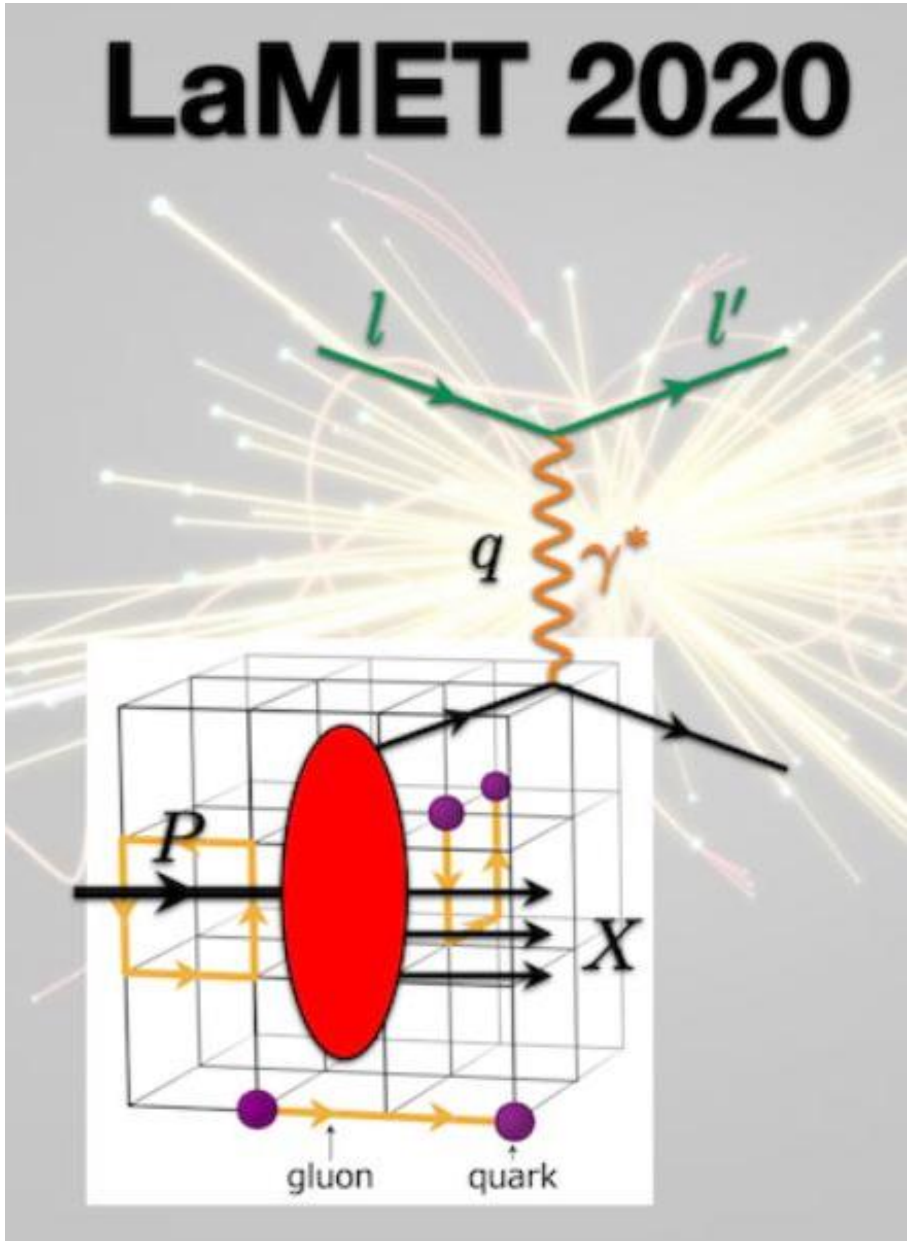


Matching for the twist-3 PDFs $g_T(x), \quad e(x)$ **&** $h_L(x)$**:** Success or failure?

Shohini Bhattacharya

11 September 2020

In Collaboration with
Krzysztof Cichy (Adam Mickiewicz U.)
Martha Constantinou (Temple U.)
Andreas Metz (Temple U.)
Aurora Scapellato (Adam Mickiewicz U.)
Fernanda Steffens (Bonn U.)

Outline

- **Why twist-3 PDFs?**
- **Quasi-PDF approach**
- **Matching: warming up!**
- **Perturbative corrections for** $g_T(x)\,, \quad g_{T,\mathrm{Q}}(x)$
- **Perturbative corrections for** $e(x)\,, e_{\mathrm{Q}}(x) \quad h_L(x)\,, h_{L,\mathrm{Q}}(x)$

Based on:

- **S.B., Cichy, Constantinou, Metz, Scapellato, Steffens:** arXiv:2006.12347
- **S.B., Cichy, Constantinou, Metz, Scapellato, Steffens:** Phys. Rev. D **102**, 034005 (2020), arXiv:2005.10939

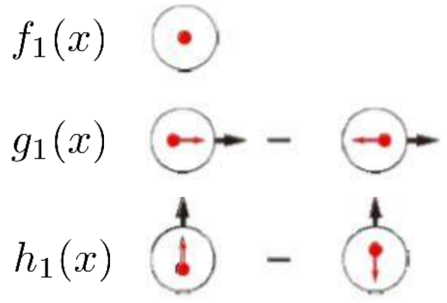
Why twist-3 PDFs?

Twist-2

Order of contribution: $\mathcal{O}(1)$

PDFs	Dirac structure
$f_1(x)$	$\Gamma = \gamma^+$
$g_1(x)$	$\Gamma = \gamma^+\gamma_5$
$h_1(x)$	$\Gamma = i\sigma^{i+}\gamma_5$

Density interpretation:



Twist-3

Order of contribution: $\mathcal{O}(1/\mathrm{Q})$

Jaffe, Ji (PRL 67, 552)/ Jaffe, Ji (Nucl. Phys. B 375, 527)

PDFs	Dirac structure
$e(x)$	$\Gamma = 1$
$g_T(x)$	$\Gamma = \gamma_\perp^i\gamma_5$
$h_L(x)$	$\Gamma = i\sigma^{+-}\gamma_5$

No density interpretation:

Twist-3

$\downarrow$

qgq correlation

Burkardt (arXiv: 0810.3589)

$\int dx\, x^2 g_T(x) \;\rightarrow \perp$ **force**

$\int dx\, x^2 e(x) \;\rightarrow \perp$ **force**

Quasi-PDF approach

Light-cone (standard) correlator $-1 \leq x \leq 1$

$$F^{[\Gamma]}(x) \;=\; \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **<u>Cannot</u> be computed on Euclidean lattice**

Correlator for quasi-PDFs (Ji, 2013) $-\infty \leq x \leq \infty$

$$F_{\mathrm{Q}}^{[\Gamma]}(x;P^3) \;=\; \frac{1}{2}\int \frac{dz^3}{2\pi}\, e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **<u>Can</u> be computed on Euclidean lattice**

- **Quasi-PDF approach made it possible to directly extract light-cone PDFs from lattice QCD**
- **Quasi-PDFs & light-cone PDFs have different UV behavior: difference dealt via perturbative matching within LAMET (Ji, 2014)**

Warming up!

1) Matching formula studied for twist-2:

$$q_{\mathrm{Q}}(x;P^3)=\int_{-1}^{+1}\frac{d\,y}{|y|}C\left(\frac{x}{y}\right)q(y)+\mathcal{O}\left(\frac{M^2}{(P^3)^2}\right)$$

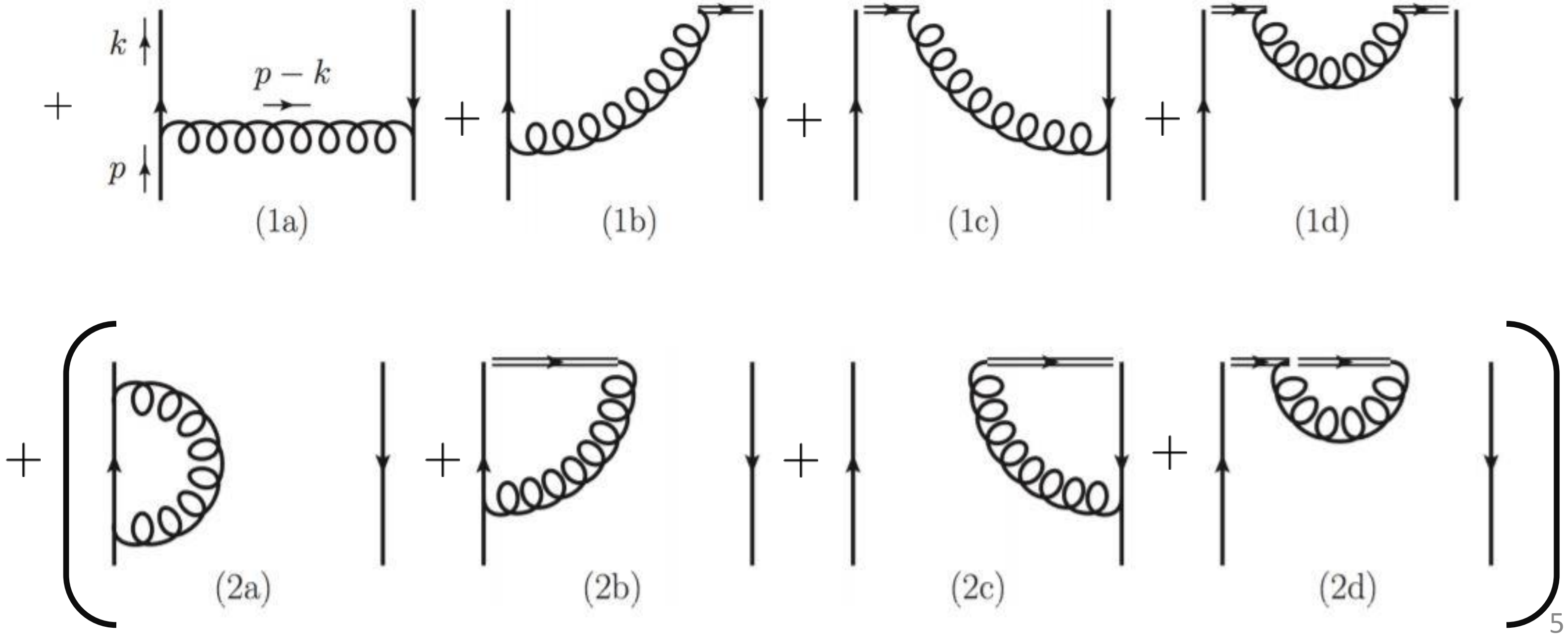
(Scale dependence omitted)

(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/
Izubuchi, Ji, Jin, Stewart, Zhao, 2018/
Ma, Qiu (2018)/ Chen, Wang, Zhu (2020)/
Li, Ma, Qiu (2020))

2) Perturbative corrections to 1-loop:

1-loop corrections $= \delta(1-x) +$

(Feynman Gauge)



Warming up!

3) Matching kernel:

$$\boxed{C(x)=\delta(1-x)+\underbrace{\frac{\alpha_sC_F}{2\pi}\left[\widetilde{\Gamma}(x)-\Gamma(x)\right]}_{\text{Real-corrections}}+\underbrace{\frac{\alpha_sC_F}{2\pi}\delta(1-x)\left[\widetilde{\Pi}-\Pi\right]}_{\text{Virtual-corrections}}}$$

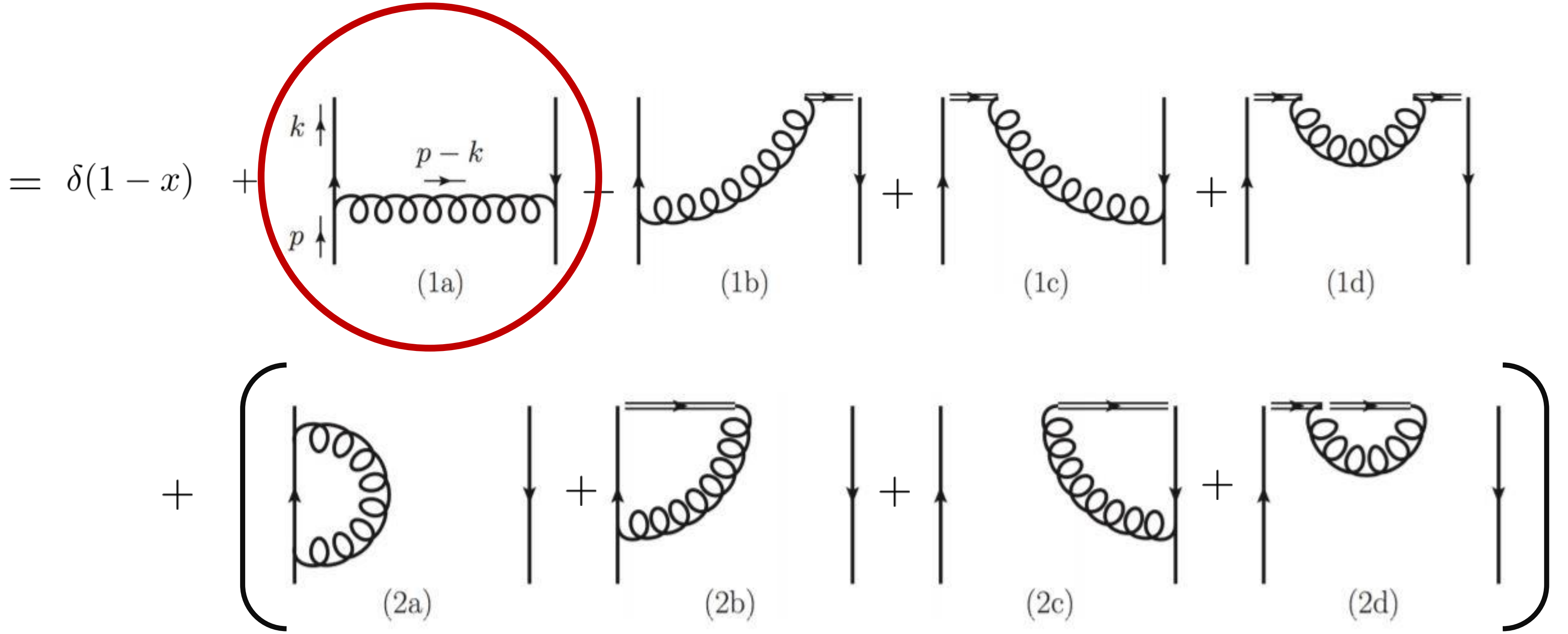
- **Essence of such a factorization formula is the IR finiteness of the kernel**
- **Set up for calculation:**

i. **Feynman Gauge** **ii.** **UV :** $\int^{\infty} d^2k_\perp \longrightarrow \epsilon_{\text{UV}}$ **iii.** **IR :** $\int_0 d^2k_\perp \longrightarrow \begin{cases} m_q \neq 0 \\ \epsilon_{\text{IR}} \\ m_g \neq 0 \end{cases}$

Focus of this talk will be diagram (1a)/ ladder diagram

1-loop corrections

(Feynman Gauge)



Ladder diagram: origin of new features at twist-3

Case 1: g_T & $g_{T,\mathbf{Q}}$

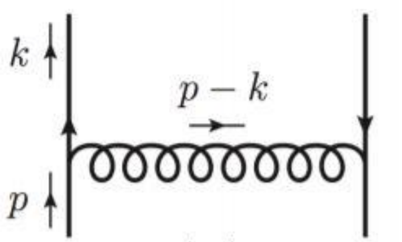
Light-cone $g_T(x)$

Definition of $g_T(x)$:

$$\frac{M}{P^+} S_\perp^i \, g_T(x) = \Phi^{[\gamma^i \gamma_5]}$$

$(M,\, S_\perp^i,\, P^+)$: hadron attributes

Quark Target Model (QTM)



(1a)

$$\boxed{\frac{m_q}{p^+} \, s_\perp^i \, g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu\,(\not{k} + m_q)\,\gamma^i \gamma_5\,(\not{k} + m_q)\,\gamma^\mu\right]}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}\,\delta\!\left(x - \frac{k^+}{p^+}\right)\frac{1}{p^+}}$$

- **One cannot set m_q to zero at the start in QTM calculations**
- **Extract linear terms in m_q & then set $m_q = 0$ unless it is used as the IR regulator**

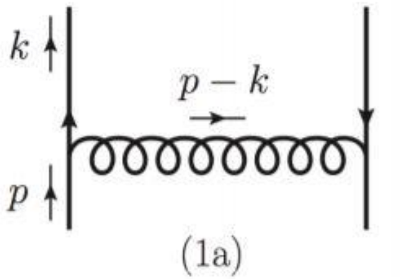
Light-cone $g_T(x)$

Definition of $g_T(x)$:

$$\frac{M}{P^+} S_\perp^i \, g_T(x) = \Phi^{[\gamma^i \gamma_5]}$$

$(M,\, S_\perp^i,\, P^+)$: hadron attributes

Quark Target Model (QTM)



(1a)

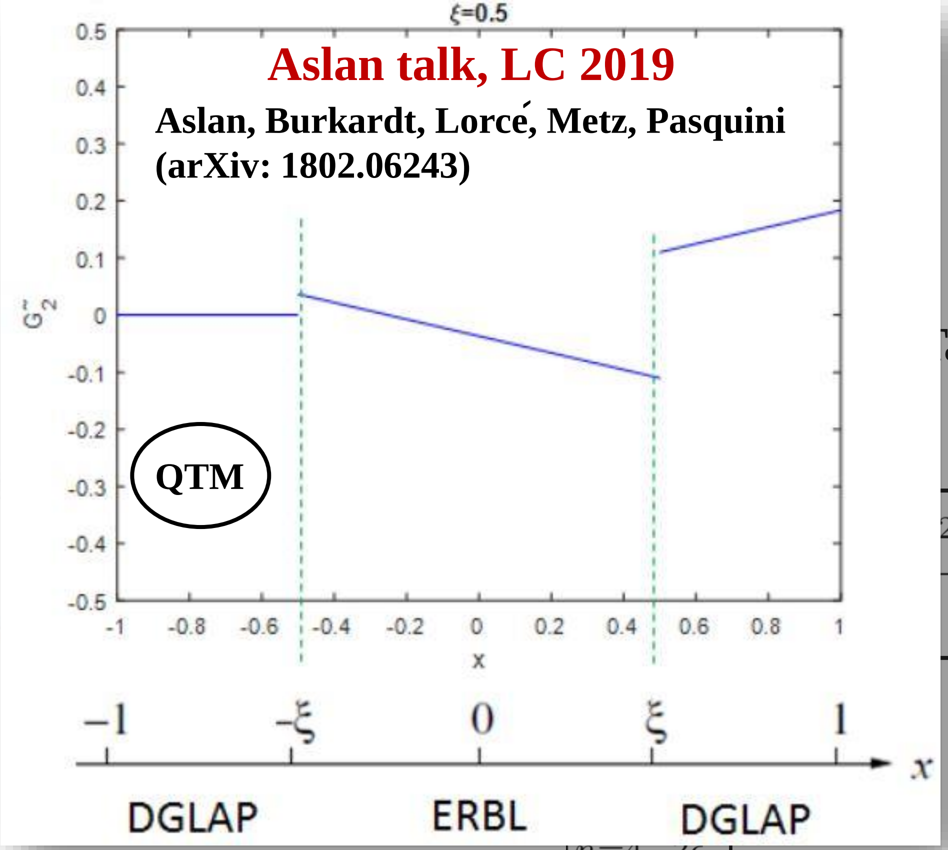
$$\boxed{\frac{m_q}{p^+} s_\perp^i \, g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu \left(\not{k} + m_q\right) \gamma^i \gamma_5 \left(\not{k} + m_q\right) \gamma^\mu\right]}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)} \, \delta\!\left(x - \frac{k^+}{p^+}\right) \frac{1}{p^+}}$$

Trace algebra $\Big|_{n=4-2\epsilon}$

Power of k^- : not present at twist-2

$$g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon}}{(2\pi)^n} p^+ \int_{-\infty}^{\infty} d^{n-2} k_\perp dk^- dk^+ \frac{\overbrace{2p^+k^-}+ \ldots}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)} \, \delta\!\left(x - \frac{k^+}{p^+}\right) \frac{1}{p^+}$$

Light-cone $g_T(x)$



Aslan talk, LC 2019

Aslan, Burkardt, Lorcé, Metz, Pasquini (arXiv: 1802.06243)

QTM

$(M,\, S^i_\perp,\, P^+)$: hadron attributes

arget Model (QTM)

k

p

$$\boxed{{}^{2\epsilon} g_{\mu\nu} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k} + m_q)\,\gamma^i\gamma_5\,(\not{k} + m_q)\,\gamma^\mu\right]}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}\,\delta\!\left(x - \frac{k^+}{p^+}\right)\frac{1}{p^+}}$$

$\big|_{n=4-2\epsilon}$

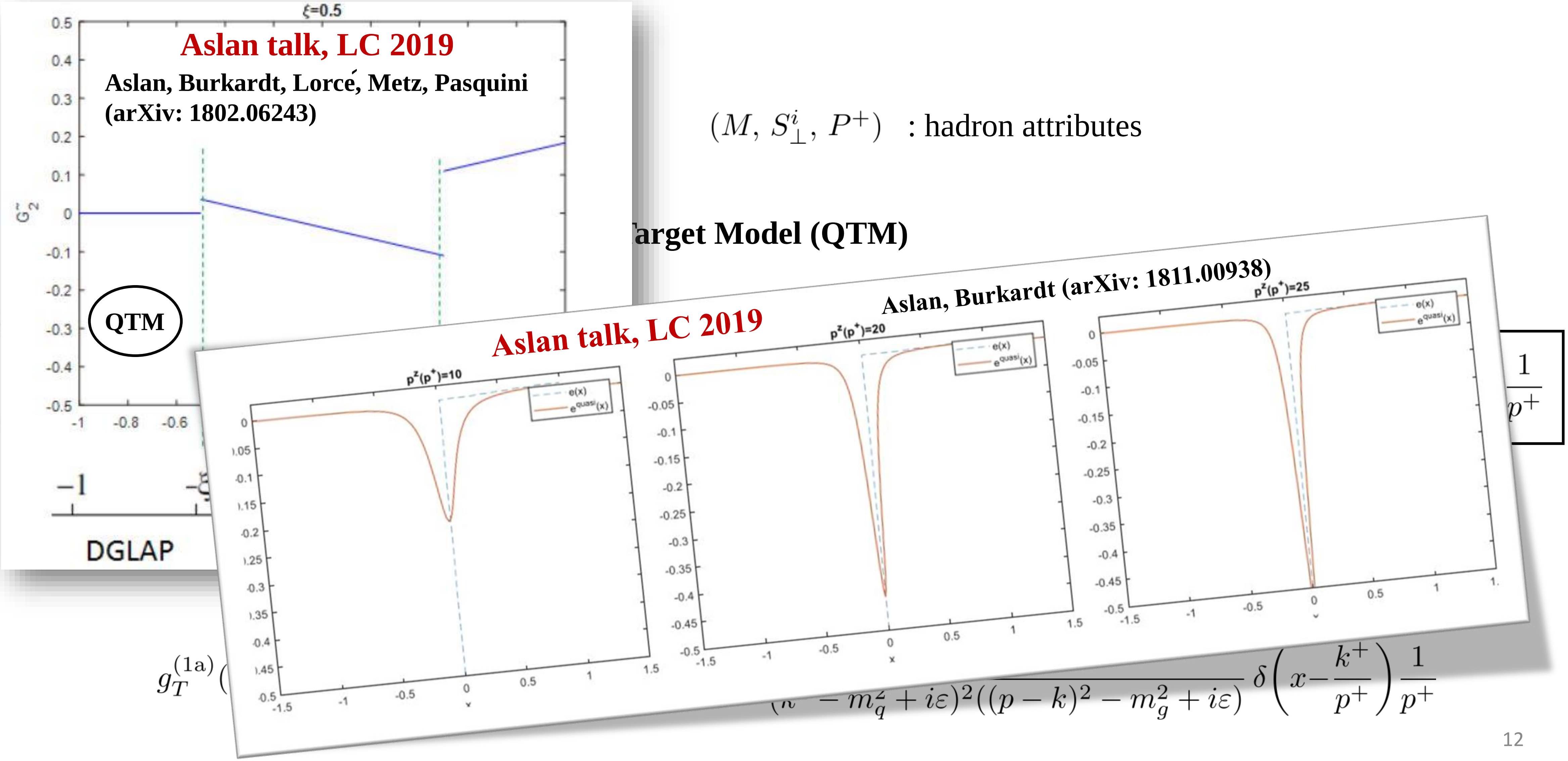
Power of k^- : not present at twist-2

$$g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon}}{(2\pi)^n} p^+ \int_{-\infty}^{\infty} d^{n-2}k_\perp dk^- dk^+ \frac{\overbrace{2p^+k^-} + \ldots}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}\,\delta\!\left(x - \frac{k^+}{p^+}\right)\frac{1}{p^+}$$

Light-cone $g_T(x)$

$(M,\, S_\perp^i,\, P^+)$: hadron attributes

arget Model (QTM)



Aslan talk, LC 2019

Aslan, Burkardt, Lorcé, Metz, Pasquini (arXiv: 1802.06243)

Aslan talk, LC 2019

Aslan, Burkardt (arXiv: 1811.00938)

k

p

-1

DGLAP

$\frac{1}{p^+}$

$g_T^{(1a)}($ … $-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{k^+}{p^+}\right)\frac{1}{p^+}$

Close look …

$$\boxed{\textbf{Term:}\quad \frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}}$$

i. **Cancellation of gluon propagator:**

$$\frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)} \xrightarrow{k^- = -\frac{(p-k)^2-m_g^2}{2(1-x)p^+} - \frac{(k_\perp^2+m_g^2)}{2(1-x)p^+} + \frac{m_q^2}{2p^+}} \overbrace{\frac{1}{(k^2-m_q^2+i\varepsilon)^2}}^{\text{Singular term}} + \ldots$$

Close look …

Term: $\dfrac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$

i. Cancellation of gluon propagator:

$$\frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)} \xrightarrow{k^- = -\frac{(p-k)^2-m_g^2}{2(1-x)p^+} - \frac{(k_\perp^2+m_g^2)}{2(1-x)p^+} + \frac{m_q^2}{2p^+}} \overbrace{\frac{1}{(k^2-m_q^2+i\varepsilon)^2}}^{\text{Singular term}} + \ldots$$

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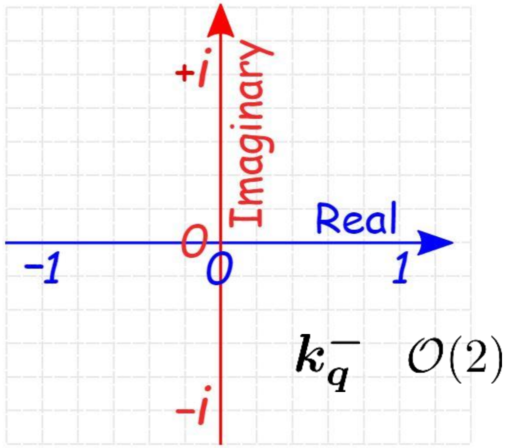
ii. Result after $\int dk^-$:

Yan (Phys. Rev. D 7, 1780)/ Burkardt (arXiv: 9505226)/ Aslan, Burkardt (arXiv: 1811.00938) …



Close look …

$$\boxed{\textbf{Term:}\quad \frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}}$$

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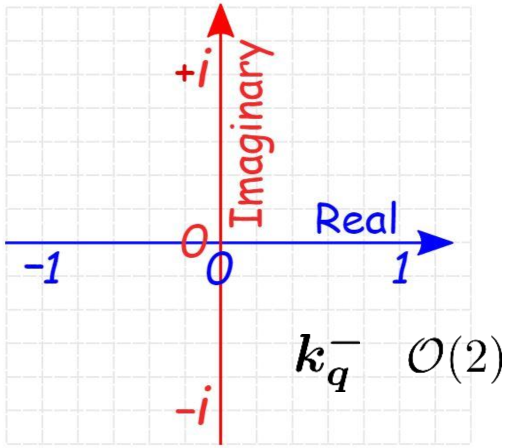

$$\int_{-\infty}^{\infty} \frac{dk^-}{(k^2-m_q^2+i\varepsilon)^2}$$

Close look …

Term: $$\frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$$

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---

ii. Result after $\int dk^-$:

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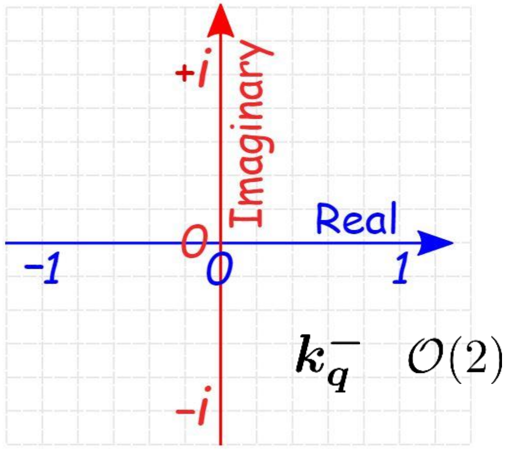

$$\int_{-\infty}^{\infty}\frac{dk^-}{(k^2-m_q^2+i\varepsilon)^2} = \begin{cases} \boldsymbol{k^+ \neq 0:} \quad \int_{-\infty}^{\infty}\frac{dk^-}{(2k^+k^- - k_\perp^2 - m_q^2+i\varepsilon)^2} = 0 \\ \\ \end{cases}$$

Close look …

Term: $$\frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$$

i. Cancellation of gluon propagator:

$$\frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)} \xrightarrow{k^- = -\frac{(p-k)^2-m_g^2}{2(1-x)p^+} - \frac{(k_\perp^2+m_g^2)}{2(1-x)p^+} + \frac{m_q^2}{2p^+}} \overbrace{\frac{1}{(k^2-m_q^2+i\varepsilon)^2}}^{\text{Singular term}} + \dots$$

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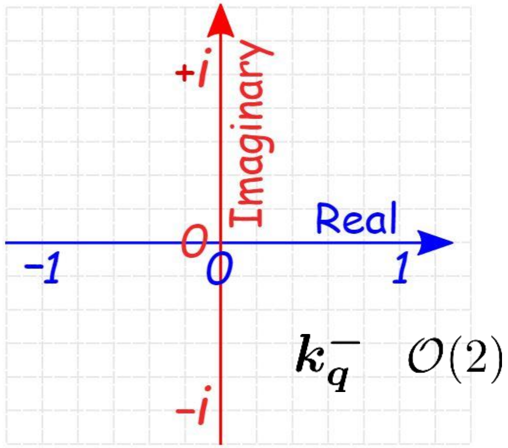
ii. Result after $\int dk^-$:

Yan (Phys. Rev. D 7, 1780)/ Burkardt (arXiv: 9505226)/ Aslan, Burkardt (arXiv: 1811.00938) …



$$\int_{-\infty}^{\infty} \frac{dk^-}{(k^2-m_q^2+i\varepsilon)^2} = \begin{cases} \boldsymbol{k^+ \neq 0}: & \int_{-\infty}^{\infty} \frac{dk^-}{(2k^+k^- - k_\perp^2 - m_q^2 + i\varepsilon)^2} = 0 \\ \boldsymbol{k^+ = 0}: & \int_{-\infty}^{\infty} \frac{dk^-}{(k_\perp^2 + m_q^2 - i\varepsilon)^2} = \text{linear divergence} \end{cases}$$

Close look …

$$\boxed{\textbf{Term:}\quad \frac{2p^+k^-}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}}$$

i. Cancellation of gluon propagator:

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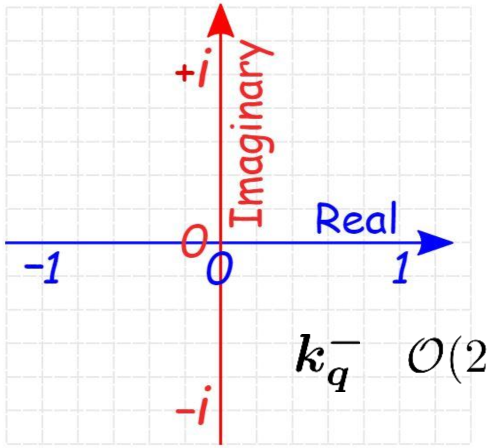
ii. Result after $\int dk^-$: **Yan (Phys. Rev. D 7, 1780)/ Burkardt (arXiv: 9505226)/ Aslan, Burkardt (arXiv: 1811.00938) …**



k_q^- $\mathcal{O}(2)$

$$\int_{-\infty}^{\infty}\frac{dk^-}{(k^2-m_q^2+i\varepsilon)^2} = \begin{cases} \boldsymbol{k^+\neq 0}: & \int_{-\infty}^{\infty}\frac{dk^-}{(2k^+k^- - k_\perp^2 - m_q^2+i\varepsilon)^2} = 0 \\ \boldsymbol{k^+ = 0}: & \int_{-\infty}^{\infty}\frac{dk^-}{(k_\perp^2+m_q^2-i\varepsilon)^2} = \text{linear divergence} \end{cases}$$

$$\boxed{\therefore \quad \int_{-\infty}^{\infty}\frac{dk^-}{(k^2-m_q^2+i\varepsilon)^2} = \frac{i\pi}{k_\perp^2+m_q^2}\delta(k^+)}$$

Zero modes

Close look …

Singular term: $\dfrac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. Result after $\int d^{\,n-2}k_\perp$: $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \displaystyle\int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}$

Close look …

Singular term: $\dfrac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. **Result after** $\int d^{\,n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon}\displaystyle\int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}}\,\frac{1}{(k_\perp^2 + m_q^2)}$

i. $m_q \neq 0$

Close look …

Singular term: $\frac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. **Result after $\int d^{n-2}k_\perp$:** $g^{(1a)}_{T(\mathrm{s})}(x) = -\alpha_s C_F \delta(x) \underbrace{(4-n)}_{\propto\, \epsilon_{\mathrm{UV}}} \mu^{2\epsilon} \int \underbrace{\frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}}_{\propto\, \frac{1}{\epsilon_{\mathrm{UV}}}}$

i. $m_q \neq 0$

Close look …

Singular term: $\frac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. Result after $\int d^{n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{m_q \neq 0} = -\frac{\alpha_s C_F}{2\pi}\delta(x)$

Close look …

Singular term: $\dfrac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. Result after $\int d^{\,n-2}k_\perp$ **:** $$g^{(1a)}_{T(\mathrm{s})}(x) = -\alpha_s C_F \delta(x)\, \underbrace{(4-n)\, \mu^{2\epsilon} \int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + \cancel{m_q^2})}}$$

$$\delta(x)\,\epsilon_{\mathrm{UV}} \frac{1}{\epsilon_{\mathrm{UV}}} - \delta(x)\,\epsilon_{\mathrm{IR}} \frac{1}{\epsilon_{\mathrm{IR}}}$$

i. $m_q \neq 0$	$g^{(1a)}_{T(\mathrm{s})}\big\vert_{m_q \neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	

Close look …

Singular term: $\dfrac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. Result after $\int d^{n-2}k_\perp$: $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \displaystyle\int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + \cancel{m_q^2})}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{m_q \neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{\epsilon_{\mathrm{IR}}} = 0$

- **IR dependence of zero modes**

Close look …

Singular term: $\dfrac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. **Result after $\int d^{n-2}k_\perp$:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{m_q\neq 0} = -\frac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{\epsilon_{\mathrm{IR}}} = 0$
iii. $m_g \neq 0$	

- **IR dependence of zero modes**

Close look …

Singular term: $\dfrac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

iii. Result after $\int d^{n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \dfrac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \dfrac{1}{(k_\perp^2 + m_q^2)}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{m_q \neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{\epsilon_{\mathrm{IR}}} = 0$
iii. $m_g \neq 0$	

- **IR dependence of zero modes**
- **Working with** $m_g \neq 0$ **is an issue at twist-3:** **IR divergence unattended for the singular term! First time at twist-3!**

Close look …

Singular term: $\dfrac{1}{(k^2 - m_q^2 + i\varepsilon)^2}$

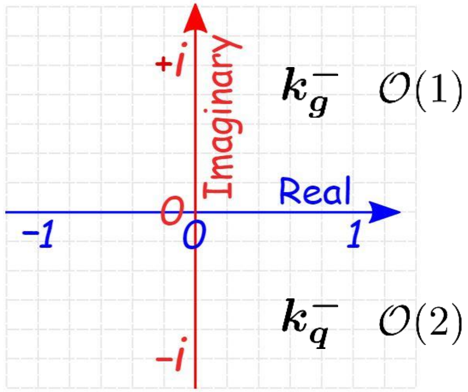
iii. Result after $\int d^{n-2}k_\perp$: $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \dfrac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \dfrac{1}{(k_\perp^2 + m_q^2)}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\|_{m_q \neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	$g_{T(\mathrm{s})}^{(1a)}\big\|_{\epsilon_{\mathrm{IR}}} = 0$
iii. $m_g \neq 0$	$g_{T(\mathrm{s})}^{(1a)}(x) = \begin{cases} g_{T(\mathrm{s})}^{(1a)}(x)\big\|_{m_q \neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x) \\ g_{T(\mathrm{s})}^{(1a)}(x)\big\|_{\epsilon_{\mathrm{IR}}} = 0 \end{cases}$

- **IR dependence of zero modes**
- **Working with $m_g \neq 0$ is an issue at twist-3: IR divergence unattended for the singular term! First time at twist-3!**
- **Consider two practical options:**

1. **Retain m_q in $g_{T(\mathrm{s})}$**
2. **Do DR for $\int_0 d^{n-2}k_\perp$ in $g_{T(\mathrm{s})}$**
3. **Work with $m_g \neq 0$ for $g_{T(\mathrm{c})}$**

Results for canonical part:



- **Starting expression**

$$g_{T(\mathrm{c})}(x) \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^-}{2\pi} \frac{2k^2 + 2k_\perp^2 + 2m_q^2 - (4-n)m_g^2}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}$$

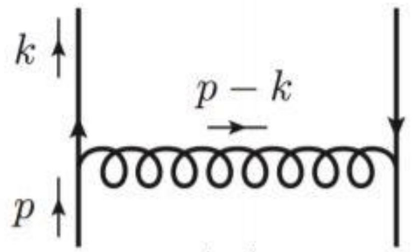
- k^- **poles on both sides of real axis: usual machinery for twist-2**

i. $m_q \neq 0$	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\left(x\,\mathcal{P}_{\mathrm{UV}} + x\ln\frac{\mu_{\mathrm{UV}}^2}{(1-x)^2 m_q^2} + \frac{x^2-2x-1}{1-x}\right)$
ii. $m_g \neq 0$	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\vert_{m_g} = \frac{\alpha_s C_F}{2\pi}\left(x\,\mathcal{P}_{\mathrm{UV}} + x\ln\frac{\mu_{\mathrm{UV}}^2}{x m_g^2} + (1-x)\right)$
iii. ϵ_{IR}	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\vert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\left(x\,(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}}) + x\ln\frac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right)$

$$\mathcal{P}_{\mathrm{UV/IR}} = \frac{1}{\epsilon_{\mathrm{UV/IR}}} + \ln 4\pi - \gamma_E$$

Light-cone $g_T(x)$

General structure for the ladder-diagram result



(1a)

$$g_T^{(1a)} = g_{T(\mathrm{s})}^{(1a)} + g_{T(\mathrm{c})}^{(1a)}$$

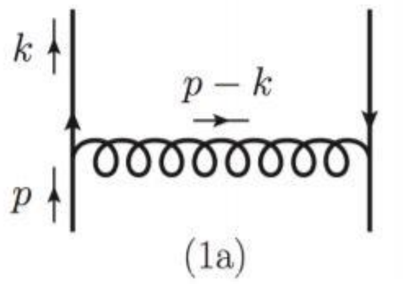
Singular term

$\propto \delta(x)$

(IR scheme-dependent)

Canonical term

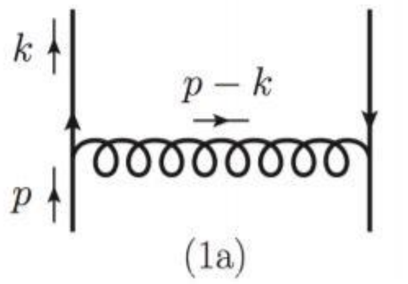
Quasi $g_{T,\mathrm{Q}}(x)$



(1a)

$$\frac{m_q}{\boldsymbol{p^3}} g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{\boldsymbol{k^3}}{\boldsymbol{p^3}}\right)\frac{1}{\boldsymbol{p^3}}$$

Quasi $g_{T,\mathrm{Q}}(x)$



(1a)

$$\frac{m_q}{\boldsymbol{p^3}} g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\left(x-\frac{\boldsymbol{k^3}}{\boldsymbol{p^3}}\right)\frac{1}{\boldsymbol{p^3}}$$

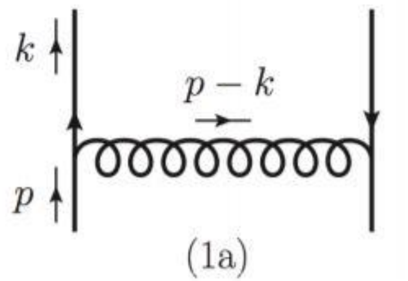
Split into singular & canonical parts

$$g_{T,\mathrm{Q}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2-m_q^2+i\varepsilon)^2} + \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{2k^2+2k_\perp^2+2m_q^2-(4-n)m_g^2}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$$

$g_{T,\mathrm{Q(s)}}^{(1a)}$

$g_{T,\mathrm{Q(c)}}^{(1a)}$

Quasi $g_{T,\mathrm{Q}}(x)$



$$\frac{m_q}{\boldsymbol{p^3}}\, g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4}\int_{-\infty}^{\infty}\frac{d^n k}{(2\pi)^n}\,\frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{\boldsymbol{k^3}}{\boldsymbol{p^3}}\right)\frac{1}{\boldsymbol{p^3}}$$

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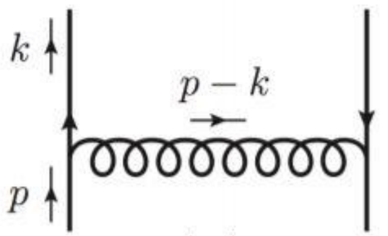
$$\frac{(4-n)}{(k_\perp^2+x^2p_3^2+m_q^2)^{3/2}}$$

$g_{T,\mathrm{Q(s)}}^{(1a)}$

$g_{T,\mathrm{Q(c)}}^{(1a)}$

Quasi $g_{T,\mathrm{Q}}(x)$



(1a)

$$\frac{m_q}{p^3}\, g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{i g^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4}\int_{-\infty}^{\infty}\frac{d^n k}{(2\pi)^n}\,\frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{k^3}{p^3}\right)\frac{1}{p^3}$$

Split into singular & canonical parts

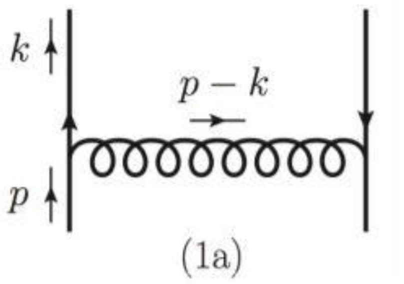
$$g_{T,\mathrm{Q}} \approx \alpha_s C_F \mu^{2\epsilon}\int\frac{d^{n-2}k_\perp}{(2\pi)^{n-2}}\int\frac{dk^0}{2\pi}\,\frac{(4-n)}{(k^2-m_q^2+i\varepsilon)^2} + \alpha_s C_F \mu^{2\epsilon}\int\frac{d^{n-2}k_\perp}{(2\pi)^{n-2}}\int\frac{dk^0}{2\pi}\,\frac{2k^2+2k_\perp^2+2m_q^2-(4-n)m_g^2}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$$

$g_{T,\mathrm{Q(s)}}^{(1a)}$

$$\frac{(4-n)}{(k_\perp^2+x^2p_3^2+m_q^2)^{3/2}} \quad \propto \epsilon$$

$g_{T,\mathrm{Q(c)}}^{(1a)}$

Quasi $g_{T,\mathrm{Q}}(x)$



$$\frac{m_q}{p^3}\, g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4}\int_{-\infty}^{\infty}\frac{d^n k}{(2\pi)^n}\,\frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{k^3}{p^3}\right)\frac{1}{p^3}$$

Split into singular & canonical parts

$$\xcancel{g_{T,\mathrm{Q}} \approx \alpha_s C_F \mu^{2\epsilon}\int\frac{d^{n-2}k_\perp}{(2\pi)^{n-2}}}\int\frac{dk^0}{2\pi}\,\frac{(4-n)}{(k^2-m_q^2+i\varepsilon)^2} + \alpha_s C_F \mu^{2\epsilon}\int\frac{d^{n-2}k_\perp}{(2\pi)^{n-2}}\int\frac{dk^0}{2\pi}\,\frac{2k^2+2k_\perp^2+2m_q^2-(4-n)m_g^2}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$$

$$\frac{(4-n)}{(k_\perp^2+x^2p_3^2+m_q^2)^{3/2}} \propto \epsilon$$

$g_{T,\mathrm{Q(s)}}^{(1a)}$

$g_{T,\mathrm{Q(c)}}^{(1a)}$

Singular part drops out!

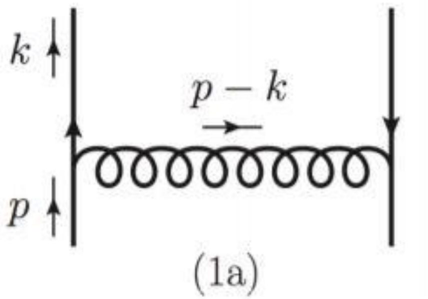
Results for canonical part:

i. $m_q \neq 0$	$g^{(1\mathrm{a})}_{T,\mathrm{Q(c)}}(x)\Big\|_{m_q} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\ \ln\frac{x}{x-1} - 1 & x>1 \\ x\ \ln\frac{4xp_3^2}{(1-x)m_q^2} + 1 - 2x + \frac{2}{x-1} & 0<x<1 \\ x\ \ln\frac{x-1}{x} + 1 & x<0 \end{cases}$
ii. $m_g \neq 0$	$g^{(1\mathrm{a})}_{T,\mathrm{Q(c)}}(x)\Big\|_{m_g} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\ \ln\frac{x}{x-1} - 1 & x>1 \\ x\ \ln\frac{4(1-x)p_3^2}{m_g^2} + 1 - 2x & 0<x<1 \\ x\ \ln\frac{x-1}{x} + 1 & x<0 \end{cases}$
iii. ϵ_{IR}	$g^{(1\mathrm{a})}_{T,\mathrm{Q(c)}}(x)\Big\|_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\ \ln\frac{x}{x-1} - 1 & x>1 \\ x\ \ln\frac{4x(1-x)p_3^2}{\mu_{\mathrm{IR}}^2} - x - x\,\mathcal{P}_{\mathrm{IR}} & 0<x<1 \\ x\ \ln\frac{x-1}{x} + 1 & x<0 \end{cases}$

Quasi $g_{T,\mathrm{Q}}(x)$

General structure for the ladder-diagram result

$\therefore$



(1a)

$$g_{T,\mathrm{Q}}^{(1a)} = \cancel{g_{T,\mathrm{Q(s)}}^{(1a)}} + g_{T,\mathrm{Q(c)}}^{(1a)}$$

Singular term **Canonical term**

Agreement in the IR between light-cone & quasi

Coefficient of zero-modes IR finite

$$g_{T(\mathrm{s})}^{(1a)}(x) = \begin{cases} g_{T(\mathrm{s})}^{(1a)}(x)\big|_{m_q\neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x) \\ \\ g_{T(\mathrm{s})}^{(1a)}(x)\big|_{\epsilon_{\mathrm{IR}}} = 0 \end{cases}$$

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$g^{(1\mathrm{a})}_{T(\mathrm{c})}(x)\Big\vert_{m_q} = \dfrac{\alpha_s C_F}{2\pi}\left(x\,\mathcal{P}_{\mathrm{UV}} \boxed{+x\ln\dfrac{\mu^2_{\mathrm{UV}}}{(1-x)^2 m_q^2}} + \dfrac{x^2-2x-1}{1-x}\right)$	$g^{(1\mathrm{a})}_{T,\mathrm{Q}(\mathrm{c})}(x)\Big\vert_{m_q} = \dfrac{\alpha_s C_F}{2\pi}\begin{cases} x\ \ln\frac{x}{x-1} - 1 & x>1 \\ \boxed{x\ \ln\frac{4xp_3^2}{(1-x)m_q^2}} + 1 - 2x + \frac{2}{x-1} & 0<x<1 \\ x\ \ln\frac{x-1}{x} + 1 & x<0 \end{cases}$
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$g^{(1\mathrm{a})}_{T(\mathrm{c})}(x)\Big\vert_{\epsilon_{\mathrm{IR}}} = \dfrac{\alpha_s C_F}{2\pi}\left(x\,\mathcal{P}_{\mathrm{UV}} \boxed{-\,x\,\mathcal{P}_{\mathrm{IR}}} + x\ln\dfrac{\mu^2_{\mathrm{UV}}}{\mu^2_{\mathrm{IR}}}\right)$	$g^{(1\mathrm{a})}_{T,\mathrm{Q}(\mathrm{c})}(x)\Big\vert_{\epsilon_{\mathrm{IR}}} = \dfrac{\alpha_s C_F}{2\pi}\begin{cases} x\ \ln\frac{x}{x-1} - 1 & x>1 \\ x\ \ln\frac{4x(1-x)p_3^2}{\mu^2_{\mathrm{IR}}} - x \boxed{-\,x\,\mathcal{P}_{\mathrm{IR}}} & 0<x<1 \\ x\ \ln\frac{x-1}{x} + 1 & x<0 \end{cases}$

Agreement in the IR between light-cone & quasi

- **Other diagrams can be calculated just like in the twist-2 case**

- **Diagram by diagram the IR poles exactly match between $g_T(x)$ & $g_{T,\mathrm{Q}}(x)$: heart of quasi-PDF approach**

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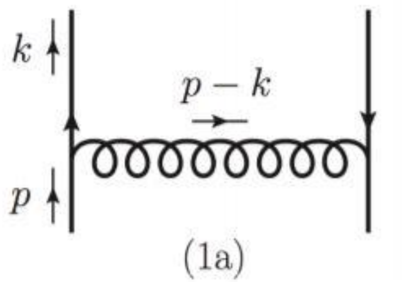
- **Matching kernel can be extracted diagram by diagram**

<u>Example</u>:



$$\begin{aligned} C^{(1a)}(x) &= \delta(1-x) + \tilde{q}^{(1a)}(x) - q^{(1a)}(x) \\ &= \delta(1-x) + C^{(1a)}_{(\mathrm{s})}(x) + C^{(1a)}_{(\mathrm{c})}(x) \end{aligned}$$

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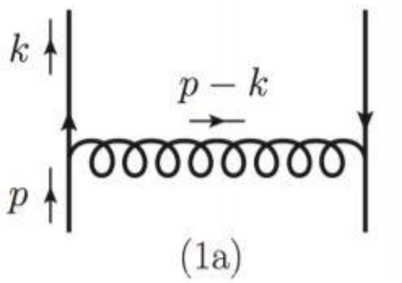

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Canonical part of kernel independent of IR regulator (like twist-2)

$$C^{(1a)}_{(\mathrm{c})}(x)\Big|_{m_q, m_g, \epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi} \begin{cases} x \ \ln \frac{x}{x-1} - 1 & x > 1 \\ x \ \ln \frac{4x(1-x)p_3^2}{\mu^2} - x & 0 < x < 1 \\ x \ \ln \frac{x-1}{x} + 1 & x < 0 \end{cases}$$

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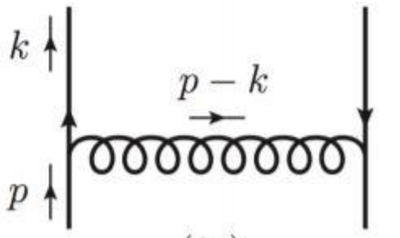


(1a)

$$
\begin{aligned}
C^{(1a)}(x) &= \delta(1-x) + \tilde{q}^{(1a)}(x) - q^{(1a)}(x) \\
&= \delta(1-x) + C^{(1a)}_{(\mathrm{s})}(x) + C^{(1a)}_{(\mathrm{c})}(x)
\end{aligned}
$$

Canonical part of kernel independent of IR regulator (like twist-2)

$$
C^{(1a)}_{(\mathrm{c})}(x)\Big|_{m_q, m_g, \epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi} \begin{cases} x \ \ln \frac{x}{x-1} - 1 & x > 1 \\ x \ \ln \frac{4x(1-x)p_3^2}{\mu^2} - x & 0 < x < 1 \\ x \ \ln \frac{x-1}{x} + 1 & x < 0 \end{cases}
$$

Singular part of kernel: dependent on IR regulator (new at twist-3)

$$
C^{(1a)}_{(\mathrm{s})}(x) = \begin{cases} C^{(1a)}_{(\mathrm{s})}(x)\Big|_{m_q \neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x) \\ C^{(1a)}_{(\mathrm{s})}(x)\Big|_{\epsilon_{\mathrm{IR}}} = 0 \end{cases}
$$

Matching in $\overline{\text{MS}}$

DR

$$
\begin{aligned}
C_{\overline{\text{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right)\Big|_{\epsilon_{\text{IR}}} &= \delta(1-\xi) \\
&+ \frac{\alpha_s C_F}{2\pi}
\begin{cases}
\left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi > 1 \\
\left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\
\left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi < 0
\end{cases} \\
&+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2} + \frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right)
\end{aligned}
$$

If DR to singular terms for $m_g \neq 0$:

$$C_{\overline{\text{MS}}}\Big|_{m_g} = C_{\overline{\text{MS}}}\Big|_{\epsilon_{\text{IR}}}$$

$\boldsymbol{m_q \neq 0}$

$$
\begin{aligned}
C_{\overline{\text{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right)\Big|_{m_q} &= \delta(1-\xi) \\
&+ \frac{\alpha_s C_F}{2\pi}
\begin{cases}
\left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi > 1 \\
\left[\boldsymbol{\delta(\xi)} + \frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\
\left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi < 0
\end{cases} \\
&+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{1}{2} + \frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right)
\end{aligned}
$$

If $m_q \neq 0$ to singular terms for $m_g \neq 0$:

$$C_{\overline{\text{MS}}}\Big|_{m_g} = C_{\overline{\text{MS}}}\Big|_{m_q}$$

Matching in $\overline{\mathrm{MS}}$

$$\approx \frac{3}{2}\ln\xi$$

DR

$$\begin{cases} C_{\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right)\Big|_{\epsilon_{\mathrm{IR}}} = \delta(1-\xi) \\ \quad + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{\xi}{1-\xi}+\frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi>1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2}+\frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi}-\frac{\xi}{1-\xi}+\frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi<0 \end{cases} \\ \quad + \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2}+\frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right) \end{cases}$$

Problems with $\overline{\mathrm{MS}}$:

i. **Convolution integrals are divergent : $\frac{3}{2}\ln\xi$ divergence**

ii. **Mismatch in norm:** $\int_{-\infty}^{\infty}\tilde{q}^{\overline{\mathrm{MS}}}(x,\mu,p^3) \neq \int_0^1 q^{\overline{\mathrm{MS}}}(x,\mu)$

Matching in $\overline{\mathrm{MS}}$

$$\approx \frac{3}{2}\ln\xi$$

DR

$$\left\{\begin{aligned} C_{\overline{\mathrm{MS}}}\left(\xi,\frac{\mu^2}{p_3^2}\right)\Big|_{\epsilon_{\mathrm{IR}}} &= \delta(1-\xi) \\ &+ \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{\xi}{1-\xi}+\frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi>1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2}+\frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi}-\frac{\xi}{1-\xi}+\frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi<0 \end{cases} \\ &+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2}+\frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right) \end{aligned}\right.$$

Problems with $\overline{\mathrm{MS}}$:

i. **Convolution integrals are divergent :** $\frac{3}{2}\ln\xi$ **divergence**

ii. **Mismatch in norm:** $\int_{-\infty}^{\infty}\tilde{q}^{\overline{\mathrm{MS}}}(x,\mu,p^3) \neq \int_0^1 q^{\overline{\mathrm{MS}}}(x,\mu)$

Introduce $\mathrm{M}\overline{\mathrm{MS}}$ scheme: **Alexandrou et. al. (arXiv: 1902.00587)**

i. **Subtract divergence outside physical region**

ii. **Impose:** $\int_{-\infty}^{\infty}\tilde{q}^{\mathrm{M}\overline{\mathrm{MS}}}(x,\mu,p^3) = \int_0^1 q^{\overline{\mathrm{MS}}}(x,\mu)$

Matching in $\mathrm{M}\overline{\mathrm{MS}}$

DR

$$
\left\{
\begin{aligned}
C_{\mathrm{M}\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right)\Big|_{\epsilon_{\mathrm{IR}}} &= \delta(1-\xi) \\
&+ \frac{\alpha_s C_F}{2\pi}
\begin{cases}
\left[\frac{-\xi^2+2\xi+1}{1-\xi}\ \ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ & \xi > 1 \\
\left[\frac{-\xi^2+2\xi+1}{1-\xi}\ \ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\
\left[\frac{-\xi^2+2\xi+1}{1-\xi}\ \ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0
\end{cases}
\end{aligned}
\right.
$$

Matching implemented in lattice QCD **(S.B, Cichy, Constantinou, Metz, Scapellato, Steffens:** arXiv:2004.04130**)**

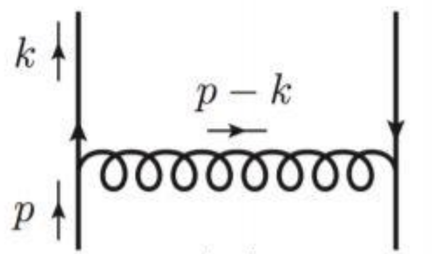
Results are encouraging

See Krzysztof's talk today for lattice results of $g_T(x)$

Case 2:

$$\begin{cases} e \\ h_L \end{cases} \quad \& \quad \begin{cases} e_{\mathbf{Q}} \\ h_{L,\mathbf{Q}} \end{cases}$$

General structure for the ladder-diagram result



(1a)

$$\left\{\begin{array}{l} e^{(1a)} \\ \\ h_L^{(1a)} \end{array}\right. = \left\{\begin{array}{l} e^{(1a)}_{(\mathrm{s})} \\ \\ h^{(1a)}_{L(\mathrm{s})} \end{array}\right. + \left\{\begin{array}{l} e^{(1a)}_{(\mathrm{c})} \\ \\ h^{(1a)}_{L(\mathrm{c})} \end{array}\right.$$

Trouble-maker term for both light-cone & quasi-PDF results

Light-cone results

Singular term: $e_{(\mathrm{s})}^{(1a)}(x) = -h_{L(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F\, \delta(x)\,(2-n)\ \mu^{2\epsilon} \int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}}\, \frac{1}{(k_\perp^2 + m_q^2)}$

Light-cone results

$$-2(1-\epsilon)$$

Singular term:

$$e_{(\mathrm{s})}^{(1a)}(x) = -h_{L(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F\, \delta(x) \boxed{(2-n)}\, \mu^{2\epsilon} \int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}$$

Light-cone results

$$-2(1-\epsilon)$$

Singular term: $$e_{(\mathrm{s})}^{(1a)}(x) = -h_{L(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F\,\delta(x)\,(2-n)\,\mu^{2\epsilon}\int\frac{d^{n-2}k_\perp}{(2\pi)^{n-2}}\,\frac{1}{(k_\perp^2+m_q^2)}$$

i. $m_q \neq 0$	$e_{(\mathrm{s})}^{(1a)}(x)\big\rvert_{m_q} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu_{\mathrm{UV}}^2}{m_q^2} - 1\right)$
ii. ϵ_{IR}	$e_{(\mathrm{s})}^{(1a)}(x)\big\rvert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right)$
iii. $m_g \neq 0$	$e_{(\mathrm{s})}^{(1a)}(x) = \begin{cases} e_{(\mathrm{s})}^{(1a)}(x)\big\rvert_{m_q} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu_{\mathrm{UV}}^2}{m_q^2} - 1\right) \\ e_{(\mathrm{s})}^{(1a)}(x)\big\rvert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right) \end{cases}$

- **Zero modes are unavoidable**

Light-cone results

Singular term: $$e^{(1a)}_{(\mathrm{s})}(x) = -h^{(1a)}_{L(\mathrm{s})}(x) = -\alpha_s C_F\,\delta(x)\,\overset{-2(1-\epsilon)}{(2-n)}\,\mu^{2\epsilon}\int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}}\,\frac{1}{(k_\perp^2+m_q^2)}$$

i. $m_q \neq 0$	$e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2_{\mathrm{UV}}}{m_q^2} - 1\right)$
ii. ϵ_{IR}	$e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu^2_{\mathrm{UV}}}{\mu^2_{\mathrm{IR}}}\right)$
iii. $m_g \neq 0$	$e^{(1a)}_{(\mathrm{s})}(x) = \begin{cases} e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2_{\mathrm{UV}}}{m_q^2} - 1\right) \\ e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu^2_{\mathrm{UV}}}{\mu^2_{\mathrm{IR}}}\right) \end{cases}$

- **Zero modes are unavoidable**
- **IR-dependent prefactors of the zero modes**

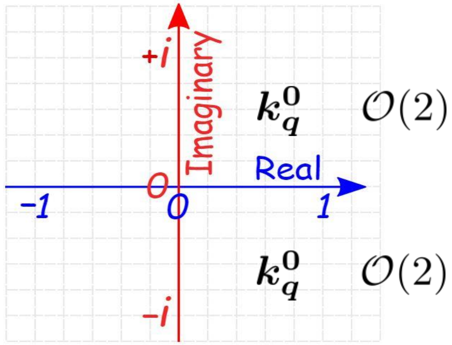
Quasi results

Singular term: $$e_{\mathrm{Q(s)}}(x) = -h_{L,\mathrm{Q(s)}}(x) \approx \alpha_s C_F\, p^3\, \mu^{2\epsilon} \int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{\overset{-2(1-\epsilon)}{\boxed{(2-n)}}}{(k^2 - m_q^2 + i\varepsilon)^2}$$

Quasi results

Singular term: $$e_{\mathrm{Q(s)}}(x) = -h_{L,\mathrm{Q(s)}}(x) \approx \alpha_s C_F\, p^3\, \mu^{2\epsilon} \int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{\boxed{(2-n)}}{(k^2 - m_q^2 + i\varepsilon)^2}$$

i. $\int dk^0$ **:**

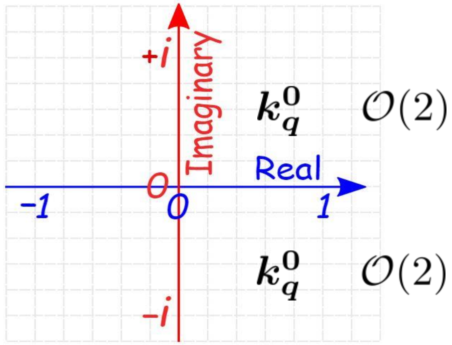


$$\int_{-\infty}^{\infty} \frac{dk^0}{(k^2 - m_q^2 + i\varepsilon)^2} \approx \frac{1}{(k_\perp^2 + x^2 p_3^2 + m_q^2)^{3/2}}$$

Quasi results

Singular term: $$e_{\mathrm{Q(s)}}(x) = -h_{L,\mathrm{Q(s)}}(x) \approx \alpha_s C_F\, p^3\, \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{\boxed{(2-n)}}{(k^2 - m_q^2 + i\varepsilon)^2}$$

i. $\int dk^0$ **:**



$$\int_{-\infty}^{\infty} \frac{dk^0}{(k^2 - m_q^2 + i\varepsilon)^2} \approx \frac{1}{(k_\perp^2 + x^2 p_3^2 + m_q^2)^{3/2}}$$

ii. $\int d^2k_\perp$ **:**

$$\int \frac{d^2k_\perp}{(2\pi)^2} \frac{p^3}{(k_\perp^2 + x^2 p_3^2 + m_q^2)} = \frac{1}{2\pi} \frac{p^3}{\sqrt{x^2 p_3^2 + m_q^2}}$$

$$\Big\downarrow \; \text{twist expansion}$$

$$= \frac{1}{2\pi} \begin{cases} \dfrac{1}{x} & x > 1 \\ \dfrac{1}{x} & 0 < x < 1 \\ -\dfrac{1}{x} & x < 0 \end{cases}$$

Light-cone PDF	**Quasi-PDF**
$\delta(x)$	$\frac{1}{\lvert x\rvert} \quad -\infty < x < \infty$

Comparison of singular terms

Light-cone PDF	**Quasi-PDF**
$e^{(1a)}_{(\mathrm{s})}(x) = \begin{cases} e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{m_q} = \dfrac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\dfrac{\mu_{\mathrm{UV}}^2}{m_q^2} - 1\right) \\ e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = \dfrac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\dfrac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right) \end{cases}$	$e^{(1a)}_{\mathrm{Q(s)}}(x) = \dfrac{\alpha_s C_F}{2\pi}\begin{cases} \dfrac{1}{x} & x > 1 \\ \dfrac{1}{x} & 0 < x < 1 \\ -\dfrac{1}{x} & x < 0 \end{cases}$

Comparison of singular terms

Light-cone PDF	Quasi-PDF
$e^{(1a)}_{(\mathrm{s})}(x) = \begin{cases} e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2_{\mathrm{UV}}}{m_q^2} - 1\right) \\ e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu^2_{\mathrm{UV}}}{\mu^2_{\mathrm{IR}}}\right) \end{cases}$	$e^{(1a)}_{\mathrm{Q(s)}}(x) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1}{x} & x > 1 \\ \frac{1}{x} & 0 < x < 1 \\ -\frac{1}{x} & x < 0 \end{cases}$

- **Singular terms exhibit IR divergence:** $1/x$ **pole as** $x \to 0$

Comparison of singular terms

Light-cone PDF	Quasi-PDF
$e^{(1a)}_{(\mathrm{s})}(x) = \begin{cases} e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2_{\mathrm{UV}}}{m_q^2} - 1\right) \\ e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu^2_{\mathrm{UV}}}{\mu^2_{\mathrm{IR}}}\right) \end{cases}$	$e^{(1a)}_{\mathrm{Q(s)}}(x) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1}{x} & x > 1 \\ \frac{1}{x} & 0 < x < 1 \\ -\frac{1}{x} & x < 0 \end{cases}$

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Comparison of singular terms

Light-cone PDF	**Quasi-PDF**
$e^{(1a)}_{(\mathrm{s})}(x) = \begin{cases} e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2_{\mathrm{UV}}}{m_q^2} - 1\right) \\ e^{(1a)}_{(\mathrm{s})}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu^2_{\mathrm{UV}}}{\mu^2_{\mathrm{IR}}}\right) \end{cases}$	$e^{(1a)}_{\mathrm{Q(s)}}(x) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1}{x} & x > 1 \\ \frac{1}{x} & 0 < x < 1 \\ -\frac{1}{x} & x < 0 \end{cases}$

- **Singular terms exhibit IR divergence:** $1/x$ **pole as** $x \to 0$
- **Mismatch in the IR behavior between the** x **dependent** $e_{(\mathrm{s})}(x),\ h_{L(\mathrm{s})}(x)$ **&** $e_{\mathrm{Q(s)}}(x),\ h_{L,\mathrm{Q(s)}}(x)$
- **Potential problem with matching**

Comparison of singular terms

Light-cone PDF	**Quasi-PDF**
$e^{(1a)}_{(\mathrm{s})}(x) = \begin{cases} e^{(1a)}_{(\mathrm{s})}(x)\big\|_{m_q} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2_{\mathrm{UV}}}{m_q^2} - 1\right) \\ e^{(1a)}_{(\mathrm{s})}(x)\big\|_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu^2_{\mathrm{UV}}}{\mu^2_{\mathrm{IR}}}\right) \end{cases}$	$e^{(1a)}_{\mathrm{Q(s)}}(x) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1}{x} & x > 1 \\ \frac{1}{x} & 0 < x < 1 \\ -\frac{1}{x} & x < 0 \end{cases}$

- **Singular terms exhibit IR divergence:** $1/x$ **pole as** $x \to 0$
- **Mismatch in the IR behavior between the** x **dependent** $e_{(\mathrm{s})}(x),\ h_{L(\mathrm{s})}(x)$ **&** $e_{\mathrm{Q(s)}}(x),\ h_{L,\mathrm{Q(s)}}(x)$
- **Potential problem with matching**
- **Cross-check: agreement of norm**

$$\int dx\, e_{\mathrm{Q(s)}}(x) = \int dx\, e_{(\mathrm{s})}(x)\,; \quad \int dx\, h_{L,\mathrm{Q(s)}}(x) = \int dx\, h_{L(\mathrm{s})}(x)$$

S.B., Cocuzza, Metz (arXiv: 1903.05721)

Summary

- Calculated twist-3 light-cone PDFs $g_T(x)$, $e(x)$ & $h_L(x)$ & their quasi versions in QTM
- Ladder diagram and zero modes:
  - Zero modes may or may not show up in light-cone $g_T(x)$ (IR scheme dependence)
  - Corresponding terms drop out in $g_{T,\mathrm{Q}}(x)$
  - IR poles agree between $g_T(x)$ & $g_{T,\mathrm{Q}}(x)$ for all diagrams: Matching is possible

  - Zero modes bound to show up in $e(x)$ & $h_L(x)$: $e(x),\, h_L(x) \to \delta(x)$
  - Corresponding terms bound to show up in $e_{\mathrm{Q}}(x)$ & $h_{L,\mathrm{Q}}(x)$: $e_{\mathrm{Q}}(x),\, h_{L,\mathrm{Q}}(x) \to \frac{1}{|x|}$
  - Mismatch in the IR between $e(x)$ & $e_{\mathrm{Q}}(x)$ as well as $h_L(x)$ & $h_{L,\mathrm{Q}}(x)$
  - Potential problem with matching

Back-Up Slides

LC $g_T(x)$

$$
\begin{aligned}
g_T^{(1\mathrm{b})}(x)\Big|_{m_g} &= \frac{\alpha_s C_F}{2\pi}\,\frac{1+x}{2(1-x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu_{\mathrm{UV}}^2}{x m_g^2}\right),\\
g_T^{(1\mathrm{b})}(x)\Big|_{m_q} &= \frac{\alpha_s C_F}{2\pi}\,\frac{1+x}{2(1-x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu_{\mathrm{UV}}^2}{(1-x)^2 m_q^2}\right),\\
g_T^{(1\mathrm{b})}(x)\Big|_{\epsilon_{\mathrm{IR}}} &= \frac{\alpha_s C_F}{2\pi}\,\frac{1+x}{2(1-x)}\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right).
\end{aligned}
$$

$$
\begin{aligned}
g_T^{(2\mathrm{a})}\Big|_{m_g} &= \frac{\partial\Sigma(p)}{\partial \not{p}}\Big|_{m_g} = -\frac{\alpha_s C_F}{2\pi}\int_0^1 dy\, y\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu_{\mathrm{UV}}^2}{y m_g^2} - 1\right),\\
g_T^{(2\mathrm{a})}\Big|_{m_q} &= \frac{\partial\Sigma(p)}{\partial \not{p}}\Big|_{m_q} = -\frac{\alpha_s C_F}{2\pi}\int_0^1 dy\,(1-y)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu_{\mathrm{UV}}^2}{(1-y)^2 m_q^2} - \frac{1+y^2}{(1-y)^2}\right),\\
g_T^{(2\mathrm{a})}\Big|_{\epsilon_{\mathrm{IR}}} &= \frac{\partial\Sigma(p)}{\partial \not{p}}\Big|_{\epsilon_{\mathrm{IR}}} = -\frac{\alpha_s C_F}{2\pi}\int_0^1 dy\, y\left(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}} + \ln\frac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right)
\end{aligned}
$$

Quasi $g_{T,\mathrm{Q}}(x)$

$$g^{(1\mathrm{b})}_{T,\mathrm{Q}}(x)\Big|_{m_g} = \frac{\alpha_s C_F}{2\pi}\,\frac{1+x}{2(1-x)}\begin{cases}\ln\dfrac{x}{x-1} & x>1\\[2ex] \ln\dfrac{4(1-x)p_3^2}{m_g^2} & 0<x<1\\[2ex] \ln\dfrac{x-1}{x} & x<0\,,\end{cases}$$

$$g^{(1\mathrm{b})}_{T,\mathrm{Q}}(x)\Big|_{m_q} = \frac{\alpha_s C_F}{2\pi}\,\frac{1+x}{2(1-x)}\begin{cases}\ln\dfrac{x}{x-1} & x>1\\[2ex] \ln\dfrac{4xp_3^2}{(1-x)m_q^2} & 0<x<1\\[2ex] \ln\dfrac{x-1}{x} & x<0\,,\end{cases}$$

$$g^{(1\mathrm{b})}_{T,\mathrm{Q}}(x)\Big|_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\,\frac{1+x}{2(1-x)}\begin{cases}\ln\dfrac{x}{x-1} & x>1\\[2ex] \ln\dfrac{4x(1-x)p_3^2}{\mu_{\mathrm{IR}}^2}-\mathcal{P}_{\mathrm{IR}} & 0<x<1\\[2ex] \ln\dfrac{x-1}{x} & x<0\,.\end{cases}$$

Quasi $g_{T,\mathrm{Q}}(x)$

$$g_{T,\mathrm{Q}}^{(\mathrm{1d})}(x) \;=\; \frac{\alpha_s C_F}{2\pi}\begin{cases} \dfrac{1}{1-x} & x>1 \\ \dfrac{1}{x-1} & 0<x<1 \\ \dfrac{1}{x-1} & x<0 \end{cases}$$

$$g_{T,\mathrm{Q}}^{(\mathrm{2a})}\Big|_{m_g} = \frac{\partial\Sigma(p)}{\partial \not{p}}\Big|_{m_g} = -\frac{\alpha_s C_F}{2\pi}(1-\epsilon_{\mathrm{UV}})C(\epsilon_{\mathrm{UV}})\left(\frac{p^3}{\mu_{\mathrm{UV}}}\right)^{-2\epsilon_{\mathrm{UV}}}\int dy \begin{cases} y^{-2\epsilon_{\mathrm{UV}}}\left(y\ln\dfrac{y}{y-1}-1\right) & y>1 \\ y^{-2\epsilon_{\mathrm{UV}}}\left(y\ln\dfrac{4(1-y)p_3^2}{m_g^2}+1-2y\right) & 0<y<1 \\ (-y)^{-2\epsilon_{\mathrm{UV}}}\left(y\ln\dfrac{y-1}{y}+1\right) & y<0\,, \end{cases}$$

Quasi $g_{T,\mathrm{Q}}(x)$

$$g_{T,\mathrm{Q}}^{(2\mathrm{a})}\Big|_{m_q} = \frac{\partial\Sigma(p)}{\partial \not{p}}\Big|_{m_q} = -\frac{\alpha_s C_F}{2\pi} C(\epsilon_{\mathrm{UV}}) \left(\frac{p^3}{\mu_{\mathrm{UV}}}\right)^{-2\epsilon_{\mathrm{UV}}} \int dy \begin{cases} (1-\epsilon_{\mathrm{UV}})\, y^{-2\epsilon_{\mathrm{UV}}} \left((1-y)\ln\dfrac{y}{y-1} + 1\right) & y > 1 \\ y^{-2\epsilon_{\mathrm{UV}}} \left((1-\epsilon_{\mathrm{UV}})(1-y)\ln\dfrac{4yp_3^2}{(1-y)m_q^2}\right. & \\ -(1-\epsilon_{\mathrm{UV}})\dfrac{2y^2-5y+1}{1-y} & \\ \left.-\left(1-\dfrac{\epsilon_{\mathrm{UV}}}{2}\right)\dfrac{4y}{1-y}\right) & 0 < y < 1 \\ (1-\epsilon_{\mathrm{UV}})\,(-y)^{-2\epsilon_{\mathrm{UV}}} \left((1-y)\ln\dfrac{y-1}{y} - 1\right) & y < 0\,, \end{cases}$$

Quasi $g_{T,\mathrm{Q}}(x)$

$$g^{(2\mathrm{a})}_{T,\mathrm{Q}}\Big|_{\epsilon_{\mathrm{IR}}} = \frac{\partial\Sigma(p)}{\partial \not{p}}\Big|_{\epsilon_{\mathrm{IR}}} = -\frac{\alpha_s C_F}{2\pi}(1-\epsilon_{\mathrm{UV}})C(\epsilon_{\mathrm{UV}})\left(\frac{p^3}{\mu_{\mathrm{UV}}}\right)^{-2\epsilon_{\mathrm{UV}}} \int dy \begin{cases} y^{-2\epsilon_{\mathrm{UV}}}\biggl(y\ln\dfrac{y}{y-1}-1\biggr) & y>1 \\ y^{-2\epsilon_{\mathrm{UV}}}\biggl(y\ln\dfrac{4y(1-y)p_3^2}{\mu_{\mathrm{IR}}^2} & \\ +1-y-y\,\mathcal{P}_{\mathrm{IR}}\biggr) & 0<y<1 \\ (-y)^{-2\epsilon_{\mathrm{UV}}}\biggl(y\ln\dfrac{y-1}{y}+1\biggr) & y<0\,. \end{cases}$$