LAMET 2020

Towards calculating B-meson distribution amplitude on the lattice

Shuai Zhao

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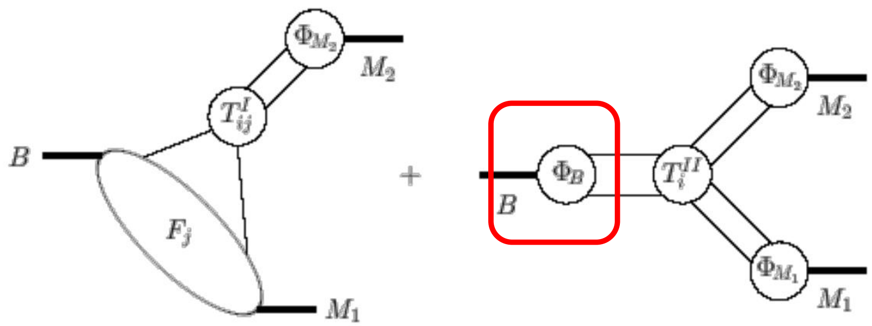
Based on:
2006.05663 with A. Radyushkin;
1908.09933 with W.Wang, Y.M. Wang and J.Xu

B-meson light-cone distribution amplitude (LCDA)

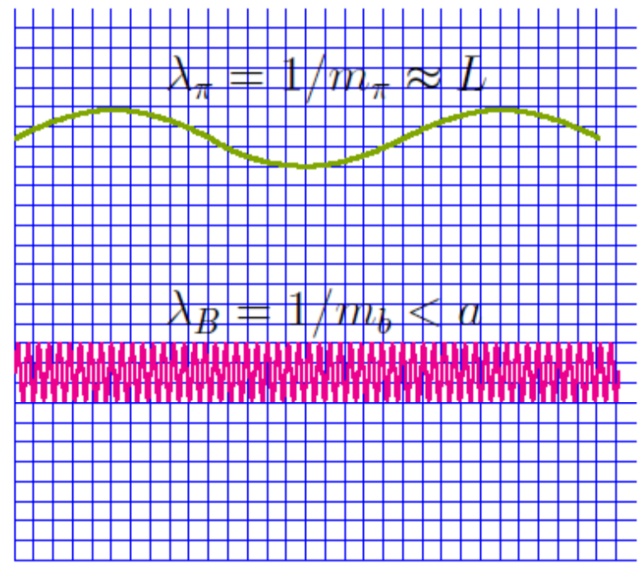
- B-meson exclusive decay provides important information for understanding the CP violation



- LCDA plays dominate role in factorization theorem for B-meson exclusive decay/QCD (SCET) sum rules.

  Beneke,Buchalla,Neubert,Sachrajda, 1999;
  Bauer, Pirjol, Stewart, 2001

- Heavy quark effective theory (HQET)



From Michele Della Morte

B-meson LCDA

- The light-cone HQET matrix element   Grozin, Neubert, 1997

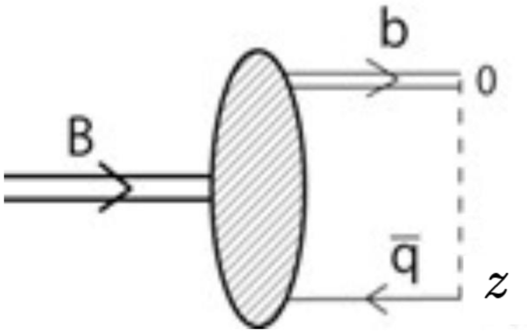
$$\langle 0|\bar{q}_\beta(z)\,[z,0]\,h_{v\alpha}(0)|\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\,\tilde{\phi}_B^+(t,\mu) + \frac{\tilde{\phi}_B^-(t,\mu)-\tilde{\phi}_B^+(t,\mu)}{t}\,\not{z}\right\}\gamma_5\right]_{\alpha\beta}$$

- Evolution equation $t \equiv z\cdot v$

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu}{\omega} + \gamma_+(\alpha_s)\right]\phi_B^+(\omega,\mu) - \omega\int_0^\infty d\eta\,\Gamma_+(\omega,\eta,\alpha_s)\,\phi_B^+(\eta,\mu)\,.$$

Lange, Neubert, 2003; Braun, Ji, Manasov, 2019

- Inverse moment & logarithmic moment are important for phenomenology
- Not related to local operator
- No local OPE   Braun, Ivanov, Korchemsky, 2003



Ioffe-time distribution amplitude (ITDA)

- It was proposed by Ji that parton physics can be studied on the lattice from LaMET. Ji, 2013,2014; Ji et al,2020

- Matrix element of off-lightcone HQET operator

$$\langle 0 | \bar{q}(z) S(z,0) \gamma^\mu \gamma_5 h_v(0) | \overline{B}(v) \rangle$$
$$= iF(\mu) \left[v^\mu M_{B,v}(\nu, -z^2, \mu) + z^\mu M_{B,z}(\nu, -z^2, \mu) \right]$$

$\nu \equiv v \cdot z$: Ioffe-time in HQET

Braun,Gornicki, Mankiewicz,1994;Radyushkin,2017; SZ,Radyushkin,2020

- Decay constant $\langle 0 | \bar{q}(0) \gamma^\mu \gamma_5 h_v(0) | \overline{B}(v) \rangle = i v^\mu F(\mu)$

- LCDA

$$\phi_B^+(\omega, \mu) = \frac{v^+}{2\pi} \int_{-\infty}^{\infty} dz^- e^{-i\omega v^+ z^-} \mathcal{I}_B^+(v^+ z^-, \mu).$$

- Quasi-DA Kawamura, Tanaka, 2018; Wang,Wang,Xu,SZ,2019

$$\widetilde{\phi}_B^+(\omega, v_3, \mu) = \frac{|v_3|}{2\pi} \int_{-\infty}^{\infty} dz_3 e^{i\omega v_3 z_3} M_B(-v_3 z_3, z_3^2, \mu)$$

B-meson quasi-DA

- Hard-collinear factorization formula

$$\tilde{\phi}_B^+(\xi, v_3, \mu) = \int_0^\infty d\omega H(\xi, \omega, v_3, \mu)\phi_B^+(\omega, \mu) + O\left(\frac{\Lambda_{\rm QCD}}{\xi v_3}\right)$$

$$\begin{aligned}
H(\xi,\,\omega,\,n_z\cdot v,\,\mu) \;=\; & \delta(\xi-\omega) + \frac{\alpha_s(\mu)\,C_F}{4\,\pi}\left\{\frac{1}{\omega-\xi}\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\theta(-\xi)\,\theta(\omega)\right.\\
& +\left\{\frac{1}{\omega-\xi}\left[3-2\left(1+\frac{2\,\xi}{\omega}\right)\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\left(\ln\left(\frac{\omega-\xi}{\xi}\right)+1\right)\right]\right\}_\oplus\theta(\xi)\,\theta(\omega-\xi)\\
& +\left\{\frac{1}{\xi-\omega}\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\xi-\omega)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\right\}_\oplus\theta(\omega)\,\theta(\xi-\omega)\\
& \left.+2\left[\ln^2\frac{\mu}{n_z\cdot v\,\xi}-3\,\ln\frac{\mu}{n_z\cdot v\,\xi}+f(a)\right]\delta(\xi-\omega)\right\},
\end{aligned}$$

Wang,Wang,Xu,SZ,2019

- The plus prescription is defined by

$$\{\mathcal{F}(\xi,\omega)\}_\oplus = \mathcal{F}(\xi,\omega) - \delta(\xi-\omega)\int_0^{a\,\xi} dt\,\mathcal{F}(\xi,t)$$

- The function f(a)

$$\begin{aligned}
f(a) \;=\; & \ln\frac{a^2}{4(a-1)^3}\,\ln\frac{\mu}{n_z\cdot v\,\xi}+\ln(a-1)\,\ln\frac{8\,(a-1)}{a}\\
& +{\rm Li}_2(1-a)+\ln a\,\ln\left(\frac{a}{4}\right)-\frac{1}{2}\ln(a-1)\\
& +\ln(8\,a)+\ln^2 2+\frac{\pi^2}{8}-3
\end{aligned}$$

- large noise-to-signal ratio

$$\frac{noise}{signal} \propto \exp(x_0\Delta), \qquad \Delta = E_{stat} - m_\pi$$

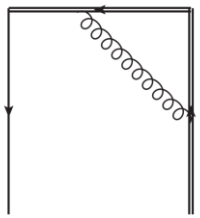
- Needs nonperturbative renormalization to achieve the continuum limit.

One-loop hard contribution in coordinate space

- UV: Polyakov regularization: a
  IR: light-quark mass: m

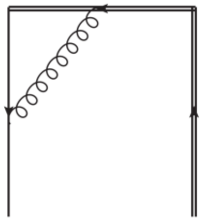
Link self energy: $$\Gamma_\Sigma(z,a) = \frac{\alpha_s C_F}{2\pi}\left(-\frac{\pi}{a}\sqrt{-z^2} + \ln\frac{-z^2}{a^2} + 2\right) + \mathcal{O}(z^2)$$

Chen, Ji, Zhang, 2016;
Radyushkin,2017

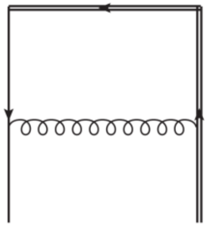


$$O^\mu(z,0;v) = \frac{\alpha_s C_F}{2\pi}\bar{q}(z)\gamma^\mu\gamma_5 h_v(0) \times \Big[\ln a^2(\ln 2iv\cdot z - \frac{1}{2}\ln(-z^2)) + \frac{1}{4}\ln^2(-z^2) - \ln^2 2iv\cdot z - \frac{\pi^2}{6}\Big] + \mathcal{O}(z^2).$$

Cusp singularity



$$O^\mu(z,0;v) = \frac{\alpha_s C_F}{2\pi}\Big\{\frac{1}{2}(\ln\frac{-z^2}{a^2} - 1)\bar{q}(z)\gamma^\mu\gamma_5 h_v(0) - \int_0^1 du\Big[\ln\frac{-z^2m^2e^{2\gamma_E}}{4}\frac{\bar{u}}{u} + \frac{\bar{u}+(2-u)\ln u^2}{u}\Big]_+ \times \bar{q}(\bar{u}z)\gamma^\mu\gamma_5 h_v(0)\Big\} + \mathcal{O}(z^2),$$



$$O^\mu(z,0;v) = -\frac{\alpha_s C_F}{2\pi}\Big\{\Big[\frac{2}{u} + \ln(iumv\cdot z e^{\gamma_E})\Big]_+ - \Big[\frac{1}{\epsilon_{\rm IR}} - 1 + \ln\frac{4\pi\mu_{\rm IR}^2 e^{-\gamma_E}}{m^2} - \ln(imv\cdot z e^{\gamma_E})\Big]\delta(u)\Big\} \times \bar{q}(\bar{u}z)\gamma^\mu\gamma_5 h_v(0) + \mathcal{O}(z^2)$$

Kawamura, Tanaka, 2018;
SZ, Radyushkin,2020

Multiplicative renormalizability

- Auxiliary field formalism

Gervais and Neveu,1980;
Ji,Zhang,Zhao,2017;Green,Jansen,Steffens;2018;Zhang et al, 2018

$$\bar{q}(z)S(z,0)\gamma^{\mu}\gamma_5 h_v(0) \longrightarrow [\bar{q}(z)\gamma^{\mu}\gamma_5 \mathcal{Q}(z)][\overline{\mathcal{Q}}(0)h_v(0)]$$

- Heavy-heavy and heavy-light currents are renormalized multiplicatively.
- The bare and renormalized operators are linked by

$$[\bar{q}(z)\gamma^{\mu}\gamma_5 h_v(0)]^R = Z(z\cdot v, z^2;\Lambda)\bar{q}(z)\gamma^{\mu}\gamma_5 h_v(0)$$

Wang, Wang, Xu, SZ, 2019

Z: UV sensitive factor
Λ: cutoff

- Rest frame B-meson ITDA cannot be used to renormalize UV because UV singularity depends on v.

Reduced B-meson ITDA

- Consider matrix element of the leading Fock state $|b(v)\bar{q}(\omega v)\rangle$

$$\langle 0|\bar{q}(z)\gamma^\mu\gamma_5 h_v(0)|b(v)\bar{q}(\omega v)\rangle = iv^\mu f(\mu) m_B(\omega\nu, -z^2)$$

$$\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 h_v(0)|b(v)\bar{q}(\omega v)\rangle = iv^\mu f(\mu)$$

- UV divergence only depends on operator (not state), then

$$\frac{F^R(\mu)}{F(\mu;a)} = \frac{f^R(\mu)}{f(\mu;a)} = Z(a)$$

$$\frac{F^R(\mu)M_B^R(\nu,-z^2)}{F(\mu;a)M_B(\nu,-z^2;a)} = \frac{f^R(\mu)m_B^R(\omega\nu,-z^2)}{f(\mu;a)m_B(\omega\nu,-z^2;a)} = Z(z\cdot v,-z^2;a)$$

Reduced B-meson ITDA

- Define reduced B-meson ITDA   SZ, Radyushkin,2020

$$\overline{M}_B(\nu,-z^2) = \frac{M_B(\nu,-z^2;a)}{m_B(\omega\nu,-z^2;a)}\bigg|_{\omega=0}$$

- One-loop correction

$$\begin{aligned} M_B(\nu,-z^2,a) &= M_B(\nu)^{(0)} \\ &+ \frac{\alpha_s C_F}{2\pi}\bigg\{\bigg[-\frac{\pi}{a}\sqrt{-z^2}+\frac{3}{2}\ln\frac{-z^2}{a^2}+2 \\ &+ \ln a^2(\ln 2i\nu - \frac{1}{2}\ln(-z^2)) - \frac{\pi^2}{6} - \ln^2 2i\nu \\ &+ \frac{1}{4}\ln^2(-z^2) + \frac{1}{2}\ln\frac{a^2}{4} - \ln i\nu\bigg] M_B(\nu)^{(0)} \\ &- \int_0^1 dw\bigg[\frac{w}{\bar{w}}\ln\frac{-z^2m^2e^{2\gamma_E}}{4} + \ln(i\bar{w}m\nu e^{\gamma_E}) + \frac{2}{\bar{w}} \\ &+ \frac{w+(2-\bar{w})\ln\bar{w}^2}{\bar{w}}\bigg]_+ M_B(w\nu)^{(0)}\bigg\} + \mathcal{O}(z^2). \end{aligned}$$

UV sensitive

$$\begin{aligned} \overline{M}_B(\nu,-z^2) &= \overline{M}_B(\nu)^{(0)} \\ &- \frac{\alpha_s C_F}{2\pi}\int_0^1 dw\bigg[\frac{w}{\bar{w}}\ln\frac{-z^2m^2e^{2\gamma_E}}{4} + \ln(i\bar{w}m\nu e^{\gamma_E}) \\ &+ \frac{2+w+(2-\bar{w})\ln\bar{w}^2}{\bar{w}}\bigg]_+ \overline{M}_B(w\nu)^{(0)} + \mathcal{O}(z^2). \end{aligned}$$

No UV singularity

Matching relation

- Lightcone ITDA SZ, 2019

$$\mathcal{I}_B^+(\nu,\mu^2) = \mathcal{I}_B^+(\nu,\mu)^{(0)}\left\{1-\frac{\alpha_s C_F}{2\pi}\left[\ln^2(i\mu\nu e^{\gamma_E})+\ln(i\mu\nu e^{\gamma_E})+\frac{5\pi^2}{24}\right]\right\}$$
$$+\frac{\alpha_s C_F}{2\pi}\int_0^1 dw\left[\frac{w}{\bar{w}}\ln\frac{\mu^2}{\bar{w}^2 m^2}-\frac{2}{\bar{w}}-\ln(i\bar{w}e^{\gamma_E}m\nu)\right]_+\mathcal{I}_B^+(w\nu,\mu)^{(0)}+\mathcal{O}(\alpha_s^2)$$

- Matching relation

$$\overline{M}_B(\nu,z_3^2) = \mathcal{I}_B^+(\nu,\mu^2)+\frac{\alpha_s C_F}{2\pi}\left\{\left[\ln^2(i\tilde{\mu}\nu)+\ln(i\tilde{\mu}\nu)+\frac{5\pi^2}{24}\right]\mathcal{I}_B^+(\nu,\mu^2)\right.$$
$$\left.-\int_0^1 du\left[\frac{u}{\bar{u}}\left(\ln\frac{z_3^2\tilde{\mu}^2}{4}+1\right)+2\frac{\ln\bar{u}}{\bar{u}}\right]_+\right\}\mathcal{I}_B^+(u\nu,\mu^2)+\mathcal{O}(\alpha_s^2) \qquad \tilde{\mu}\equiv\mu e^{\gamma_E}$$

- μ^2 evolution of position space LCDA

$$\mu\frac{d}{d\mu}\mathcal{I}_B^+(\nu,\mu) = -\frac{\alpha_s C_F}{\pi}\left\{\left[\ln(i\tilde{\mu}\nu)+\frac{1}{2}\right]\mathcal{I}_B^+(\nu,\mu^2)-\int_0^1 du\left[\frac{u}{\bar{u}}\right]_+\mathcal{I}_B^+(u\nu,\mu)\right\}$$

Kawamura, Tanaka, 2010

Summary

- We propose to evaluate B-meson LCDA on the lattice, within LaMET and pseudodistribution approaches.
- A reduced ITDA for B-meson is constructed, which is UV finite, can approach to its continuum limit on the lattice.
- A practical lattice calculation is expected.

Thank you