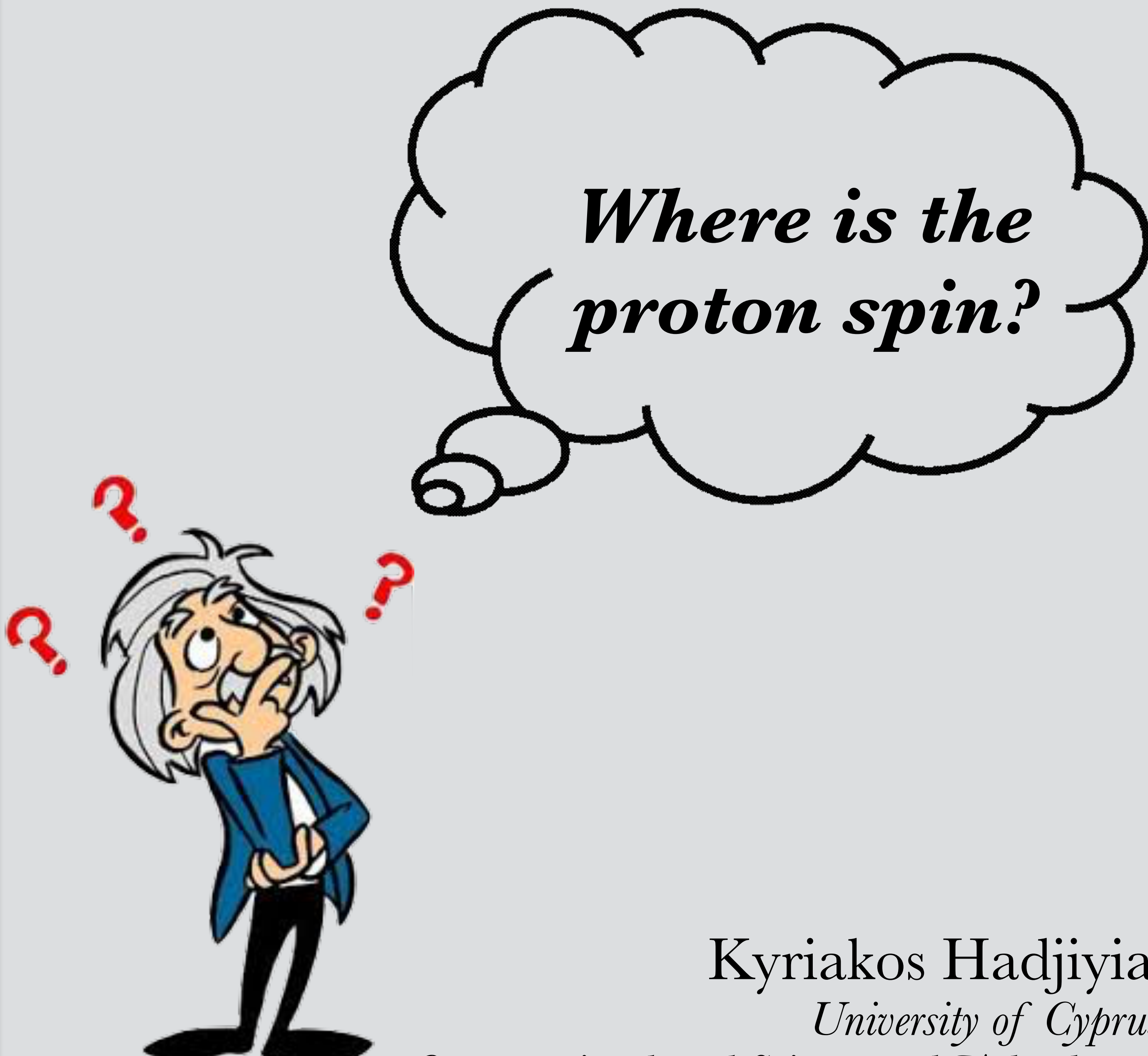
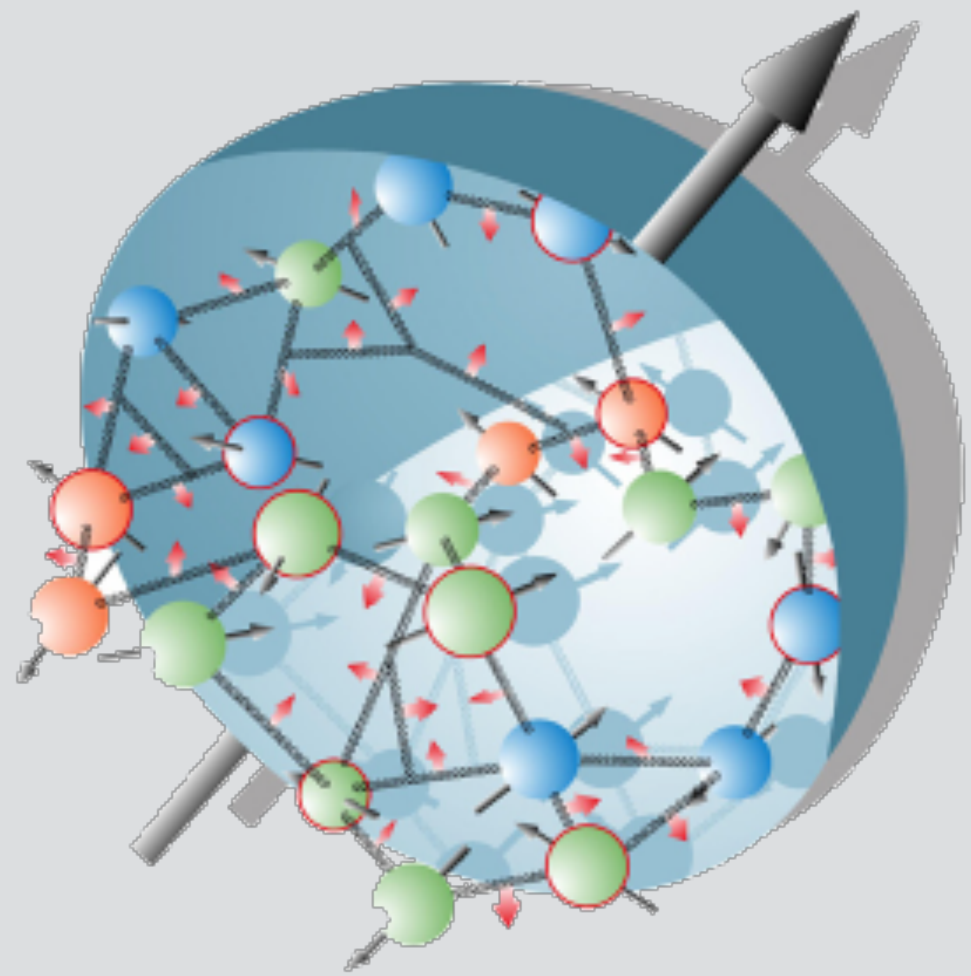
Decomposing the proton spin to quark and gluon contributions





Kyriakos Hadjiyiannakou

University of Cyprus and
Computation-based Science and Technology Research Centre (CaSToRC),
The Cyprus Institute

LaMET 2020
Sept. 7-11

In collaboration with

- C. Alexandrou (University of Cyprus, The Cyprus Institute)
- S. Bacchio (The Cyprus Institute)
- M. Constantinou (Temple University)
- J. Finkenrath (The Cyprus Institute)
- K. Jansen (NIC, DESY)
- G. Koutsou (The Cyprus Institute)
- H. Panagopoulos (University of Cyprus)
- G. Spanoudes (University of Cyprus)

Work published in:

➡ **Phys. Rev. D 101, 094513 (2020)**

➡ Phys. Rev. Lett. 119, 142002 (2017)

Proton Spin Crisis

- Up to 1980s physicists expected that quarks carry all the proton spin

Non-relat. quark model

$$\frac{1}{2}\left(\Delta u_{\mathrm{v}} + \Delta d_{\mathrm{v}}\right) = \frac{1}{2}$$

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$\Delta\Sigma_{q+}$: Quark helicity

L_{q+} : Quark orbital angular momentum

J_g : Gluon contribution to the nucleon spin

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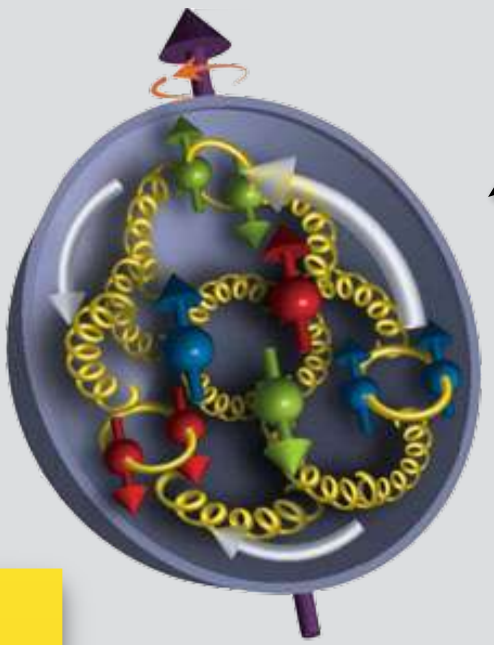

Complete picture

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta\Sigma_{q+} + L_{q+}\right) + J_g$$

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Alternative approach: Jaffe-Manohar not investigated in our study

Operators relevant to proton spin

- First moments of PDFs and GPDs are readily accessible on the lattice
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Orbital angular momentum $L^q = J^q - \frac{1}{2}\Delta\Sigma^q$

Extracting connected and disconnected contributions

Maximally twisted fermions:

→ **Automatic $\mathcal{O}(a)$ improvement**
R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007, 0306014

→ **Very attractive for hadron structure**

- Configurations Simulation by ETMC
- Dynamical quarks: N_f =2+1+1
- Lattice size: 64^3 x 128

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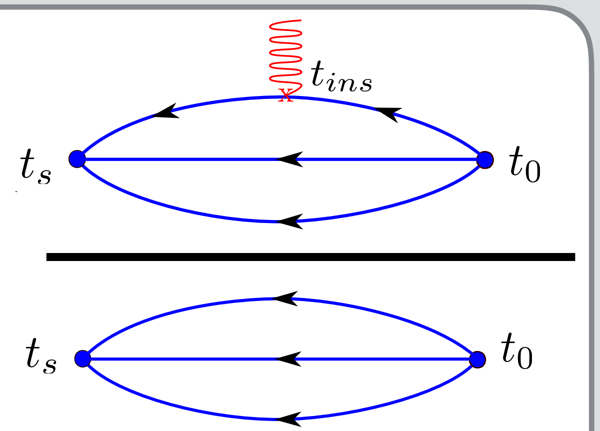
$R =$ 

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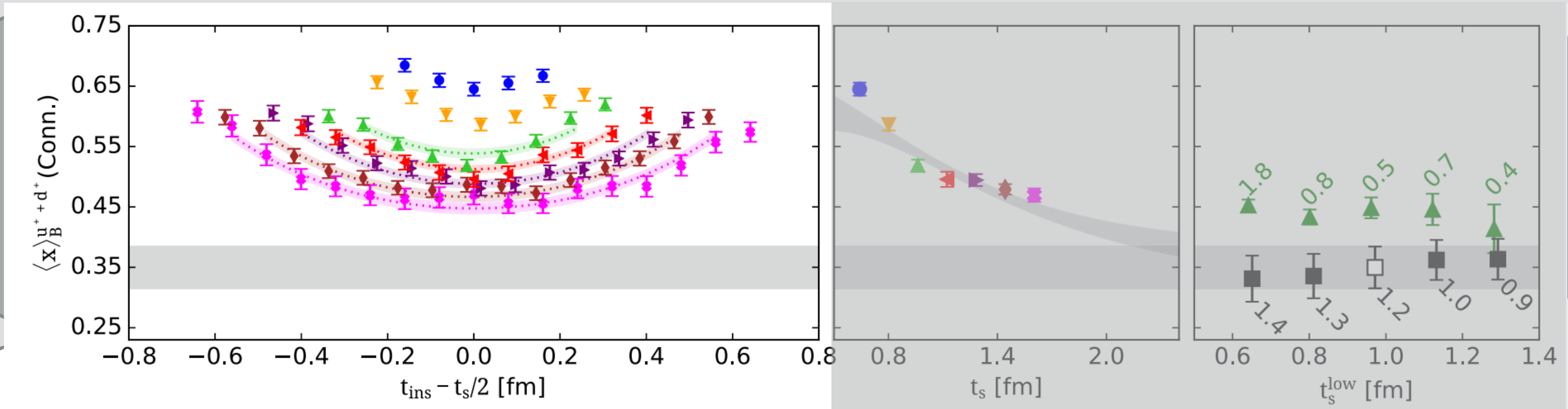

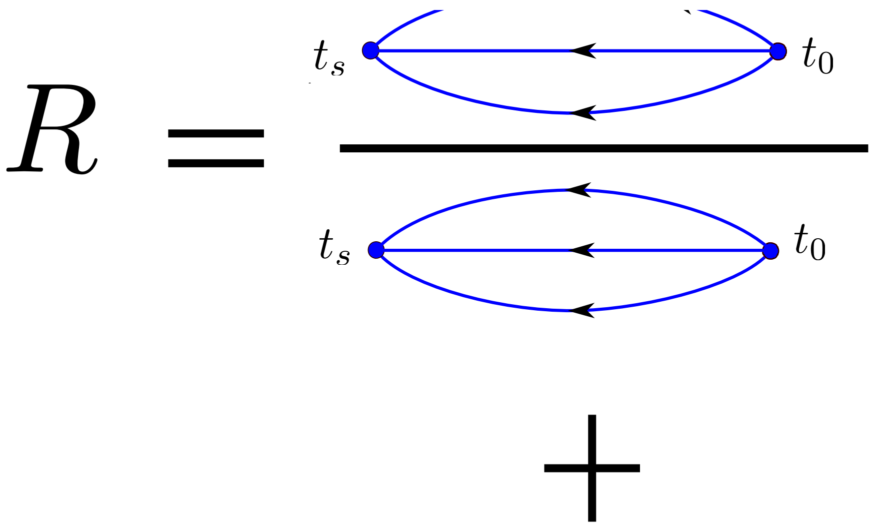

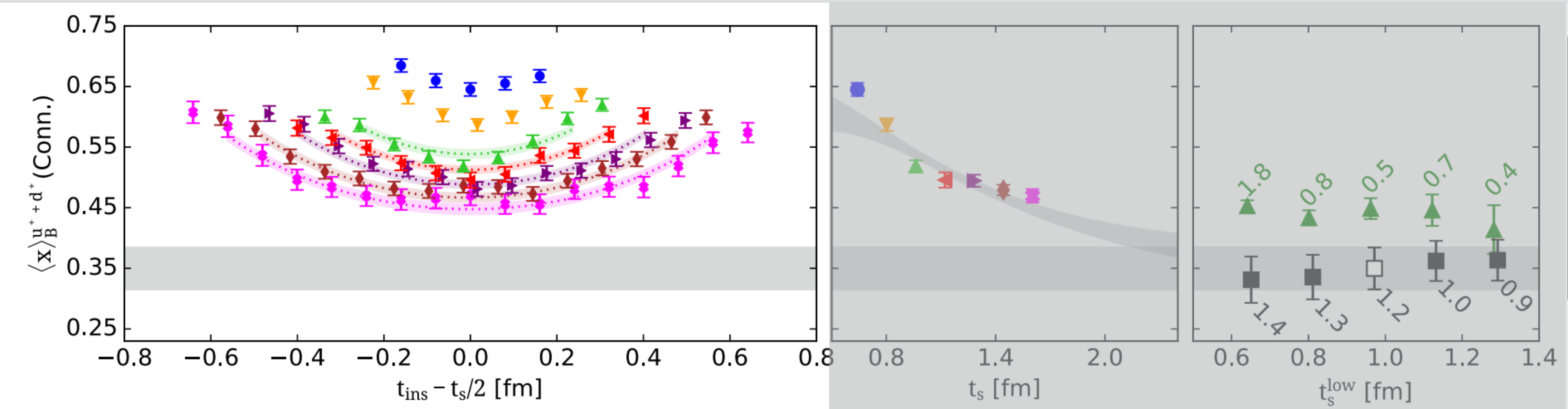
0.75
0.65
0.55
0.45
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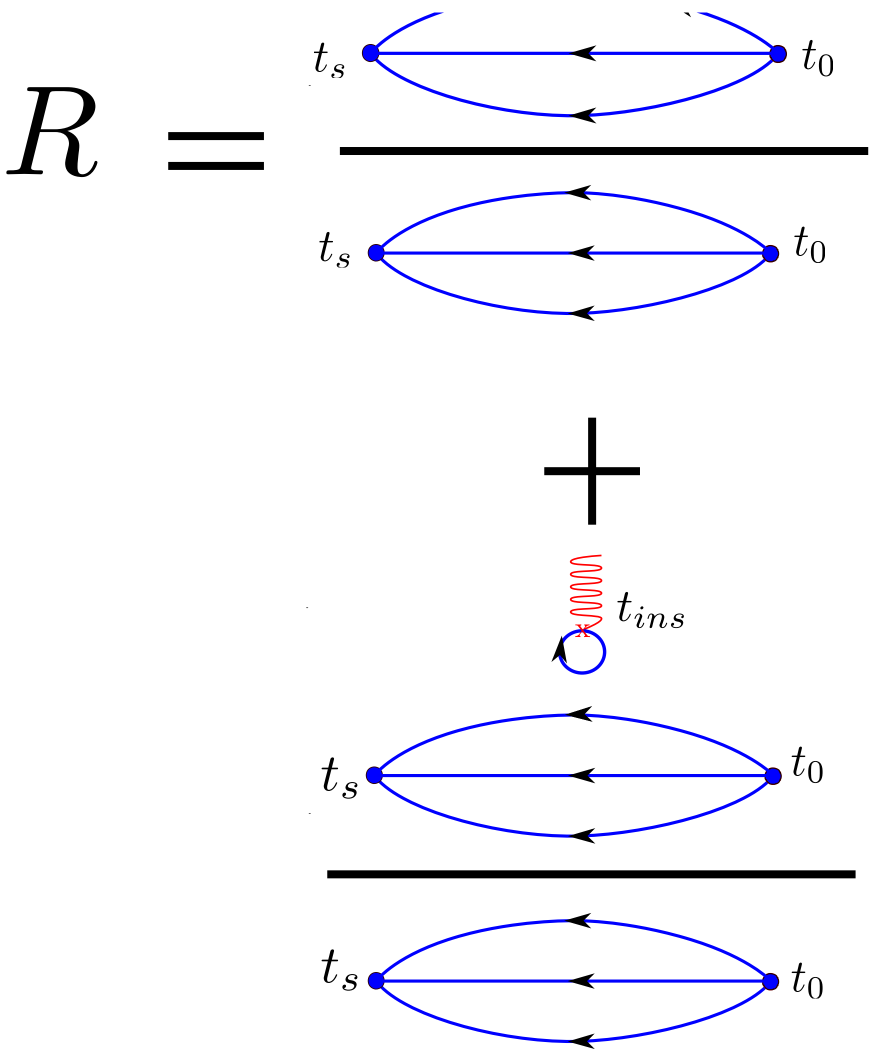
0.25
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
$\langle x \rangle_B^{u^+ + d^+}$ (Conn.)
$t_{ins} - t_s/2$ [fm]
0.8
1.4
2.0
t_s [fm]
0.6
0.8
1.0
1.2
1.4
t_s^{low} [fm]
1.8
0.8
0.5
0.7
0.4
1.4
1.3
1.2
1.0
0.9


$R =$
t_s
t_0
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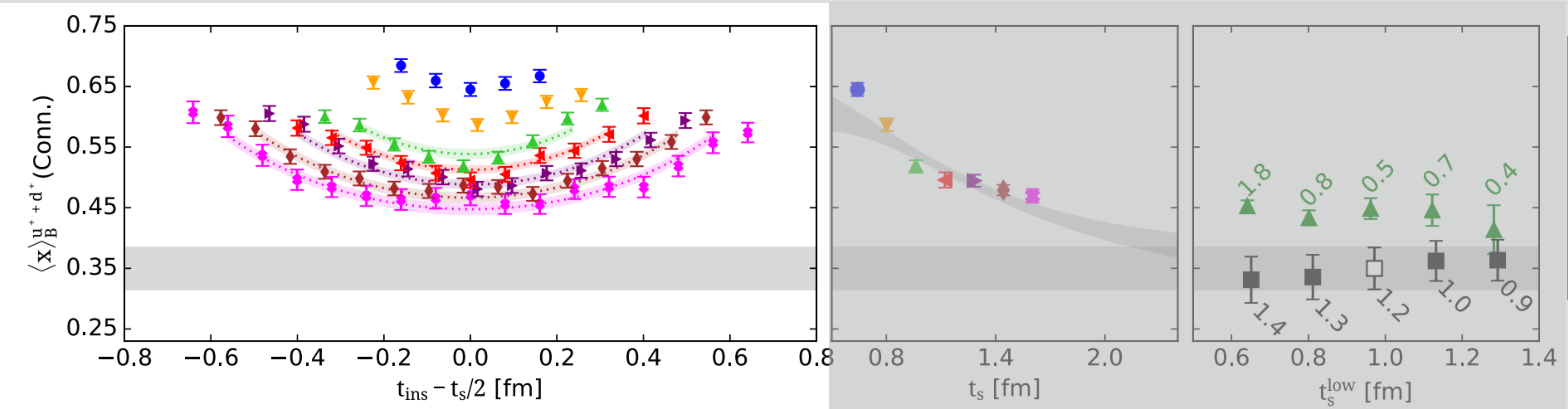

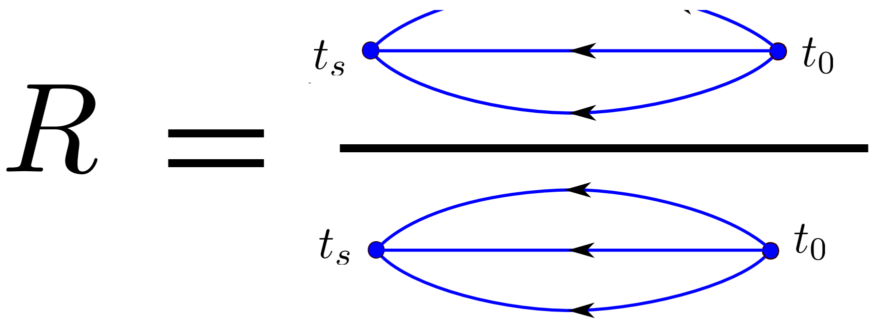

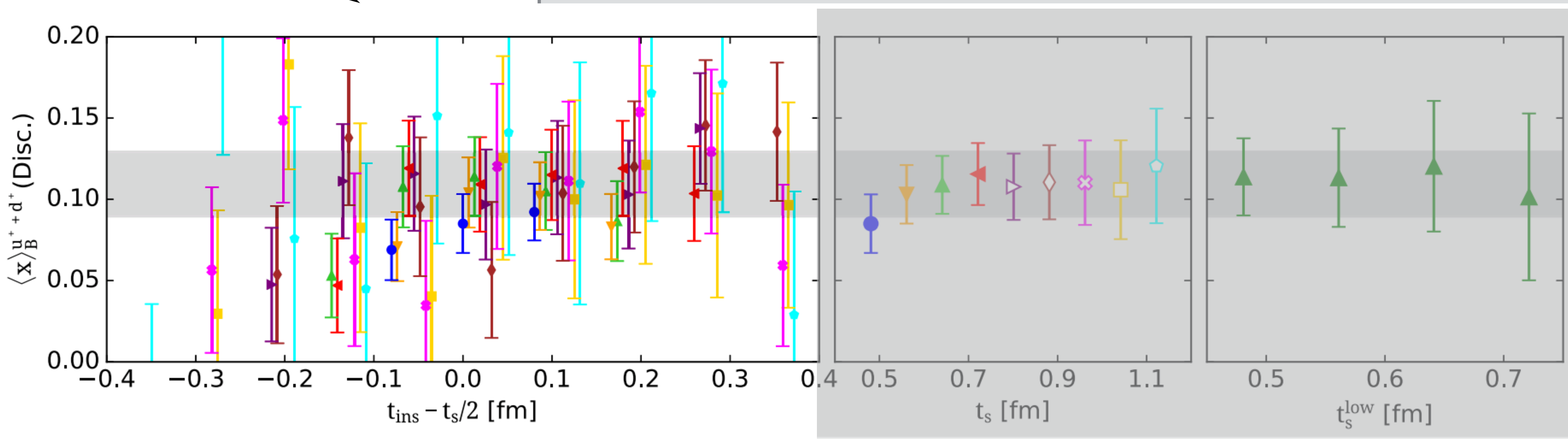

Results for quark helicity and average momentum fraction

Quantities are given in $\overline{\mathrm{MS}}$ scheme at $\mu^2 = 4\ \mathrm{GeV}^2$

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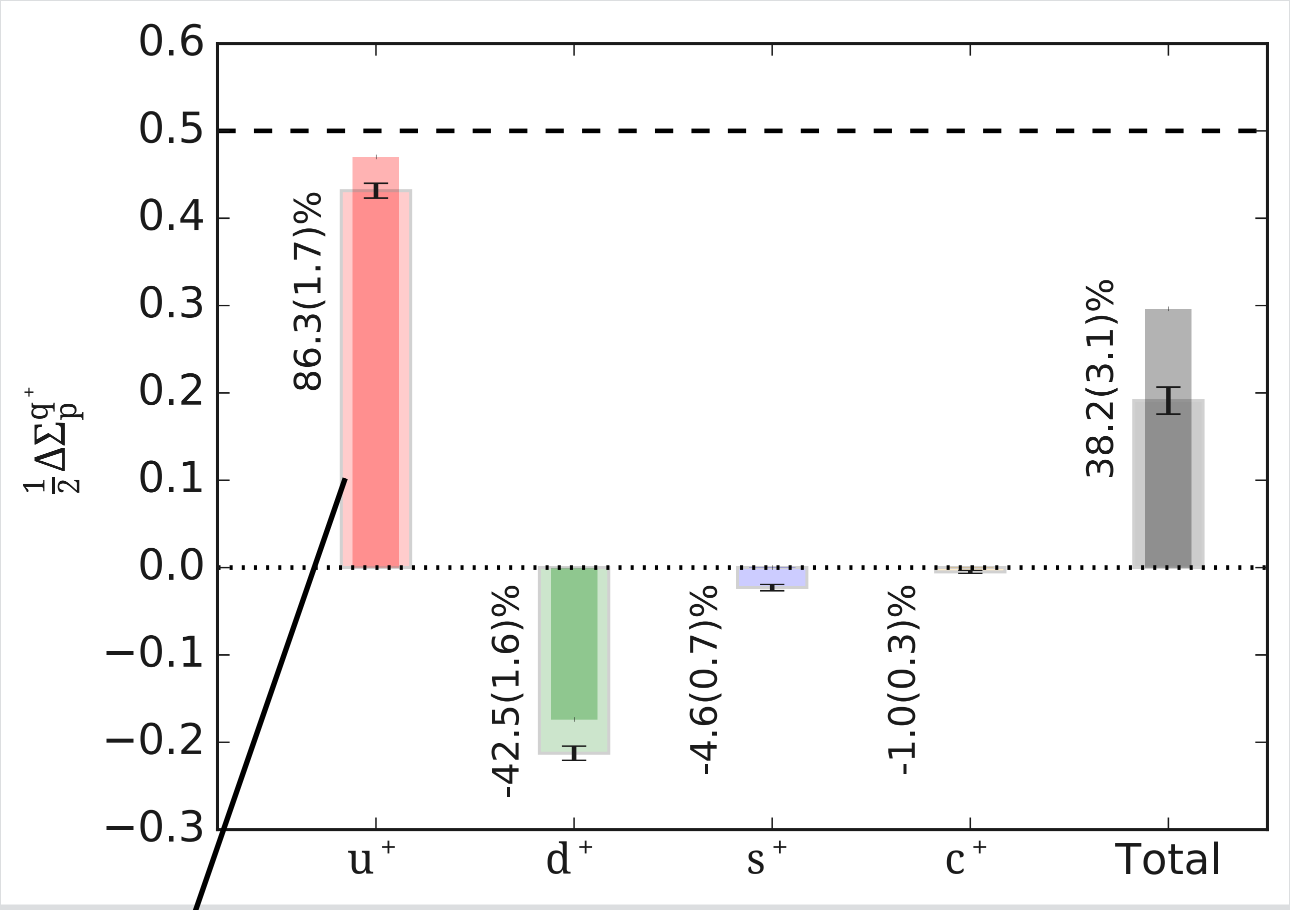
Phys. Rev. D 101, 094513 (2020)



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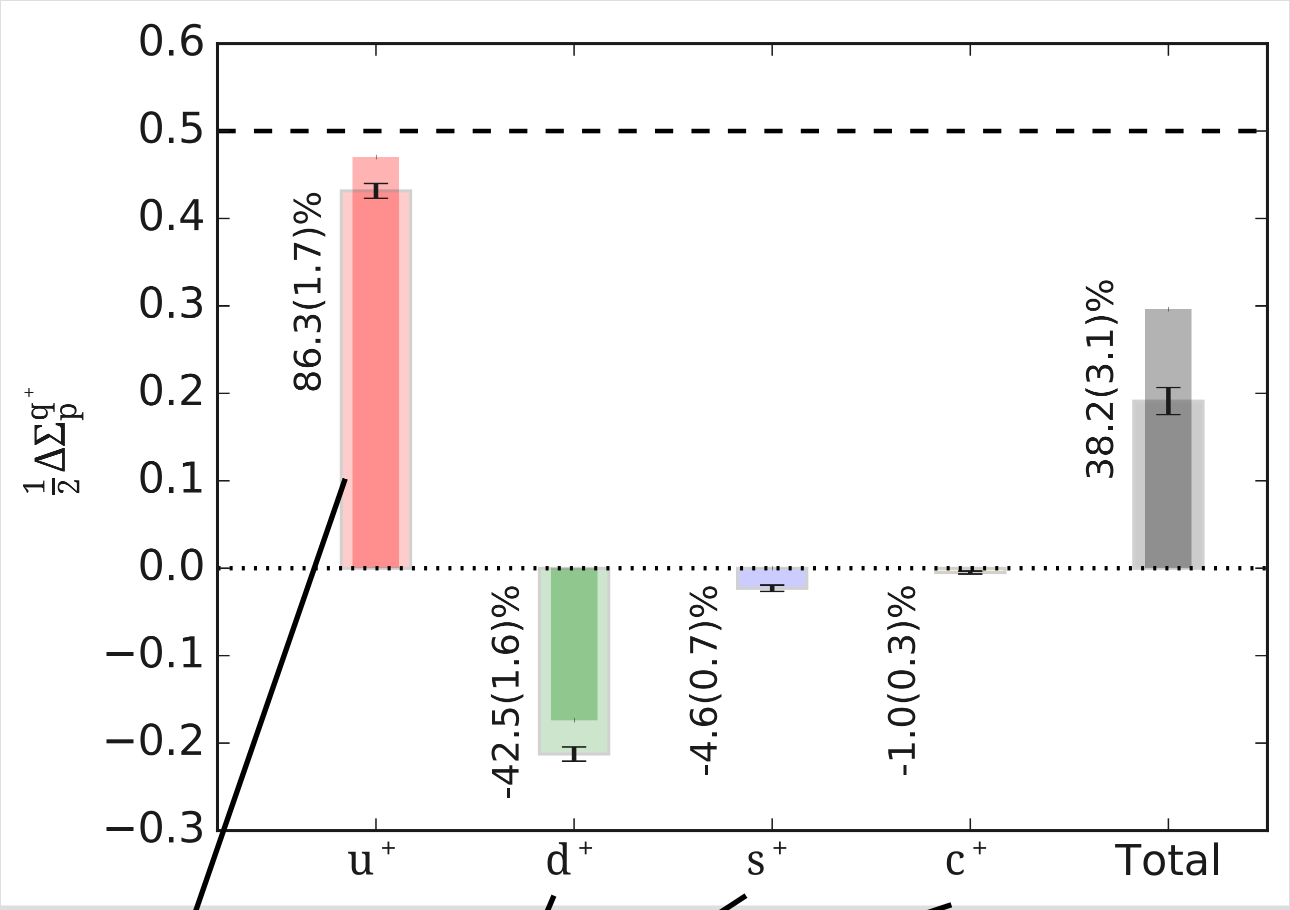

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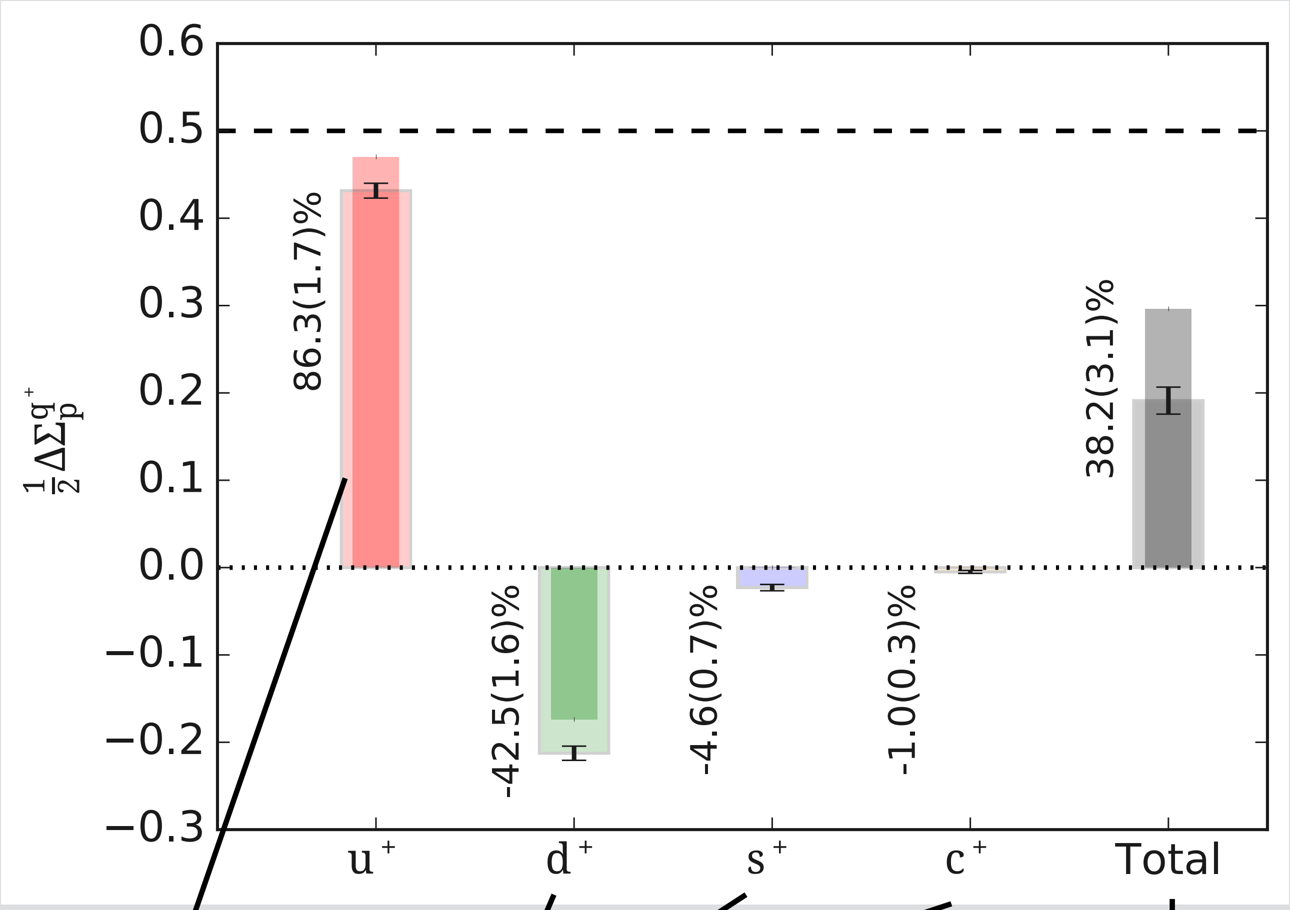

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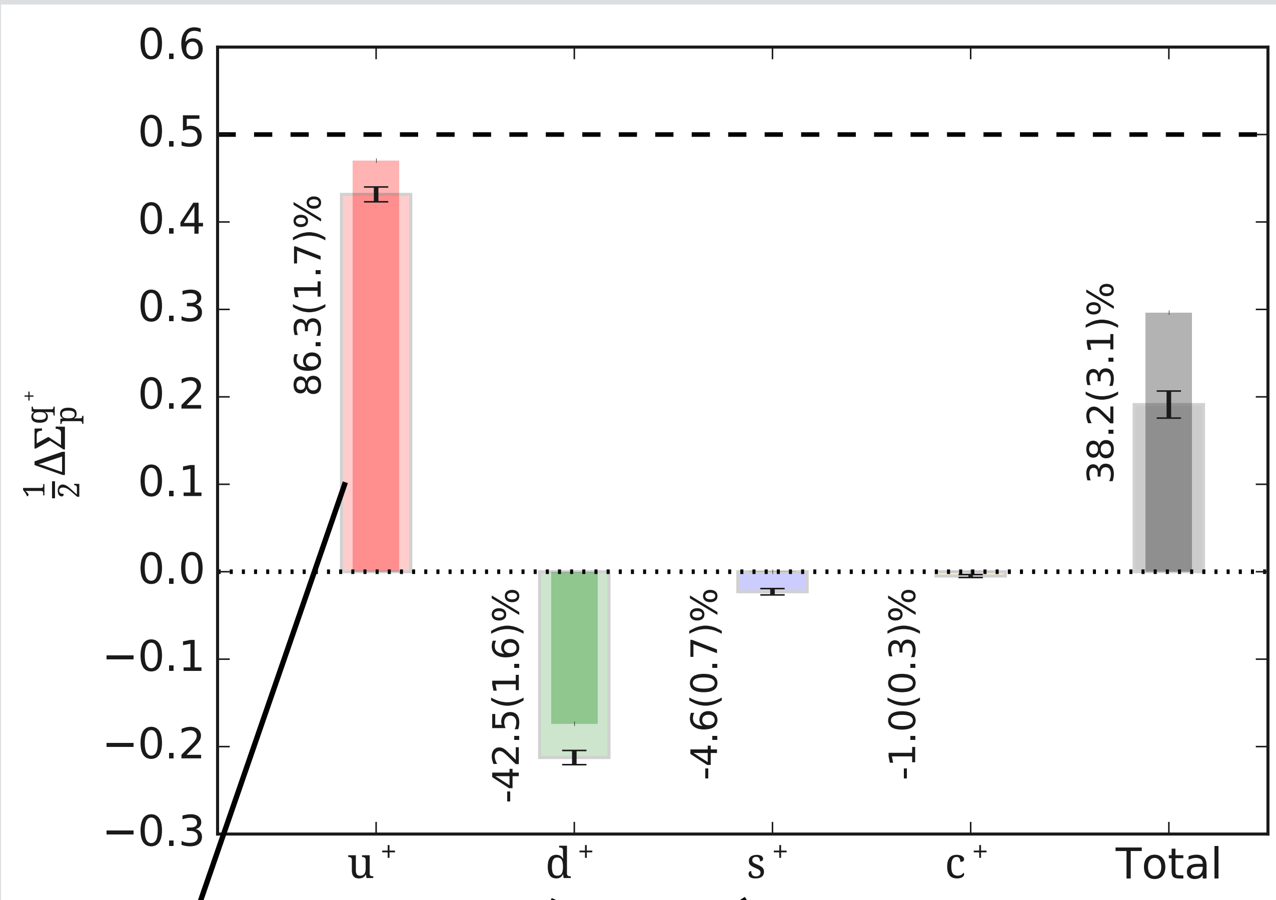

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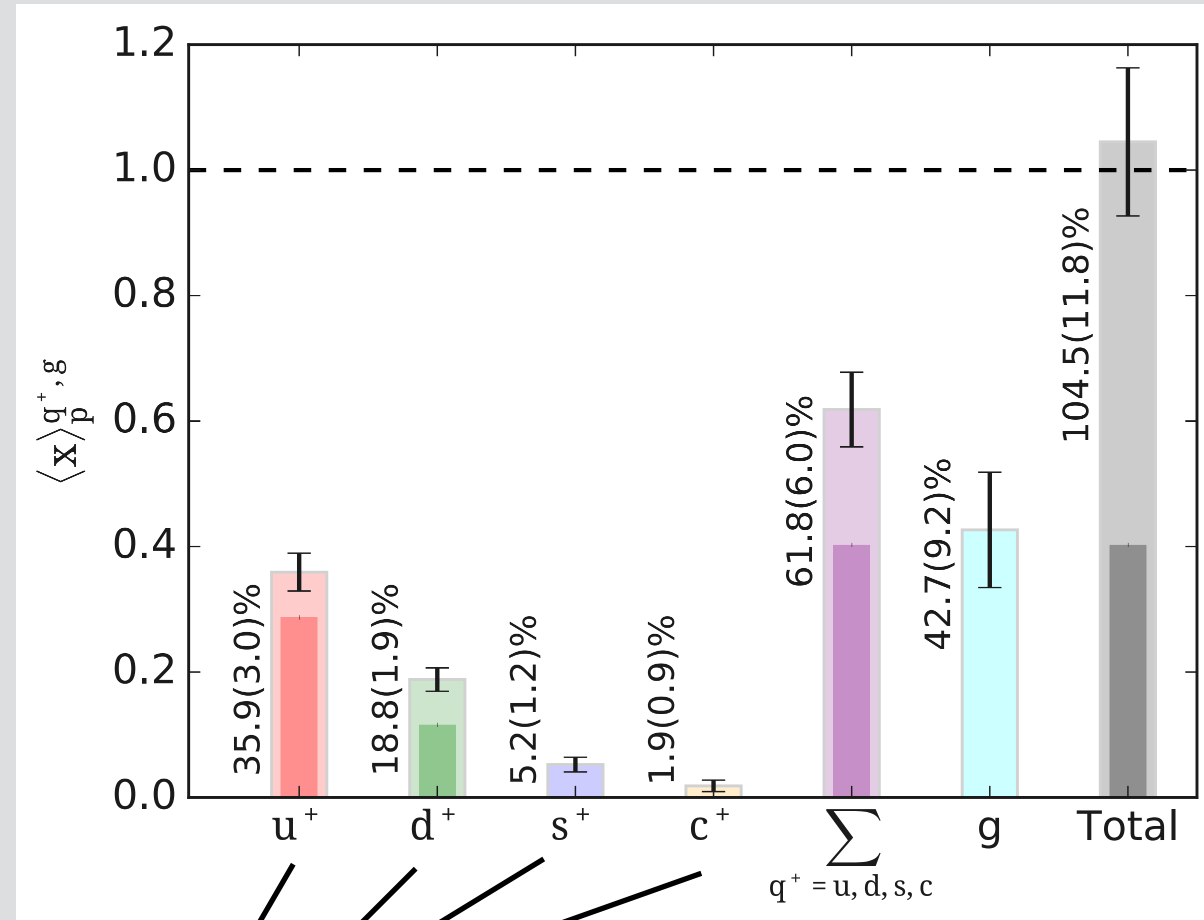

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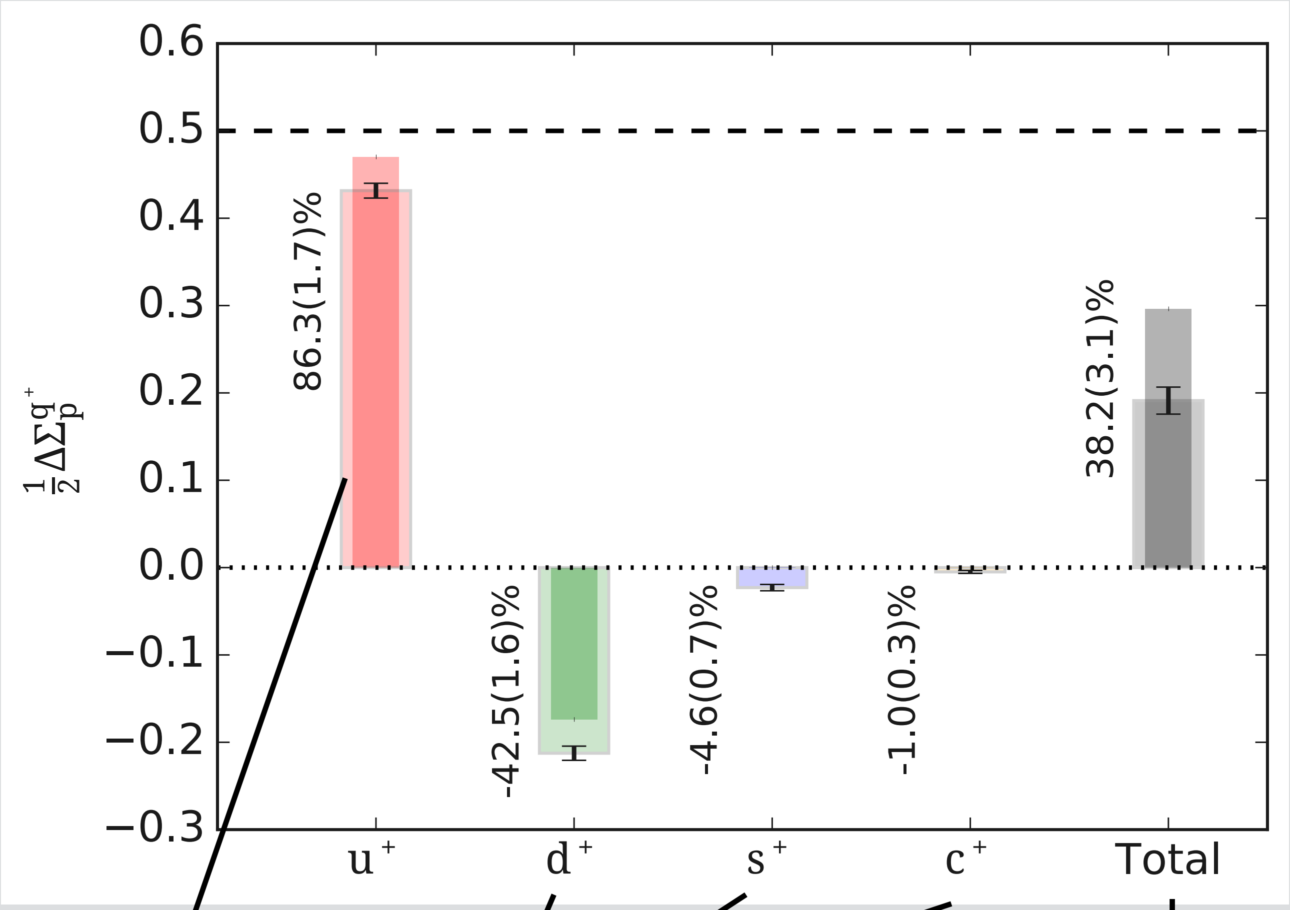

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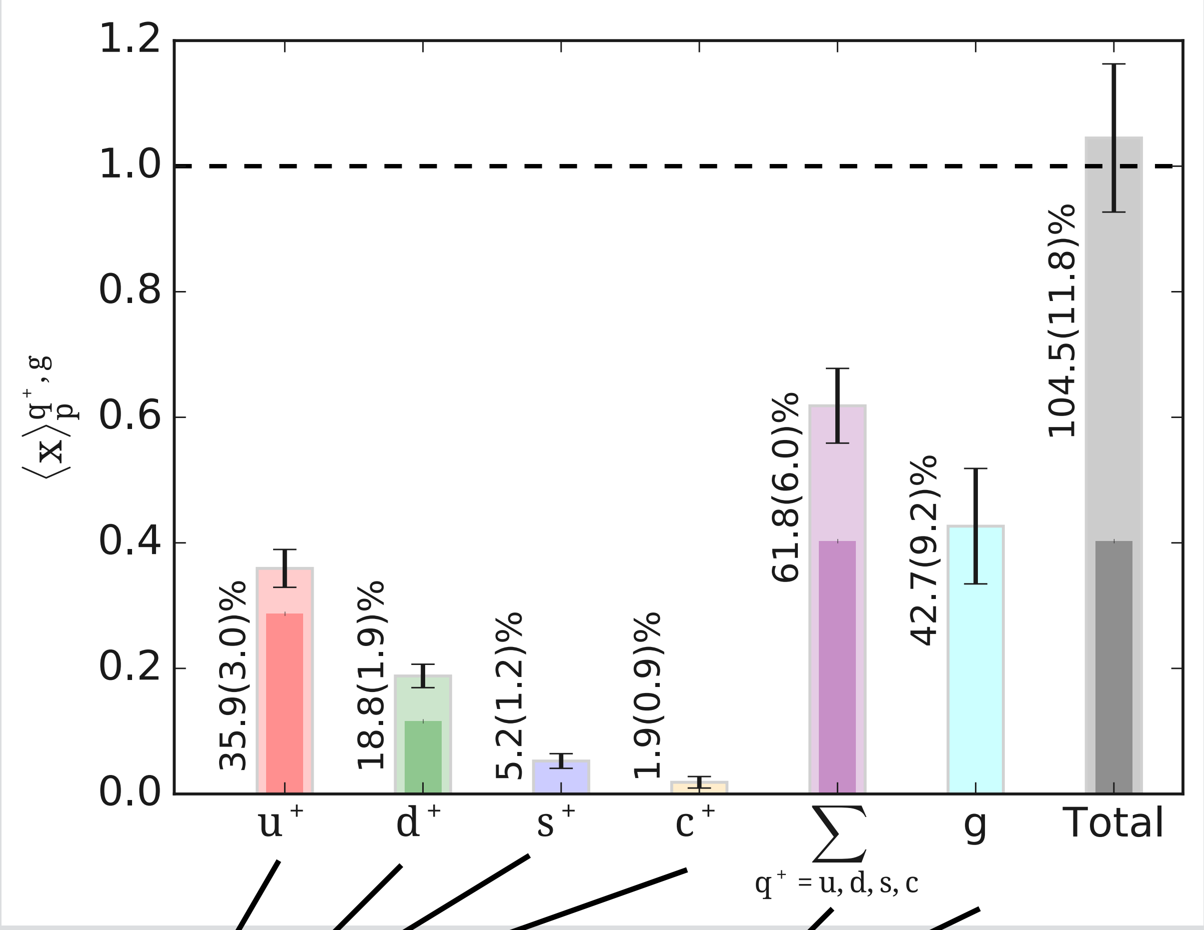

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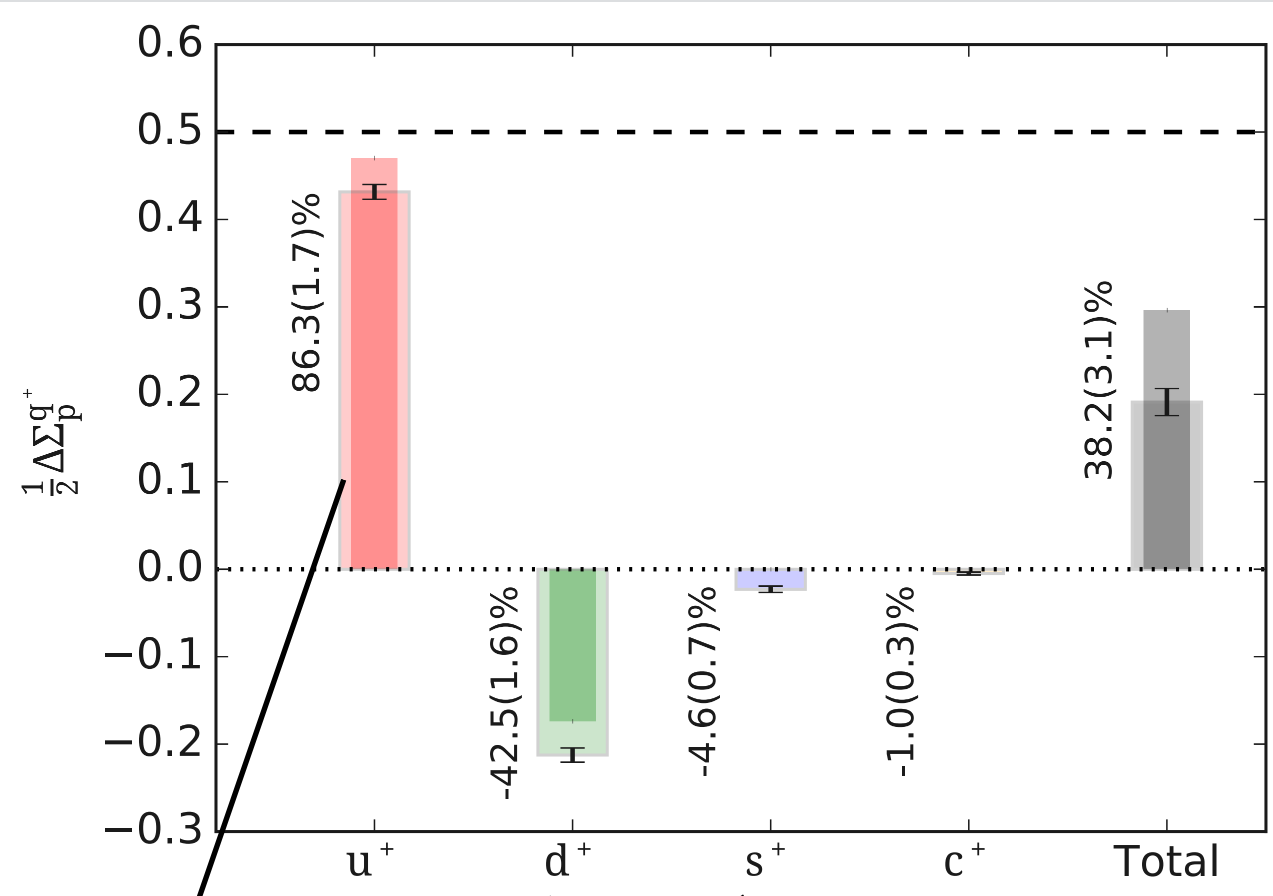

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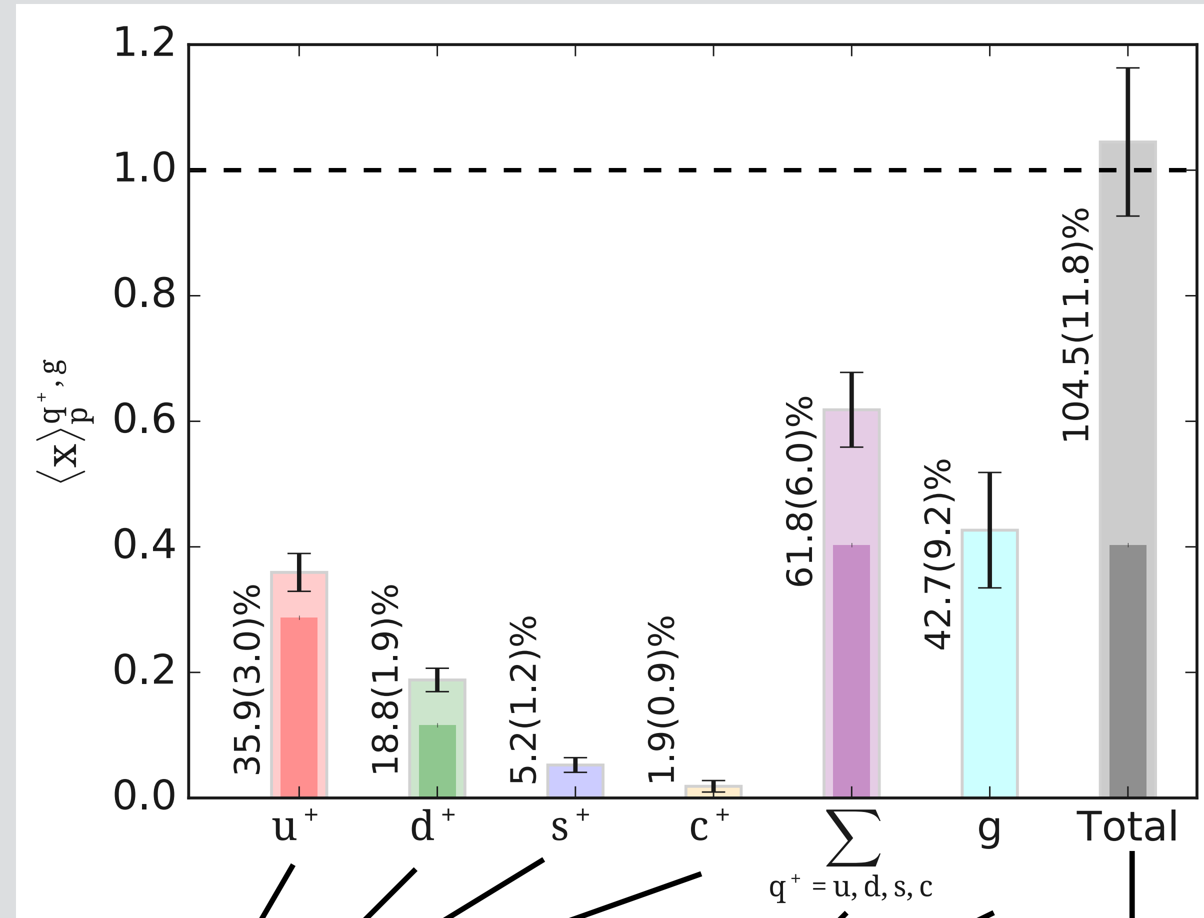

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Total is in agreement with momentum sum. Disconnected have the dominant contr.

Comparison with other studies

Our previous study
Phys. Rev. Lett. 119, 142002 (2017)

Results from χQCD
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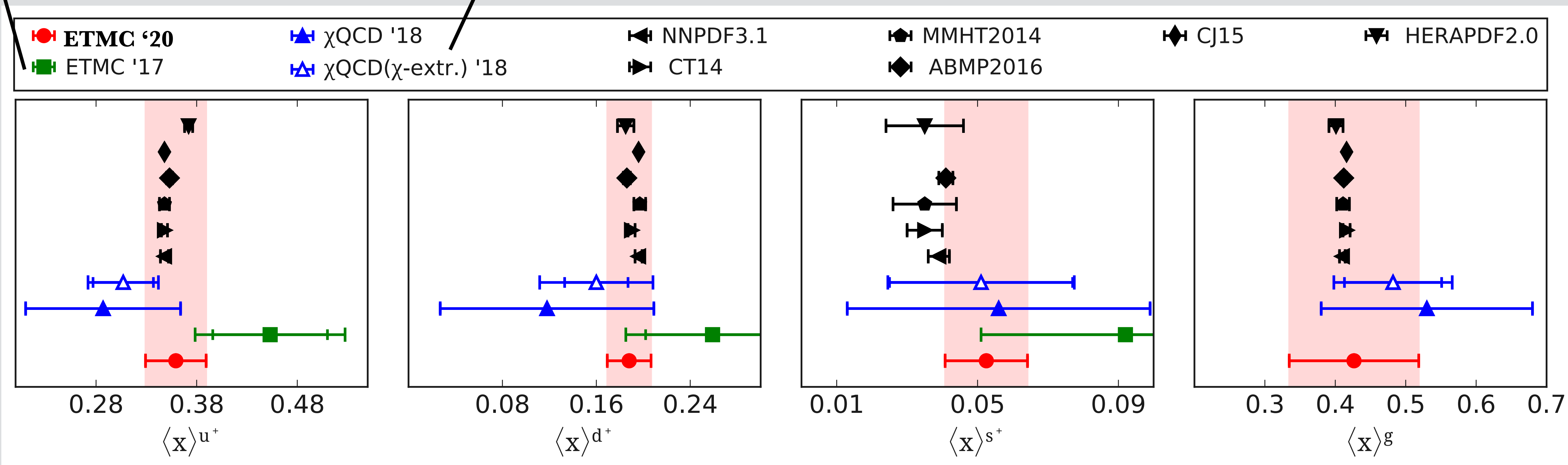
Colorful points are results from the lattice, while black points are results from phenomenology



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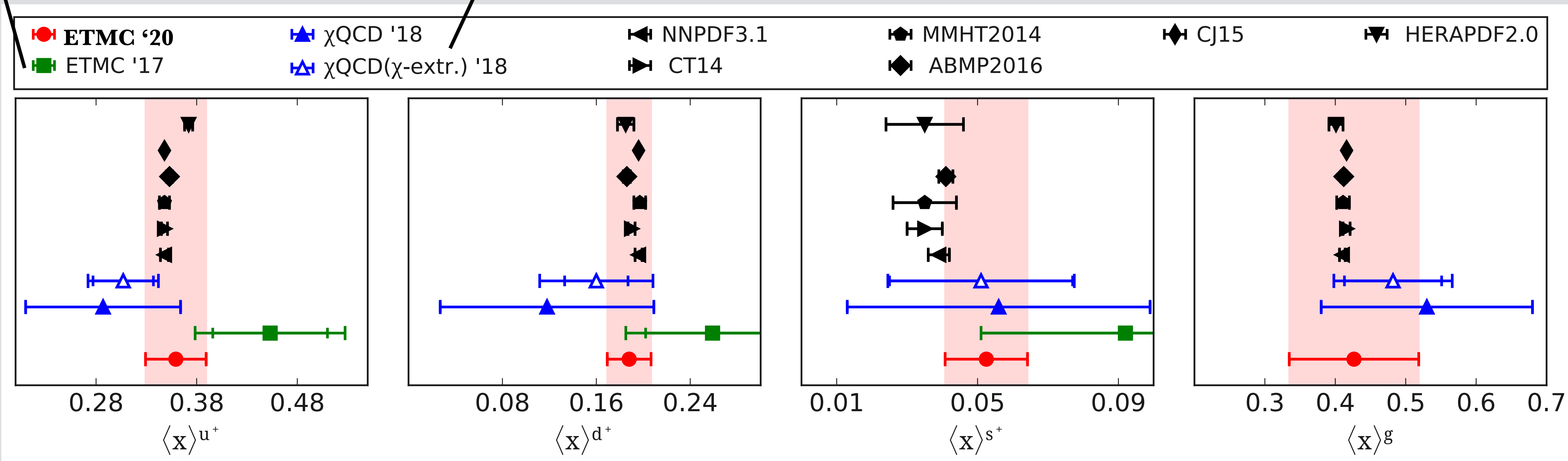

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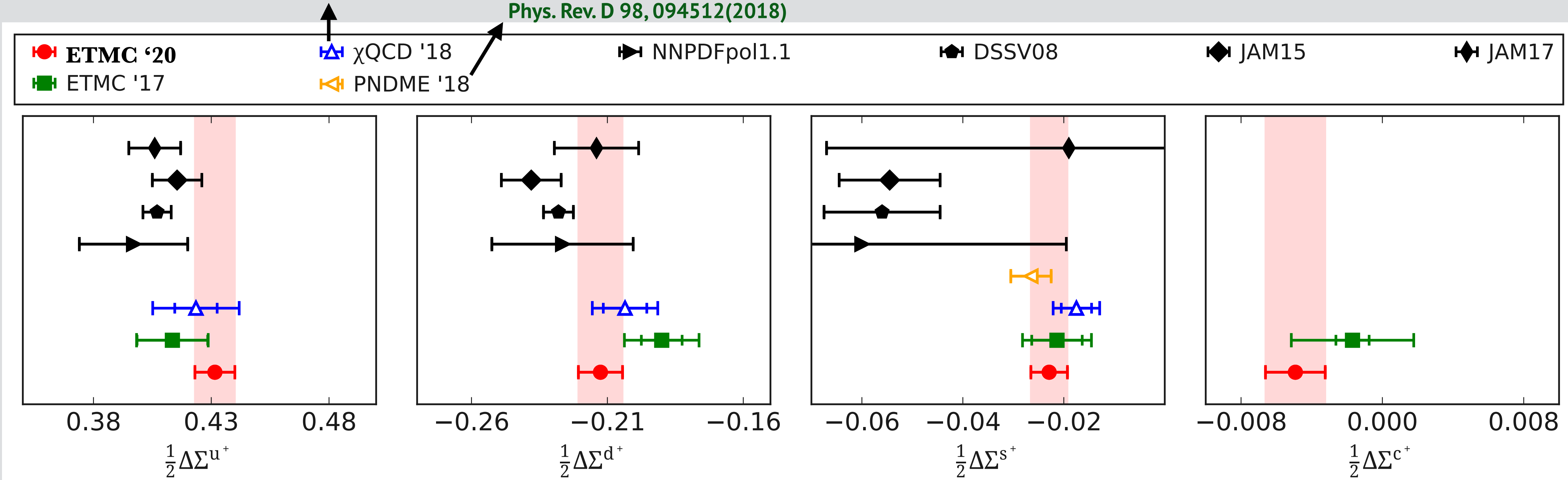
Phys. Rev. D 98, 094512(2018)



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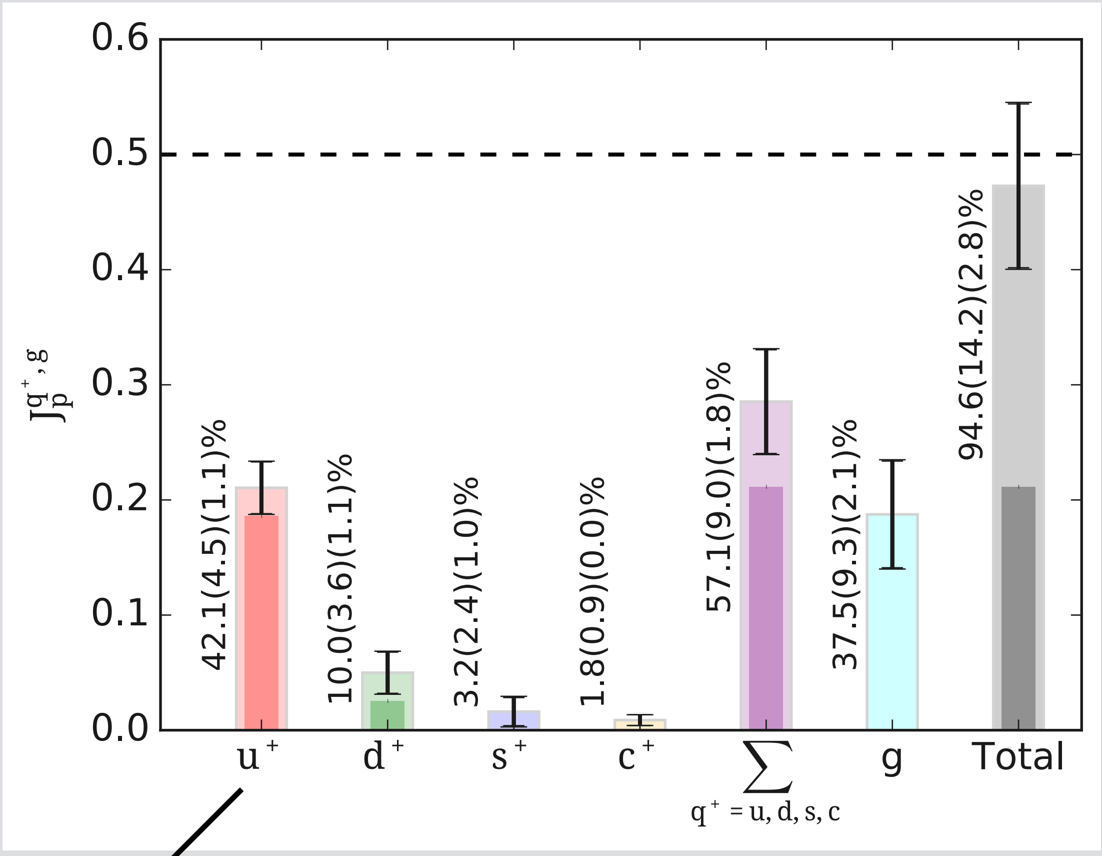

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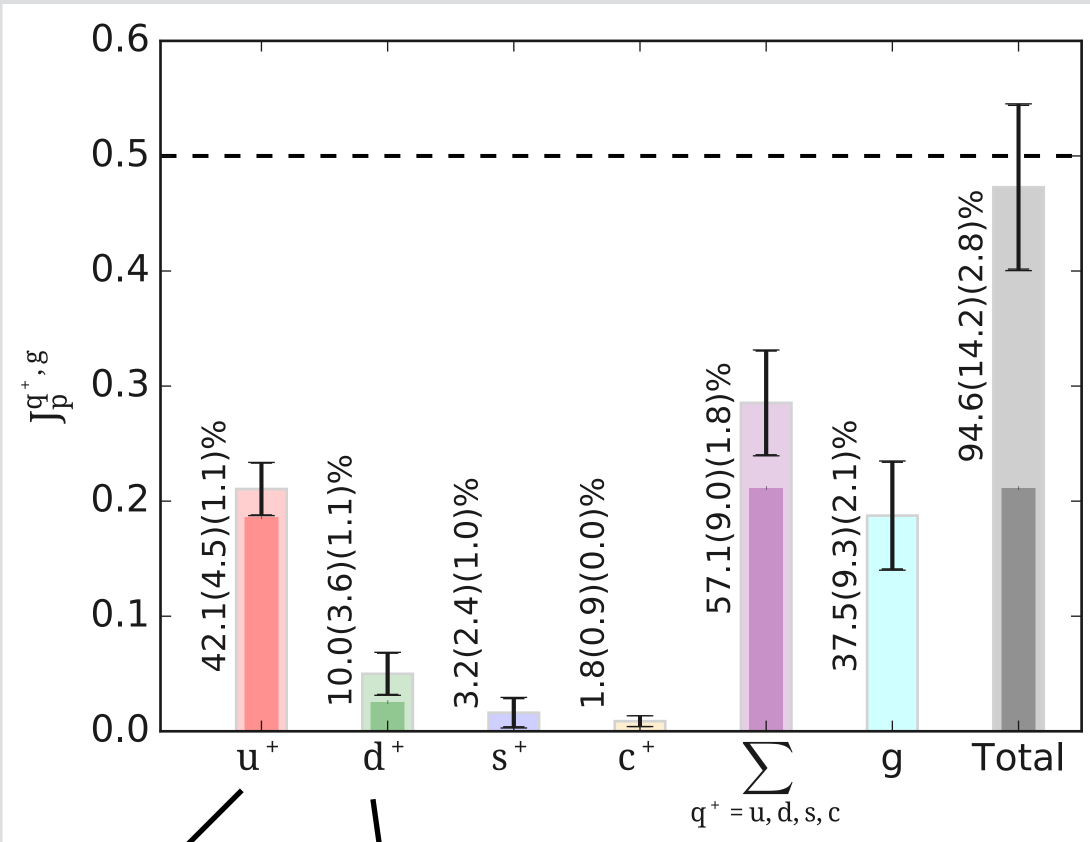

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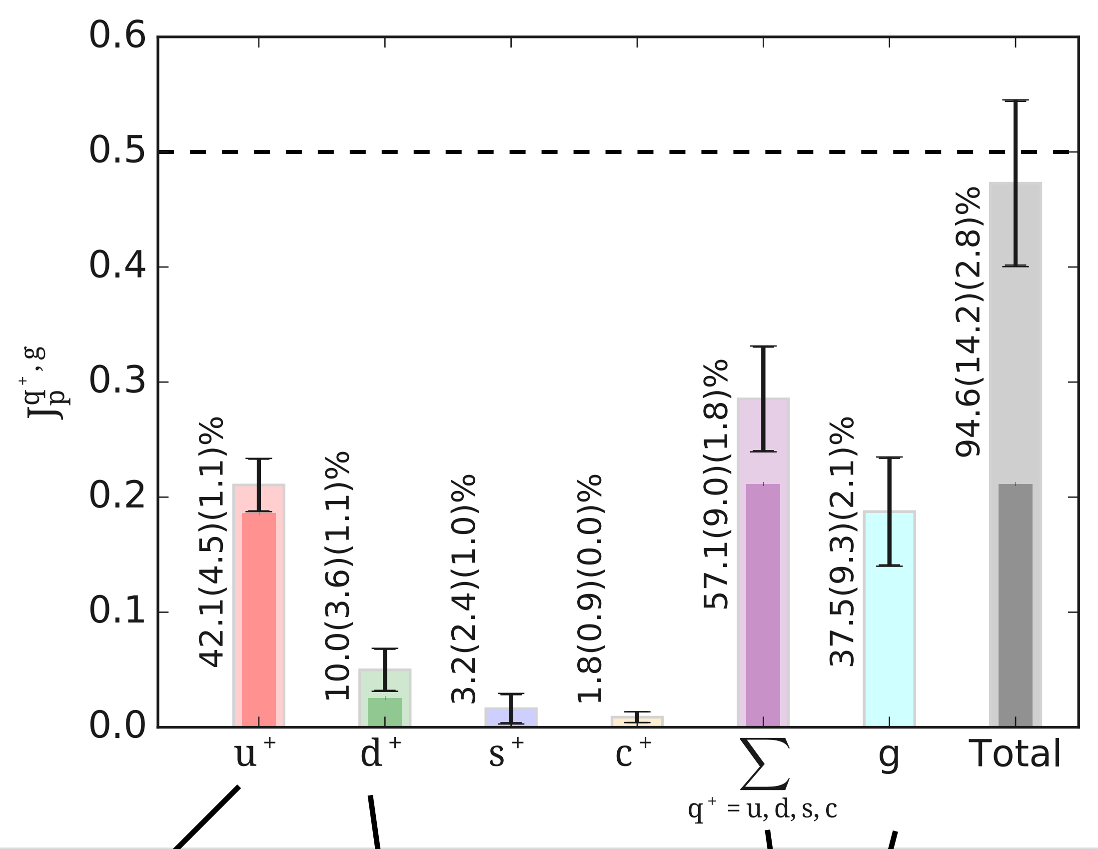

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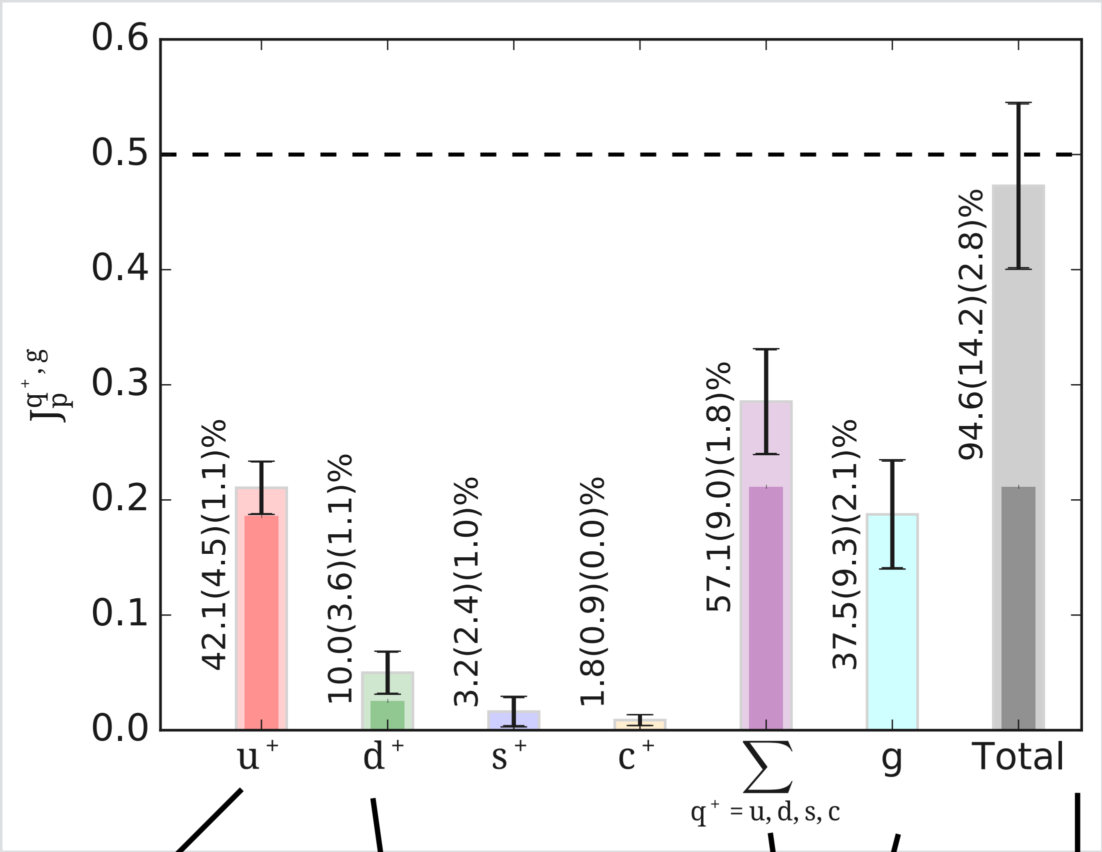

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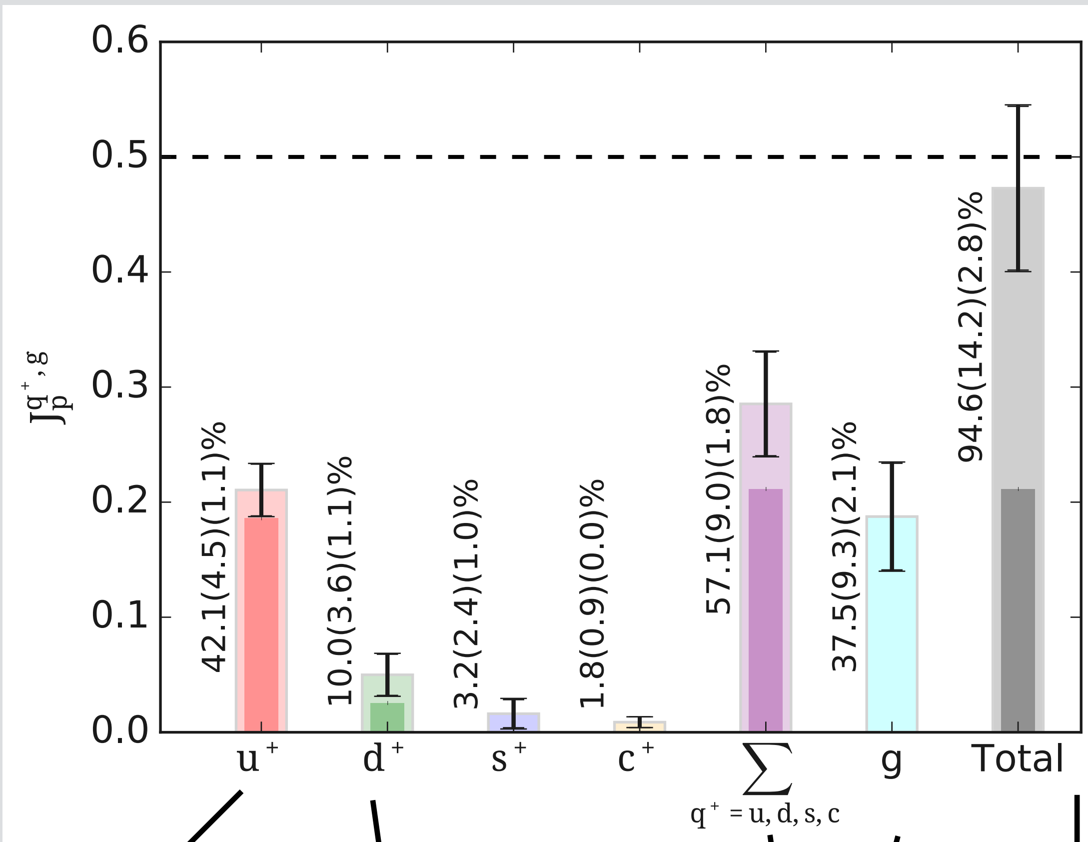
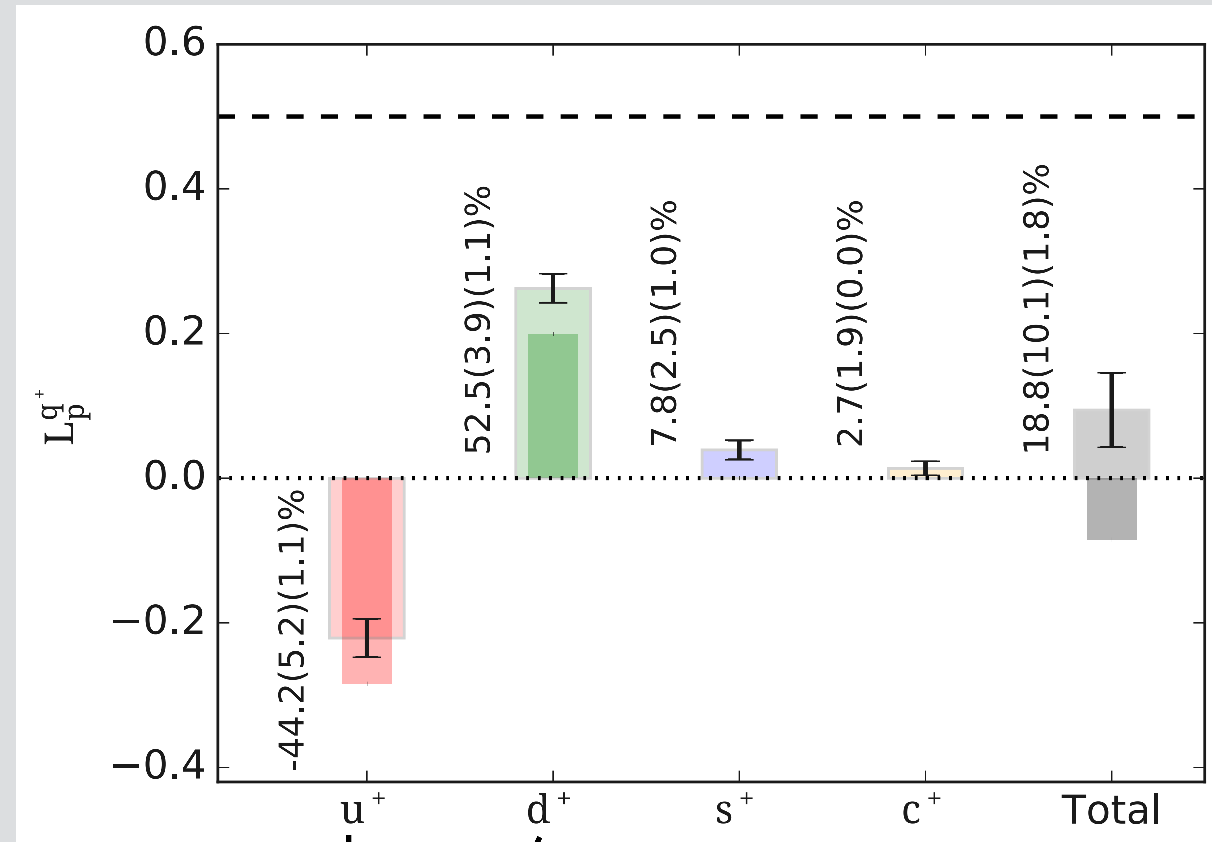


✱ Orbital Angular momentum



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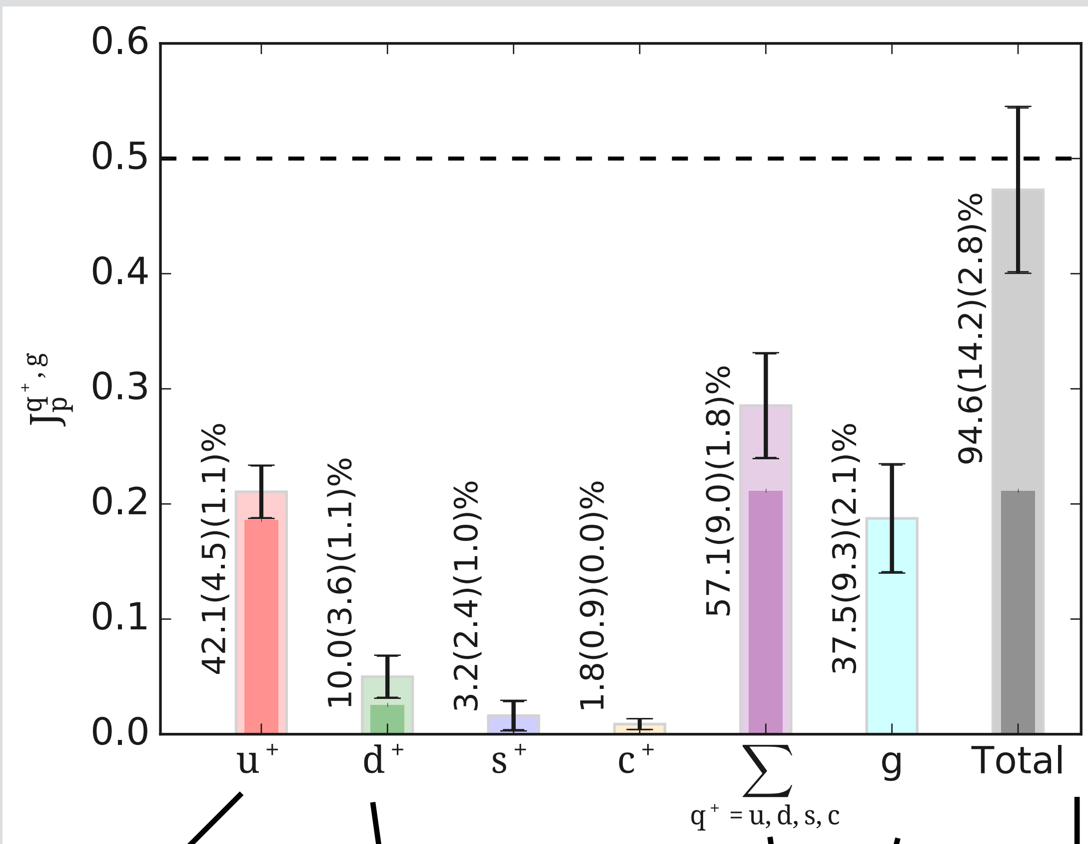

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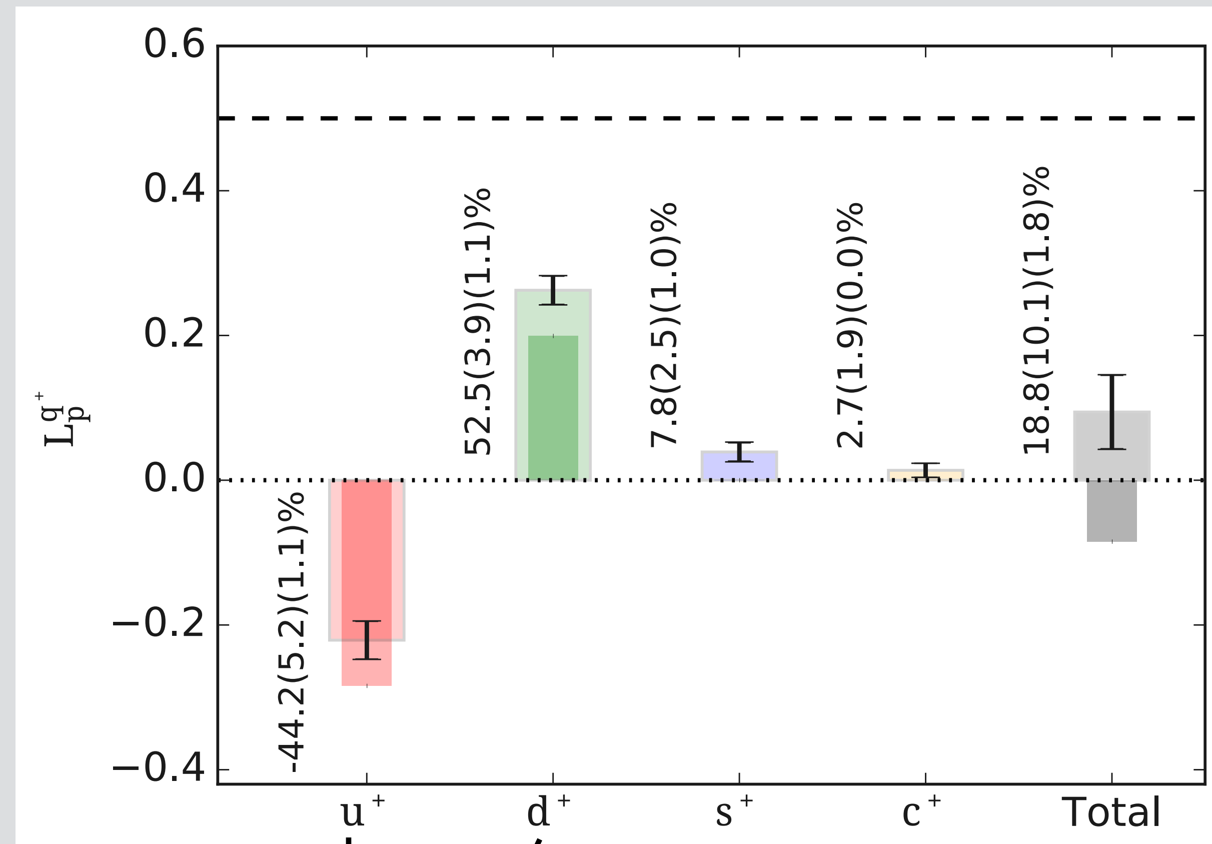

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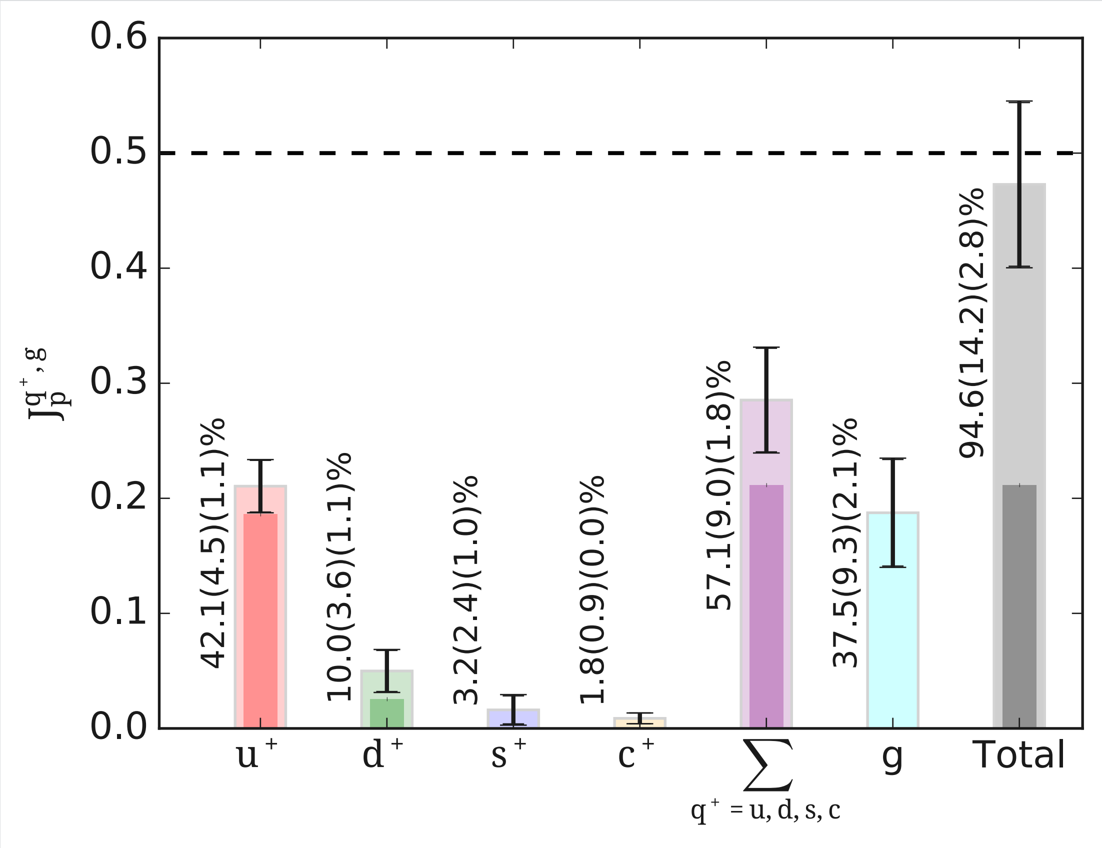
The total quark orbital angular momentum is 10-25% of the proton spin but uncertainties are high

Summary & conclusion & future plans

- A complete $N_f = 2+1+1$ LQCD study about the origin of the proton spin
  - Spin sum is verified
  - Momentum sum is verified
  - Non-perturbative renormalization
  - All disconnected contributions are included



Phys. Rev. D 101, 094513 (2020)

	$\langle x \rangle$	J	$\frac{1}{2}\Delta\Sigma$	L
u^+	0.359(30)	0.211(22)(5)	0.432(8)	-0.221(26)(5)
d^+	0.188(19)	0.050(18)(5)	-0.213(8)	0.262(20)(5)
s^+	0.052(12)	0.016(12)(5)	-0.023(4)	0.039(13)(5)
c^+	0.019(9)	0.009(5)(0)	-0.005(2)	0.014(10)(0)
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Tot.	1.045(118)	0.473(71)(14)	0.191(15)	0.094(51)(9)

Summary & conclusion & future plans

- A complete $N_f = 2+1+1$ LQCD study about the origin of the proton spin
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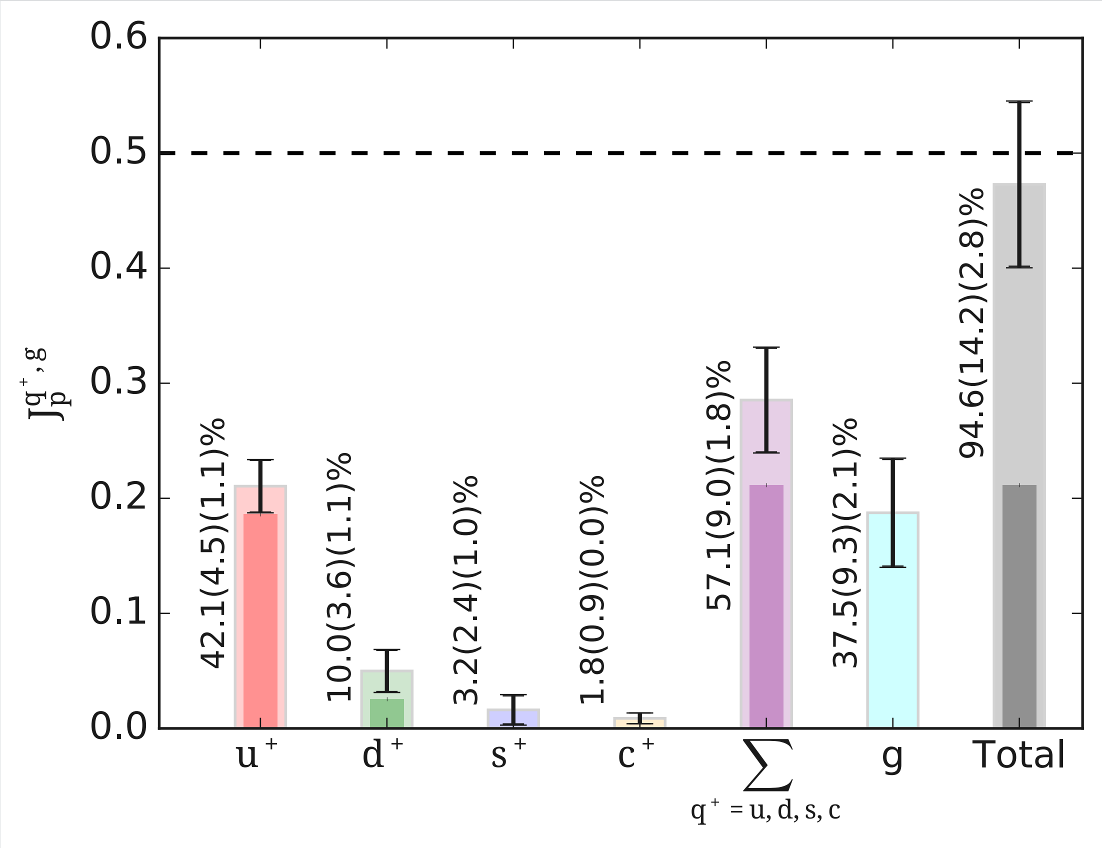
  - Momentum sum is verified
  - Non-perturbative renormalization
  - All disconnected contributions are included



Phys. Rev. D 101, 094513 (2020)

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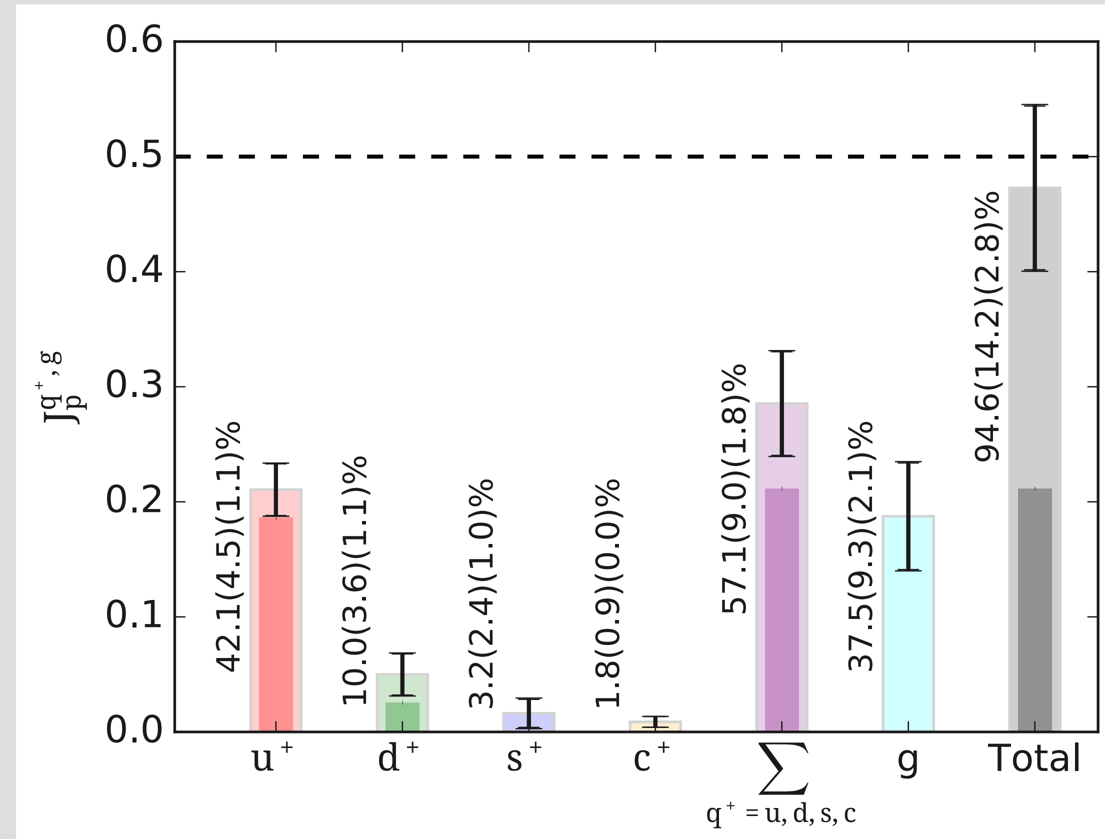

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Backup slides

Excited states analysis procedure

Excited states analysis procedure

✱ Plateau method

$$R(t_s, t_{ins}, t_0) \xrightarrow[t_s - t_{ins} \gg 1]{t_{ins} - t_0 \gg 1} \mathcal{M}\left[1 + \mathcal{O}\left(e^{-\Delta E(t_{ins}-t_0)}, e^{-\Delta E(t_s - t_{ins})}\right)\right]$$

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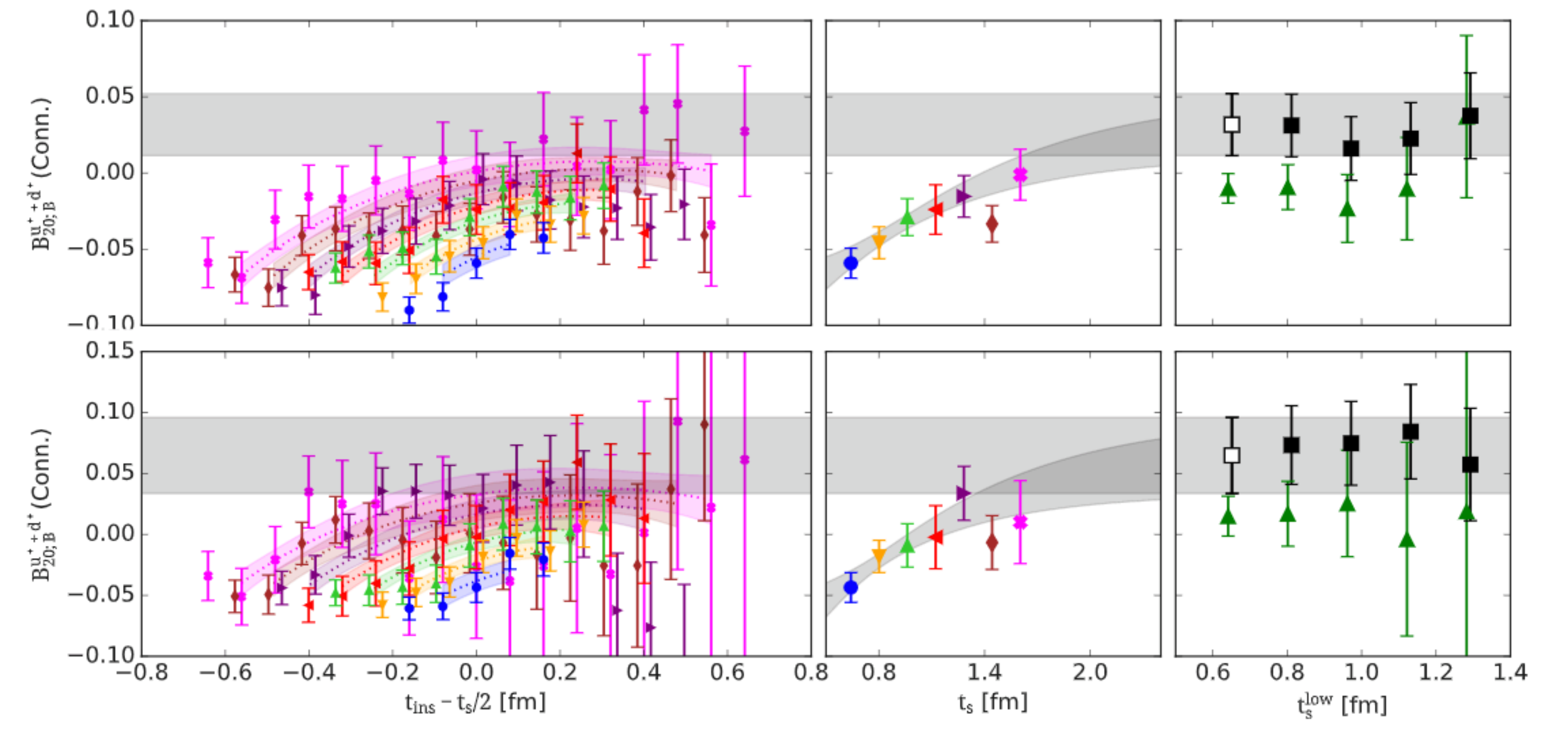
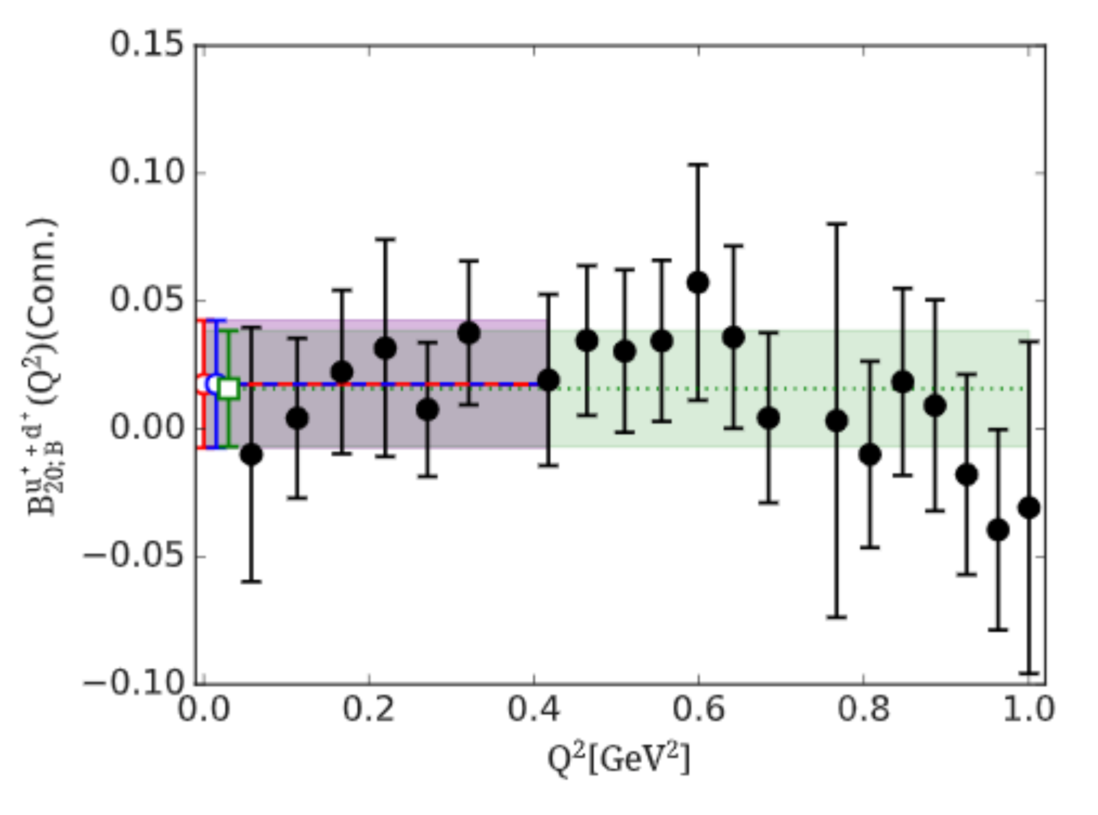
✱ Two-state fit method

$$\begin{aligned} G^{3pt} &= A_{00} e^{-E_0(t_s - t_0)} + A_{01} e^{-E_0(t_s - t_{ins})} e^{E_1(t_{ins} - t_0)} \\ &+ A_{10} e^{-E_1(t_s - t_{ins})} e^{-E_0(t_{ins} - t_0)} + A_{11} e^{-E_1(t_s - t_0)} \end{aligned}$$

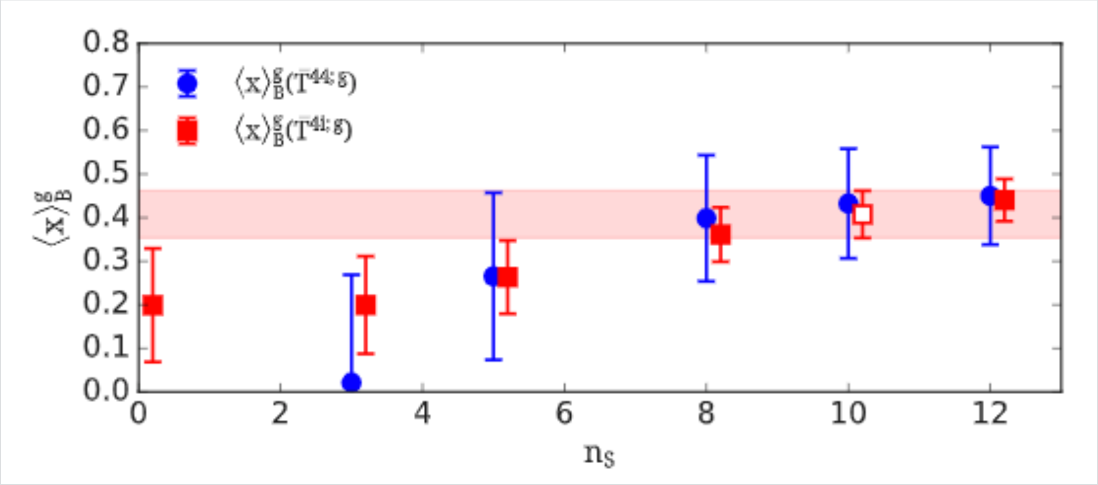
$$G^{2pt} = c_0 e^{-E_0(t_s - t_0)} + c_1 e^{-E_1(t_s - t_0)} \qquad \mathcal{M} = \frac{A_{00}}{c_0}$$

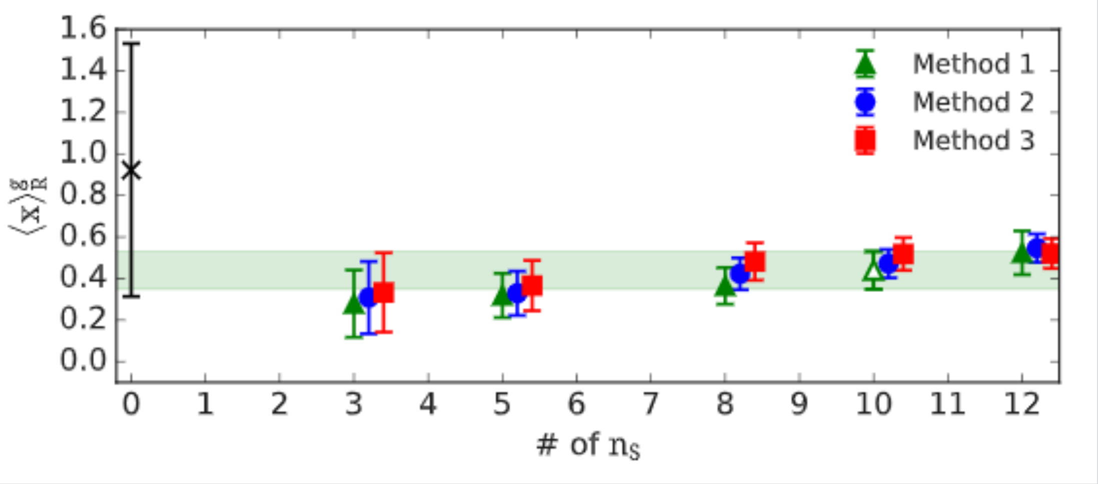
Extraction of B_{20}

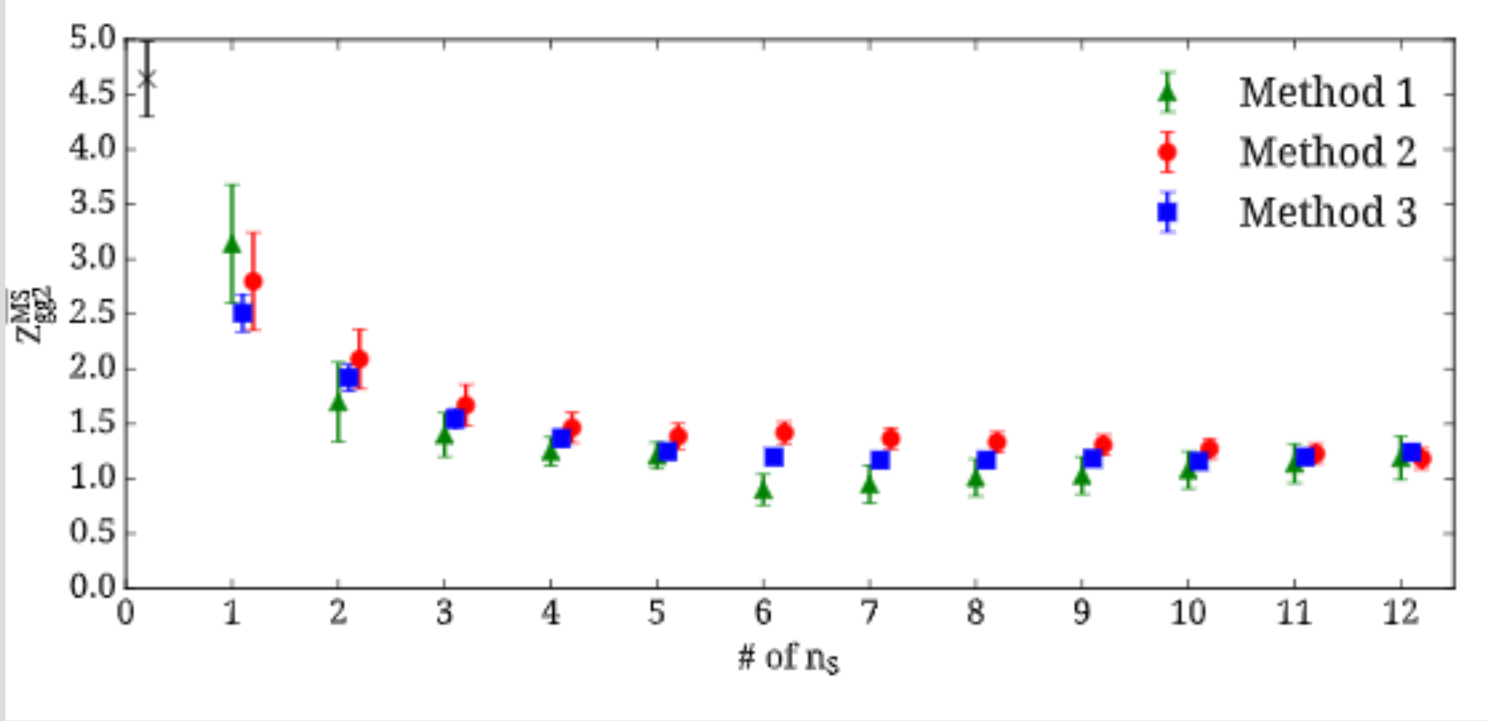




Renormalization





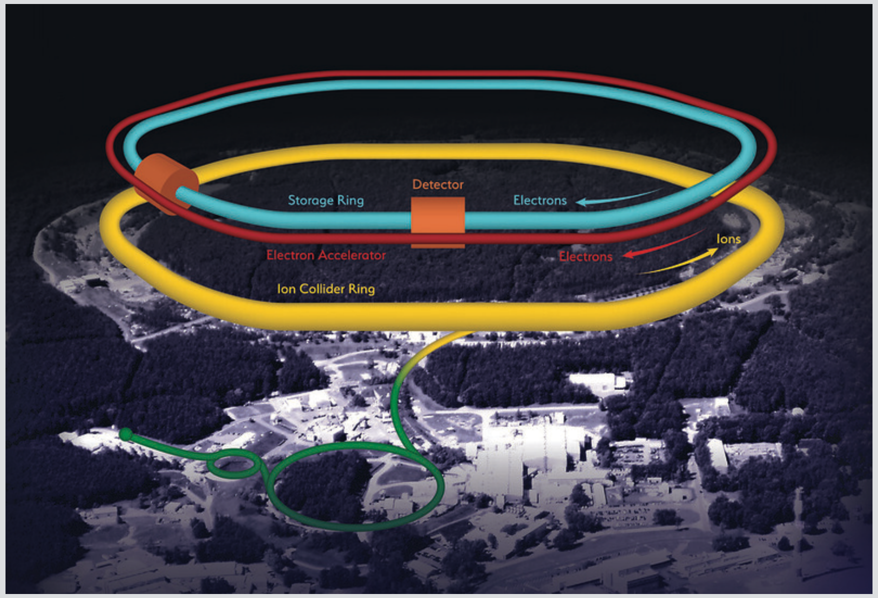


How we can see inside the proton…

How we can see inside the proton…

Experimentalists need accelerators

Electron-Ion Collider at BNL



COMPASS experiment at CERN

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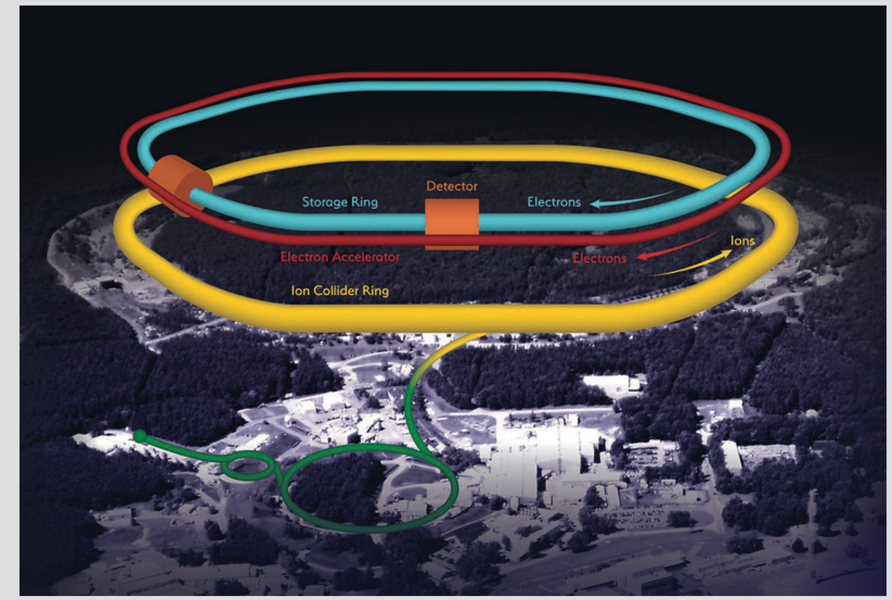

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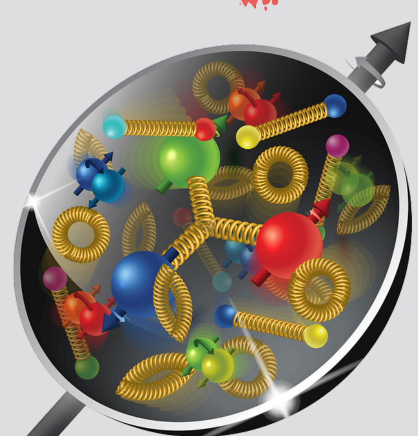

Proton spin content

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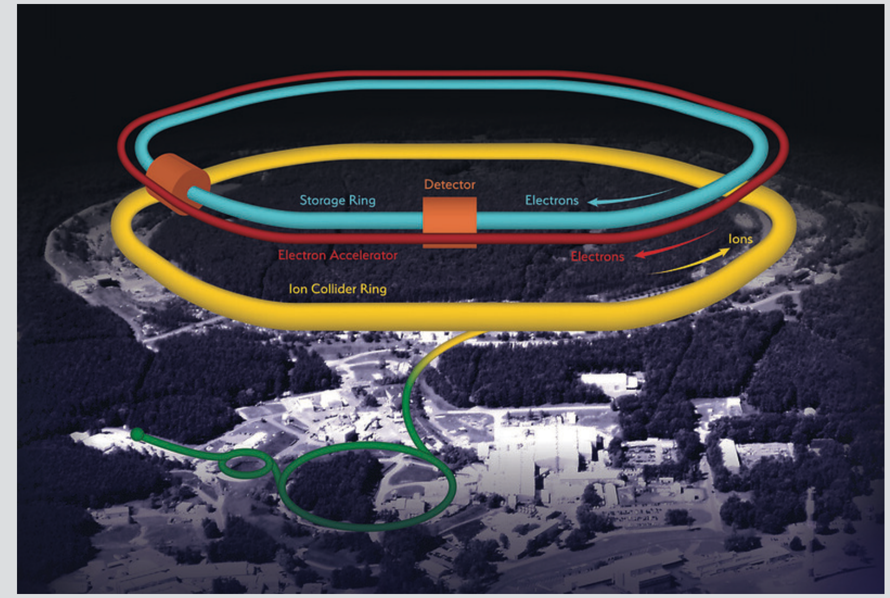

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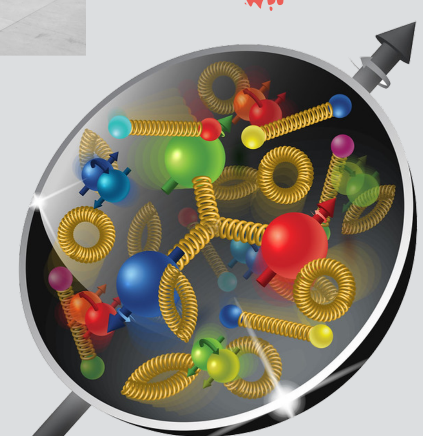

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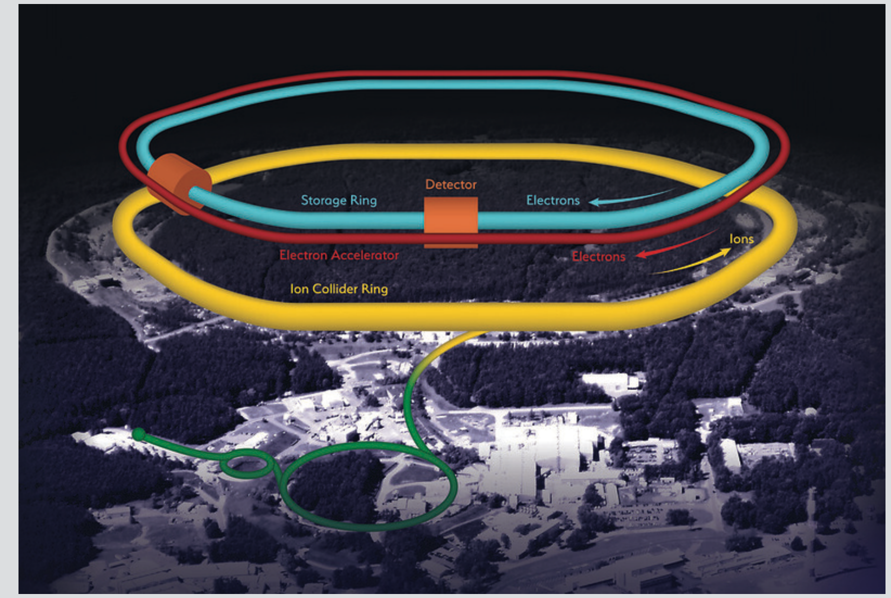

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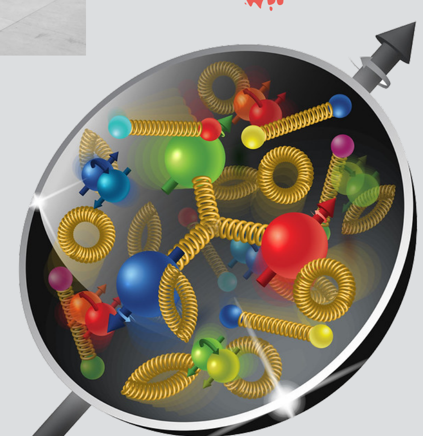
Approach we follow in this work



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