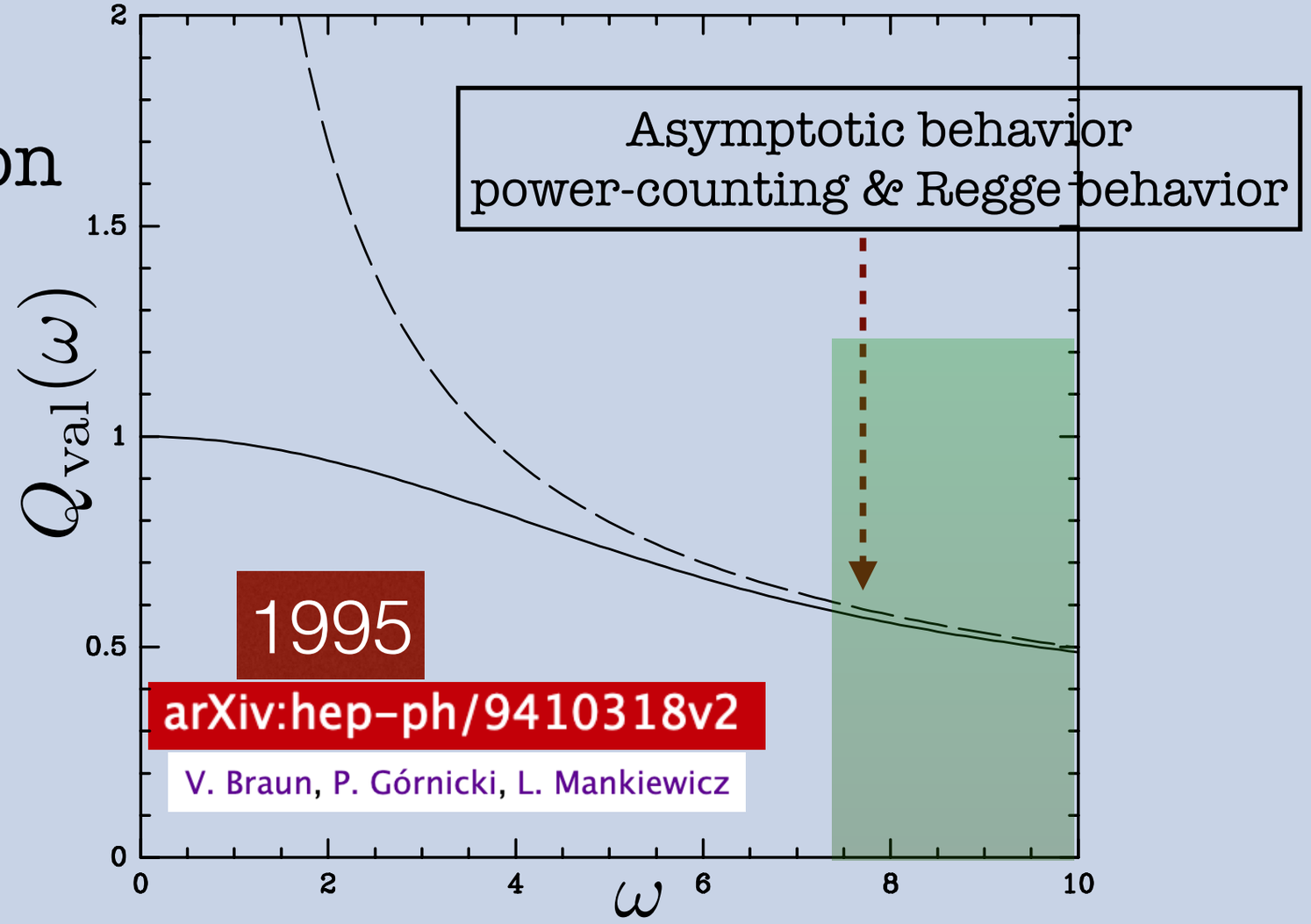
- Fourier transform of a constant signal is a delta function



- Helps producing singular behavior

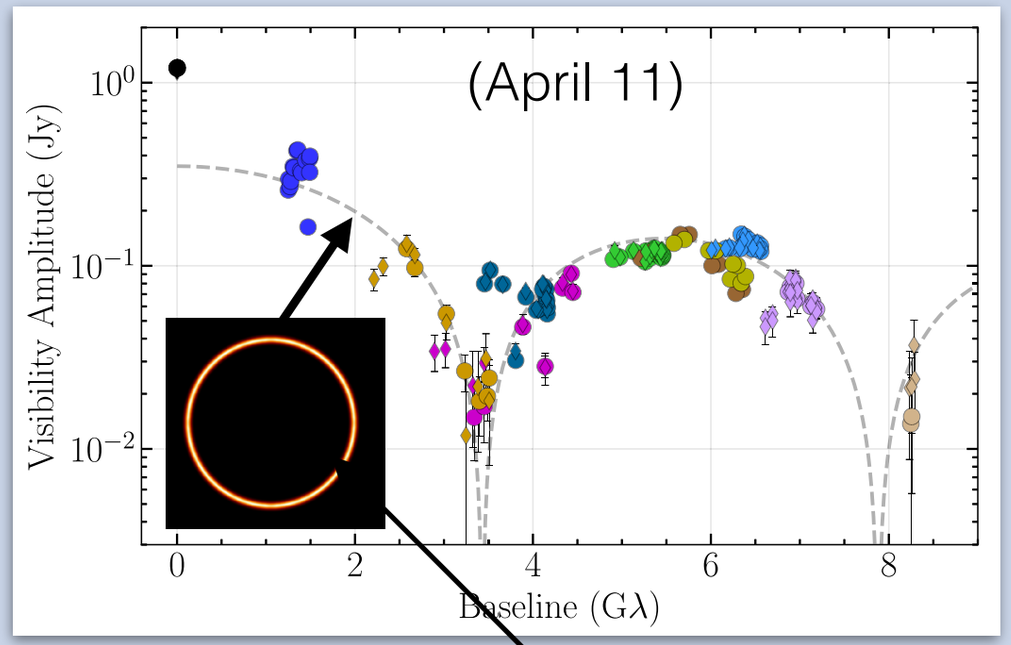
  Not a solution to the problem!

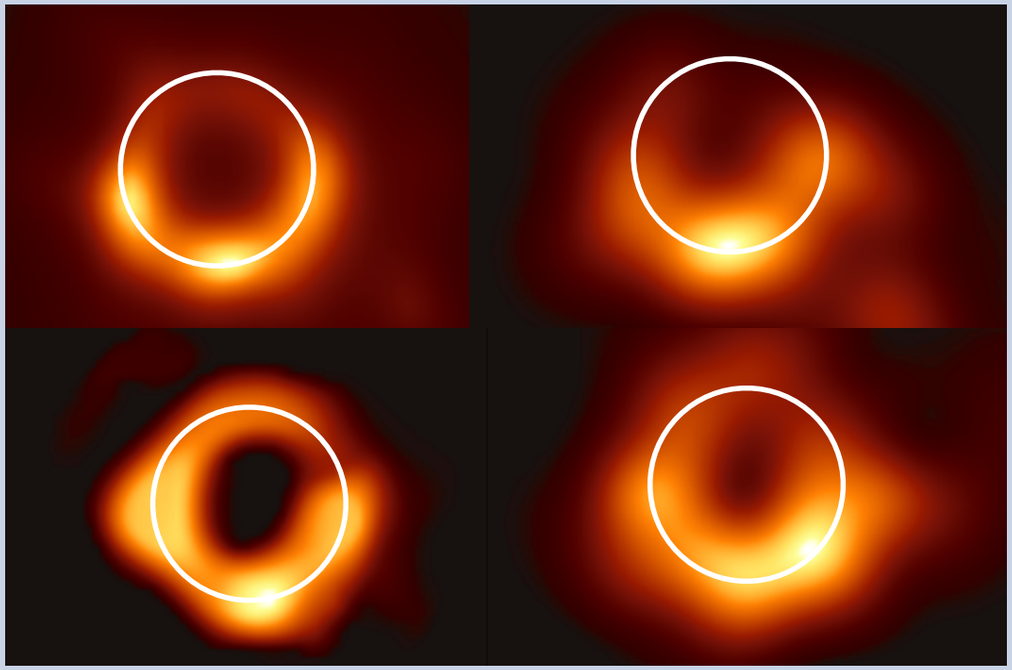
- Ioffe-time distribution



Inverse problem (A common challenge)

- Recall the black hole image





Additional information like functional forms of PDFs

- Problem associated with attempt to construct complete information from limited data

- Nothing special to do with any specific method of calculating PDF using Lattice QCD

Some observations

- arXiv:2005.12015 Fan, Gao, et al

part of the lattice qPDF lattice and the results of global analysis. This suggests that the range of applicability of 1-loop matching is perhaps limited to $z \lesssim 0.2$ fm in the case of the nucleon. It remains to be seen if this agreement gets better with the addition of higher-loop corrections, or this observed discrepancy arises because of contamination of higher-twist effects at larger z. This observation has an important implication for our ability to described the x-dependence of PDF within the LaMET framework. For example, if the 1-loop perturbative matching works only for $z \simeq 0.2$ fm, reliable calculations of nucleon PDF down to $x \simeq 0.1$ will need $P_z \gtrsim 10$ GeV.

HT and/or convergence problem of 1-loop matching?

- arXiv:2005.02102 Bhat, Cichy, et al

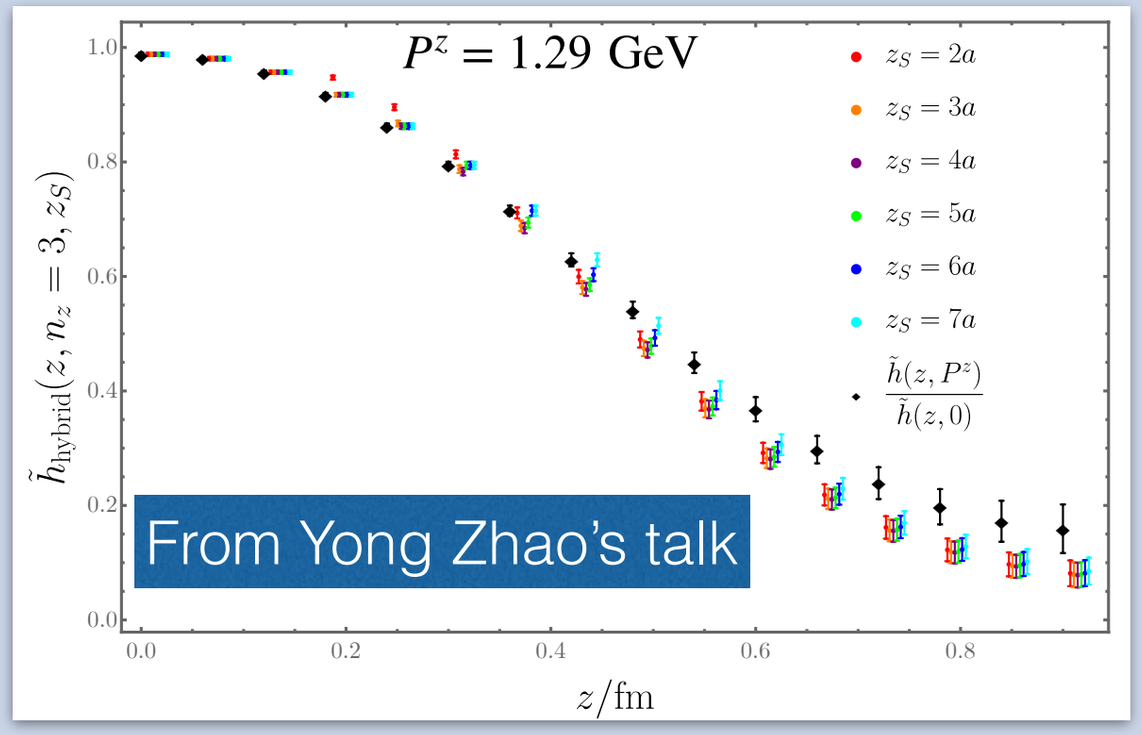
bare matrix elements as the present study, the one-loop matching effects are considerably larger. The matching for quasi-PDFs is performed in x-space and the difference between a quasi-PDF and a matched PDF are above 100% in many regions of x. Hence, it is plausible that the matching in the pseudo-distribution approach, at the level of ITDs (in ν-space), is more controlled, i.e. less subject to truncation effects.

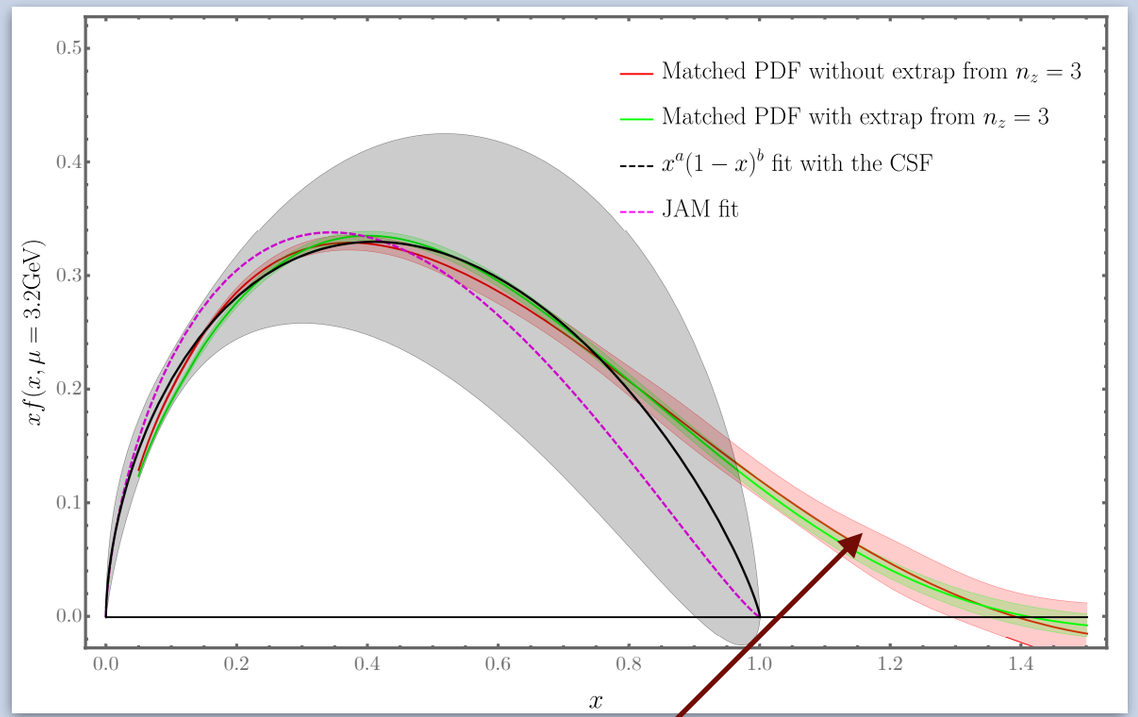
Convergence problem of matching?

- Several other observations...

Some observations

- Hybrid renormalization (Ji, et al 2020) **arXiv:2008.03886**





Possible

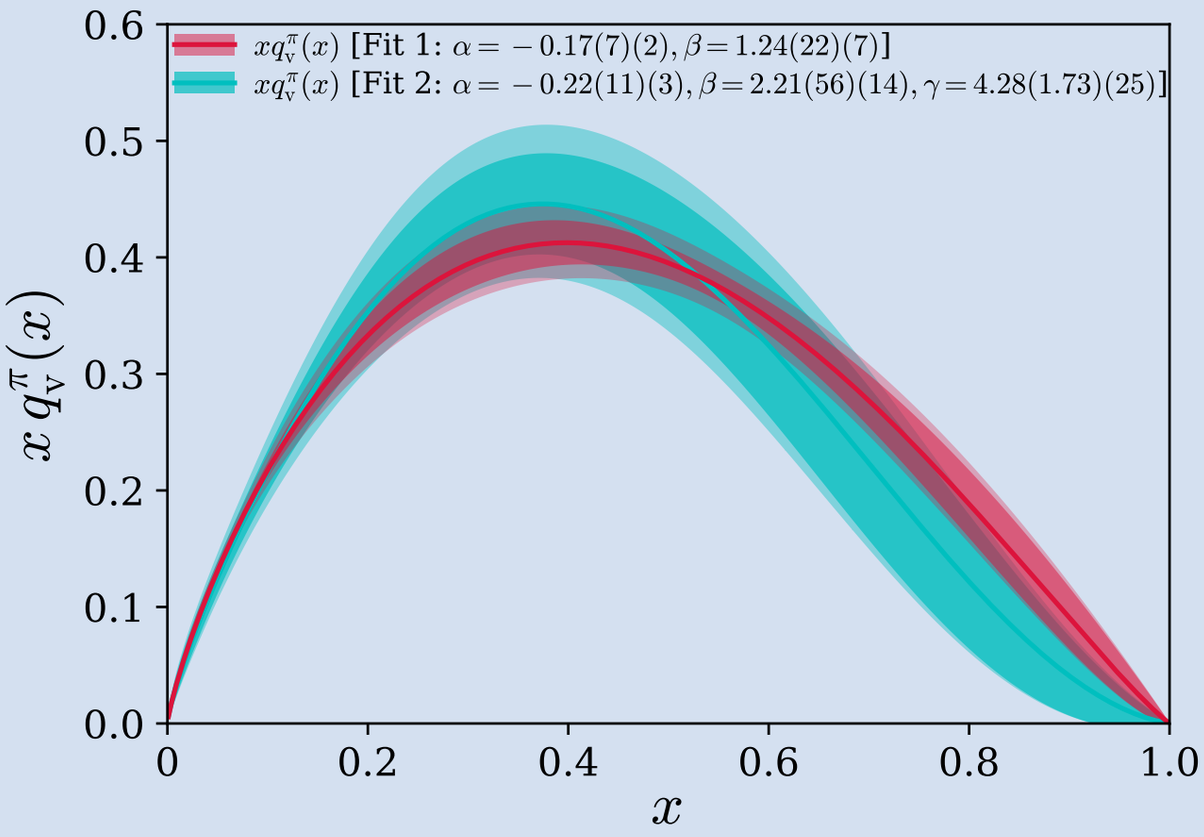
HT effect? If yes…too big?
Convergence of 1-loop matching?
Mixture of both?

- If "Hybrid renormalization" is unavoidable what about the reliability/effect on previous quasi-PDF/LaMET results?

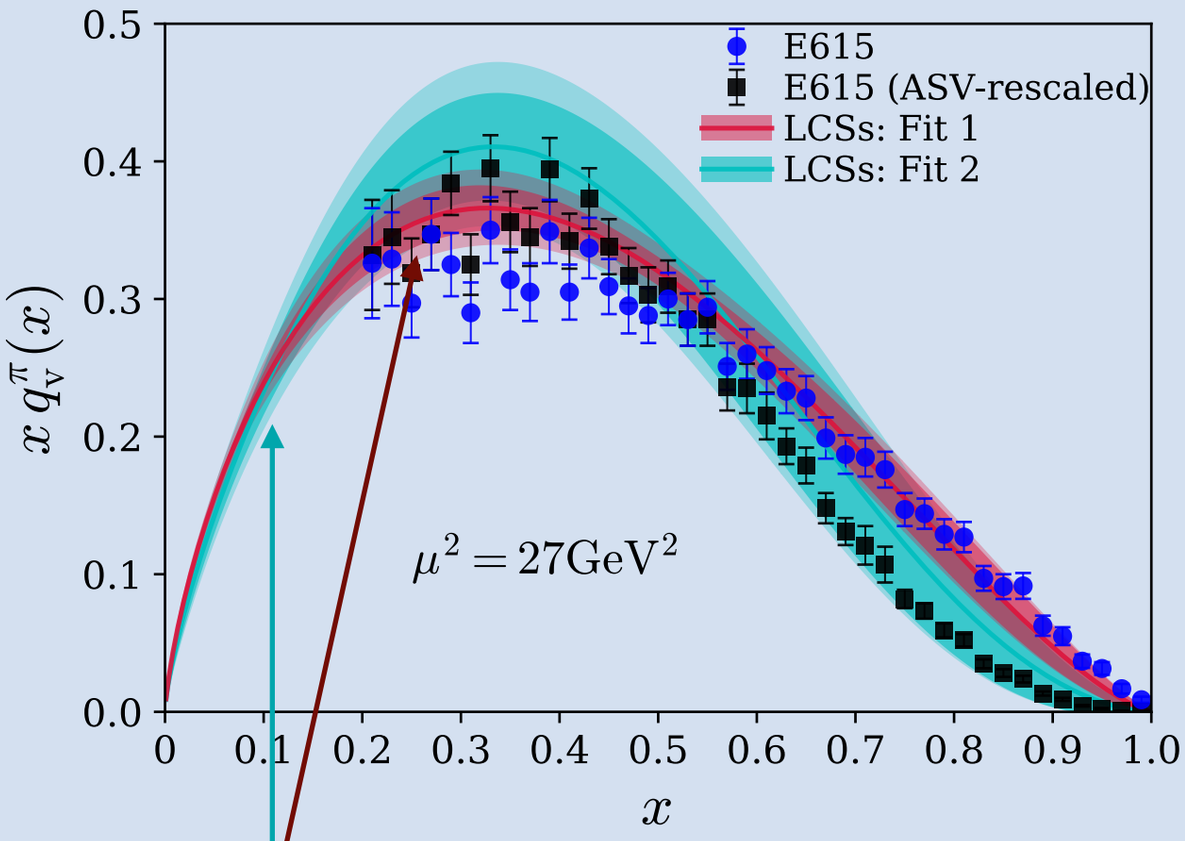
Finally … Pion PDF from LCS

- $$\sigma_{VA} = K \otimes q_{\rm v}$$
- Extraction of PDF using the pQCD expression and functional form of PDFs

$$q_{\rm v}^{\pi}(x) = \frac{x^{\alpha}(1-x)^{\beta}(1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$

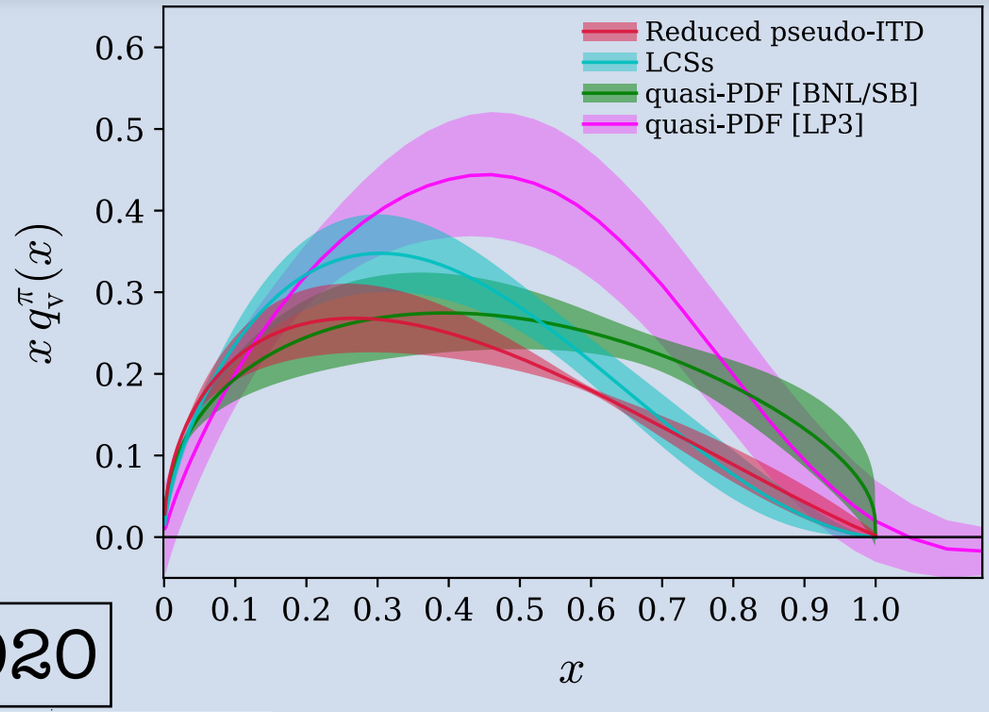


RSS, Egerer, Karpie, Edwards,Joo, Orginos, Qiu, Richards (2020)

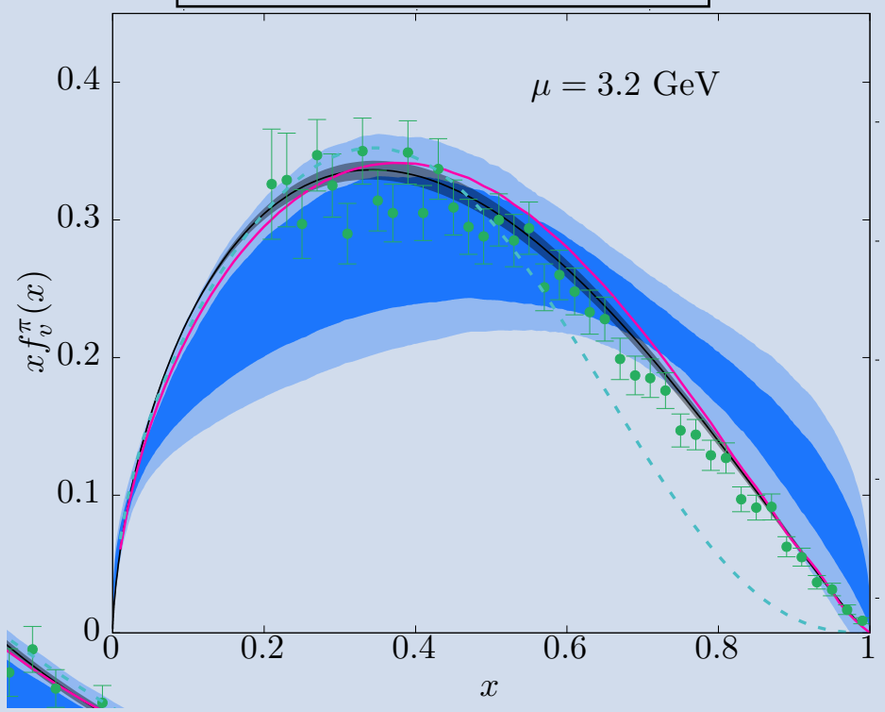


In agreement within errors in the entire x-region

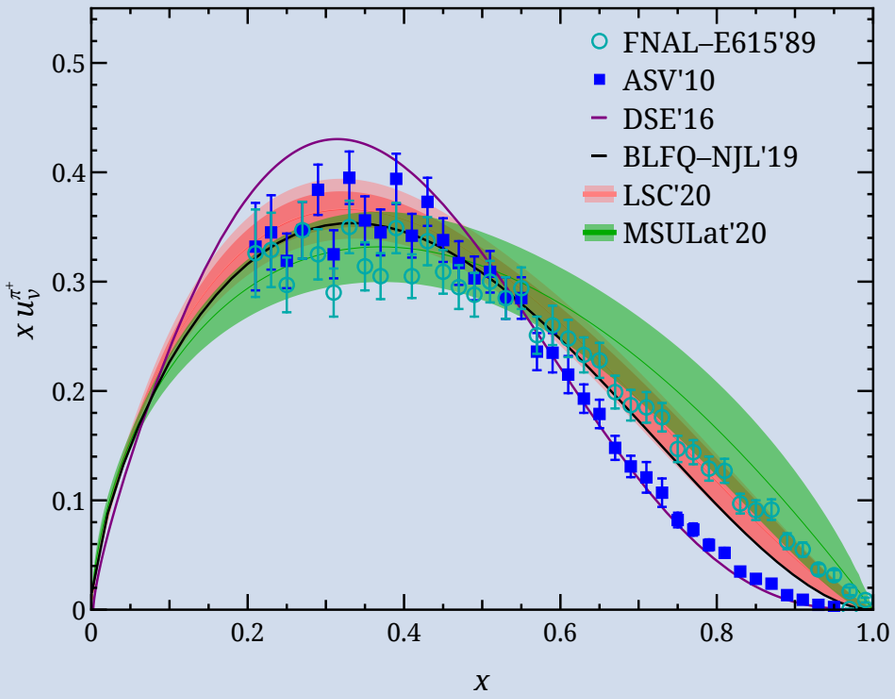
Other calculations



Gao, et al 2020



Lin, et al 2020



Beautiful collaborative work between LQCD community using different methods/formalisms !

Summary & Outlook

- LCS is a powerful tool for understanding partonic structure
  [Complementary to Hadronic tensor (Liu 1994), Position-space correlators (Braun & Mueller 2008), Quasi-PDF (Ji 2013), Pseudo-PDF (Radyushkin 2017) and experiments/global fits]

- Very stable and convergent 1-loop coordinate space matching

- Precise data around $\omega \sim 8 - 10$ (smaller lattice spacings) required to pin down $(1-x)^{\beta}$ behavior for pion

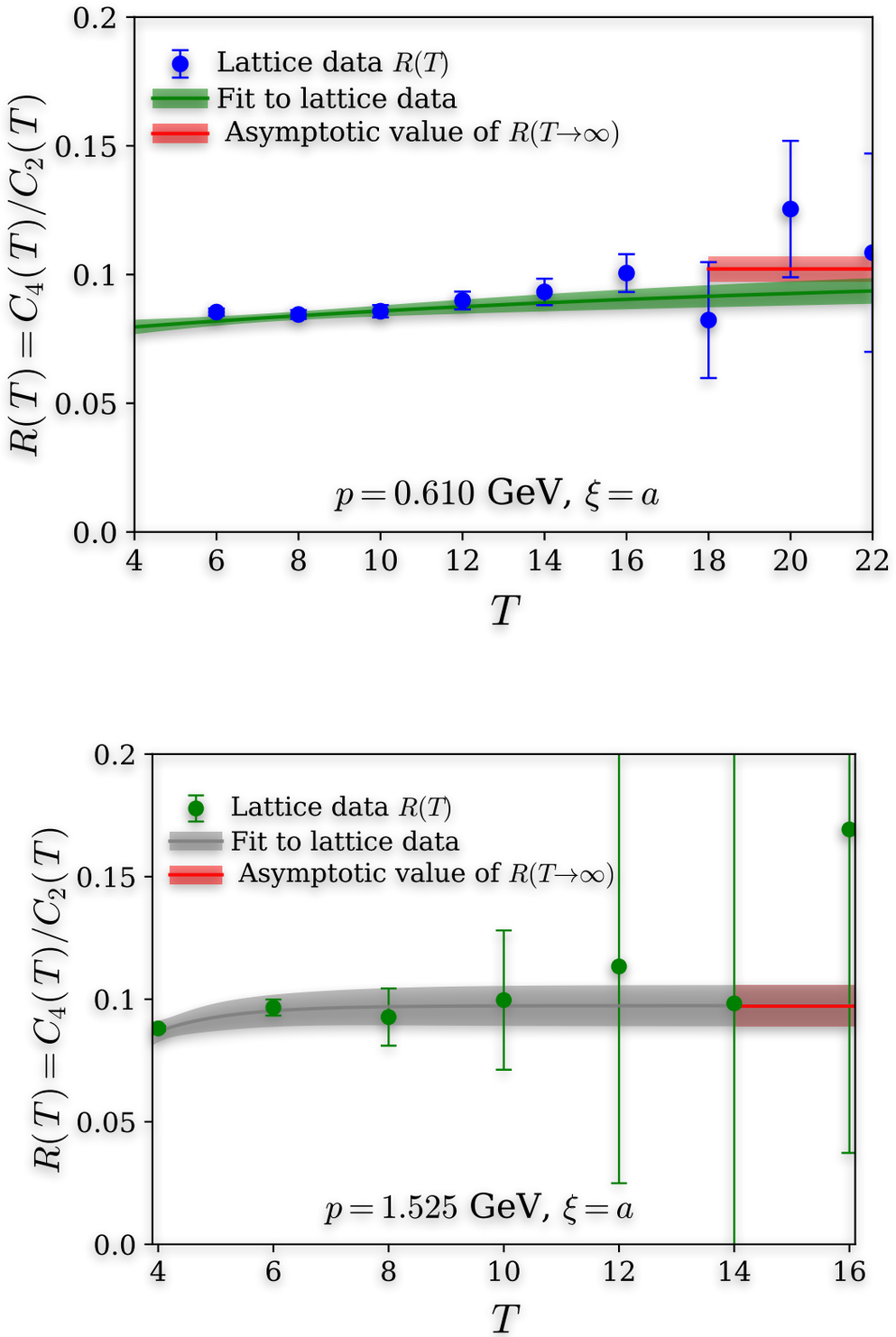
- Precise data, goodness of fits, different fitting methods will allow estimate systematics of the inverse problem (like global fitting estimates systematics in their analyses)

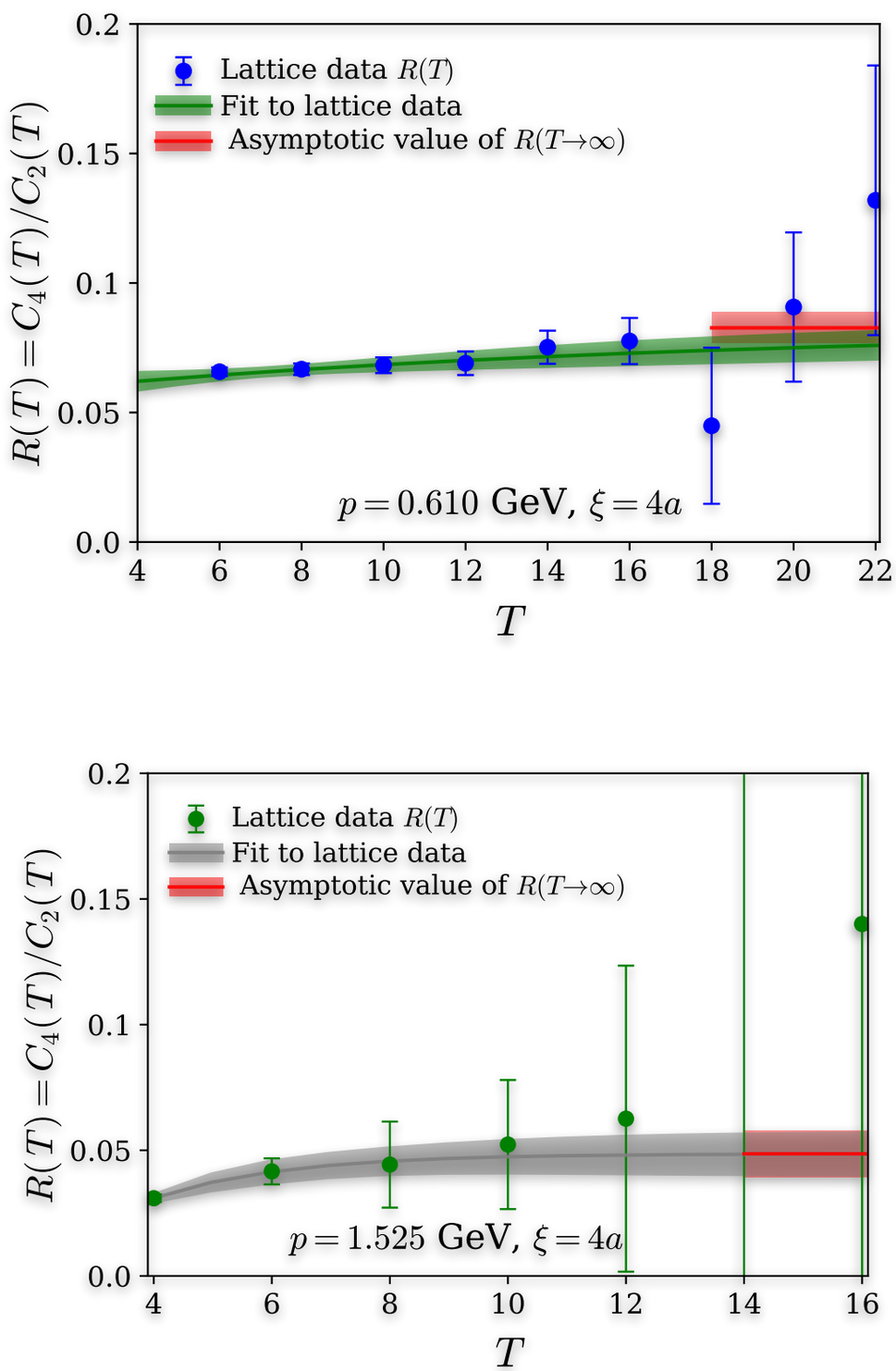
- Further theoretical development to achieve precision and convenient numerical implementation of LCS (Ma, Orginos, Qiu, & Richards)

Thank you!

Lattice Calculation

$$T_{\max} \sim 2.5 \text{ fm}$$





Quasi- vs Pseudo- vs LCS

- All integrals of Ioffe-Time Distribution Function
- Should yield same PDF after matching and systematic controls

Quasi-PDF

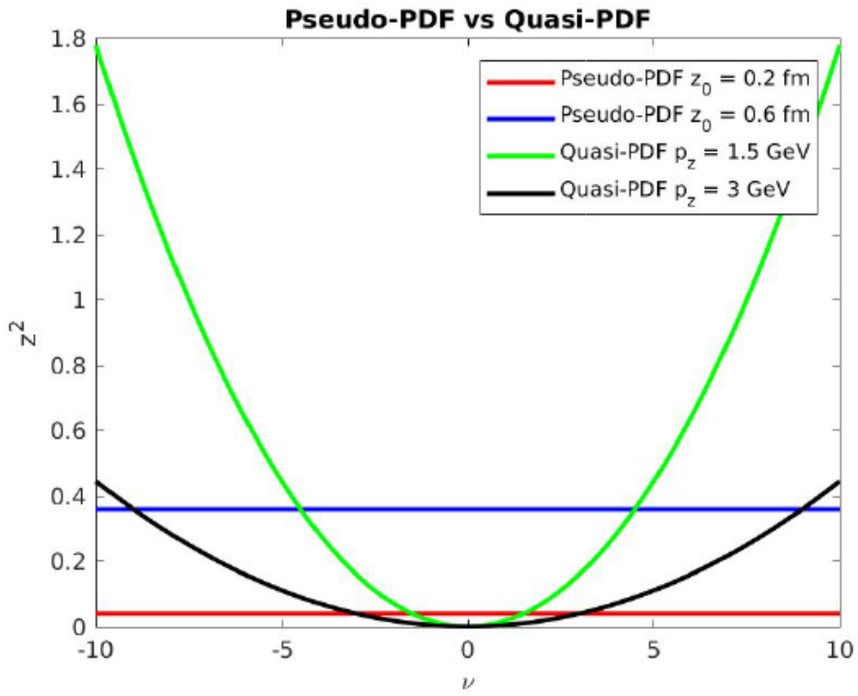
$$\mathcal{Q}(x, p_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x}\mathcal{M}(\nu, -\nu^2/p_3^2)$$

$$\mathcal{P}(x, -z_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x}\mathcal{M}(\nu, -z_3^2)$$

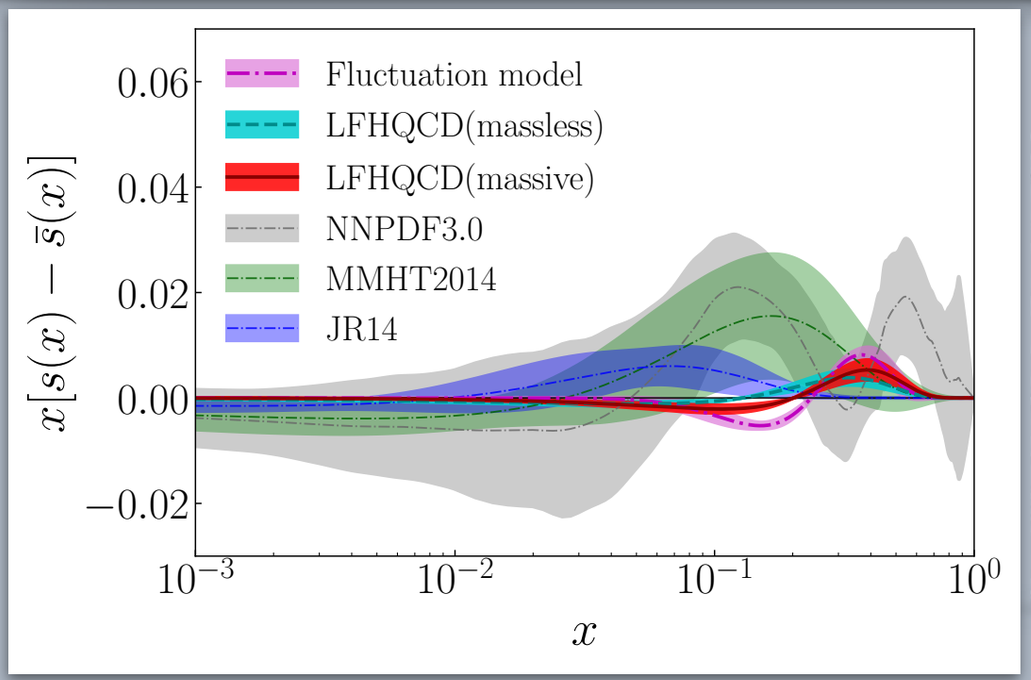
Pseudo-PDF and LCS

For pPDF + LCS, z sets short-distance scale.

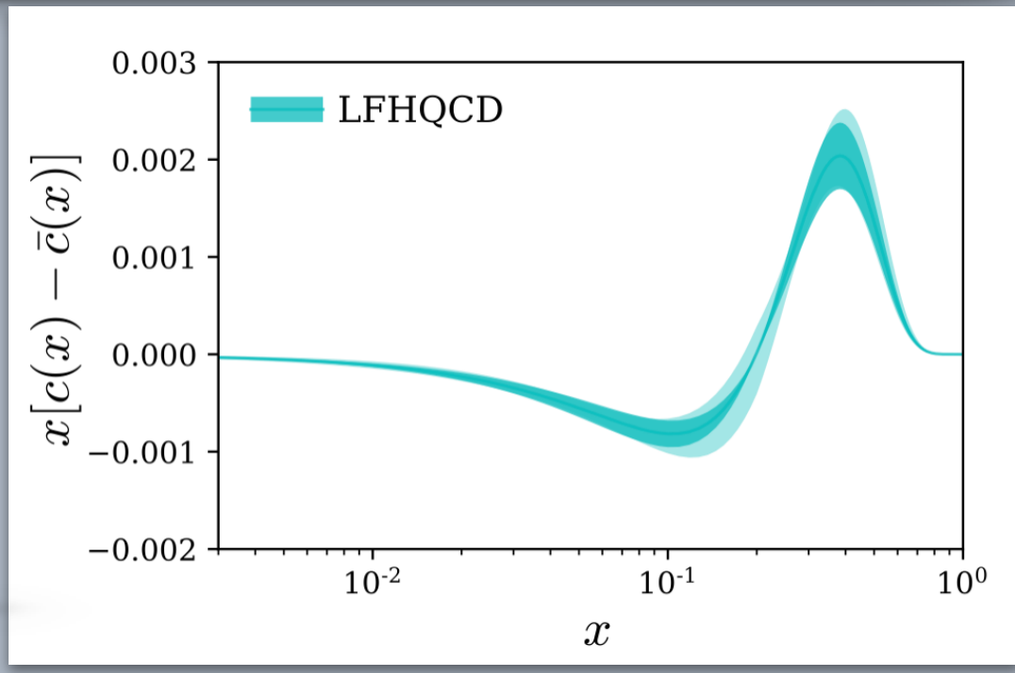
$$z << \frac{1}{\Lambda_{\mathrm{QCD}}}$$



Phenomenology (Light Front Holographic QCD)

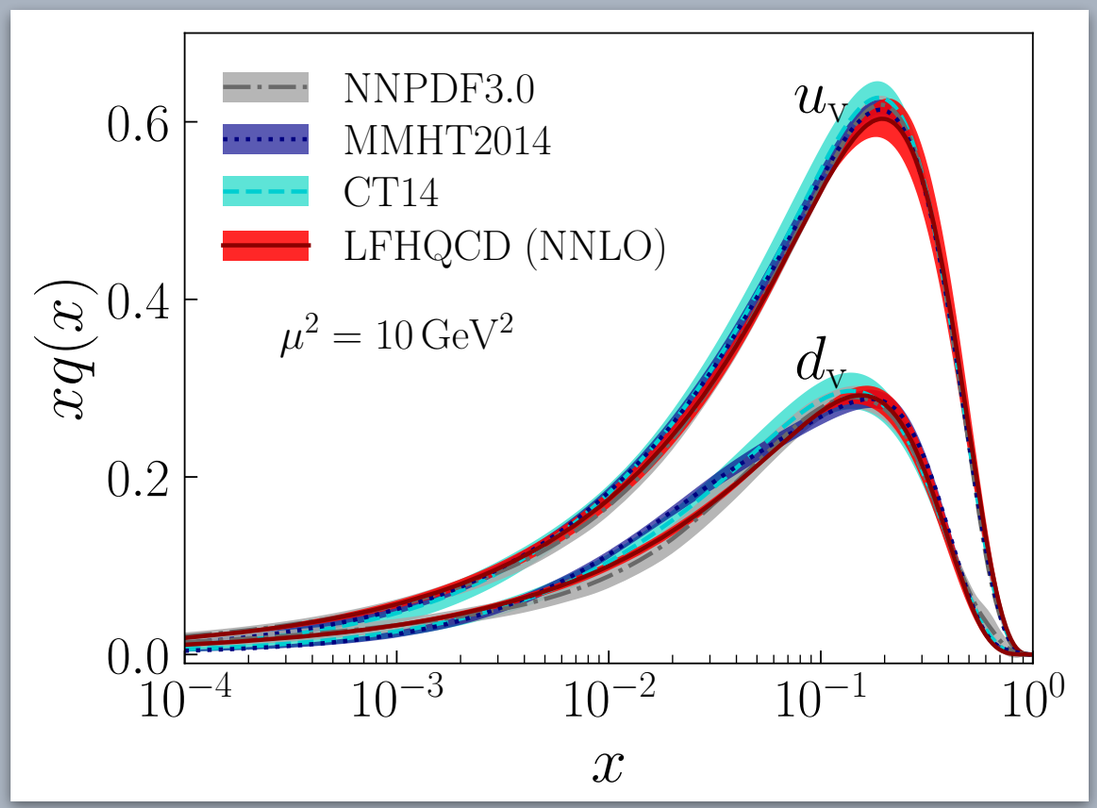


Constrained by LQCD + solved a problem of adding mass in LFHQCD

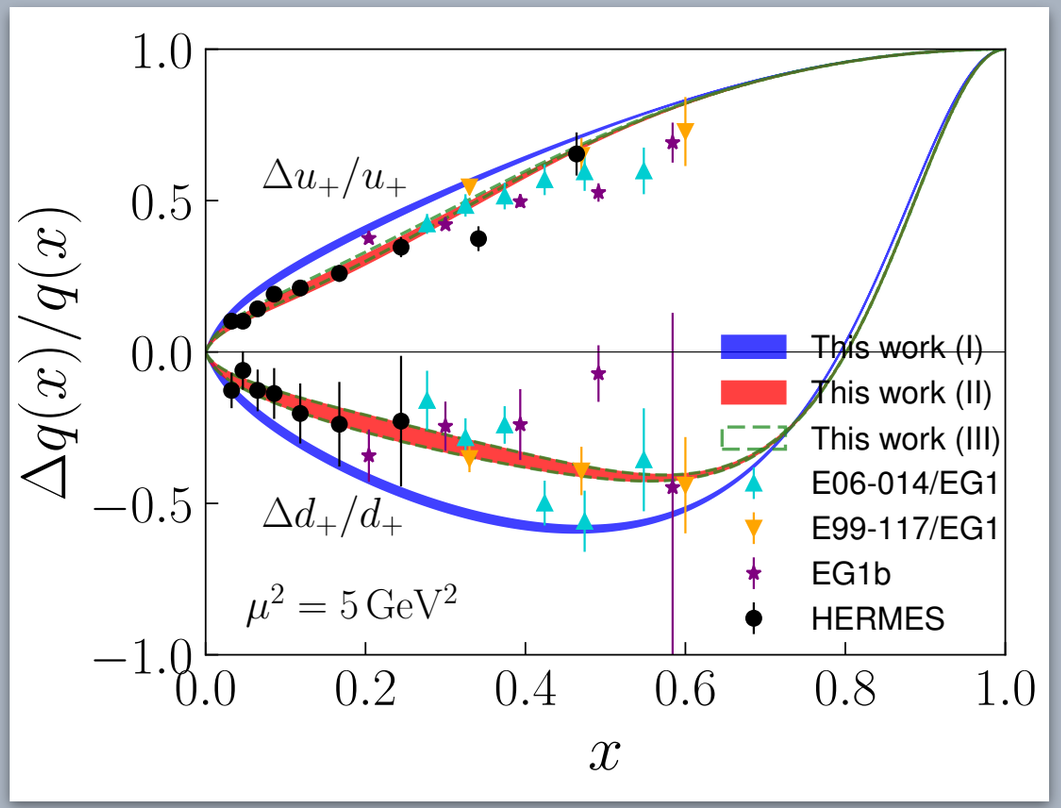


RSS, Liu, et. al. PRD 2019

RSS, Liu, et. al PLB 2020



de Teramond, Liu, RSS, PRL 2018



Liu, RSS, et. al. PRL 2020