Transverse momentum dependent factorization for lattice observables

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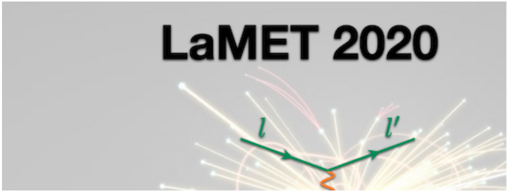
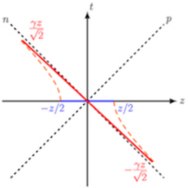
based on [A.Schäfer, AV,2002.07527]

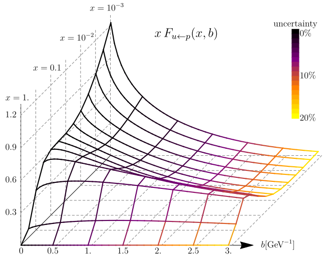


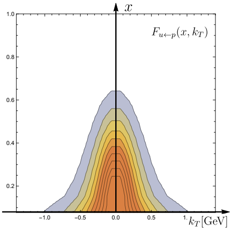


In the limit of large hadron's momentum transverse momentum dependent (TMD) factorization can be applied to certain lattice observables.

Plenty of examples for collinear factorization (sensitive to PDF and GPD)

Could one study TMDs with lattice? $\Rightarrow$ **YES**





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Collinear factorization → PDFs

$$\bar{q}(z)\gamma^\mu[z,0]q(0) = \int_0^1 du \frac{\partial}{\partial z^\mu} \underbrace{\bar{q}(uz)\, \not{z}[uz,0]q(0)}_{\text{tw-2}} (1+\alpha_s ...) + z^2[\text{tw-3/4}] + ...$$

$$\langle p|\bar{q}(z)\gamma^\mu[z,0]q(0)|p\rangle = 2p^\mu \int_{-1}^1 dx e^{ix(pz)} \underbrace{f_1(x, x(pv))}_{\text{PDF}} (1+\alpha_s ...) + z^2[\text{tw-2/3/4}] + ...$$

Collinear factorization → PDFs

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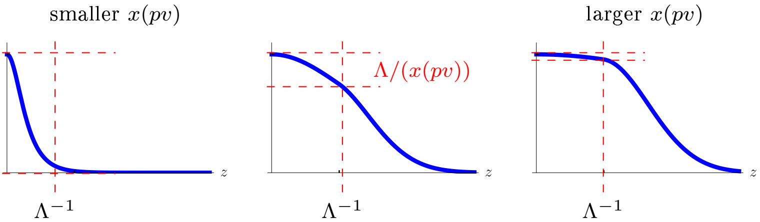
$$\langle p|\bar{q}(z)\gamma^{\mu}[z,0]q(0)|p\rangle = 2p^{\mu} \int_{-1}^{1} dx e^{ix(pz)} \underbrace{f_1(x, x(pv))}_{\text{PDF}} \color{red}{(1+\alpha_s \dots)} + z^2[\text{tw-2/3/4}] + \dots$$



Larger (pv) = smoother asymptotic $z^2 \to 0$.

One can construct plenty of such observables

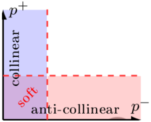
(Thursday session!)[Braun,Muller,07; Ji,13; Radyushkin, 16; Ma,Qiu,17;...]

That does not work for TMD-like observables.
There is no twist-expansion, instead there is field-mode separation

Field modes:

$$q \to q_{\text{collinear}} + q_{\text{anti-collinear}} + q_{\text{soft}} + q_{\text{hard}}$$

$$\begin{aligned} p_{\text{collinear}} &\sim \{1, \lambda^2, \lambda\} \\ p_{\text{anti-collinear}} &\sim \{\lambda^2, 1, \lambda\} \\ p_{\text{soft}} &\sim \{\lambda^2, \lambda^2, \lambda\} \end{aligned}$$



Central assumption:
fast moving hadron has only colinear fields

$$\lim_{p\to p^+\bar{n}} |p\rangle \simeq \Psi\left(q_{\text{collinear}}, A_{\text{collinear}}\right)|0\rangle$$

If the observable is “spherically-symmetric” then this approach gives collinear factorization

If there are some (finite) transverse elements then this approach gives TMD factorization

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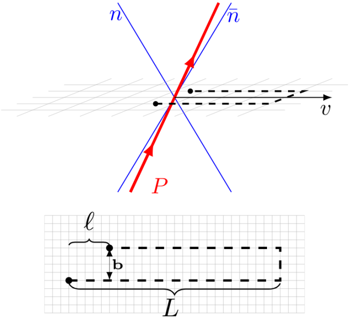


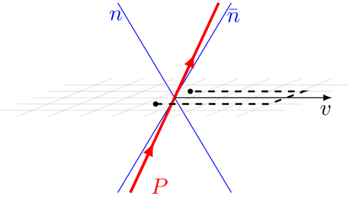
Restrictions on observable

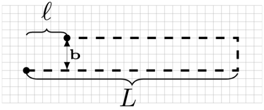
- Equal-time
- With transverse size $(bP) = 0$
- With anti-collinear modes

Simplest case:

$$W^{[\Gamma]}_{f\leftarrow h}(b;\ell,L;v,P,S) = \frac{1}{2}\langle P,S|\bar{q}_f(b+\ell v)\Gamma \times [b+\ell v, b+Lv][b+Lv, Lv][Lv,0]q_f(0)|P,S\rangle, \quad (1)$$

Γ = some Dirac structure





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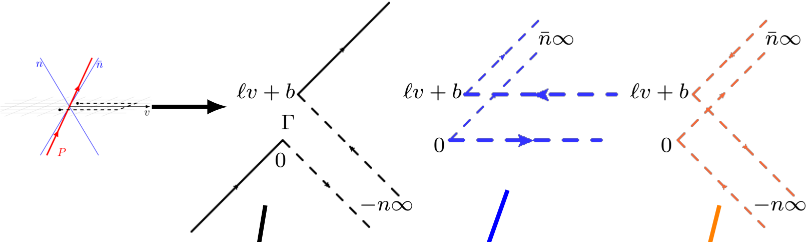
Γ = some Dirac structure

It is like DIS+(instant)jet

$$\text{At } L\to\infty \qquad [0,Lv]\to H(0) \qquad (\text{with } \mathcal{L}_{HH} = H^\dagger(ivD)H)$$

$$\text{current } J_i(x) = H^\dagger(x)q(x), \qquad \text{hadron tensor } W_{ij} = \langle P|J_i^\dagger(x)J_j(0)|P\rangle$$

Factorization is (almost) equivalent to factorization of SIDIS or DY



$$W^{[\Gamma]}_{f\leftarrow h}(b;\ell,L;v,P,S) =$$
$$\left|C_H(\hat{p}v)\right|^2 \langle P,S| \left(\bar{\xi}W_n(b+\ell v)\frac{\Gamma}{2}W_n^\dagger\xi(0)\right)\left(H^\dagger W_{\bar{n}}(0)W_{\bar{n}}^\dagger H(b+\ell v)\right)\frac{\mathrm{Tr}}{N_c}\left[Y_n^\dagger Y_{\bar{n}}(b+\ell v)Y_{\bar{n}}^\dagger Y_n(0)\right]|P,S\rangle.$$

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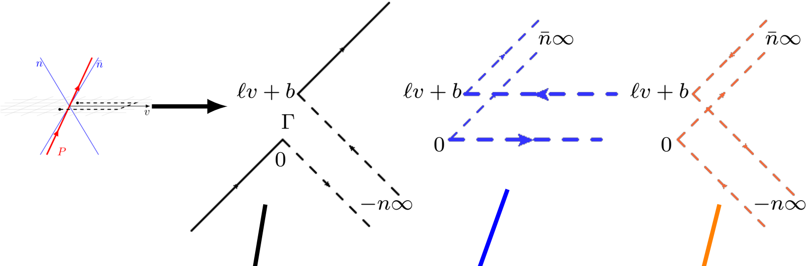

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Negelecting power corrections and accounting the overlap in the soft modes

$$W^{[\Gamma]}_{f\leftarrow h}(b;\ell,L;v,P,S) = \left|C_H(\hat{p}v)\right|^2 \widetilde{\Phi}^{[\Gamma']}_{f\leftarrow h}(b,\ell v^-;P,S)\widetilde{\Psi}(b,\ell v^+;v)\frac{S(b)}{\text{Z.b.}}$$

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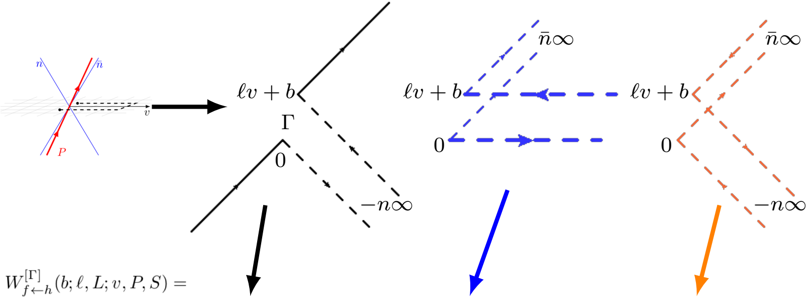
Factorization is (almost) equivalent to factorization of SIDIS or DY



$$W^{[\Gamma]}_{f\leftarrow h}(b;\ell,L;v,P,S) =$$
$$\Big|C_H(\hat p v)\Big|^2 \langle P,S| \left(\bar\xi W_n(b+\ell v)\frac{\Gamma}{2}W_n^\dagger \xi(0)\right)\left(H^\dagger W_{\bar n}(0)W_{\bar n}^\dagger H(b+\ell v)\right)\frac{\mathrm{Tr}}{N_c}\left[Y_n^\dagger Y_{\bar n}(b+\ell v)Y_{\bar n}^\dagger Y_n(0)\right]|P,S\rangle.$$

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operator of parton's momentum ($\hat p \sim xP$)
Fourier conjugated to ℓ

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Components of factorized expression (intermediate)

unsubtracted TMDPDF (in position space)

$$\widetilde{\Phi}^{[\Gamma]}_{f\leftarrow h}(b,x^-;P,S) = \langle P,S|\bar{q}(x^-n+b)[x^-n+b,-\infty n+b]\frac{\Gamma}{2}[-\infty n,0]q(0)|P,S\rangle,$$

- Rapidity divergent!
- The leading power contribution selects $\Gamma = \Gamma'$

$$\Gamma' = \frac{1}{4}\gamma^+\gamma^-\Gamma\gamma^-\gamma^+$$

Hard coefficient function [Ebert,Stewart,Zhao;1811.00026][AV,Schäfer;2002.07527]

$$|C_H|^2 = 1 + C_F\frac{\alpha_s}{4\pi}\left(-\mathbf{L}^2 + 2\mathbf{L} - 4 + \zeta_2\right) + \alpha_s^2...$$

- Independent on Γ (for leading twist Γ) $\rightarrow$ talk by S.Schindler

Components of factorized expression (intermediate)

"instant jet TMD function" (in position space)

$$\widetilde{\Psi}(b,x^+;v)=\langle 0|H^\dagger(0)[0,-\infty\bar n][-\infty\bar n+b,x^+\bar n+b]H(x^+\bar n+b)|0\rangle$$

- Rapidity divergent!

TMD soft factor

$$S(b)=\frac{\mathrm{Tr}}{N_c}\langle 0|[b,-n\infty+b][-n\infty,0][0,-\bar n\infty][-\bar n\infty+b,b]|0\rangle$$

- Rapidity divergent! Rapidity divergent!
- Z.b. = zero-bin subtractions, (in some regularizations) equals to S^2

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Recombination of rapidity divergences and final form

n-rap.div. cancel $\bar{n}$-rap.div. cancel

$$W^{[\Gamma]} = |C_H(\hat{p}v)|^2 \, \tilde{\Phi}^{[\Gamma']}(b, \ell v^-; P, S) \, \frac{S(b)}{\text{Z.b.}} \, \tilde{\Psi}(b; v) \quad + \text{power corrections}$$

$R_{\text{TMD}}(\zeta) R^{-1}_{\text{TMD}}(\zeta)$

R_{TMD} also contains a finite and non-pertrubative parts such that $\tilde{\Phi} R_{\text{TMD}} = \Phi$ is **universal TMD**.

$$R_{\text{TMD}} = \frac{1}{\sqrt{S_{\text{TMD}}}}$$

Up to minor details it coincides with [Ebert,Stewart,Zhao,19] and [Ji,Liu,Liu,19]

Recombination of rapidity divergences and final form

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$R_{\text{TMD}}(\zeta)R^{-1}_{\text{TMD}}(\zeta)$

$$W^{[\Gamma]} = |C_H\left(\frac{\hat{p}v}{\mu}\right)|^2 \; \Phi^{[\Gamma']}(b,\ell v^-;\mu,\zeta;P,S) \;\; \Psi(b;\mu,\bar{\zeta};v) \quad + \text{power corrections}$$

- $\Phi^{[\Gamma']}(b,\ell v^-;\mu,\zeta;P,S)$ is ordinary TMD distribution (e.g. $\Phi^{[\gamma^+]} = f_1 + (b\times s)Mf_{1T}^{\perp}$)
- $\Psi(b;\mu,\bar{\zeta};v)$ is composition of soft-factors; $\Psi \sim \tilde{\Psi}/\sqrt{S}$

$$\zeta\bar{\zeta} = (2\hat{p}^+v^-)^2\mu^2 \sim (2xP^+v^-)^2\mu^2$$

Up to minor details it coincides with [Ebert,Stewart,Zhao,19] and [Ji,Liu,Liu,19]

Sources and sizes of power corrections

$$W^{[\Gamma]} = |C_H\left(\frac{\hat{p}v}{\mu}\right)|^2\ \Phi^{[\Gamma']}(b,\ell v^-;\mu,\zeta;P,S)\ \ \Psi(b;\mu,\bar{\zeta};v) \quad + \textbf{power corrections}$$

- $\dfrac{P^-}{x^2P^+}$ and $\dfrac{\ell}{L}$ from collinear/anti-collinear modes separation
- $\dfrac{1}{x|b|P^+}$ from collinear/transverse modes separation
- $\dfrac{b}{L}$ from anti-collinear/transverse modes separation
- $\ell\Lambda_{\text{QCD}}$ to remove ℓ-dependence from Ψ

Factorization limit: $L\to\infty$, $P^+\to\infty$, b-fixed(non-zero), ℓ-fixed(also zero).

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$\Psi(b;\mu,\bar{\zeta};v)$ is generally unknown (see talk by Qi-An Zhang)
It cancels in the ratios with same b.
If the same v, ℓ L are taken the Wilson-line renormalization factor also cancel.

$$R = \frac{W^{[\Gamma_1]}_{f_1\leftarrow h_1}(b;\ell,L,v;P_1,S_1;\mu)}{W^{[\Gamma_2]}_{f_2\leftarrow h_2}(b;\ell,L,v;P_2,S_2;\mu)}$$
$$= \frac{|C_H\left(\frac{\hat{p}v}{\mu}\right)|^2\ \Phi^{[\Gamma_1']}_{f_1\leftarrow h_1}(b,\ell v^-;\mu,\zeta;P_1,S_1)}{|C_H\left(\frac{\hat{p}v}{\mu}\right)|^2\ \Phi^{[\Gamma_2']}_{f_2\leftarrow h_2}(b,\ell v^-;\mu,\zeta;P_2,S_2)} + \textbf{power corrections}$$

Plenty of information/tests

- Test power corrections! E.g. $\Gamma_1=\gamma^-$ and $\Gamma_2=\gamma^+$ then $R=$ **power corrections**
- ...
- Collins-Soper kernel

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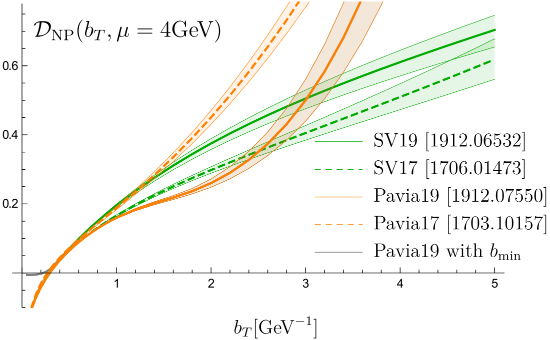
Collins-Soper kernel

- CS-kernel dictates evolution for TMD distribution
- Is a non-perturbative function
- Perturbative at small-b (known to NNLO=three-loops)
- Describes QCD vacuum properties [AV;2003.02288]
- Extracted from data

Phenomenological extractions



Extraction of Collins-Soper kernel $\mathcal{D} = -\frac{K}{2}$
Ratio at different $P_{1,2}$ and rest all the same [Ebert,Stewart,Zhao,1811.00026]

$$R_{P_1/P_2} = \frac{P_2^+}{P_1^+} \frac{\int dx_1 e^{ix_1 \ell v^- P_1^+} \left| C_H \left(\frac{x_1 v^- P_1^+}{\mu} \right) \right|^2 \Phi_{f\leftarrow h}^{[\Gamma']}(x_1, b; \mu, \zeta_1)}{\int dx_2 e^{ix_2 \ell v^- P_2^+} \left| C_H \left(\frac{x_2 v^- P_2^+}{\mu} \right) \right|^2 \Phi_{f\leftarrow h}^{[\Gamma']}(x_2, b; \mu, \zeta_2)} + \textbf{power corr.}$$

here $\zeta_1 = c_0(2|x_1 v^-|P_1^+)^2$ and $\zeta_2 = c_0(2|x_2 v^-|P_2^+)^2$.

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here $\zeta_1 = c_0(2|x_1 v^-|P_1^+)^2$ and $\zeta_2 = c_0(2|x_2 v^-|P_2^+)^2$.

$$R_{P_1/P_2} = \left(\frac{P_2^+}{P_1^+}\right)^{2\mathcal{D}(b,\mu)+1} \frac{\int dx_1 e^{ix_1 \ell v^- P_1^+} \left| C_H\left(\frac{x_1 v^- P_1^+}{\mu}\right)\right|^2 \Phi^{[\Gamma']}_{f\leftarrow h}(x_1, b)|x_1|^{-2\mathcal{D}(b,\mu)}}{\int dx_2 e^{ix_2 \ell v^- P_2^+} \left| C_H\left(\frac{x_2 v^- P_2^+}{\mu}\right)\right|^2 \Phi^{[\Gamma']}_{f\leftarrow h}(x_2, b)|x_2|^{-2\mathcal{D}(b,\mu)}} + \textbf{po}$$

- Both TMDs at the same (μ, ζ)-point (e.g. to null-evolution-line)
- Convergence problem! $\mathcal{D} > 0$

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Extraction of Collins-Soper kernel $\mathcal{D} = -\frac{K}{2}$
Ratio at different $P_{1,2}$ and rest all the same [Ebert,Stewart,Zhao,1811.00026]

To facilitate cancellation set $\ell = 0$

$$R_{P_1/P_2} = \left(\frac{P_2^+}{P_1^+}\right)^{2\mathcal{D}(b,\mu)+1} \frac{\int dx \left|C_H\left(\frac{xv^- P_1^+}{\mu}\right)\right|^2 \Phi^{[\Gamma']}_{f\leftarrow h}(x,b)|x|^{-2\mathcal{D}(b,\mu)}}{\int dx \left|C_H\left(\frac{xv^- P_2^+}{\mu}\right)\right|^2 \Phi^{[\Gamma']}_{f\leftarrow h}(x,b)|x|^{-2\mathcal{D}(b,\mu)}} + \textbf{power corr.}$$

TMDs do not cancel only due to perturbative logarithms (here $\mu = 2|v^-|\sqrt{P_1^+ P_2^+}$)

$$R_{P_1/P_2}(\ell = 0) = \left(\frac{P_2^+}{P_1^+}\right)^{2\mathcal{D}(b,\mu)+1} \mathbf{r} + \textbf{power corrections} \qquad (1)$$

$$\mathbf{r} = 1 + 4C_F\frac{\alpha_s(\mu)}{4\pi}\ln\left(\frac{P_1^+}{P_2^+}\right)\left[1 - 2M^{\Gamma}_{\ln|x|}(b,\mu)\right] + \mathcal{O}(\alpha_s^2)$$

details in [Schäfer,AV,2002.07527]

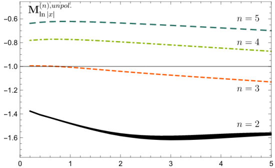
$$M^{\Gamma}_{\ln|x|}(b,\mu)=\frac{\int dx \ln|x| |x|^{-2\mathcal{D}+1}\Phi^{[\Gamma]}(x,b)}{\int dx |x|^{-2\mathcal{D}+1}\Phi^{[\Gamma]}(x,b)}$$

Numerator and denominator could diverge at $x\to 0$

- Convergent properties of the integral strongly depends on Γ
  - $\Gamma=\gamma^+$ divergent for all b.
  - $\Gamma=\gamma^+\gamma^5$ divergent for large b.
  - $\Gamma=\sigma^{\mu+}\gamma^5$ convergent.
- Phenomenological studies shows that $M_{\ln|x|}$ is slow function of b. The test on phenomenological extractions shows

$$\mathbf{r}=\mathbf{constant}+2-3\%(b).$$

for $b\sim 1-5\text{GeV}^{-1}$.



→ **talk by M.Schlemmer /Friday/**

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Conclusion

- In the **Large Momentum** regime one can apply TMD factorization for lattice observables
- The factorization theorem is more cumbersome and contaminated by extra factors and (expectedly) strong power corrections
- Some combinations (ratios) are much clearer/simpler to measure and they still give a valuable information about TMDs

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