Twist-3 PDFs from lattice QCD: $g_T(x)$

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Aurora Scapellato (Adam Mickiewicz University)
Fernanda Steffens (University of Bonn)

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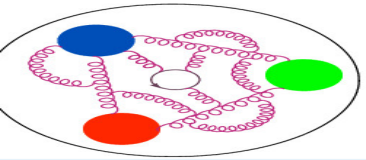

Twist-3 PDFs

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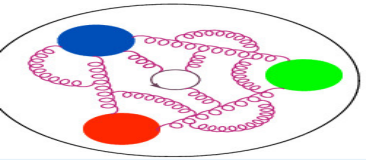
Our work
Procedure
Lattice setup
Bare ME
g_T and g_1
WW approx.
Summary

PDFs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

Leading twist: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements difficult due to their suppressed $\mathcal{O}(1/Q)$ kinematical behavior.



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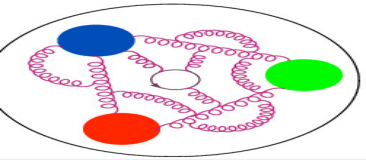
- (Shohini's talk) **matching for twist-3 PDF $g_T(x)$**
  proved factorization at 1-loop
  extracted the matching coefficient between quasi and light-cone
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005 (arXiv:2005.10939)
- (Shohini's talk) **role of zero-mode contributions for twist-3 chiral-odd PDFs $h_L(x)$ and $e(x)$**
  (importance of chiral-odd PDFs: R.L. Jaffe, X. Ji, PRL 67(1991)552, NPB 375(1992)527)
  light-cone and quasi do not fully agree in the infrared
  breakdown of matching?
  S. Bhattacharya et al., arXiv:2006.12347



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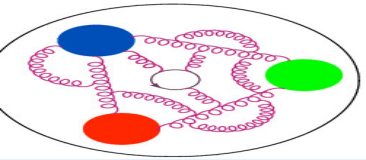
- (this talk) **lattice extraction of isovector combination $g_T^{u-d}(x)$**
  test of Wandzura-Wilczek (WW) approximation
  S. Bhattacharya et al., arXiv:2004.04130



Reminder of quasi-PDFs lattice procedure

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spatial correlation in a boosted nucleon
$\langle N(P_3)|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(P_3)\rangle$
lattice computation of bare ME

↓

renormalization
intermediate RI scheme
conversion to $\overline{\text{MMS}}$ scheme
(incl. evolution to $\mu = 2$ GeV)

↓

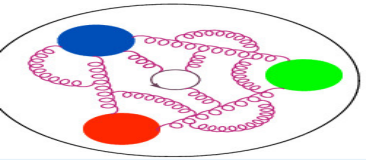
reconstruction of x-dependence
z-space$\to x$-space
Backus-Gilbert

↓

matching to light cone
$\overline{\text{MMS}} \to \overline{\text{MS}}$

↓

light-cone PDF



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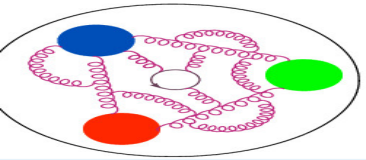
Lattice matrix element:

$$\mathcal{M}_{g_T}(P_3, z) = \langle N(P_3)|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(P_3)\rangle$$

Dirac structure: $\Gamma = \gamma^x\,\gamma^5$ or $\Gamma = \gamma^y\,\gamma^5$



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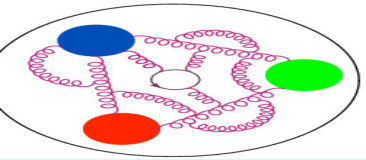
Lattice setup: S. Bhattacharya et al., arXiv:2004.04130

- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $a \approx 0.093$ fm, $m_\pi \approx 260$ MeV.
- $32^3 \times 64$, $L \approx 3$ fm, $m_\pi L \approx 4$,



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- $32^3 \times 64$, $L \approx 3$ fm, $m_\pi L \approx 4$,

- Use of momentum smearing
- Source-sink separation: $t_s = 12a$ (1.12 fm)
- Nucleon boosts: $4\pi/L$, $6\pi/L$, $8\pi/L$ (0.83, 1.25, 1.67 GeV)
- 1552, 11696, 105216 measurements

Bare matrix elements

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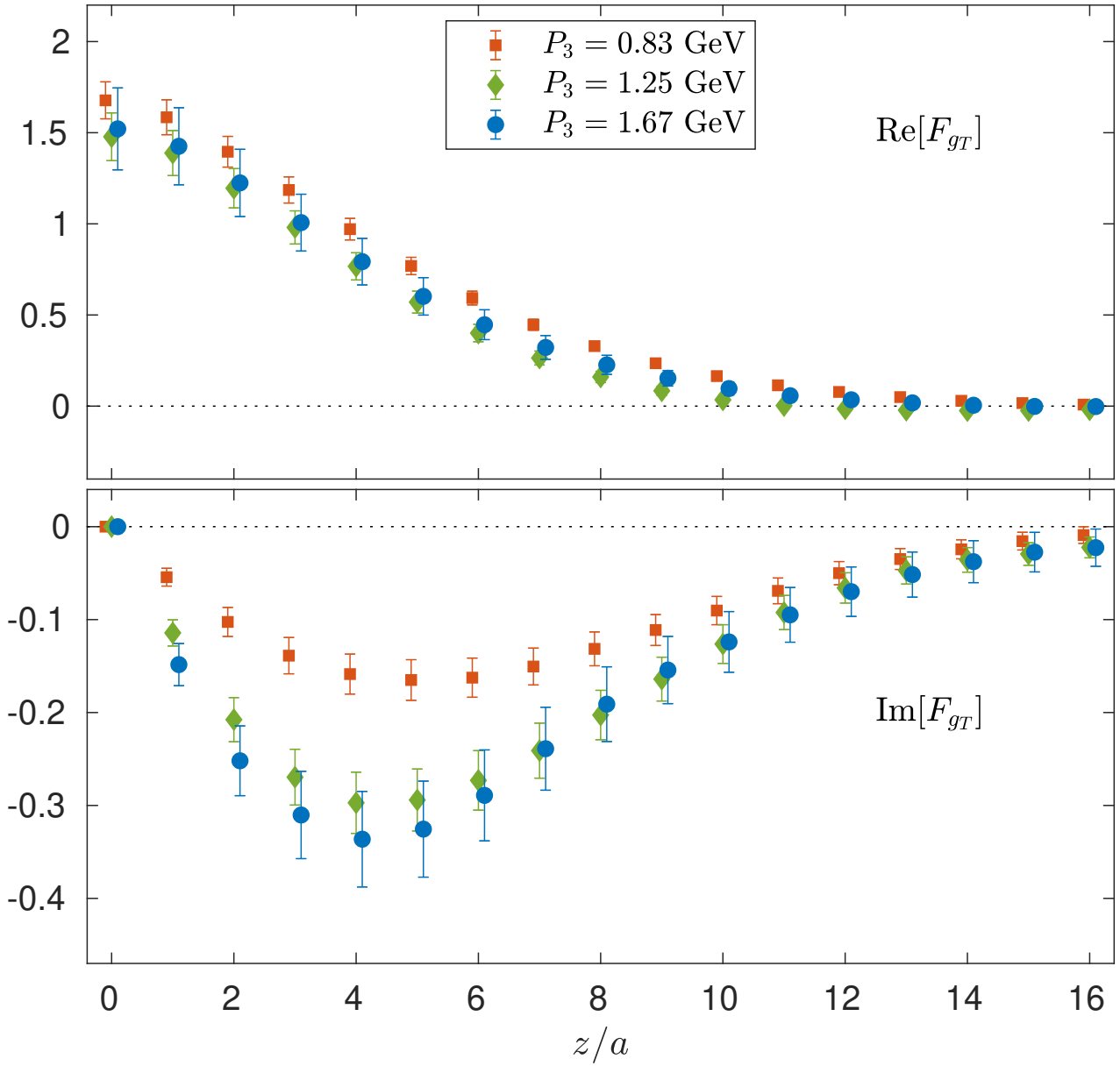
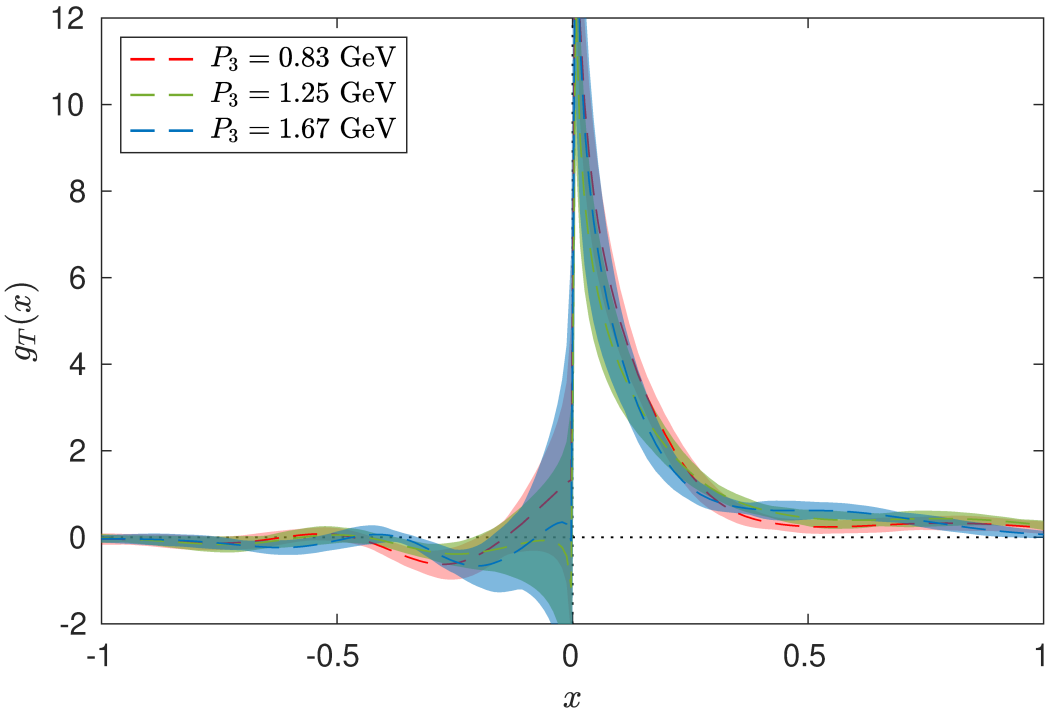


Results for g_T and g_1

Nucleon boost dependence
(after matching)
(quasi-g_T reconstructed with BG)



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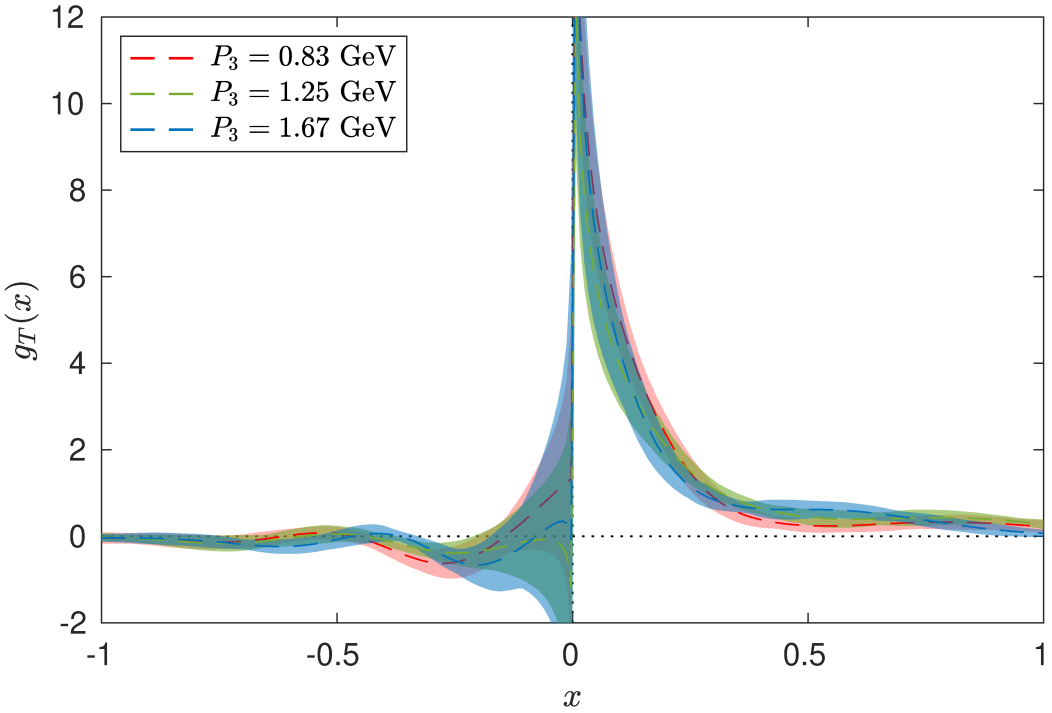
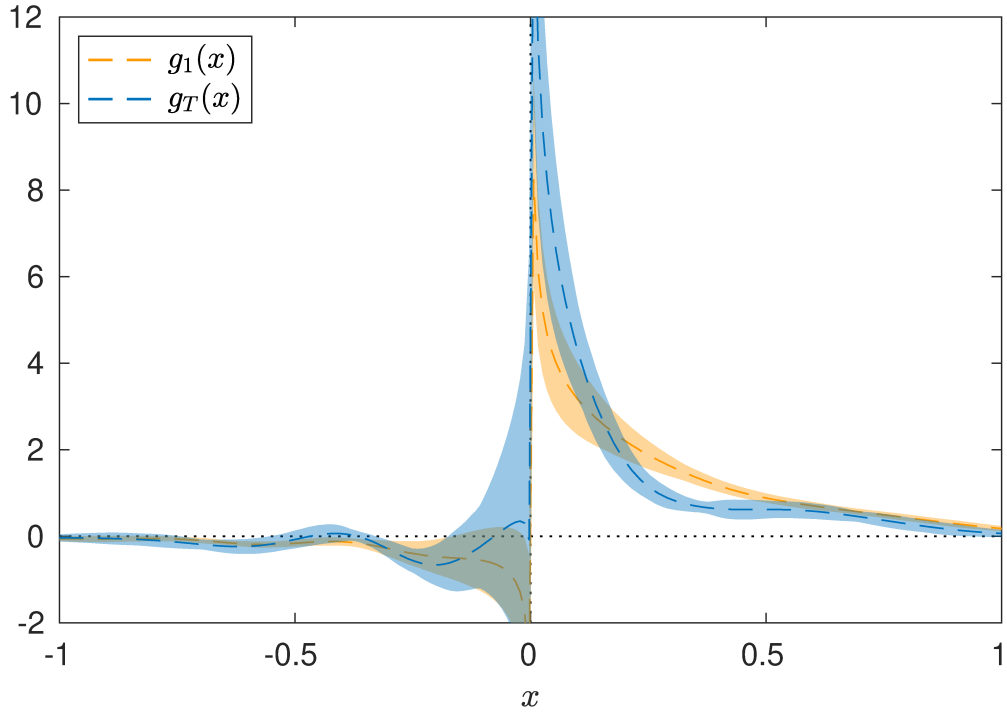


Twist-2 g_1 vs. twist-3 g_T
(at the largest boost)



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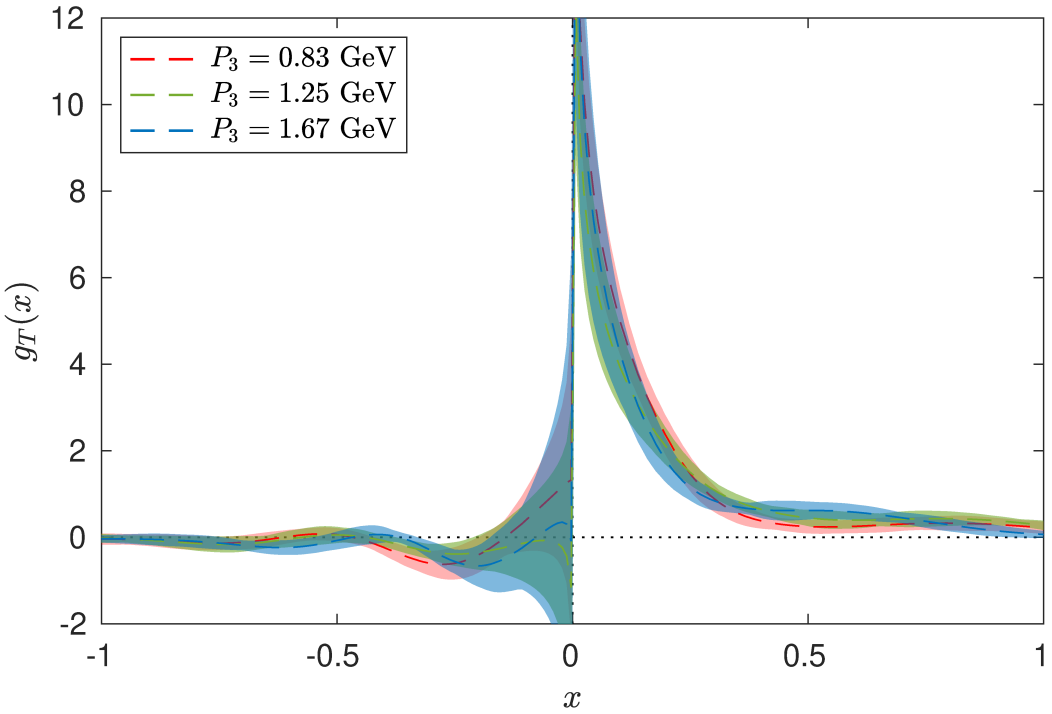

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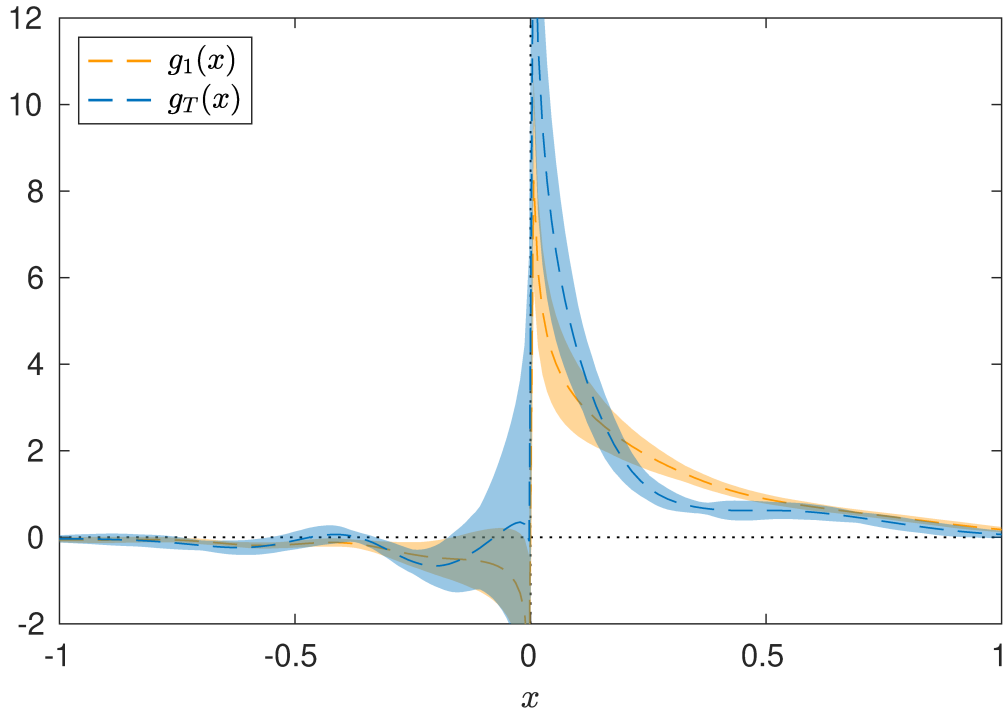

Burkhardt-Cottingham sum rule:

$$\int_{-1}^{1} dx g_T(x) = \int_{-1}^{1} dx g_T(x)$$

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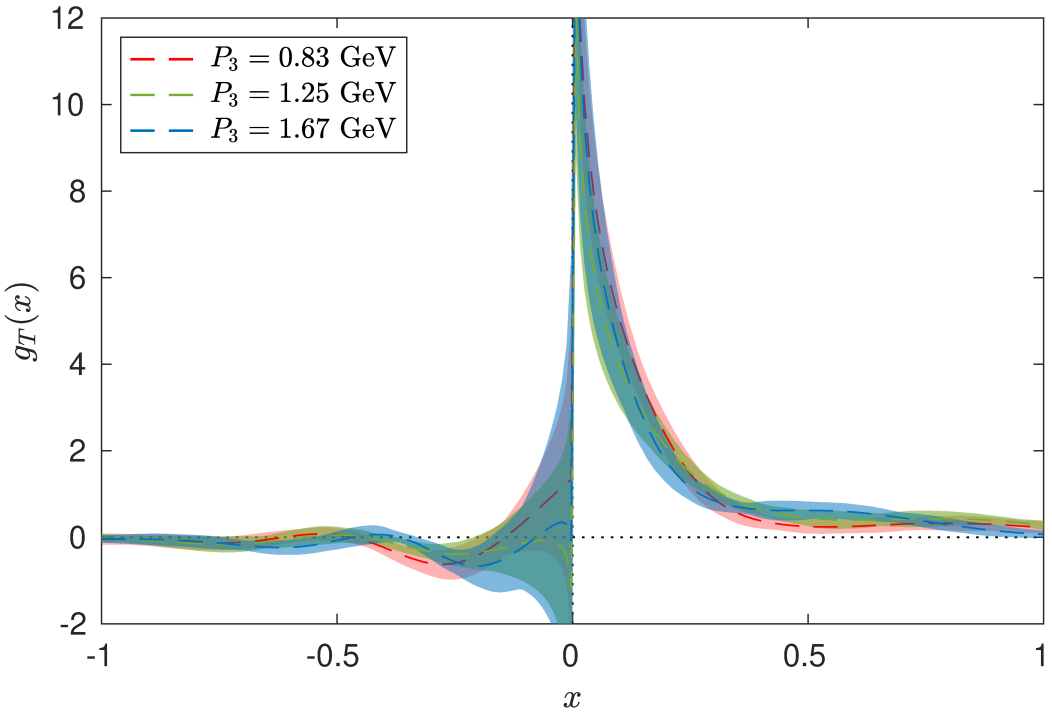

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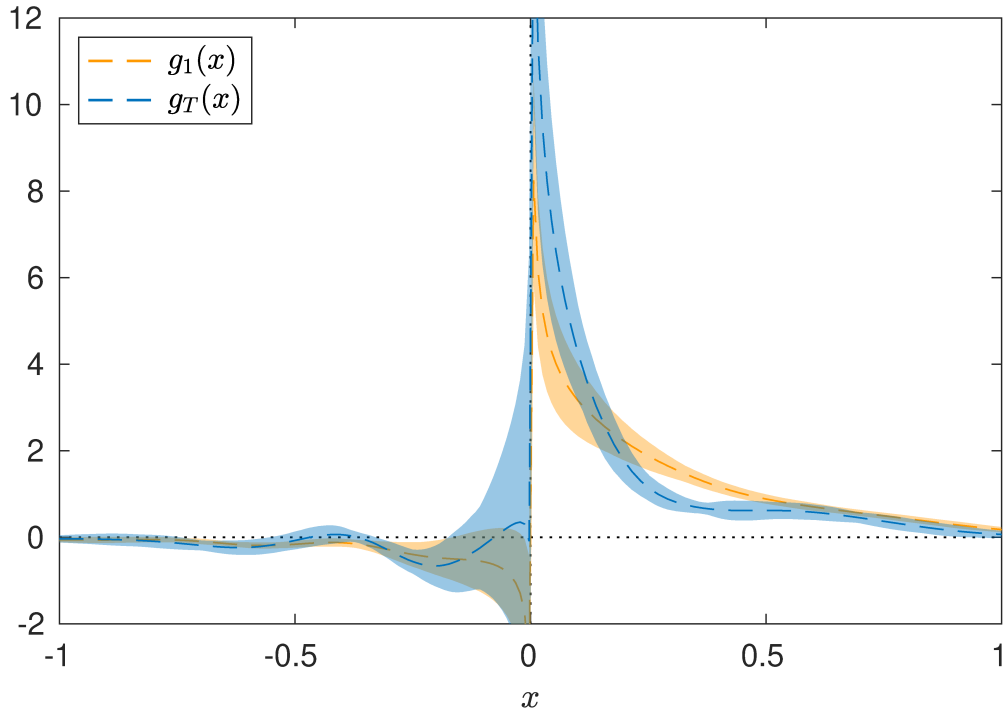

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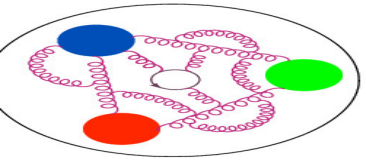
satisfied in our data.



Wandzura-Wilczek approximation

WW approximation: twist-3 $g_T(x)$ fully determined by twist-2 $g_1(x)$:

$$g_T^{\mathrm{WW}}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$

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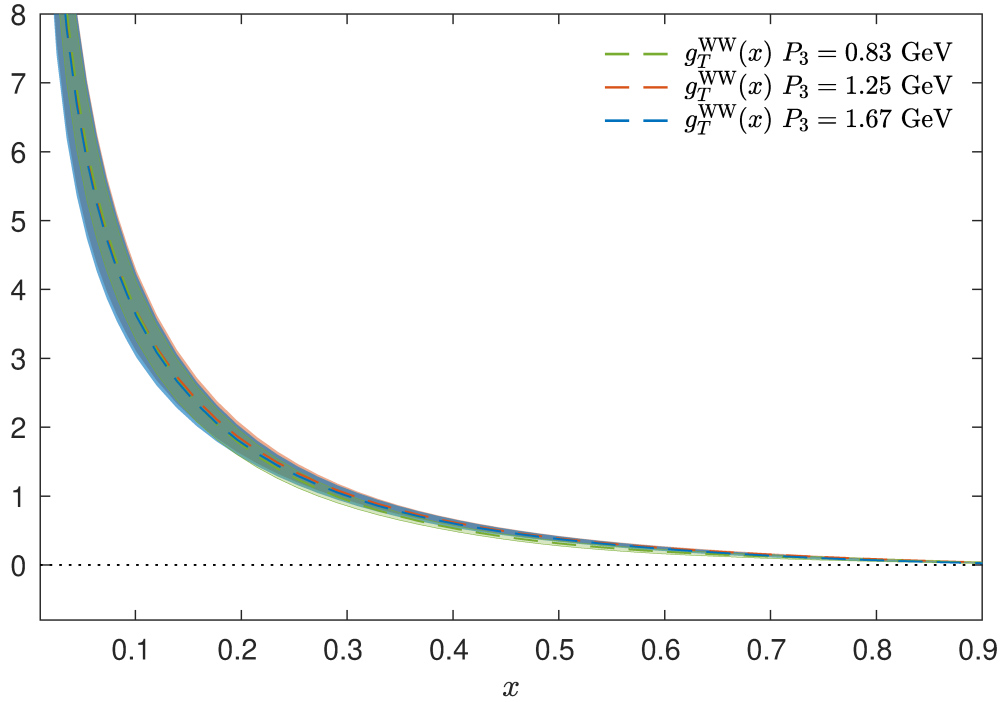

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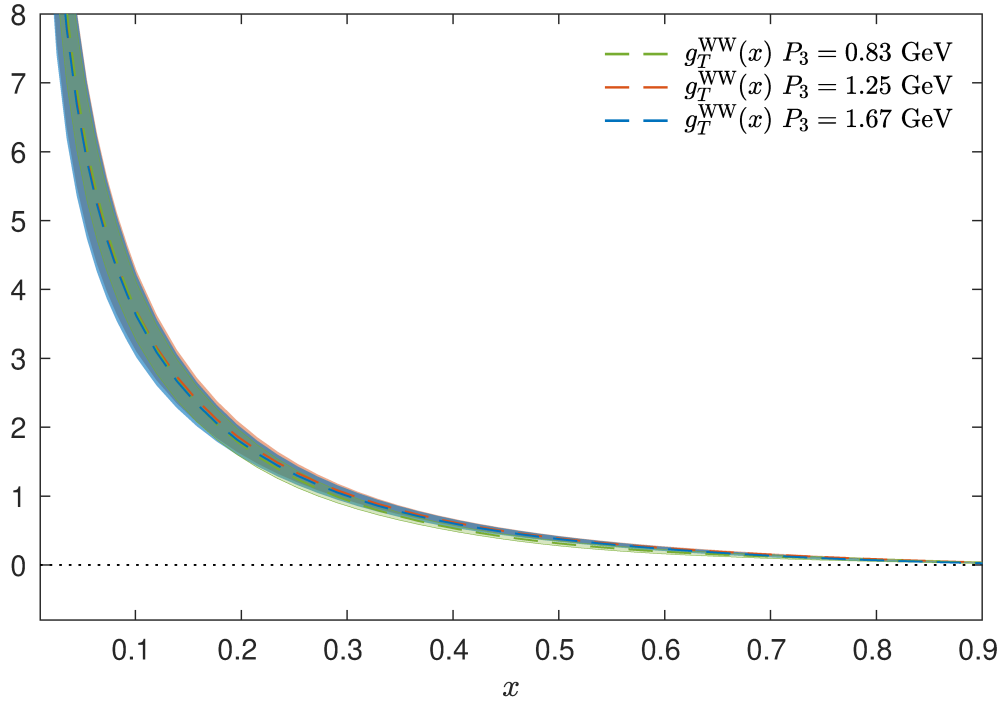

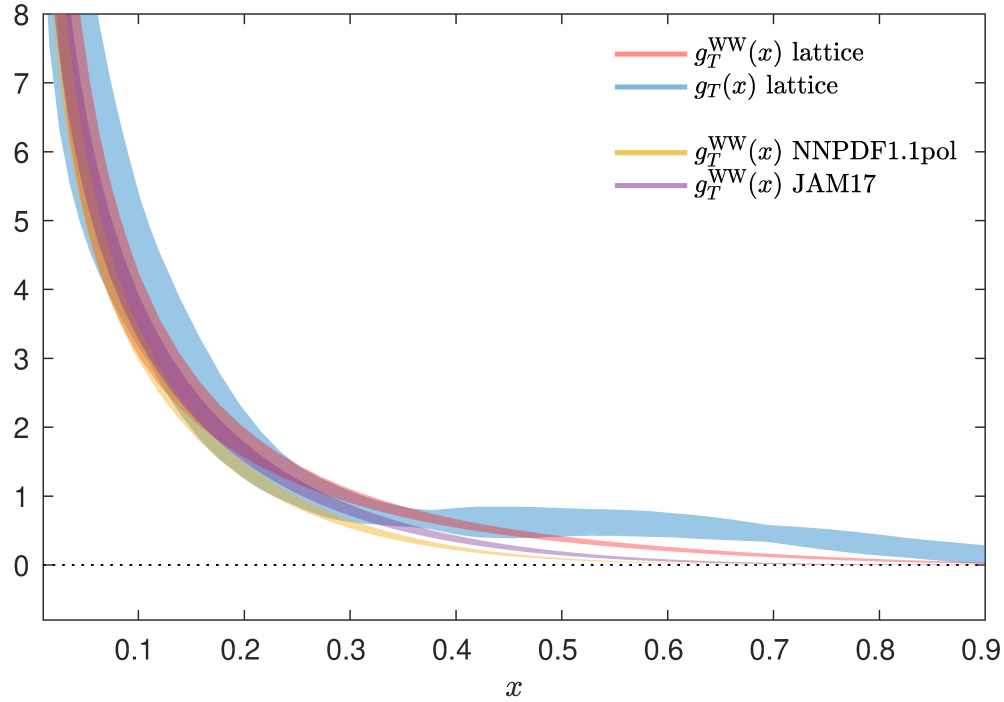

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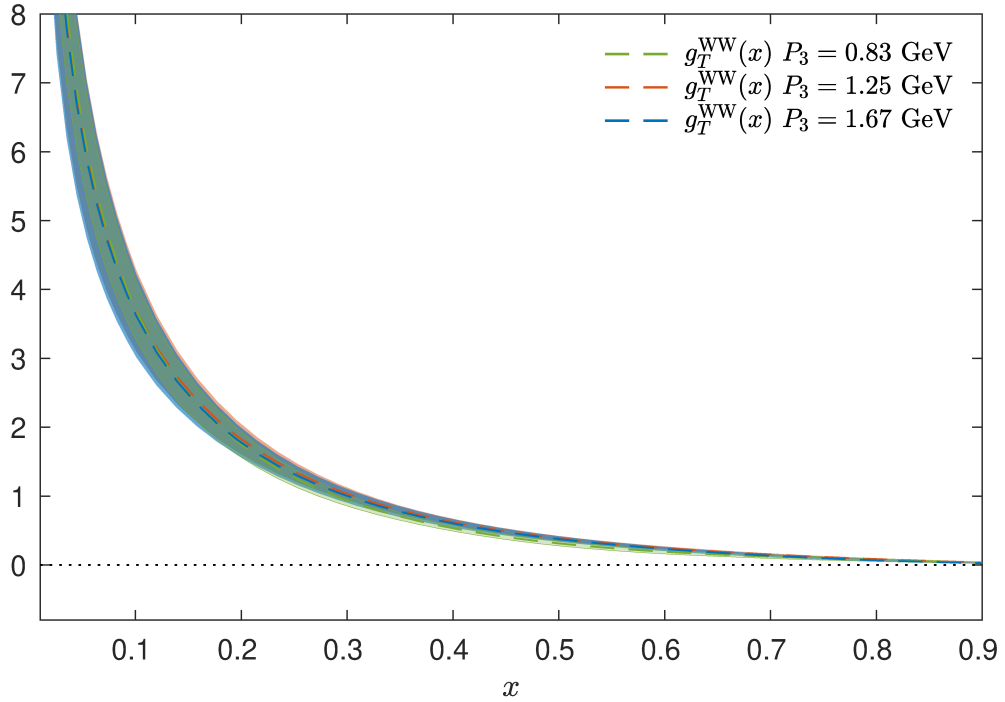

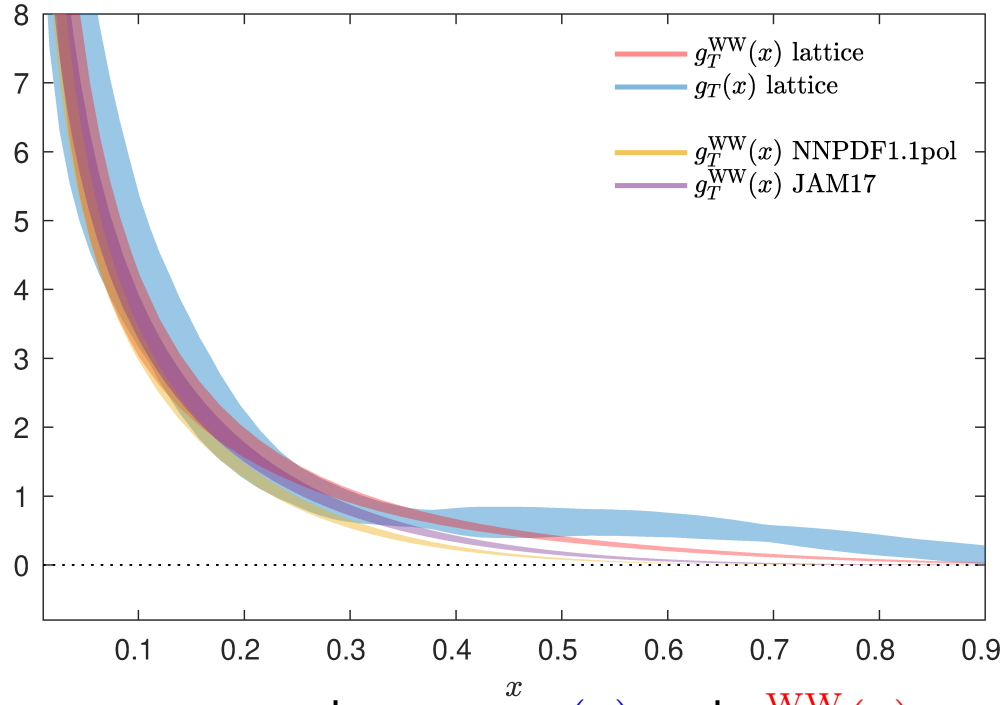

agreement between $g_T(x)$ and $g_T^{\mathrm{WW}}(x)$
for $x \lesssim 0.5$ within uncertainties

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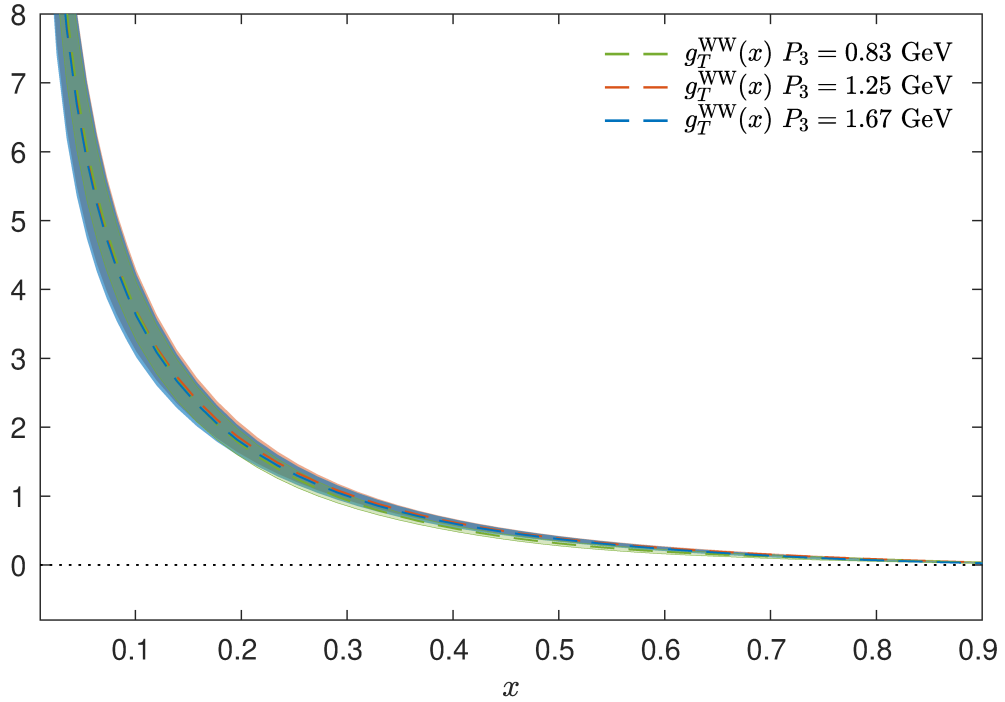

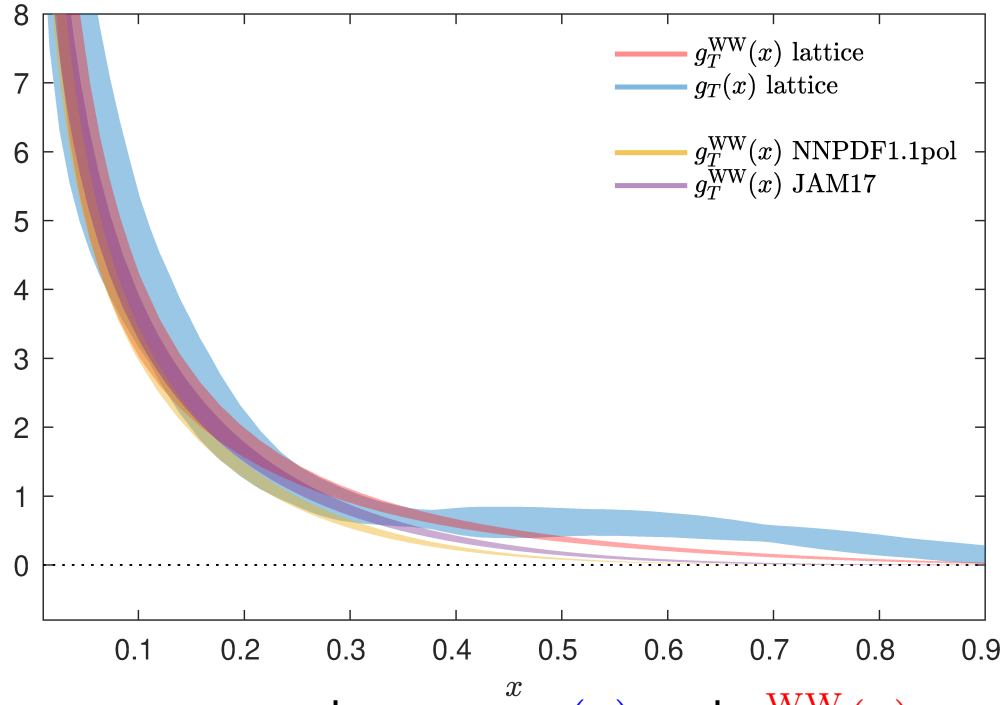

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still: possible violation up to 30-40%

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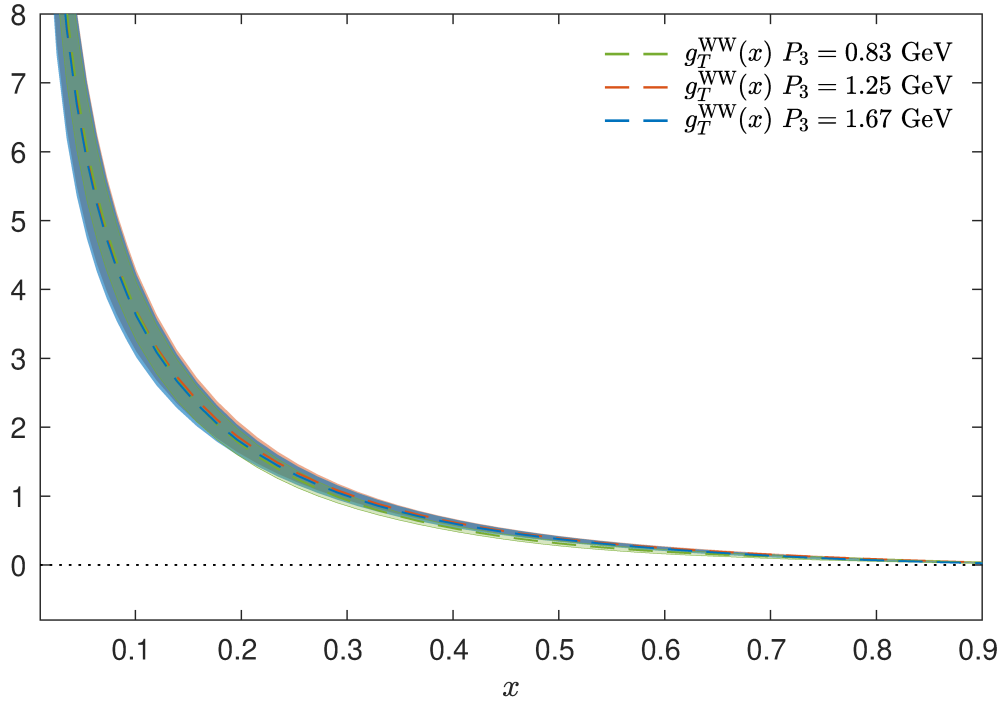

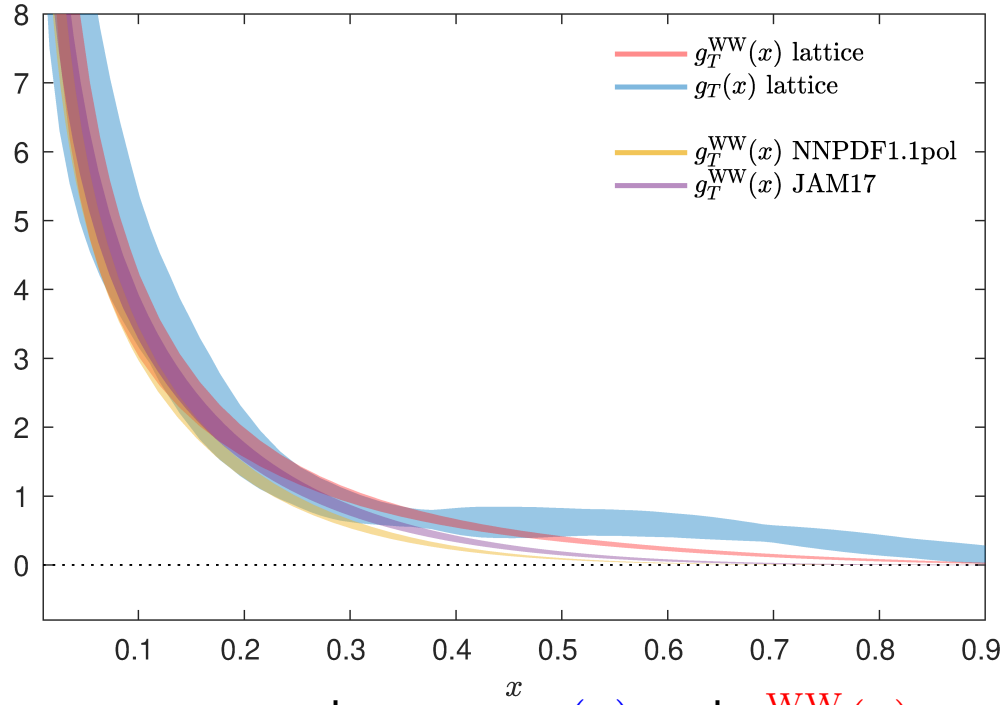

agreement between $g_T(x)$ and $g_T^{\mathrm{WW}}(x)$
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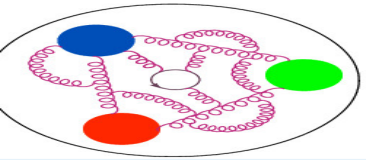
still: possible violation up to 30-40%
interestingly, similar possible violation (15-40%)
in experimental data analysis by JLab:

A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, JHEP 11 (2009) 093



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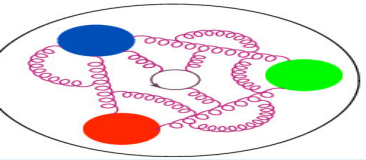

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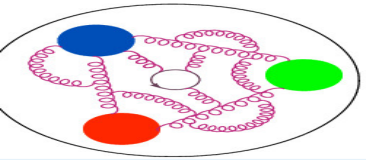

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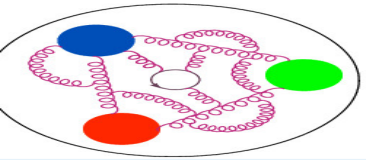

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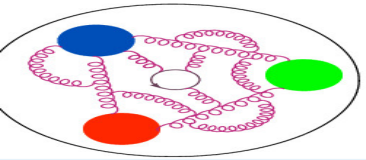

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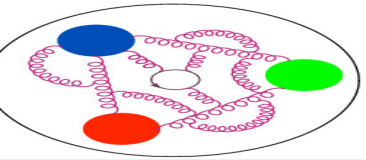

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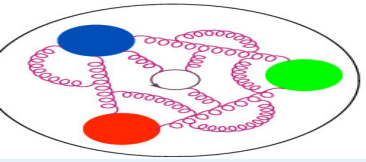
- In the future, lattice results for twist-3 can have important impact on phenomenology.



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