Determining the Nonperturbative Collins-Soper Kernel From TMD Observables Using Lattice QCD

Maximilian Schlemmer[a]

[a]University of Regensburg – Institute for Theoretical Physics

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Universität Regensburg

Outline

1. Lattice calculations
2. Fit correlators to parametrization to obtain the moments of TMDPDFs
3. Extraction of the CS-Kernel from lattice data

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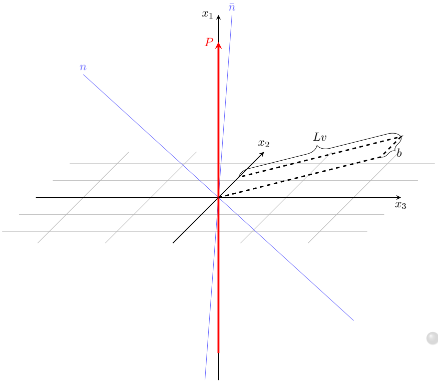
Matrix element



Musch, et. al., 1011.1213

- Nucleon Matrix Element which is sensitive to transverse momentum

$$W_f^{[\Gamma]}(b; L, v; P, S) = \frac{1}{2}\langle P, S|\bar{q}_f(b)\Gamma\,[b, b+Lv]\,[b+Lv, Lv]\,[Lv, 0]\,q(0)|P, S\rangle$$

with nuclon state $|P, S\rangle$

- Time equal matrix element $v^0 = b^0 = 0$
- b^μ is orthogonal to v^μ and P^μ therefore $b \cdot v = b \cdot P = 0$

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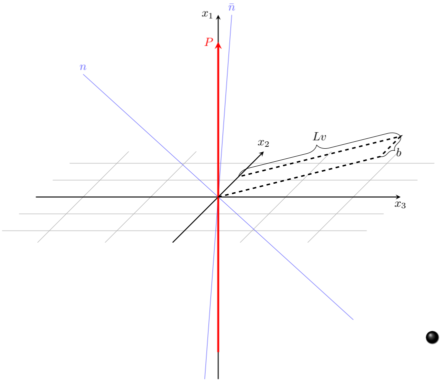

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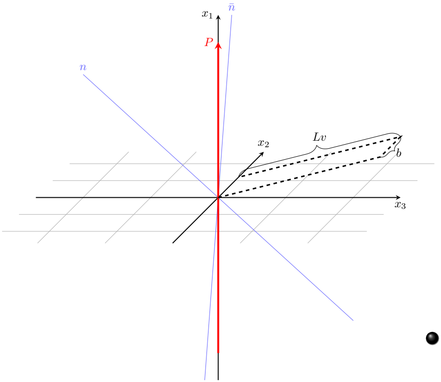

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Extract matrix element and mass

- For our analysis we require $b^+ = 0$ and $\mathbf{P}_T = \mathbf{0}$ thus

$$S = (0,0,0,1), \qquad P = (P_0, P_1, 0, 0)$$
$$b = (0,0,b_2,b_3), \qquad v = (0, v_1, v_2, v_3)$$

- To improve the lattice signal, HYP- and momentum smearing is applied
- u-d Quark channel to cancel disconnected diagrams
- We use the CLS ensembles with $N_f = 2+1$ flavors and Wilson-Clover fermions

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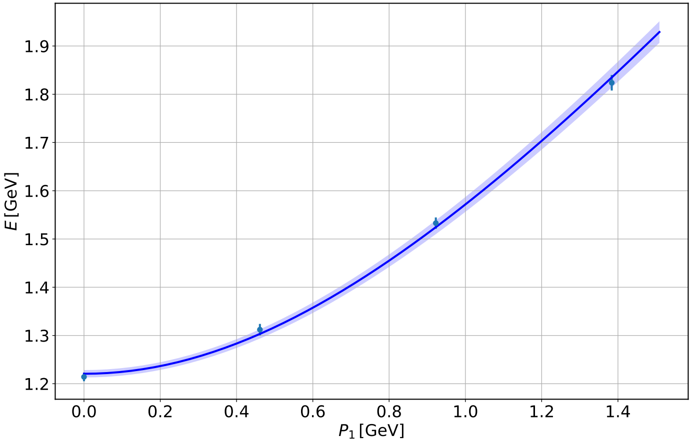
CLS ensemble H101

β	$L^3 \times T$	a	m_π	#cnfg
3.4	$32^3 \times 96$	0.084fm	422MeV	2000



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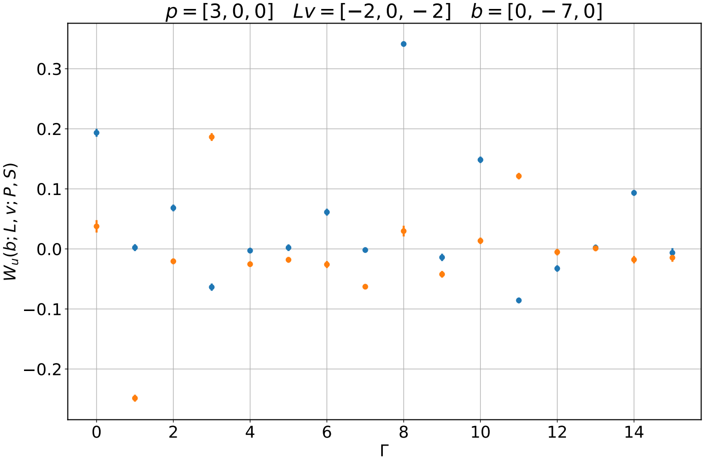

1. Lattice calculations

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Parametrization

The correlators are grouped into Lorentz-invariant products of b, P, Lv: $(P^2, b^2, (Lv)^2, Lv \cdot P)$ and parameterized, e.g. for the vector channel γ^μ as

$$
\begin{aligned}
\frac{1}{2} W_f^{[\gamma^\mu]}(b; L, v; P, S) &= P^\mu A_2 - i m_N^2 b^\mu A_3 \\
&- i m_N \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha S_\beta A_{12} + m^2 L v^\mu B_1 + m_N L \epsilon^{\mu\nu\alpha\beta} P_\nu v_\alpha S_\beta B_7 \\
&- i m_N^3 L \epsilon^{\mu\nu\alpha\beta} b_\nu v_\alpha S_\beta B_8 - m_N^3 L (b \cdot S) \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta B_9 \\
&- i m_N^3 L^2 (v \cdot S) \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta B_{10}
\end{aligned}
$$

Musch, et. al., 1111.4249

All A_i and B_i and S depend on $(P^2, b^2, (Lv)^2, Lv \cdot P)$

$$f_1(P^2, b^2, (Lv)^2, Lv \cdot P) = 2\frac{A_2 + RB_1}{S(b^2, (Lv)^2, Lv \cdot P)}$$

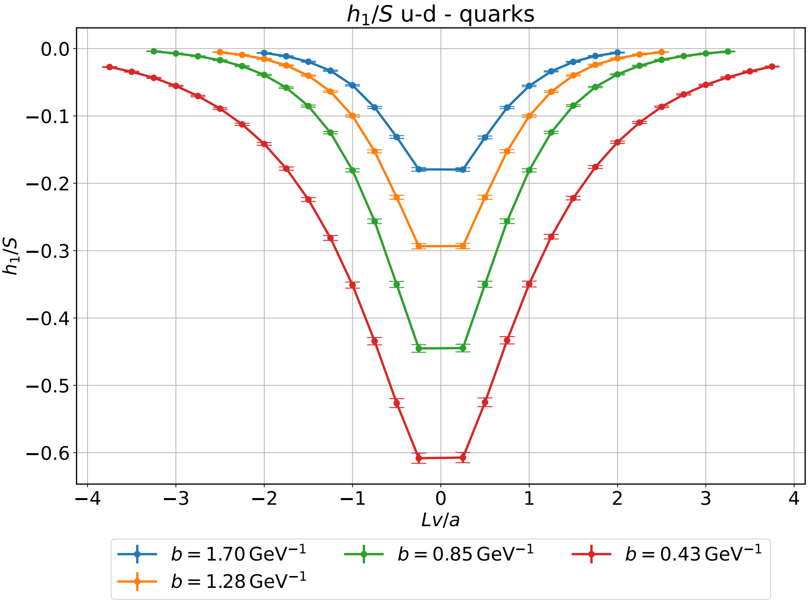
$$g_{1L}(P^2, b^2, (Lv)^2, Lv \cdot P) = 2\frac{A_7 + RB_{13}}{S(b^2, (Lv)^2, Lv \cdot P)}$$

$$h_1(P^2, b^2, (Lv)^2, Lv \cdot P) = 2\frac{A_9 + RB_{15} - \frac{1}{2}m_N^2 b^2(A_{11} - RB_{17})}{S(b^2, (Lv)^2, v \cdot P)}$$

$R(P^2, Lv \cdot P) = \frac{m^2 Lv^+}{P^+}$

Musch, et. al., 1111.4249

Obtaining moments of Fourier-transformed TMDs



Outline

$$R^{[\Gamma]}(Lv, b) = \left(\frac{P_2^+}{P_1^+}\right)^{-K(b,\mu)} r^{[\Gamma]} + \mathcal{O}(\lambda)$$
$$= \frac{W_f^{[\Gamma]}(P_1)}{W_f^{[\Gamma]}(P_2)}$$

where $r^{[\Gamma]}$ is given by

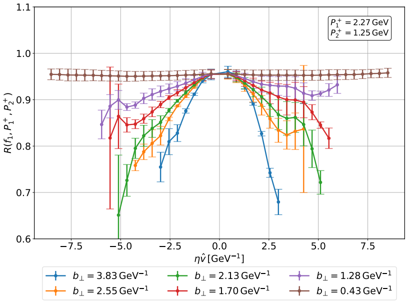
$$r^{[\Gamma]} = 1 + 4C_F \frac{\alpha_s(\mu)}{4\pi} \ln\left(\frac{P_1^+}{P_2^+}\right) \left[1 - \ln\left(\frac{4P_1^+ P_2^+ |v^-|^2}{\mu^2}\right) - 2\mathbf{M}^{[\Gamma]}_{\ln|x|}(b, \mu)\right] + \mathcal{O}(\alpha_s^2)$$

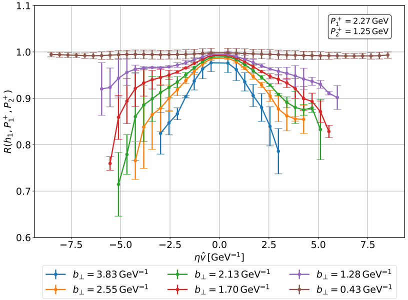
with non-perturbative $\mathbf{M}^{[\Gamma]}_{\ln|x|}(b, \mu)$

Vladimirov, Schäfer, 2002.07527

Determination of $\mathrm{M}^{(n),\Gamma}_{\ln|x|}(b,\mu)$

- Form ratios of f_1, h_1 and g_{1L} at different P^+
- Extrapolate $L \to \infty$
- Use the lattice data to determine $\mathrm{M}^{(n),\Gamma}_{\ln|x|}(b,\mu)$ at approx $1\mathrm{GeV}^{-1}$





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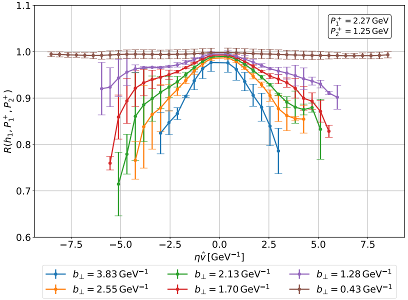

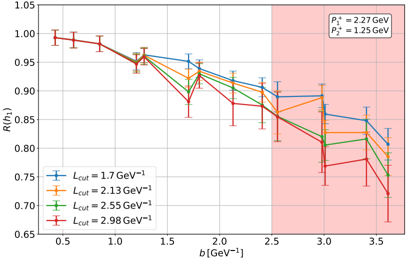

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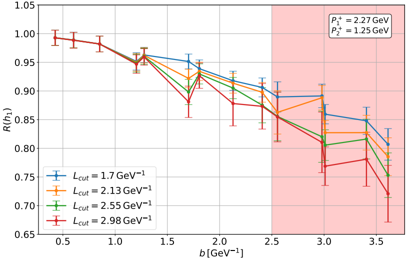

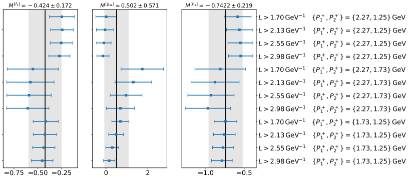

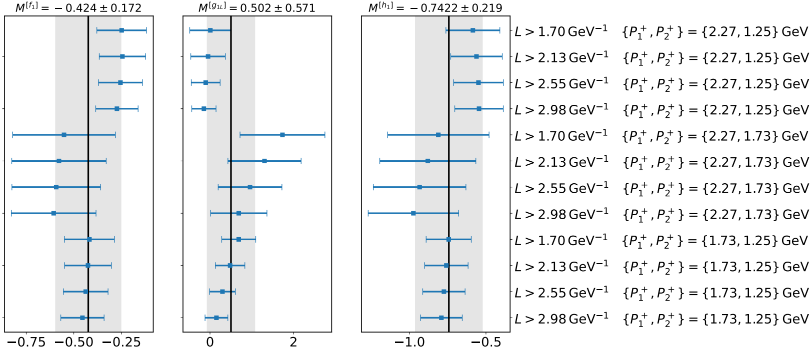

$M^{[f_1]} = -0.424 \pm 0.172$
$M^{[g_{1L}]} = 0.502 \pm 0.571$
$M^{[h_1]} = -0.7422 \pm 0.219$
−0.75 −0.50 −0.25
0 2
−1.0 −0.5
$L > 1.70\,\mathrm{GeV}^{-1}$ $\{P_1^+, P_2^+\} = \{2.27, 1.25\}$ GeV
$L > 2.13\,\mathrm{GeV}^{-1}$ $\{P_1^+, P_2^+\} = \{2.27, 1.25\}$ GeV
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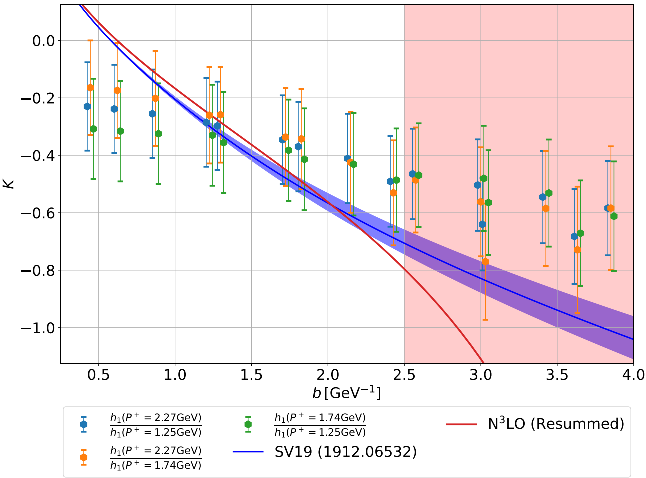
Use $\mathbf{M}^{[\Gamma]}_{\ln|x|}(b,\mu)$ to compute $r^{[\Gamma]}$ and extract K

$$K = \frac{\ln\left(\frac{r^{[\Gamma]}}{R^{[\Gamma]}}\right)}{\ln\left(\frac{P_2^+}{P_1^+}\right)}$$



Extraction of the CS-kernel K

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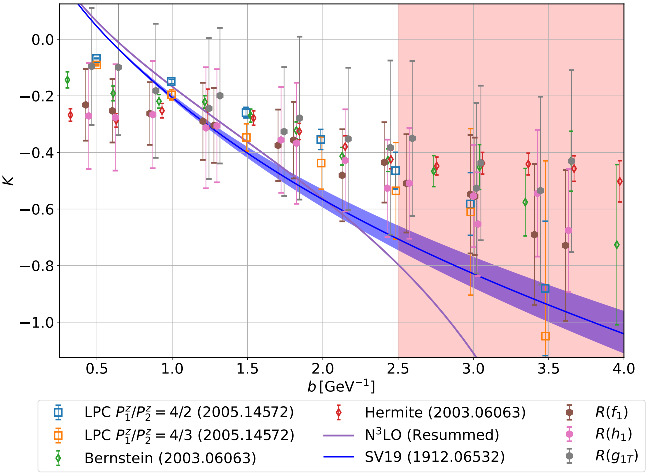

Conclusion

- Agreement with other lattice extractions, lattice model computations and pertubation theory
- Systematic errors need to be reduced, i.e. better understood
- Larger lattices need to be analyzed such that the b range can be increased where the data can be trusted

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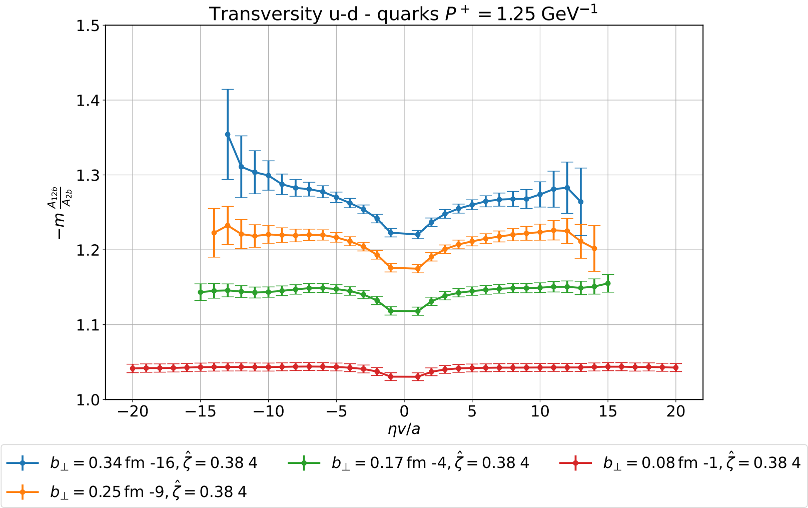
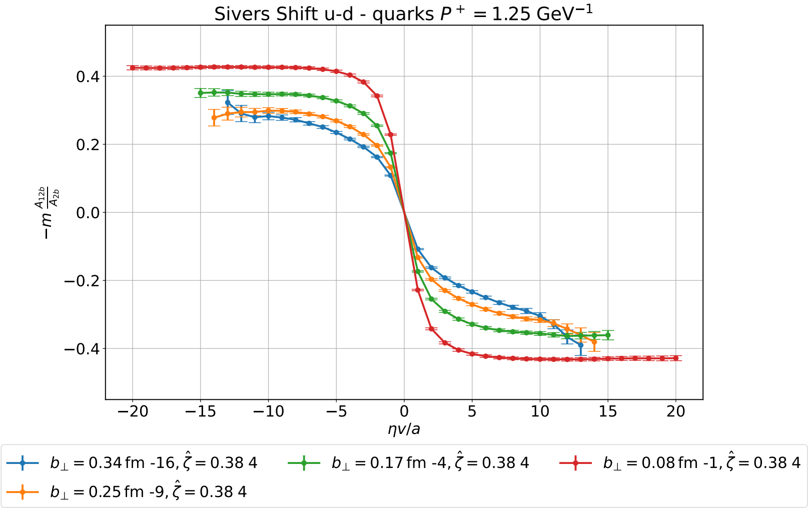
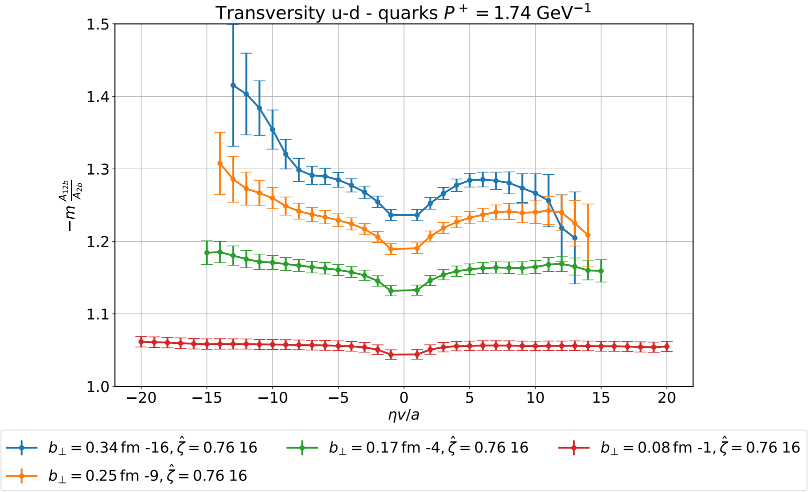
Thank you for your attention
Questions?

Other ratios




Sivers Shift u-d - quarks $P^+ = 1.25\ \mathrm{GeV}^{-1}$
$-m\frac{A_{12b}}{A_{2b}}$
0.4
0.2
0.0
−0.2
−0.4
−20
−15
−10
−5
0
5
10
15
20
$\eta v/a$
$b_\perp = 0.34$ fm -16, $\hat{\zeta} = 0.38$ 4
$b_\perp = 0.17$ fm -4, $\hat{\zeta} = 0.38$ 4
$b_\perp = 0.08$ fm -1, $\hat{\zeta} = 0.38$ 4
$b_\perp = 0.25$ fm -9, $\hat{\zeta} = 0.38$ 4



Transversity u-d - quarks $P^+ = 1.74$ GeV^{-1}
$-m\frac{A_{12b}}{A_{2b}}$
1.5
1.4
1.3
1.2
1.1
1.0
−20
−15
−10
−5
0
5
10
15
20
$\eta v/a$
$b_\perp = 0.34$ fm -16, $\hat{\zeta} = 0.76$ 16
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Other ratios