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# Lattice QCD Calculations of TMD Soft Function through Large-Momentum Effective Theory


Qi-An Zhang

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Sept.11, 2020

# OUTLINE

- **TMD Factorization and TMD Soft Function**
- **Calculate the TMD Soft Function on lattice**
- **Lattice Calculation and Numerical Results**
- **Summary and outlook**

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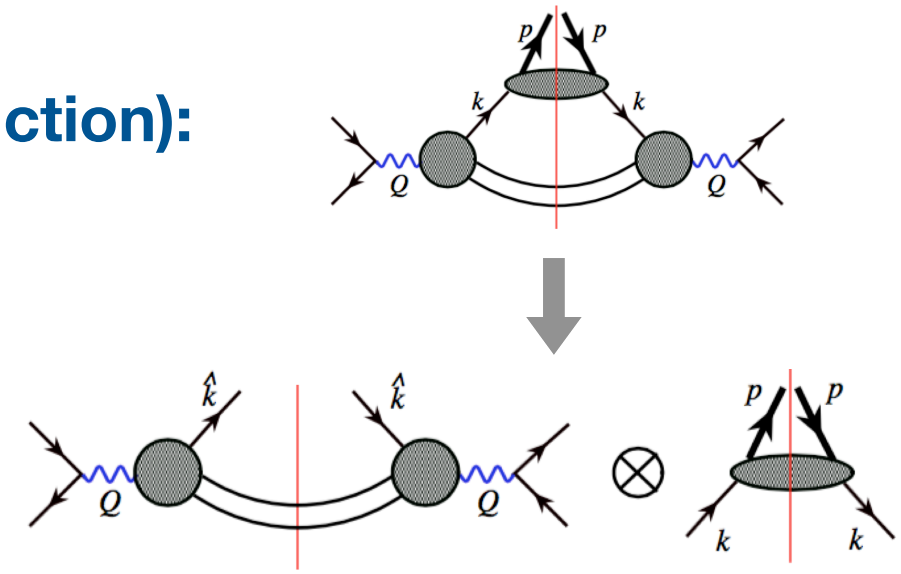
# TMD Factorization and Soft Function

- **Collinear factorization (e.g., for the DIS structure function):**

$$F_a(x, Q) = \sum_i \int_0^1 \frac{d\xi}{\xi} H_i(x/\xi, Q, \mu)\phi_{i/N}(\xi, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$



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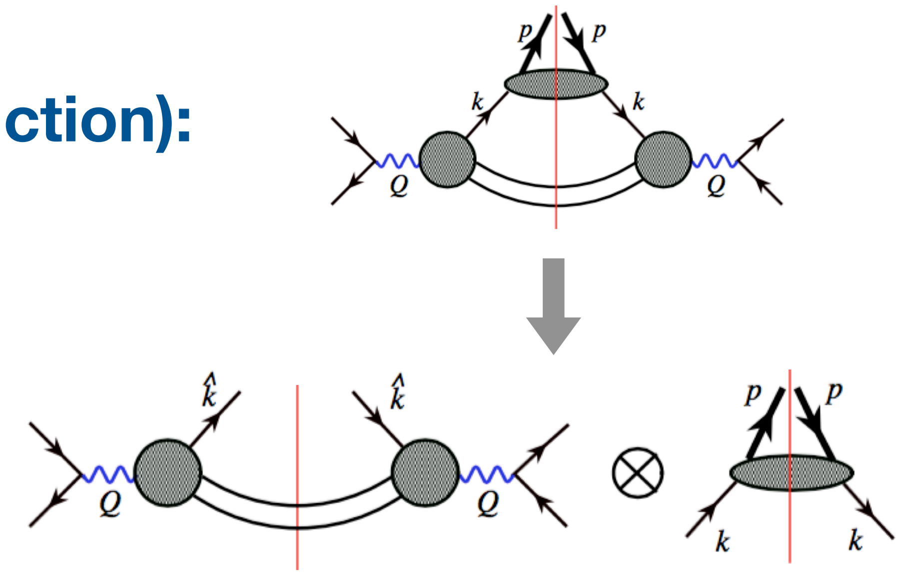
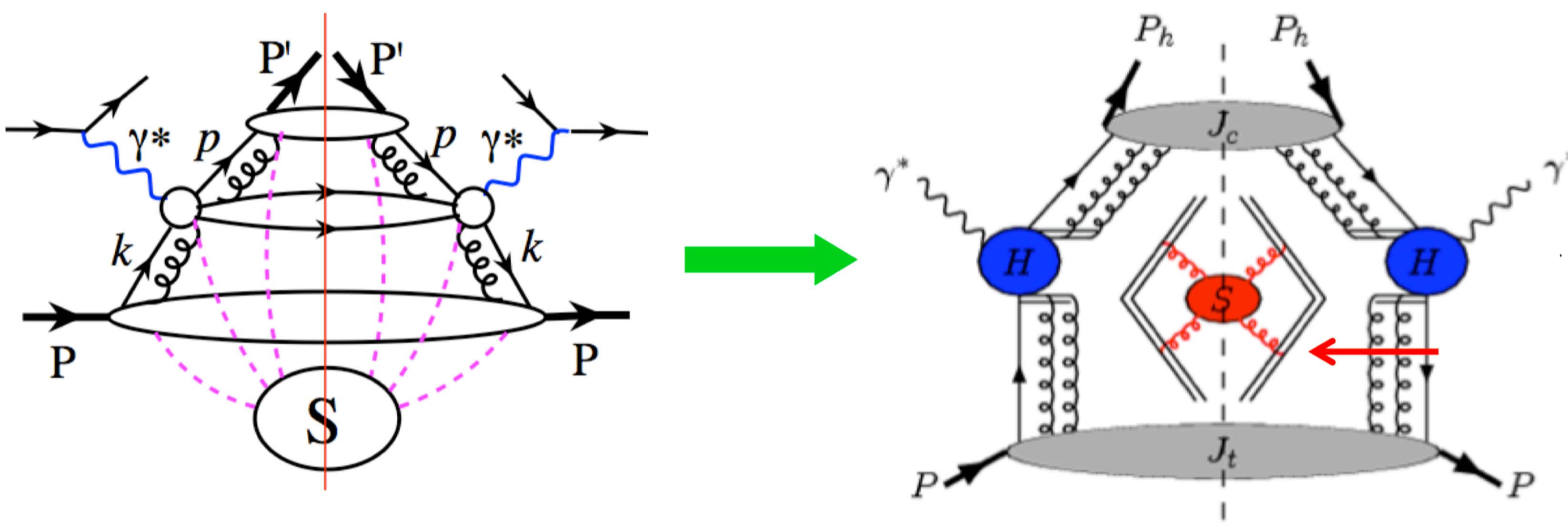



- **TMD factorization (e.g., for SIDIS cross section, $P_{h\perp} \ll Q$):**

$$\sigma_{SIDIS} = \sum_i \hat{H}(Q,\mu) \otimes f_i^{TMD}(x, k_\perp, \mu, \zeta) \otimes D_{i/N}(x', p_\perp, \mu, \zeta') \otimes S(k_{s\perp}, \mu, Y, Y') + \mathcal{O}\left(\frac{P^2_{h\perp}}{Q^2}, \frac{\Lambda^2_{QCD}}{Q^2}\right)$$



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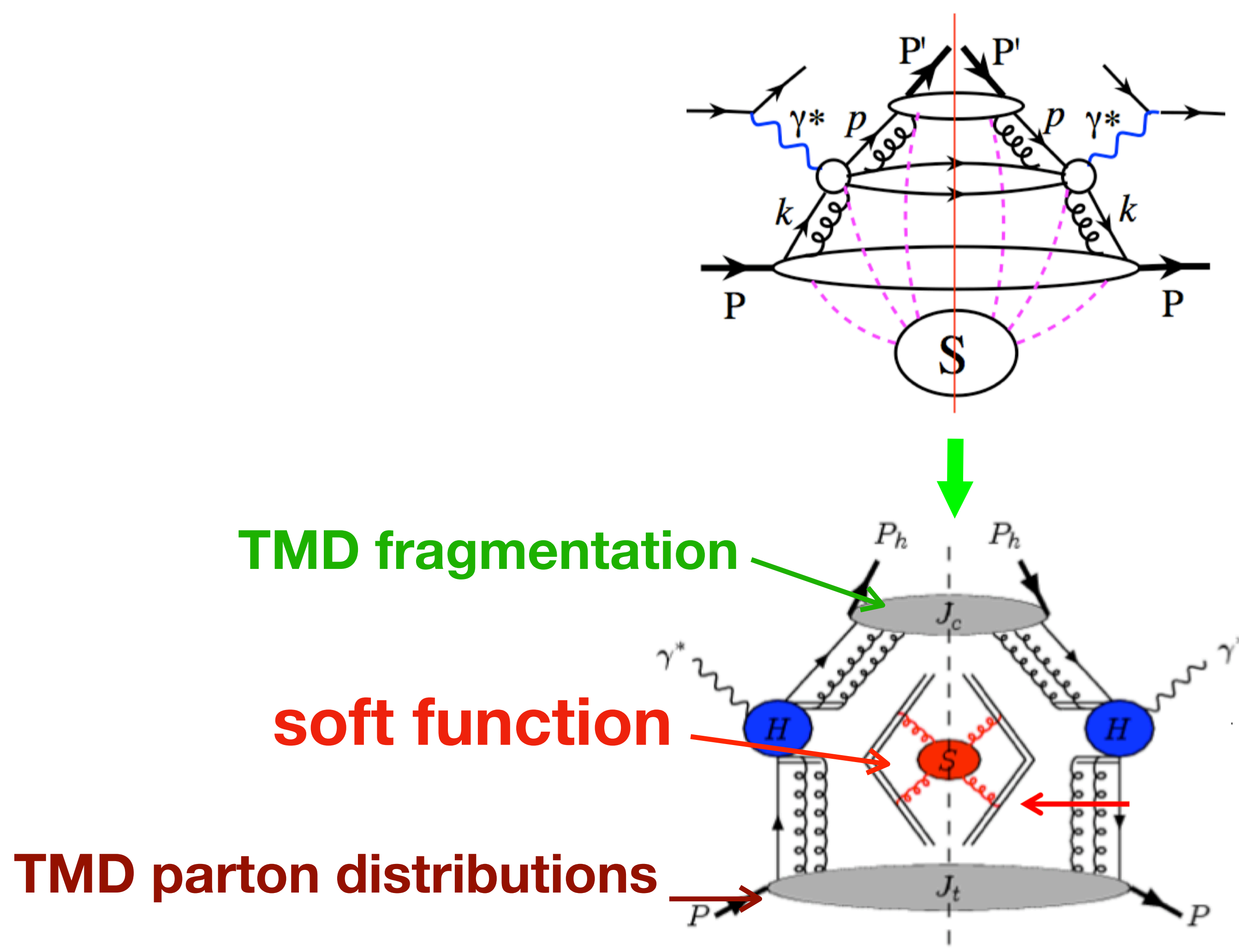

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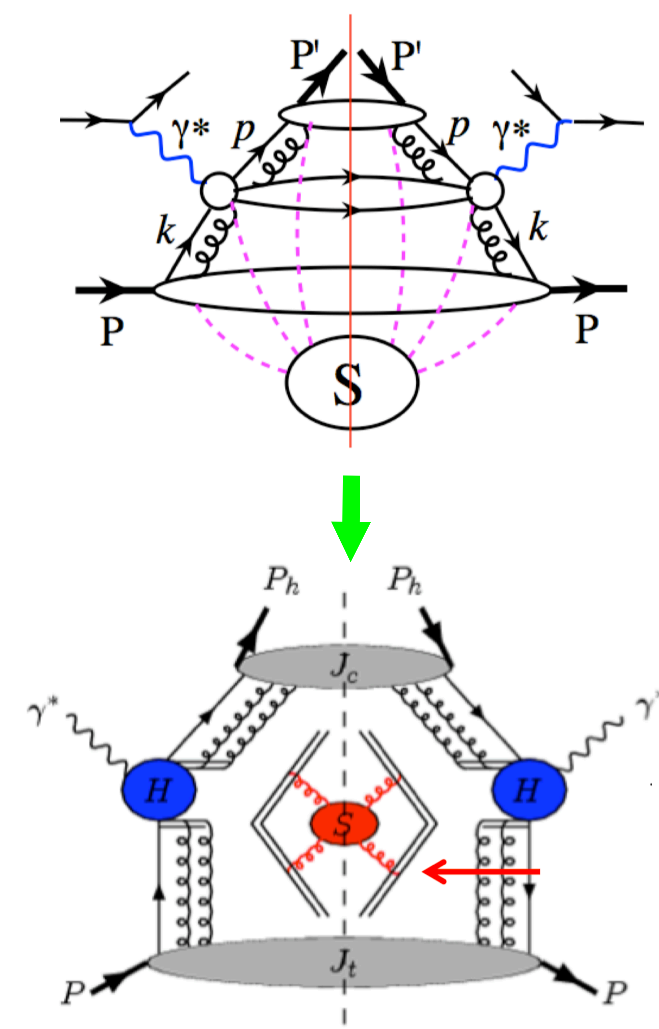
**The definition of TMDPDF:**

$$f_i^{TMD}(x, b_\perp, \mu, \zeta) = \frac{f(x, b_\perp, \mu, Y)}{\sqrt{S(b_\perp, \mu, Y, Y')}}$$

$$f(x, b_\perp, \mu, Y) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle PS | \bar{\psi}_n(\xi^-, 0, b_\perp) \gamma^+ \psi_n(0) | PS \rangle \Big|_Y$$



# TMD Soft Function

- **The TMD soft function is defined by two conjugate light-like Wilson lines:**

$$S(b_\perp, \mu, Y, Y') = \frac{1}{N_c} \mathrm{tr} \langle 0 | \bar{\mathcal{T}} \left[ U^\dagger_{n^+}(-\infty, \vec{b}_\perp)_{Y'} U^\dagger_{n^-}(\pm\infty, \vec{b}_\perp)_Y \right]$$
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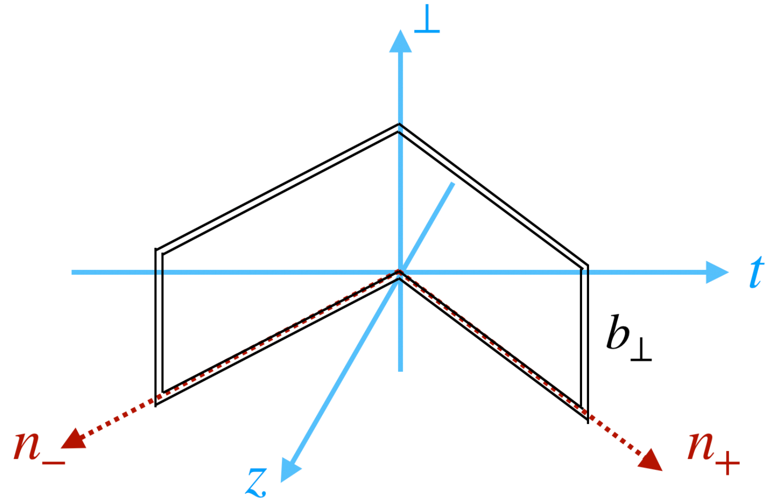

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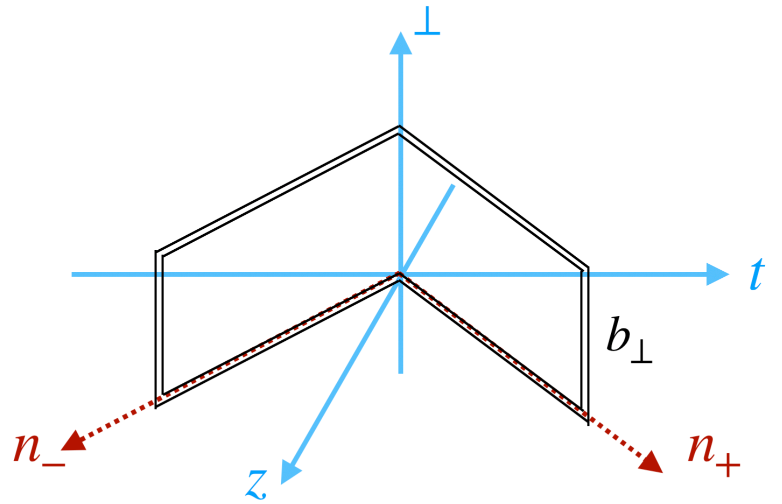


- **The intrinsic, rapidity independent, soft function $S_I(b_\perp, \mu)$:**

$$S(b_\perp, \mu, Y, Y') = e^{(Y+Y')K(b_\perp,\mu)} S_I^{-1}(b_\perp, \mu)$$

- X. Ji, Y. Liu, and Y.-S. Liu, (2019), arXiv:1910.11415 [hep-ph];
- J. Collins, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 32, 1 (2011).

**Rapidity evolution can be described by the Collins-Soper kernel $K(b_\perp, \mu)$**

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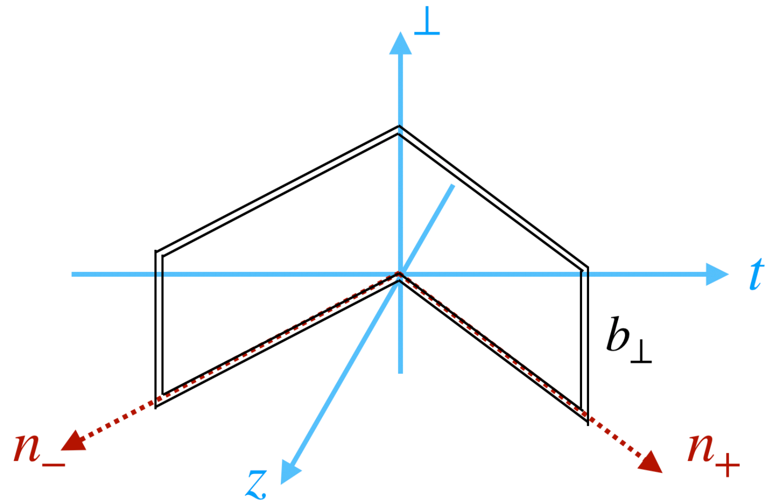


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**Rapidity evolution can be described by the Collins-Soper kernel $K(b_\perp,\mu)$**

The calculation of the intrinsic soft function paves the way towards the **first principle predictions** of physical cross sections for, e.g., SIDIS and Drell-Yan processes at small transverse momentum.

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# Calculate the TMD Soft Function on lattice

**QCD Soft Function from Large-Momentum Effective Theory on Lattice**

Xiangdong Ji,[1,2] Yizhuang Liu,[1,*] and Yu-Sheng Liu[1]

[1] *Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China*
[2] *Department of Physics, University of Maryland, College Park, MD 20742, USA*

(Dated: October 28, 2019)

We study Euclidean lattice calculation of t
lightcone directions in the framework of large
verse momentum dependent (TMD) soft funct
as the form factor of a pair of color sources t
be calculated using lattice heavy-quark effecti
torization of a large-momentum light-meson f
can also be used to extract the soft function c

**Transverse-Momentum-Dependent PDFs from Large-Momentum Effective Theory**

Xiangdong Ji,[1,2] Yizhuang Liu,[1,*] and Yu-Sheng Liu[1]

[1] *Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China*
[2] *Department of Physics, University of Maryland, College Park, MD 20742, USA*

(Dated: November 12, 2019)

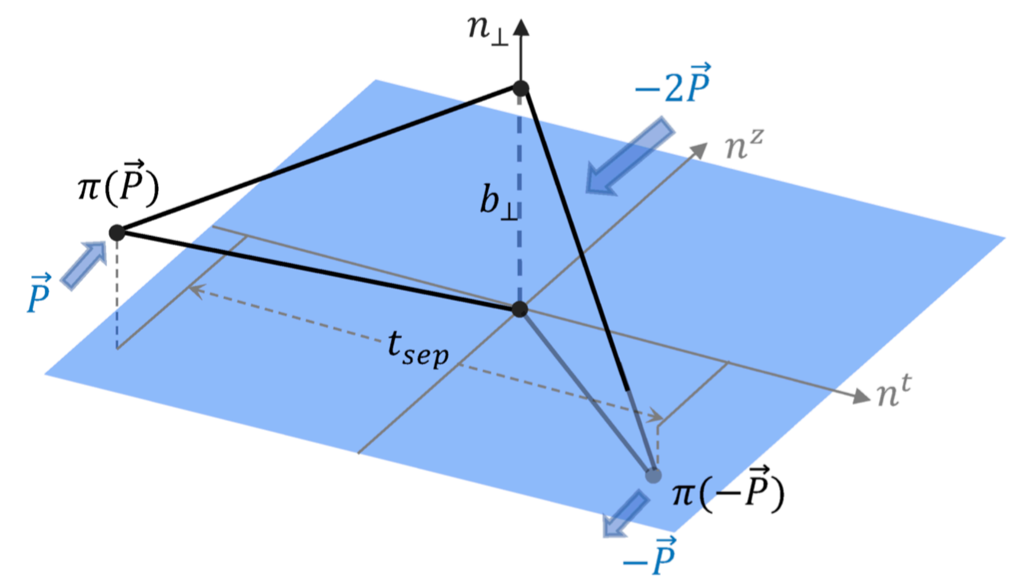
We show that the lightcone transverse-momentum-dependent parton distribution functions (TMDPDFs), important for describing high-energy scattering processes such as Drell-Yan and semi-inclusive deep-inelastic scattering with observed small transverse momentum, can be obtained from Euclidean lattice QCD calculations in the framework of large-momentum effective theory (LaMET). We present a LaMET factorization of the Euclidean quasi distributions in terms of the physical TMDPDFs at leading order in $(1/P^z)^2$ expansion, with the matching coefficient solved from a renormalization group equation. We demonstrate implementation strategies on lattice with finite-length gauge links and nonperturbative renormalization. The rapidity evolution for quasi-TMDPDF and the rapidity regularization independent factorization scheme are also discussed.

- **Ji, Xiangdong and Liu, Yizhuang and Liu, Yu-Sheng, Nucl. Phys. B 955 (2020), 115054;**
- **Ji, Xiangdong and Liu, Yizhuang and Liu, Yu-Sheng, arXiv: hep-ph/1911.03840.**

# Calculate the TMD Soft Function on lattice

1. Define a large-momentum **form factor** of a non-singlet light pseudo-scalar meson:

$$F(b_\perp, P^z) = \langle \pi(-P^z) \,|\, \left(\bar{q}_1 \Gamma q_1\right)(b_\perp)\left(\bar{q}_2 \Gamma q_2\right)(0) \,|\, \pi(P^z) \rangle$$



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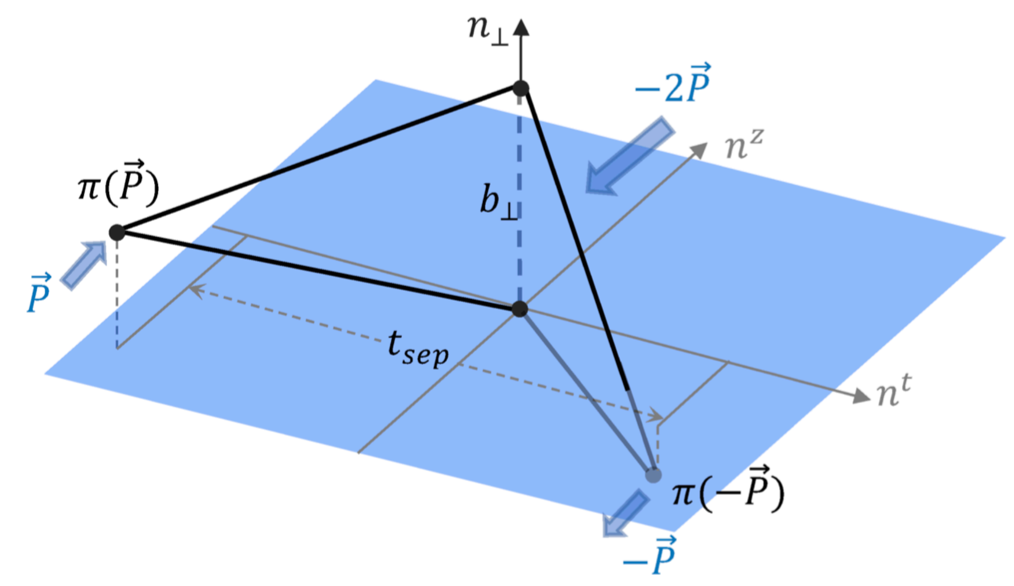



2. For large $P^z$, the **form factor** can be factorized into the **quasi-TMDWF** and the **intrinsic soft function** in the framework of **large-momentum effective theory (LaMET)**:

$$F(b_\perp, P^z) = S_I(b_\perp) \int_0^1 dx dx' H(x, x', P^z) \Phi^\dagger(x', b_\perp, -P^z) \Phi(x, b_\perp, P^z)$$

**The perturbative kernel** $H(x, x', P^z) = \dfrac{1}{2N_c} + \mathcal{O}(\alpha_s)$

# Calculate the TMD Soft Function on lattice

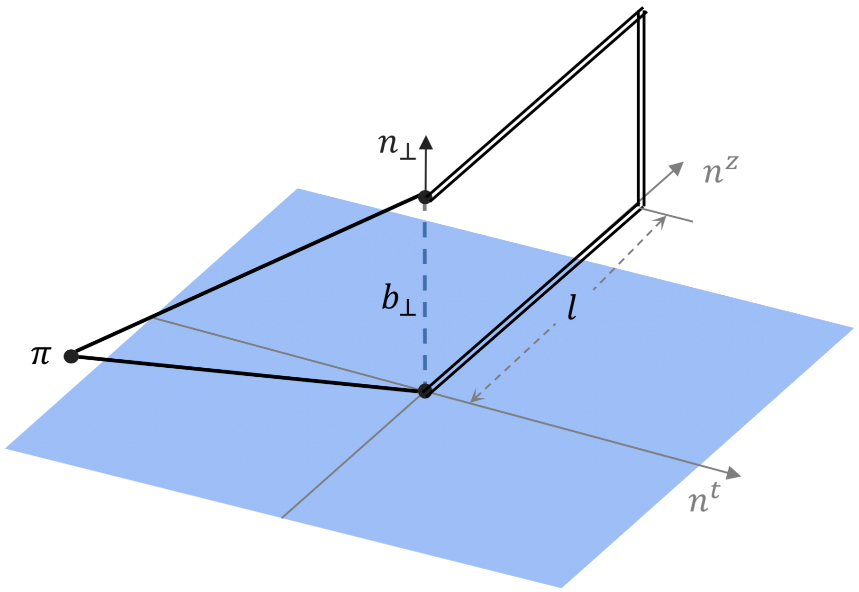
**3. The subtracted quasi-TMDWF in coordinate space:**

$\Gamma_\Phi = \gamma^t \gamma_5$

$l$**: length of the Wilson line**

$$\phi\left(z, b_\perp, P^z\right) = \lim_{\ell \to \infty} \frac{\phi_\ell\left(z, b_\perp, P^z, \ell\right)}{\sqrt{Z_E\left(2\ell, b_\perp\right)}}$$

$$= \lim_{\ell \to \infty} \frac{\left\langle 0 \left| \bar{q}_1\left(\frac{z}{2}n^z + \vec{b}\right)\Gamma_\Phi \mathscr{W}(\vec{b}, \ell) q_2\left(-\frac{z}{2}n^z\right)\right| \pi(\vec{P})\right\rangle}{\sqrt{Z_E\left(2\ell, b_\perp\right)}}$$




- $\mathscr{W}(\vec{b}, \ell)$ **is the spacelike staple-shaped gauge link.**

- $Z_E(2l, b_\perp)$ is the vacuum expectation value of a rectangular spacelike Wilson loop, which removes the **pinch-pole singularity** and **Wilson-line self-energy** in quasi-TMDWF.

# Calculate the TMD Soft Function on lattice

4. **The intrinsic soft function at leading order:**

$$S_I\left(b_\perp\right) = \frac{2N_c F\left(b_\perp, P^z\right)}{\left|\phi\left(0, b_\perp, P^z\right)\right|^2} + \mathcal{O}\left(\alpha_s, (1/P^z)^2\right)$$

The intrinsic soft function in the $\overline{\mathrm{MS}}$ **scheme:**

$$S_{I,\overline{\mathrm{MS}}}\left(b_\perp, \mu\right) = \left(\frac{S_I\left(b_\perp, 1/a\right)}{S_I\left(b_{\perp,0}, 1/a\right)}\right) S_{I,\overline{\mathrm{MS}}}\left(b_{\perp,0}, \mu\right) = \frac{F\left(b_\perp, P^z\right)}{F\left(b_{\perp,0}, P^z\right)} \frac{\left|\phi\left(0, b_{\perp,0}, P^z\right)\right|^2}{\left|\phi\left(0, b_\perp, P^z\right)\right|^2} + \mathcal{O}\left(\alpha_s, (1/P^z)^2\right)$$

calculable on lattice

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# Lattice Set Up

We do the lattice QCD calculation of the intrinsic soft function on the **A654 configurations** (generated by CLS collaboration):

| $\beta$ | $L^3 \times T$ | a (fm) | $c_{sw}$ | $\kappa_l^{\text{sea}}$ | $m_\pi^{\text{sea}}$(MeV) |
|---|---|---|---|---|---|
| 3.34 | $24^3 \times 48$ | 0.098 | 2.06686 | 0.13675 | 333 |
| | | | $N_{cfg}$ | $\kappa_l^v$ | $m_\pi^v$ (MeV) |
| | | | 868 | 0.13622 | 547 |

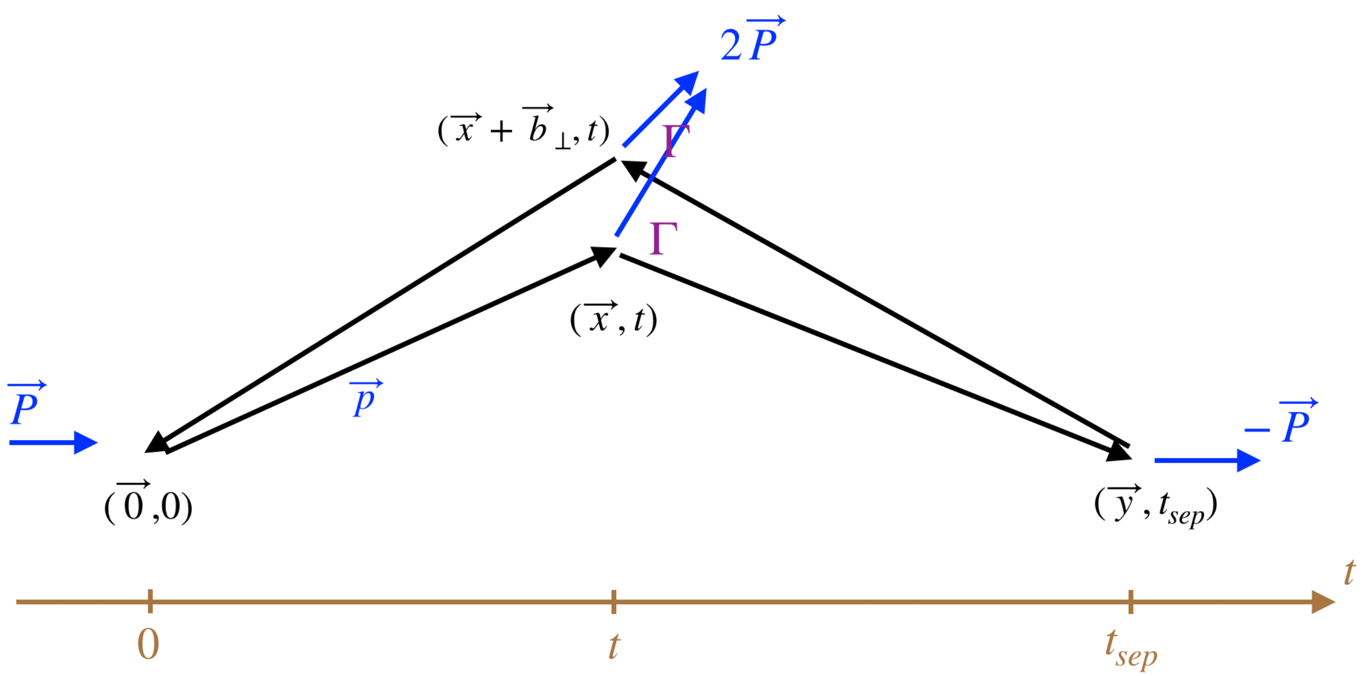
- 2+1flavor clover fermions and tree-level Symanzik gauge action;
- Coulomb gauge fixed **wall source propagators** for both initial and final states;
- $P^z = 1.05,\ 1.58,\ 2.11\text{GeV}$;
- Use $m_\pi = 547\text{MeV}$ instead of $333\text{MeV}$ to get a **better signal**;
- Physically, the soft function becomes **independent** of the meson mass for large momentum $P^z$.

# Numerical Results

The 2-point and 3-point correlation functions can be calculated on lattice:

$$C_2(b_\perp, P^z; p^z, l, t) = \frac{A_w(p^z)A_p}{2E} e^{-Et} \phi_l(0, b_\perp, P^z, l) \left(1 + c_0 e^{-\Delta E t}\right)$$

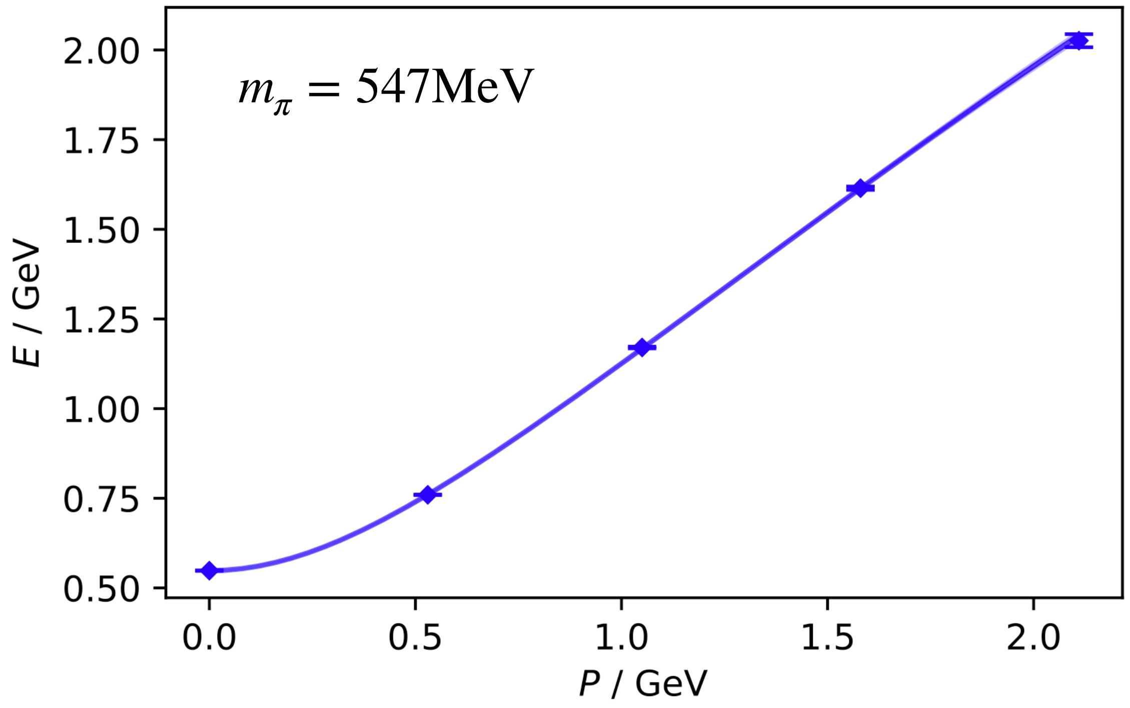
$$C_3(b_\perp, P^z; p^z, t_{sep}, t) = \frac{A_w(p^z)^2}{(2E)^2} e^{-Et_{sep}} \left[F(b_\perp, P^z) + c_1 \left(e^{-\Delta E t} + e^{-\Delta E(t_{sep}-t)}\right) + c_2 e^{-\Delta E t_{sep}}\right]$$




The quasi-TMDWF and form factor can be obtained from a **joint fit** of the 2-point and 3-point correlation functions.

# Numerical Results

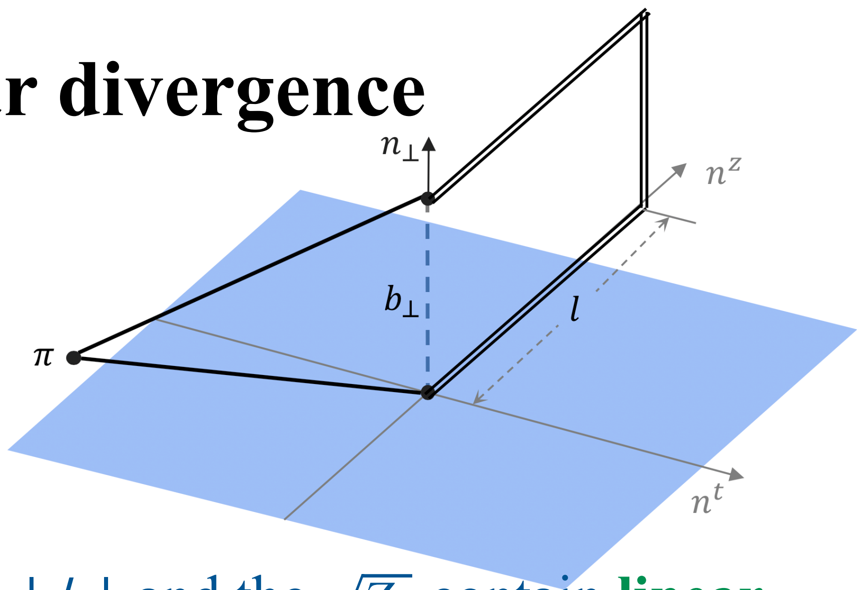
- **Simulation checks:** The dispersion relation of the pion state



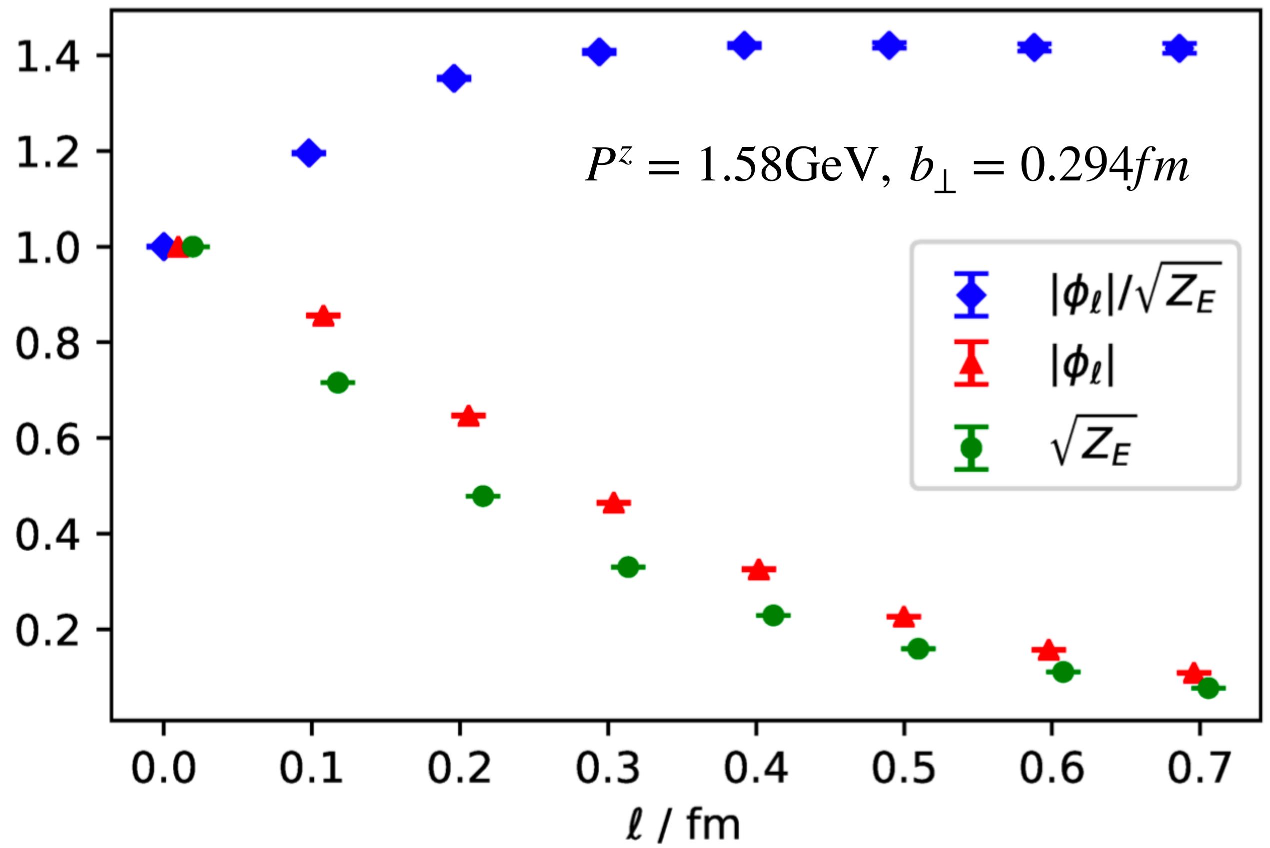

- $E_\pi = \sqrt{m_\pi^2 + c_1 P^2 + c_2 a^2 P^4}$;
- Fit results: $c_1 = 0.9945(40)$ and $c_2 = -0.0053(5)$;
- **Consistent** with the dispersion relation of pion in the continuum limit with error.

# Numerical Results

- $l$-dependence of quasi-TMDWF and Linear divergence




$$\phi\left(z, b_\perp, P^z\right) = \lim_{\ell\to\infty} \frac{\phi_\ell\left(z, b_\perp, P^z, \ell\right)}{\sqrt{Z_E\left(2\ell, b_\perp\right)}}$$



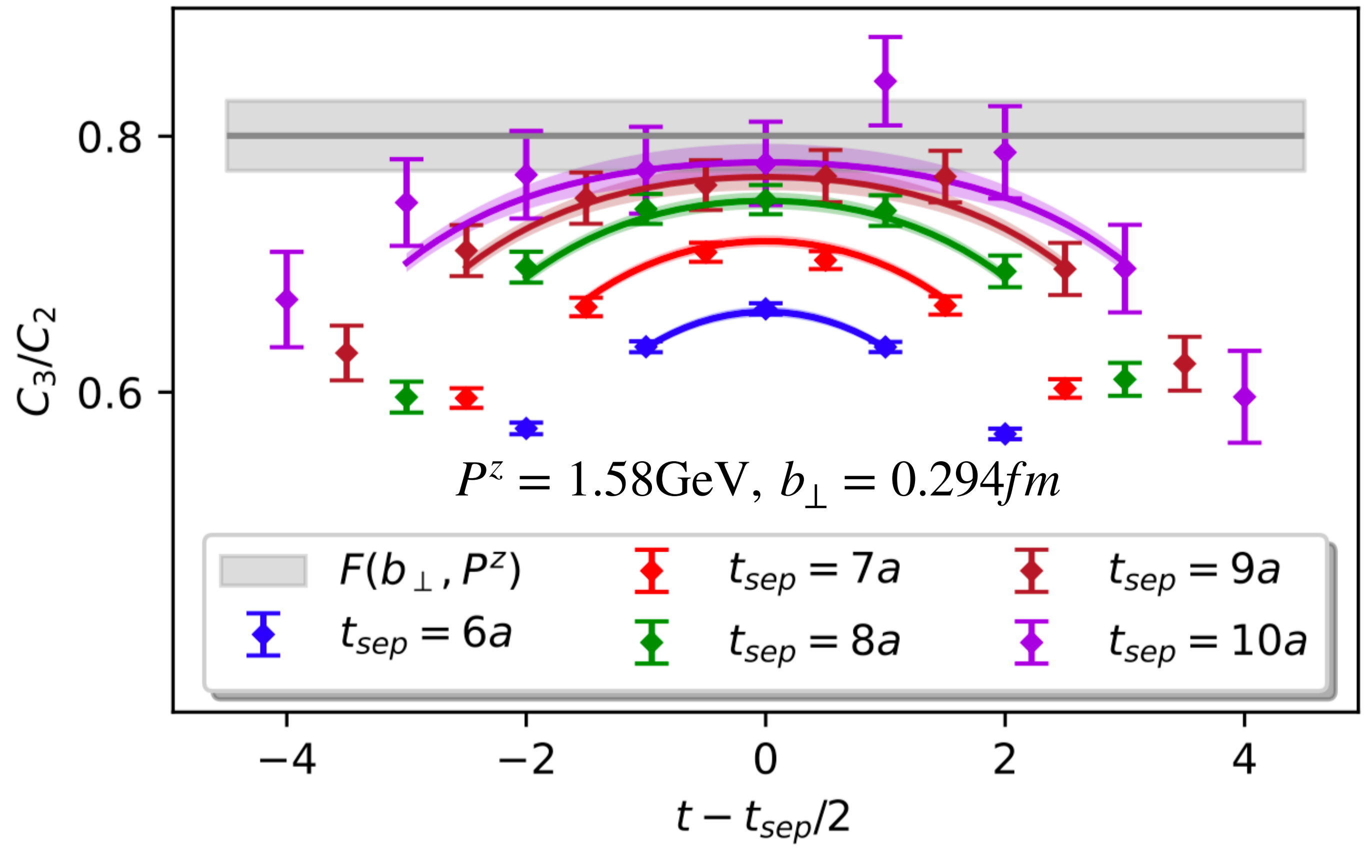

- Both the $|\phi_l|$ and the $\sqrt{Z_E}$ contain **linear divergence;**
- Linear divergence **cancelled** at large-$l$;
- $\phi$ is **length independent** when $l > 0.4fm$;
- We use $l = 7a = 0.686fm$ as asymptotic results for all cases in our calculation to **remove the linear divergence**.

# Numerical Results

- **Joint fit results of the form factor**

$$C_2(b_\perp, t) \sim \langle O_\pi(t_{sep})\bar{O}_\pi(0)\rangle = \frac{A_w A_p}{2E} e^{-Et}\phi_l\left(1 + c_0 e^{-\Delta E t}\right)$$

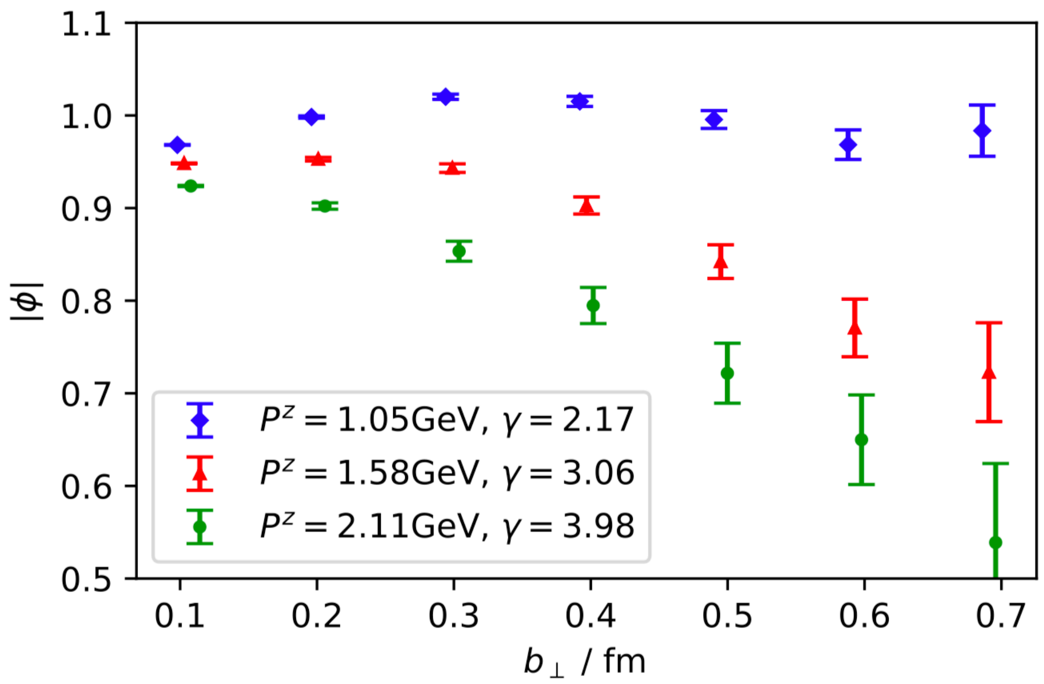
$$C_3(b_\perp, t) \sim \langle O_\pi(t_{sep})O_\Gamma(t)\bar{O}_\pi(0)\rangle = \frac{A_w^2}{(2E)^2} e^{-Et_{sep}}\left[F + c_1\left(e^{-\Delta E t} + e^{-\Delta E(t_{sep}-t)}\right) + c_2 e^{-\Delta E t_{sep}}\right]$$




- Fitted data at $t_{sep} = 6 \sim 10a$;
- $F(b_\perp, P^z)$ corresponds to the **ground state contribution at $t_{sep} \to \infty$**;
- $\chi^2/\mathrm{d.o.f} = 0.6$, our data in general **agree with** the predicted fit function.

# Numerical Results

- **Joint fit results of the quasi-TMDWF**




- Upper panel is the $P^z$-dependence of quasi-TMDWF;
- The $P^z$-dependence in $|\phi|$ is related to the **Collin-Soper kernel**;

# Numerical Results

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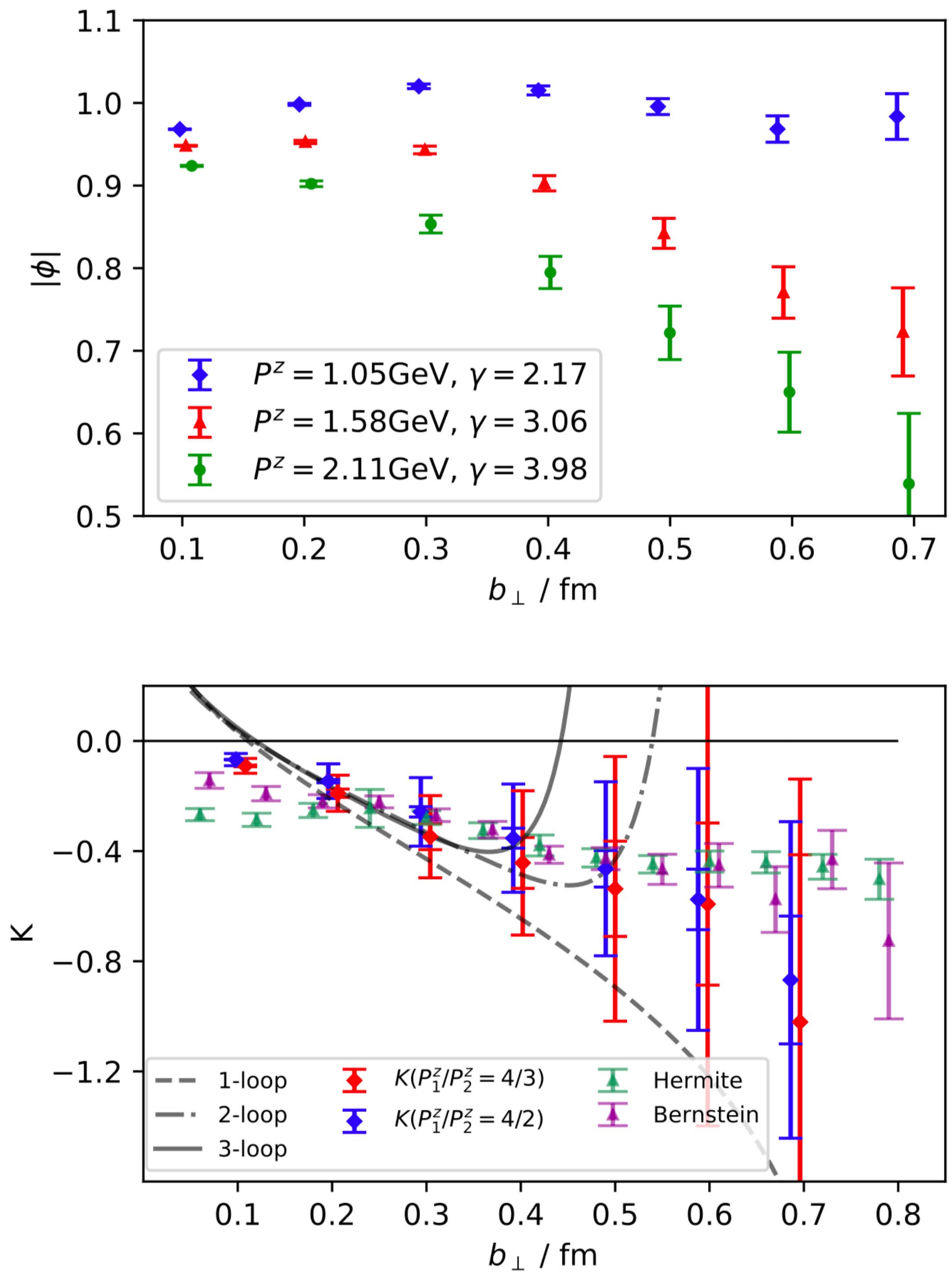



- Upper panel is the $P^z$-dependence of quasi-TMDWF;
- The $P^z$-dependence in $|\phi|$ is related to the **Collin-Soper kernel**;

$$K(b_\perp, \mu) = \frac{1}{\ln\left(P_1^z/P_2^z\right)} \ln \left| \frac{\phi\left(0, b_\perp, P_1^z\right)}{\phi\left(0, b_\perp, P_2^z\right)} \right| + \mathcal{O}\left(\alpha_s, \gamma^{-2}\right)$$

- Lower panel shows the extracted Collins Soper kernel, compare with **perturbative calculations up to 3-loops*** and results from **quenched lattice calculations of TMDPDF****.
- Our results are **consistent with** perturbative calculations (at small $b_\perp$ ) and results from the TMDPDF.

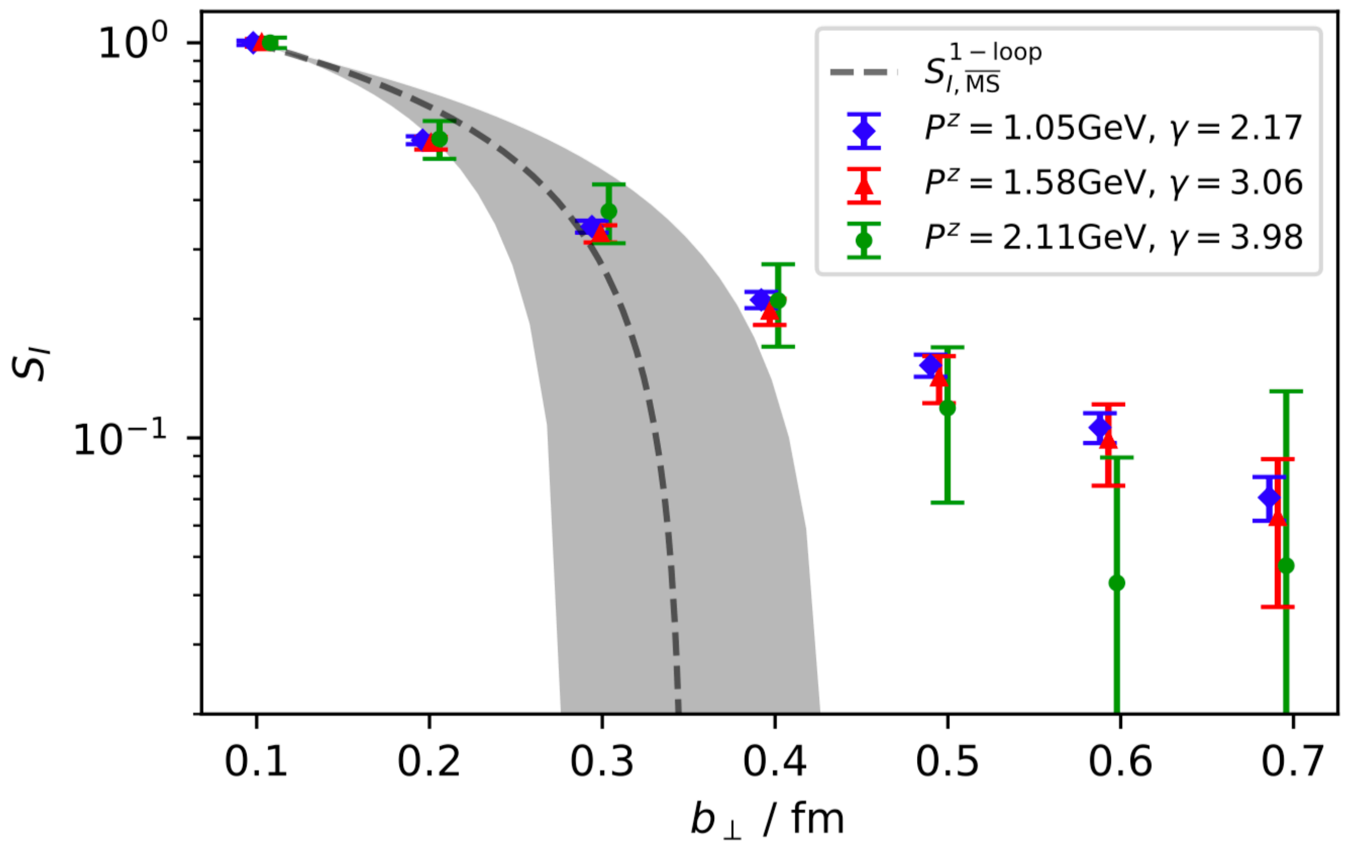
*** Li and Zhu, Phys. Rev. Lett.118(2017)2, 022004;**

**** Shanahan and Wagman and Zhao, arXiv:hep-lat/2003.06063.**

# Numerical Results

- **Joint fit results of the intrinsic soft function**

$$S_{I,\overline{\rm MS}}\left(b_\perp,\mu\right) = \frac{F\left(b_\perp, P^z\right)}{F\left(b_{\perp,0}, P^z\right)} \frac{\left|\phi\left(0,b_{\perp,0},P^z\right)\right|^2}{\left|\phi\left(0,b_\perp,P^z\right)\right|^2} + \mathcal{O}\left(\alpha_s, (1/P^z)^2\right)$$




- With different $P^z$ , the results are **consistent** with each other, demonstrating that the **asymptotic limit** is stable within errors;
- The systematic uncertainty from the **operator mixing** has been taken into account;
- The dashed curve shows the result of the **1-loop calculation** with the strong coupling constant $\alpha_s\left(1/b_\perp\right)$, and the shaded band corresponds to the scale uncertainty of $\alpha_s$:

$$S_{I,\overline{\rm MS}}\left(b_\perp,\mu\right) = 1 - \frac{\alpha_s C_F}{\pi} \ln \frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}} + \mathcal{O}\left(\alpha_s\right)$$

# Summary and Outlook

- This work present an **exploratory** lattice calculation of the intrinsic soft function;
- The Collin-Soper kernel obtained from our quasi-TMDWF agrees with the perturbative results and previous quenched lattice calculations;
- Our results of the intrinsic soft function are almost **independent** of the hadron momentum, and **consistent with** the 1-loop perturbative calculation;
- This work paves the way towards the **first principle predictions** of physical cross sections for, e.g., Drell-Yan and Higgs productions at small transverse momentum.

Thank you !

# Heatmap for the NPR factor and operator mixing effective of our result with $P^z = 1.58\text{GeV},\ b_\perp = \{0,\ 0.294,\ 0.588\}\text{fm}$