Extraction of NNLO PDFs from Lattice QCD Calculations

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Based On ZYL, Ma, Qiu, 2006.12370

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Renormalized Operator

- Quark correlation operator: Ji, 1305.1539

$$\mathcal{O}_q^{\nu,b}(\xi,\mu^2,\delta) = \overline{\psi}_q(\xi)\,\gamma^\nu\,\Phi^{(f)}(\{\xi,0\})\,\psi_q(0)\Big|_{\mu^2,\delta}$$

- Multiplicatively renormalization: Ji, Zhang, Zhao, 1706.08962

Ishikawa, Ma, Qiu, Yoshida, 1707.03107 Green, Jansen, Steffens, 1707.07152

$$\mathcal{O}_q^{\nu,\mathrm{RS}}(\xi) = \mathcal{O}_q^{\nu,b}(\xi,\mu^2,\delta)/Z^{\mathrm{RS}}(\xi^2,\mu^2,\delta)$$

- Non-perturbative renormalization schemes:

$$Z^{\mathrm{RS}}(\xi^2,\mu^2,\delta) = \frac{\langle \mathrm{RS}|\hat{n}\cdot\mathcal{O}_q^b(\xi,\mu^2,\delta)|\mathrm{RS}\rangle}{\langle \mathrm{RS}|\hat{n}\cdot\mathcal{O}_q^b(\xi,\mu^2,\delta)|\mathrm{RS}\rangle^{(0)}}$$

for example: $|\mathrm{RS}\rangle = |h(p)\rangle_{p^2=-\mu_R^2}$ Stewart, Zhao, 1709.04933

$|\mathrm{RS}\rangle = |\Omega\rangle$: single scale Braun, Vladimirov, Zhang, 1810.00048

Factorization

Ji, 1305.1539, 1404,6680

- Quasi-PDFs: $\tilde{F}^{\rm RS}_{q_{ik}/h}(y, \frac{\mu^2}{p_z^2}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{p^0} F^{0,{\rm RS}}_{q_{ik}/h}(\omega, -\frac{\omega^2\mu^2}{p_z^2}) e^{-iy\omega}$

$$\tilde{F}^{\rm RS}_{q_{ik}/h}(y, \frac{\mu^2}{p_z^2}) = \frac{1}{R^{\rm RS}} \int_{-1}^{1} \frac{dx}{|x|} f_{q_{ik}/h}(x, \mu^2) \tilde{K}(\frac{y}{x}, \frac{\mu^2}{x^2 p_z^2}) + O(\frac{\Lambda^2_{\rm QCD}}{p_z^2})$$

Radyushkin, 1705.01488

- Pseudo-PDFs: $\hat{F}^{\rm RS}_{q_{ik}/h}(y, \xi^2\mu^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{p^0} F^{0,{\rm RS}}_{q_{ik}/h}(\omega, \xi^2\mu^2) e^{-iy\omega}$

$$\hat{F}^{\rm RS}_{q_{ik}/h}(y, \xi^2\mu^2) = \frac{1}{R^{\rm RS}} \int_{-1}^{1} \frac{dx}{|x|} f_{q_{ik}/h}(x, \mu^2) \hat{K}(\frac{y}{x}, \xi^2\mu^2) + O(\xi^2\Lambda^2_{\rm QCD})$$

Ma, Qiu, 1404.6860, 1709.03018

- Lattice cross sections:

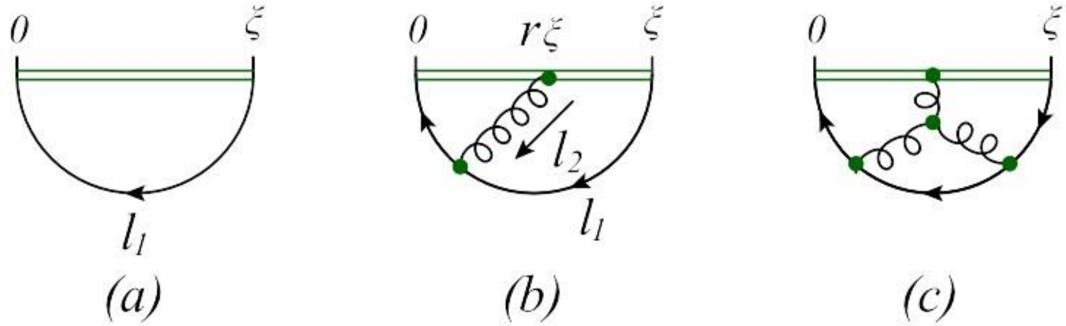
$$F^{\nu,{\rm RS}}_{q/h}(\omega, \xi^2\mu^2) = \langle h(p)|\mathcal{O}^{\nu,{\rm RS}}_q(\xi)|h(p)\rangle \qquad \omega \equiv p\cdot\xi$$

$$F^{\nu,{\rm RS}}_{q_{ik}/h}(\omega, \xi^2\mu^2) = \frac{1}{R^{\rm RS}} \int_{-1}^{1} \frac{dx}{x} f_{q_{ik}/h}(x, \mu^2) K^{\nu}(x\omega, \xi^2\mu^2) + O(\xi^2\Lambda^2_{\rm QCD})$$

$$R^{\rm RS}(\xi^2, \mu^2) \equiv Z^{\rm RS}(\xi^2, \mu^2, \epsilon)/Z^{\overline{\rm MS}}(\xi^2, \mu^2, \epsilon) \qquad q_{ik} \equiv q_i - q_k$$

Renormalization Factor

- Representative diagrams for calculating Z^{vac}:



- Calculation steps:
- Fourier transformation. IBP reduction
- Solve MIs by the method of difference equations

Renormalization Factor

- Obtain $Z^{\rm vac}$ up to NNLO

$$Z_{\overline{\rm MS}} = 1 + \frac{\alpha_s S_\epsilon}{\pi\epsilon} C_F + \left(\frac{\alpha_s S_\epsilon}{\pi\epsilon}\right)^2 C_F \left\{ \left[\frac{C_F}{2} - \frac{13 C_A}{32} + \frac{n_f T_F}{8}\right] + \left[\left(-\frac{1}{8} + \frac{\pi^2}{12}\right) C_F + \left(\frac{25}{48} - \frac{\pi^2}{48}\right) C_A - \frac{n_f T_F}{6}\right]\epsilon \right\}$$

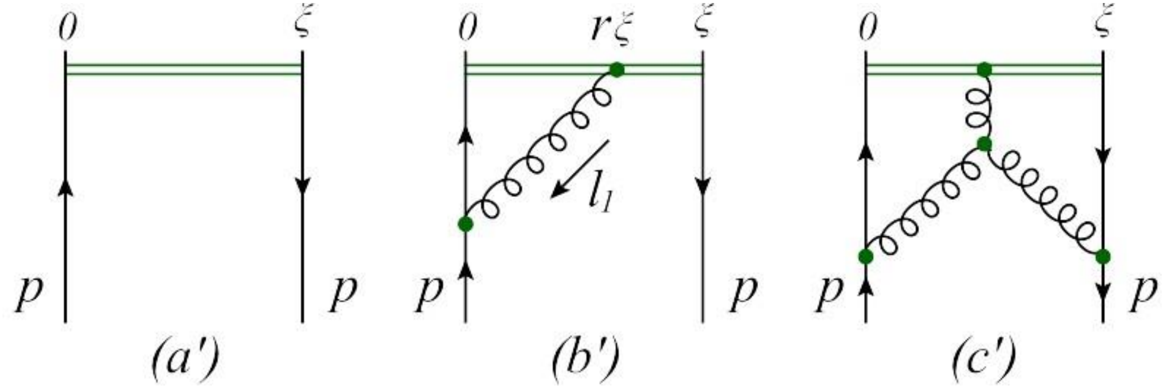
$$S_\epsilon \equiv (4\pi)^\epsilon / \Gamma(1-\epsilon)$$

$$R^{\rm vac} = 1 + \frac{\alpha_s}{\pi} C_F \left(\frac{3}{4} L + 2 + \frac{\pi^2}{3}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left\{ \left[\frac{9}{32} C_F + \frac{11}{32} C_A - \frac{1}{8} n_f T_F\right] L^2 \right.$$
$$+ \left[\left(\frac{43}{32} + \frac{5\pi^2}{12}\right) C_F + \left(\frac{75}{32} + \frac{19\pi^2}{72}\right) C_A - \left(\frac{7}{8} + \frac{\pi^2}{9}\right) n_f T_F\right] L + \left[\left(\frac{153}{128} + \frac{13\pi^2}{12} - \frac{\zeta_3}{2} + \frac{\pi^4}{90}\right) C_F \right.$$
$$\left.\left. + \left(\frac{6413}{1152} - \frac{5\pi^2}{432} - \frac{13\zeta_3}{2} - \frac{\pi^4}{90}\right) C_A - \left(\frac{589}{288} - \frac{\pi^2}{27} - 2\zeta_3\right) n_f T_F\right]\right\}$$

$$L \equiv \ln(-\xi^2\mu^2/4) + 2\gamma_E$$

Lattice Cross Sections

- Representative diagrams for calculating $F^{\nu,\mathrm{RS}}_{q_{ik}/q_i}$



- Calculation steps:
- Fourier transformation. IBP reduction
- Solve MIs by the method of differential equations. Solve boundary conditions with the help of vacuum MIs
- Obtain Taylor series of ω. Assume an ansatz and obtain analytical expressions

Matching Coefficients

- Structure of matching coefficients of LCSs:

$$K^\nu(x\omega,\xi^2\mu^2) \equiv xp^\nu\, A(x\omega,\xi^2\mu^2) + \, x\omega\frac{\xi^\nu}{-\xi^2}\, B(x\omega,\xi^2\mu^2)$$

$$iA(\omega,\xi^2\mu^2) = 2e^{i\omega} + \frac{\alpha_s}{\pi}\sum_{i=0}^{1} L^i C_F A_{i1}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \sum_{i=0}^{2} L^i C_F\left[C_F A_{i1}^{(2)} + C_A A_{i2}^{(2)} + n_f T_F A_{i3}^{(2)}\right]$$

$$A_{ij}^{(m)} = a_{ij1}^{(m)}\, e^{i\omega} + \int_0^1 dz\, \left(\frac{a_{ij2}^{(m)}(z)}{1-z}\right)_+ e^{iz\omega} + \int_{-1}^0 dz\, \left(\frac{a_{ij3}^{(m)}(z)}{1-z}\right)_+ e^{iz\omega} \qquad L \equiv \ln(-\xi^2\mu^2/4) + 2\gamma_E$$

- Examples: $a_{011}^{(2)} = -\,4\zeta_3 + \frac{\pi^2}{9} + \frac{223}{192}$

$$a_{212}^{(2)} = -\,(1+z^2)H(1;z) + \cdots \qquad a_{213}^{(2)} = 0$$

$$a_{112}^{(2)} = (3-z^2)H(1,0;z) + \cdots \qquad a_{113}^{(2)} = -\,2(1+z^2)H(-1,0;-z) + \cdots$$

$$a_{012}^{(2)} = -\,(3-z^2)H(1,1,0;z) + \cdots \qquad a_{013}^{(2)} = -\,2(5+z^2)H(-1,-1,0;-z) + \cdots$$

harmonic polylogarithms:

$$H(1;z) = -\ln(1-z)\,,\ H(1,0;z) = -\ln(1-z)\ln(z) - Li_2(z)\,,\ H(1,1,0;z) = \zeta_3 + \frac{\pi^2}{6}\ln(1-z) - Li_3(1-z)\,,\ \cdots$$

Transformations

- Pseudo-PDFs:

structure: $$\hat{K}(y,\xi^2\mu^2) = \hat{k}_1(\xi^2\mu^2)\delta(1-y) + \begin{cases} 0 & 1<y \\ \left[\hat{k}_2(y,\xi^2\mu^2)\right]_+ & 0<y<1 \\ \left[\hat{k}_3(y,\xi^2\mu^2)\right]_+ & -1\le y<0 \\ 0 & y<-1 \end{cases}$$

when $a_{ij2}^{(m)} = H(n;z)$, $\hat{K}(y,\xi^2\mu^2) = \frac{\alpha_s^m}{\pi^m} C_j \begin{cases} 0 & 1<y \\ \left[L^i \frac{H(n;y)}{1-y}\right]_+ & 0<y<1 \\ 0 & y<0 \end{cases}$

- Quasi-PDFs:

structure: $$\tilde{K}(y,\mu^2/p_z^2) = \tilde{k}_1(\mu^2/p_z^2)\delta(1-y) + \begin{cases} \left[\tilde{k}_2(y,\mu^2/p_z^2)\right]_+ & 1<y \\ \left[\tilde{k}_3(y,\mu^2/p_z^2)\right]_+ & 0<y<1 \\ \left[\tilde{k}_4(y,\mu^2/p_z^2)\right]_+ & -1\le y<0 \\ -\left[\tilde{k}_2(y,\mu^2/p_z^2)\right]_+ & y<-1 \end{cases}$$

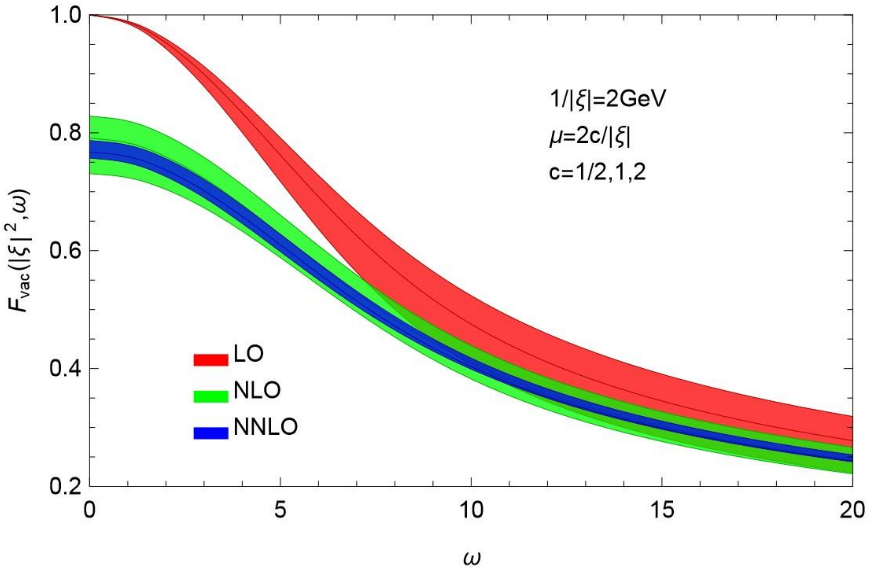
when $a_{0j2}^{(m)} = H(n;z)$, equals to psedo-PDFs

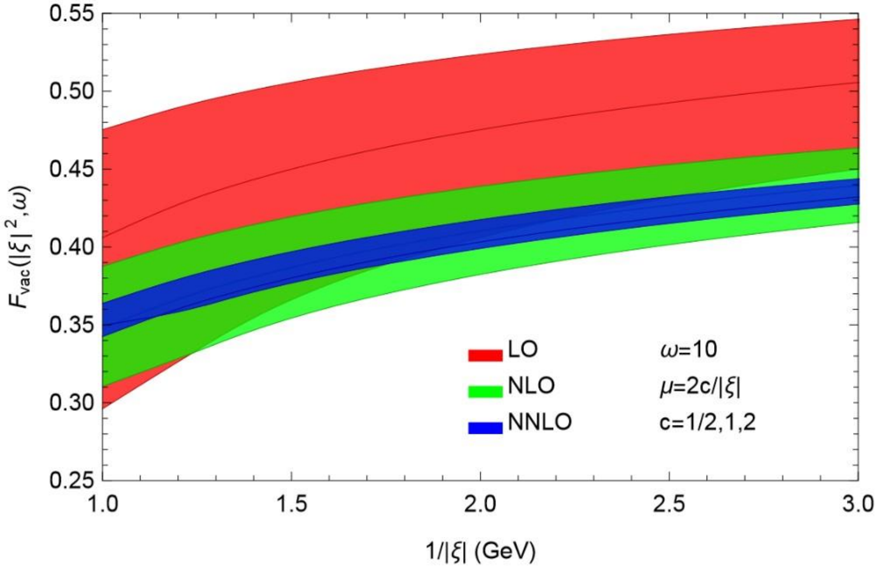
for $i \neq 0$, for example, when $a_{112}^{(2)} = H(1,0;z)$

$$\tilde{K}(y,\mu^2/p_z^2) = \frac{\alpha_s^2}{\pi^2} C_F^2 \begin{cases} \left[\frac{H(0,0,1;1/y)}{1-y} + \frac{H(1,0,1;1/y)}{1-y}\right]_+ & 1<y \\ \left[\ln(\frac{\mu^2}{4p_z^2})\frac{H(1,0;y)}{1-y} + \frac{2\zeta_3}{1-y} - \frac{3H(1,0,0;y)}{1-y} + \frac{H(1,0,1;y)}{1-y} + \frac{4H(1,1,0;y)}{1-y}\right]_+ & 0<y<1 \\ \left[\frac{2\zeta_3}{1-y} - \frac{\pi^2}{6}\frac{H(-1;-y)}{1-y} + \frac{H(-1,0,-1;-y)}{1-y} - \frac{H(-1,0,0;-y)}{1-y}\right]_+ & y<0 \end{cases}$$

Numerical Results

- Using PDFs from experimental data and matching coefficients, predict the lattice results of LCSs





Summary

- Obtain complete and analytic NNLO flavor non-singlet ($q - \bar{q}$ or $q_i - q_k$) matching coefficients for LCSs, pseudo-PDFs and quasi-PDFs

- The method can be used for gluon cases and other LCSs

Thank you!