Nucleon Ioffe-time Pseudo-distributions from Distillation

LaMET 2020
September 10, 2020

Colin Egerer

On Behalf of the
HadStruct Collaboration

Roadmap

- ➔ Pseudo-distributions
- ➔ Numerics
  - ◆ distillation & target ensemble
- ➔ Selected matrix elements
- ➔ Extracted (unpolarized) pseudo-ITD
- ➔ PDF Extraction
  - ◆ matching of coordinate space data
  - ◆ pheno. ITDs - reduction of potential biases
- ➔ Summary/Prospects

HadStruc Collaboration

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Balint Joó[2]

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Pseudo-Ioffe-time Distributions (pITDs)

A. V. Radyushkin, Phys. Rev. D96, 034025 (2017), arXiv:1705.01488 [hep-ph]

➔ A single-hadron matrix element X. Ji, Phys. Rev. Lett. 110, 262002 (2013)

pITDs

$$M^\alpha(z,p) = \langle h(p)|\,\overline{\psi}(z)\,\gamma^\alpha W(z,0;A)\,\psi(0)\,|h(p)\rangle = 2p^\alpha \mathcal{M}_\mathrm{p}(\nu,z^2) + z^\alpha \mathcal{M}_\mathrm{z}(\nu,z^2)$$

➔ Well-chosen kinematics -- access leading twist

$$p^\mu = (p^0, 0, 0, p_3) \qquad z^\mu = (0,0,0,z_3)$$

➔ A short-distance factorization to ITD

Perturbatively computable matching coefficients

$$\mathcal{M}(\nu,z^2) = \sum_{a=q,\bar{q},g} C_a(z^2\mu^2,\alpha_s) \otimes \mathcal{I}_a(\nu,\mu^2) + h.t.$$

T. Izubuchi, et al., Phys.Rev. D98 (2018) no.5, 056004
A. Radyushkin, Phys.Lett. B781 (2018) 433-442
A. Radyushkin, Phys. Rev. D 98 (2018) no.1, 014019
J.-H. Zhang, et al., Phys.Rev. D97 (2018) no.7, 074508

➔ Wilson line UV-divergent for spacelike - z^2

➔ Compute $M^\alpha(z,p)$ but instead analyze

$$\mathfrak{M}(\nu,z^2) \equiv \frac{\mathcal{M}_\mathrm{p}(\nu,z^2)}{\mathcal{M}_\mathrm{p}(0,z^2)}$$

K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017)

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- Well-chosen kinematics -- access leading twist

$$p^{\mu} = (p^0, 0, 0, p_3) \qquad z^{\mu} = (0, 0, 0, z_3)$$

C. Monahan - Tues. 10:30am
J. Karpie - Thur. 10:00am
W. Morris - Fri. 11:44am

- A short-distance factorization to ITD

Perturbatively computable matching coefficients

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Distillation - I

M. Peardon et al., Phys. Rev. D80, 054506 (2009), arXiv:0905.2160 [hep-lat]

$$J_{\sigma,n_\sigma} = e^{\sigma\nabla^2} = \sum_\lambda e^{-\sigma\lambda}\,|\lambda\rangle\langle\lambda|$$

➔ Low-mode approximation to some gauge-covariant smearing kernel (e.g. $\mathcal{D}ist = \sum_{i=1}^{N} |\lambda\rangle\langle\lambda|$)

$$-\nabla^2_{ab}\left(\vec{x},\vec{y};t\right) = 6\delta_{xy}\delta_{ab} - \sum_{j=1}^{3}\left[\tilde{U}_j\left(\vec{x},t\right)_{ab}\delta_{x+\hat{\jmath},y} + \tilde{U}_j^\dagger\left(\vec{x}-\hat{\jmath},t\right)_{ab}\delta_{x-\hat{\jmath},y}\right]$$

$$-\nabla^2\xi^{(k)} = \lambda^{(k)}\xi^{(k)}$$

➔ Define Distillation of $\mathrm{rk}\left(\mathcal{D}ist\right) = N \ll N_c \times V_3$

$$\square\left(\vec{x},\vec{y};t\right)_{ab} = \sum_{k=1}^{N}\xi_a^{(k)}\left(\vec{x},t\right)\xi_b^{(k)\dagger}\left(\vec{y},t\right)$$

Key to control of excited-states

➔ Admits extended basis of interpolators [GEVP]

- low-lying meson spectrum — R. Briceno et al., Phys.Rev.D 97 (2018) 5, 054513; J. Dudek et. al., Phys.Rev.D 88 (2013) 9, 094505; J. Dudek et al., Phys.Rev.D 87 (2013) 3, 034505
- exotic hadrons — J. Dudek, et. al., Phys. Rev.D83, 111502 (2011); L. Liu, et. al., JHEP 07, 126 (2012)

Distillation - II

"Perambulators"

$$\tau^{kl}_{\alpha\beta}(t_f, t_0) = \xi^{(k)\dagger}(t_f)\, M^{-1}_{\alpha\beta}(t_f, t_0)\, \xi^{(l)}(t_0)$$

$$\Phi^{(i,j,k)}_{\mu\nu\sigma}(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a \left(\mathcal{D}_2\xi^{(j)}\right)^b \left(\mathcal{D}_3\xi^{(k)}\right)^c (t)\, S_{\mu\nu\sigma}$$

"Elementals"

Irrep. projection

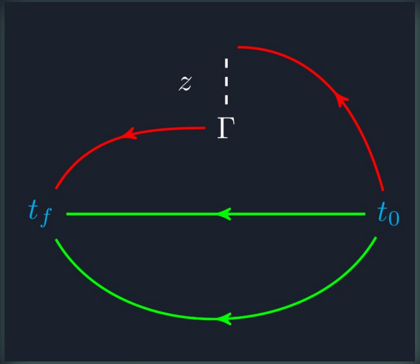
➔ Wick contract "distilled" (smeared) fields

$$C_{mn}(t) = \sum_{\vec{x},\vec{y}} \langle 0|\, \mathcal{O}_m(t,\vec{x})\, \mathcal{O}^{\dagger}_n(0,\vec{y})\, |0\rangle$$

$$\equiv \mathrm{Tr}\left[\Phi_m(t) \otimes \tau(t,0)\, \tau(t,0)\, \tau(t,0) \otimes \Phi_n(0)\right]$$

Why Distillation ?

- *factorization of correlation functions*
- *explicit momentum projections - all times*
- *reusability*

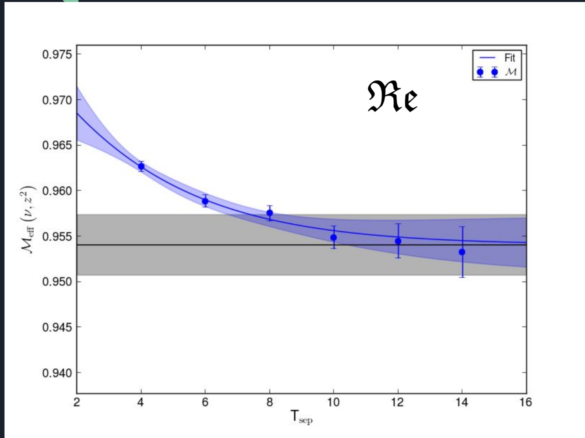


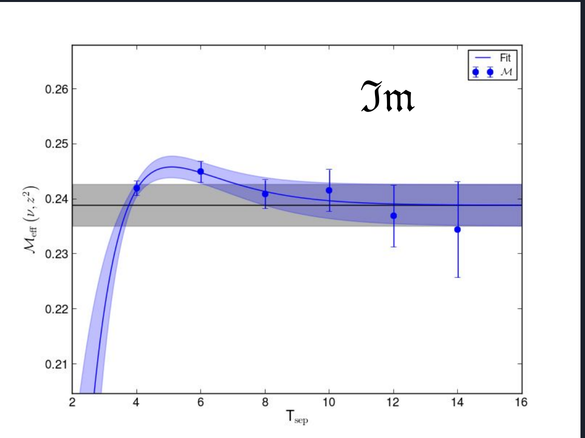
➔ Unpolarized pseudo-ITD workload

ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	N_{cfg}	N_{srcs}	N_{vec}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64

$$\tilde{\tau}^{ij}_{\text{pITD}}(t_f, t_0; \tau, \vec{z}; \vec{q}) = \boxed{\sum_{\vec{z}_0}} e^{i\vec{q}\cdot\vec{z}_0}\, \xi^{(i)\dagger}(t_f)\, D^{-1}(t_f; \vec{z}+\vec{z}_0, \tau)\, \Gamma W(\vec{z}+\vec{z}_0, \vec{z}_0; \tau)\, D^{-1}(\vec{z}_0, \tau; t_0)\, \xi^{(j)}(t_0)$$

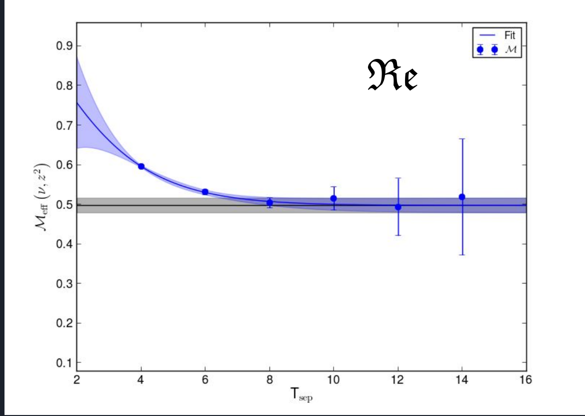
Selected Reduced Summed-Ratio Fits

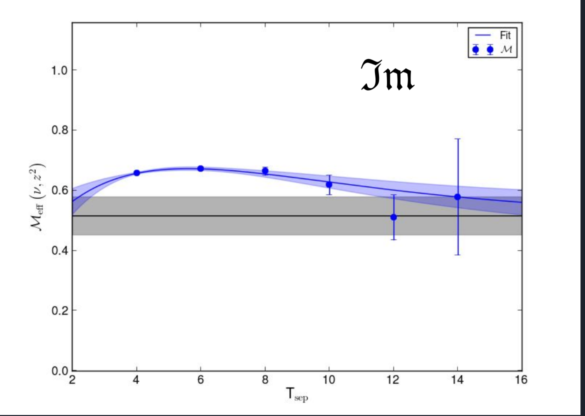




$$p_z \simeq 1.19 \text{ GeV}$$
$$[p_{lat} = 1]$$
$$z/a = 6$$

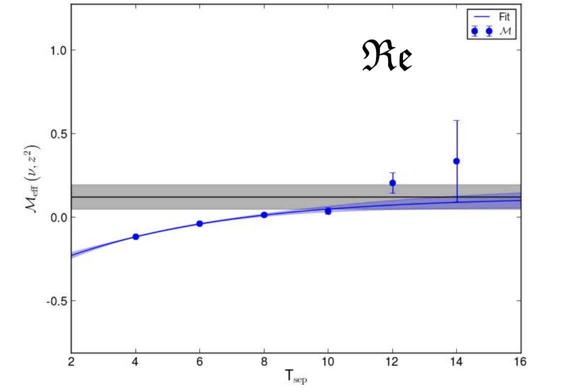
$$f(T) = \mathcal{M} + \mathcal{A}e^{-\Delta ET} + \mathcal{B}Te^{-\Delta ET}$$

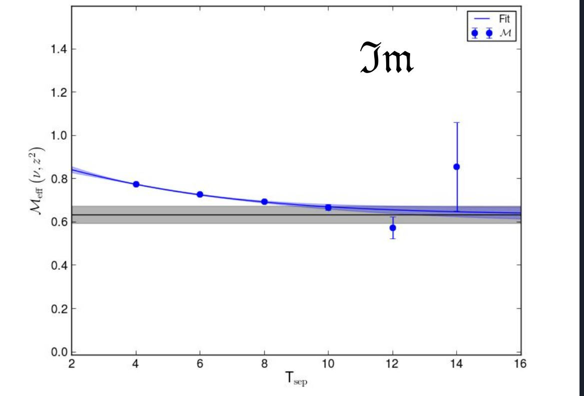




$$p_z \simeq 1.63 \text{ GeV}$$
$$[p_{lat} = 3]$$
$$z/a = 8$$

Selected (Phased) Reduced Summed-Ratio Fits

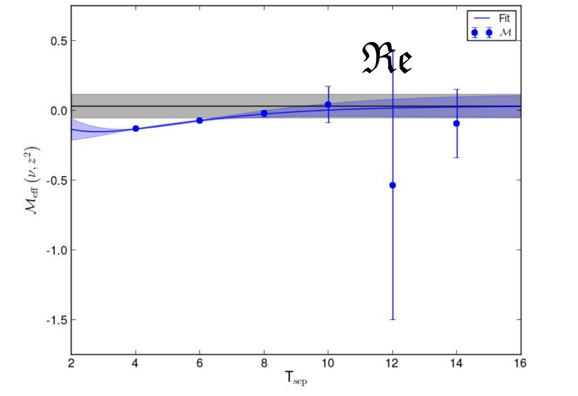


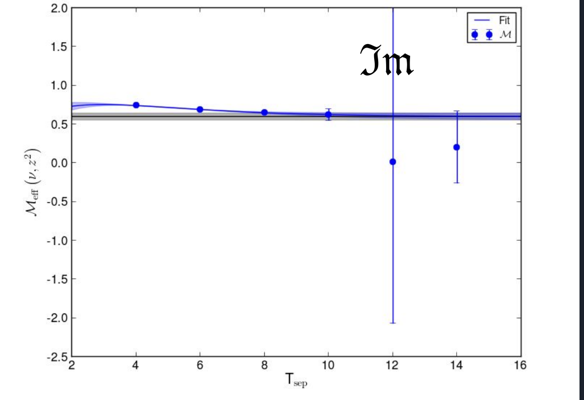


$$p_z \simeq 1.90 \text{ GeV}$$
$$[p_{lat} = 4]$$
$$z/a = 8$$

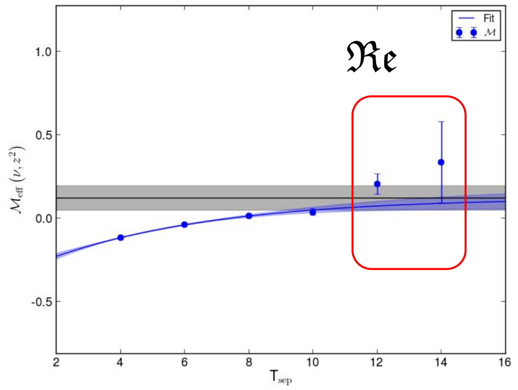
$$f(T) = \mathcal{M} + \mathcal{A}e^{-\Delta E T} + \mathcal{B}Te^{-\Delta E T}$$

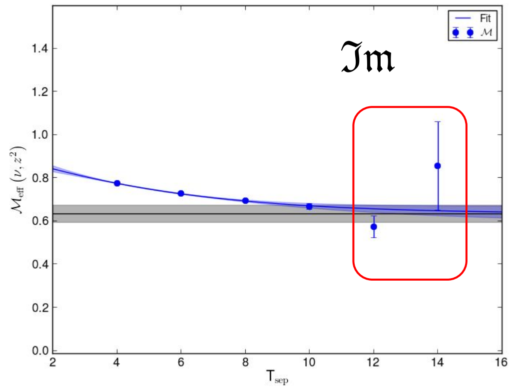
$$p_z \simeq 2.45 \text{ GeV}$$
$$[p_{lat} = 6]$$
$$z/a = 6$$





Selected (Phased) Reduced Summed-Ratio Fits





$p_z \simeq 1.90 \text{ GeV}$
$[p_{lat} = 4]$

$z/a = 8$

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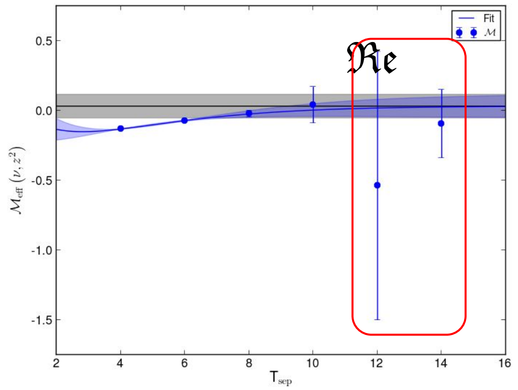
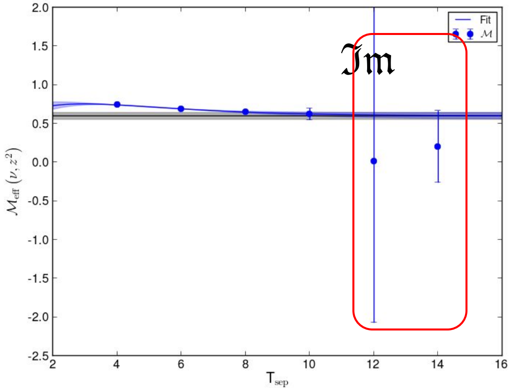
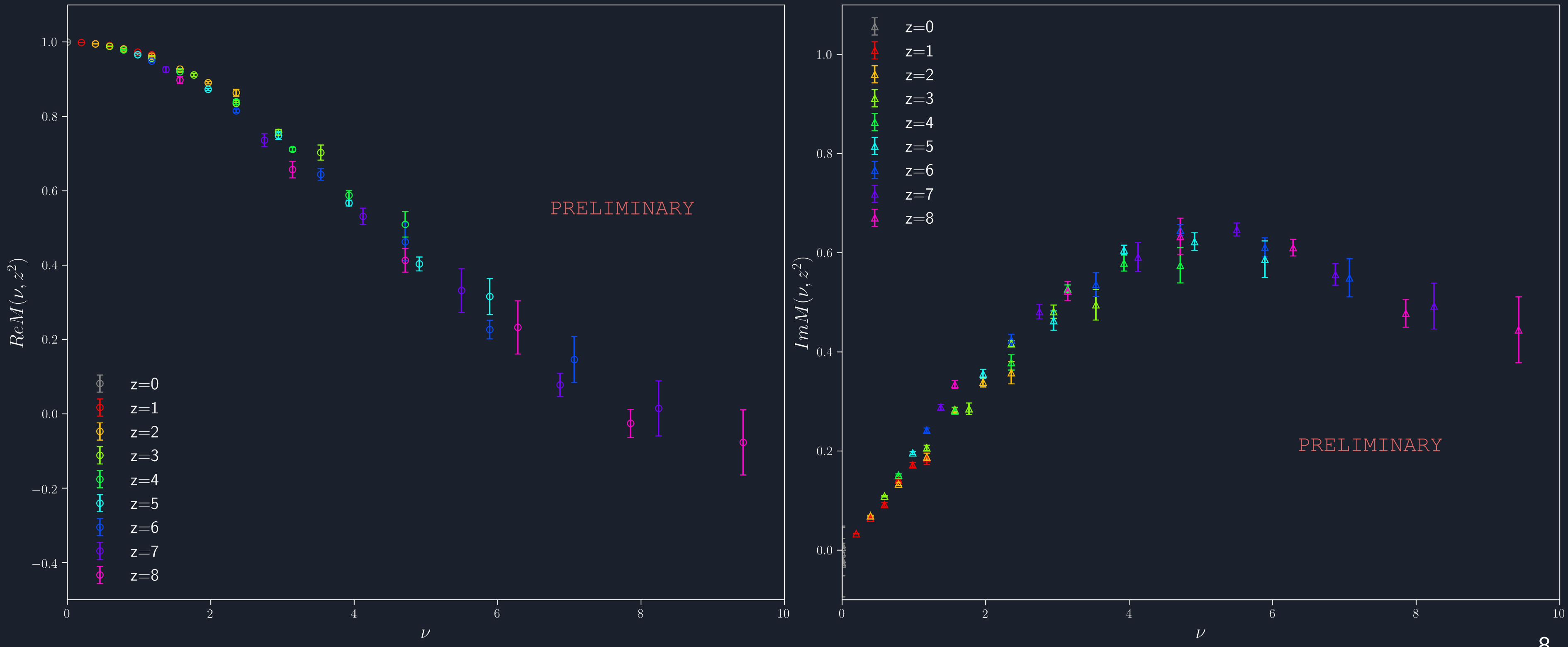
$[p_{lat} = 6]$

$z/a = 6$





Unpolarized Pseudo-ITDs



Unpolarized ITDs

PDFs from ITDs & Pseudo-ITDs

$$\mathfrak{M}\left(\nu, z^2\right) = \int_0^1 \mathrm{d}u \boxed{\mathcal{C}\left(u, z^2\mu^2, \alpha_s\left(\mu\right)\right)} \mathcal{Q}\left(u\nu, \mu^2\right) + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

$$\mathcal{C}\left(u, z^2\mu^2, \alpha_s\left(\mu\right)\right) = \delta\left(1-u\right) - \frac{\alpha_s}{2\pi} C_F \left[\ln\left(\frac{e^{2\gamma_E+1} z^2\mu^2}{4}\right) B\left(u\right) + D\left(u\right)\right]$$

- Evolve/Match pseudo-ITD to ITD
  - parametrize PDF

$$q_{\mathrm{V}}\left(x, \mu\right) = \frac{x^{\alpha}\left(1-x\right)^{\beta}\left(1 + \sum_k \lambda_k x^{(k+1)/2}\right)}{B\left(\alpha+1, \beta+1\right) + \sum_k \lambda_k B\left(\alpha+1+\frac{k+1}{2}, \beta+1\right)}$$

  - numerically fit cosine-transform to ITD
  - [method C]

- Cast ITD as cosine-transform of PDF
  - closed-form expression

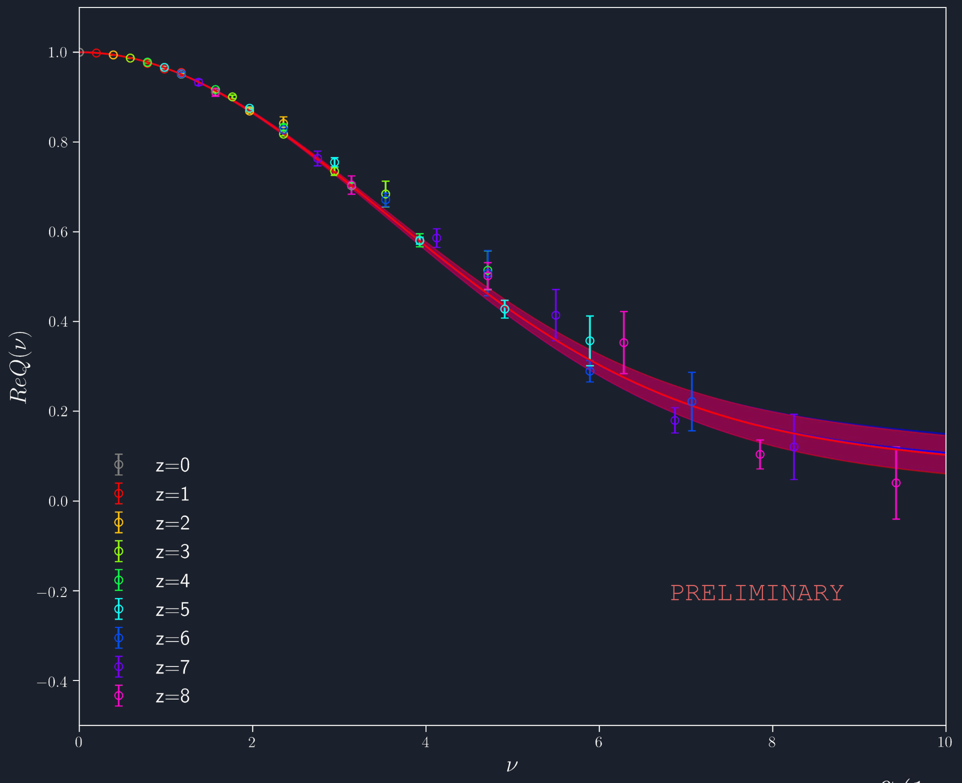
A. Radyushkin, Phys.Rev.D 100 (2019) 11, 116011
T. Izubuchi et al., Phys.Rev.D 98 (2018) 5, 056004

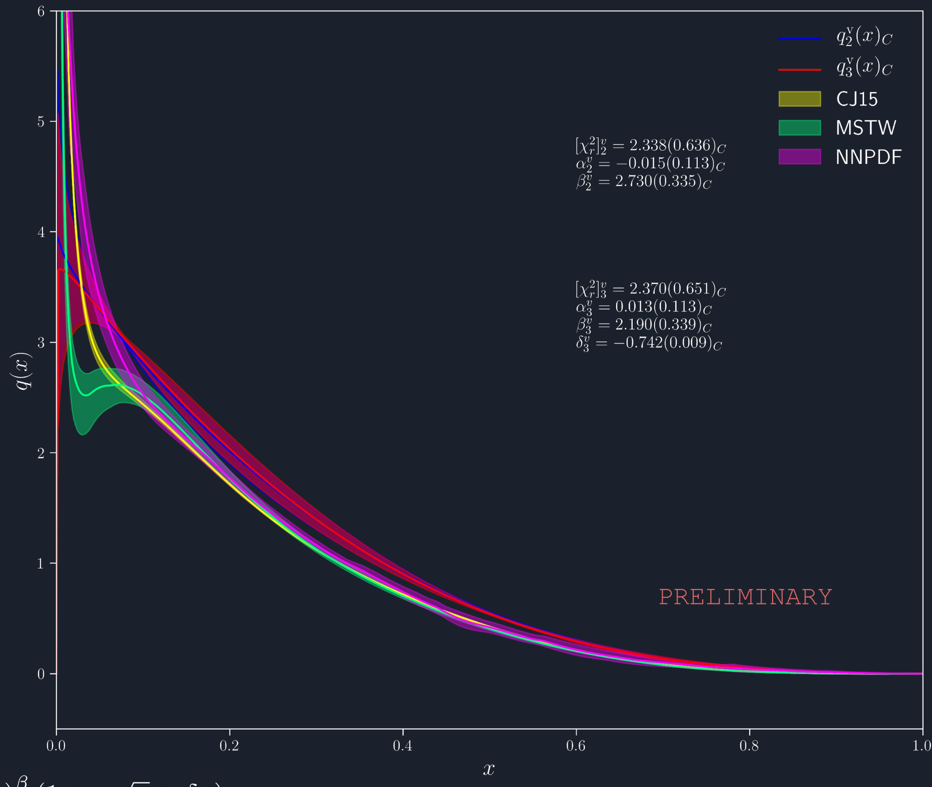
  - directly fit PDF parameters to pseudo-ITD data

Caution: $\;{}_3F_3\left(111; 222; -ix\nu\right)$

  - [method K]

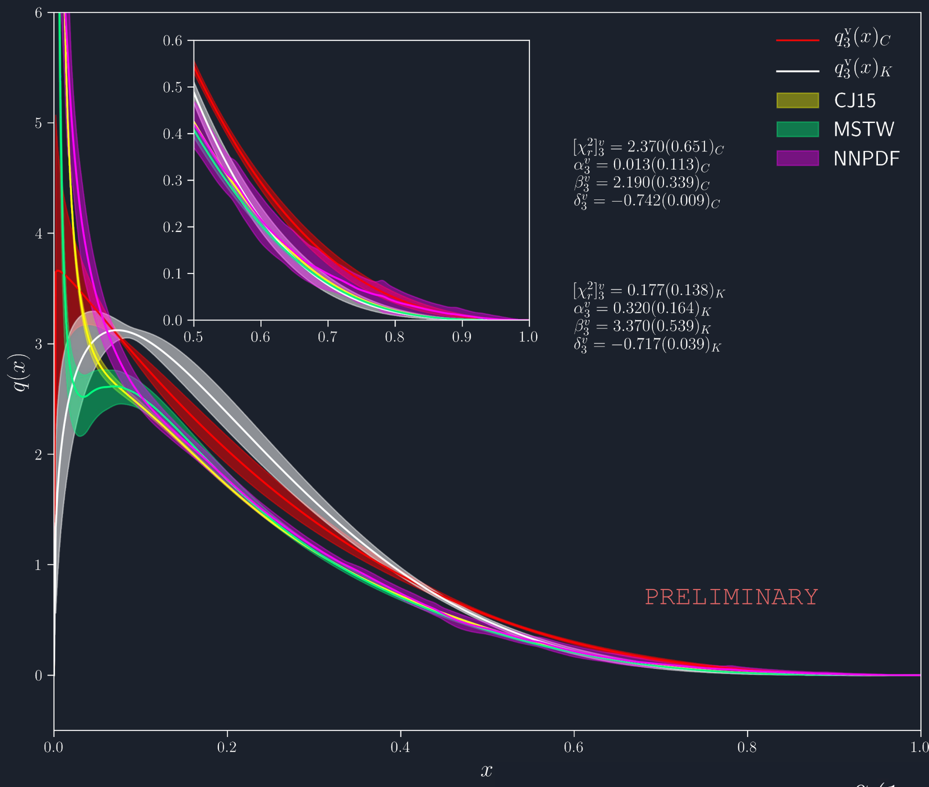
PDFs and Phenomenological Comparison

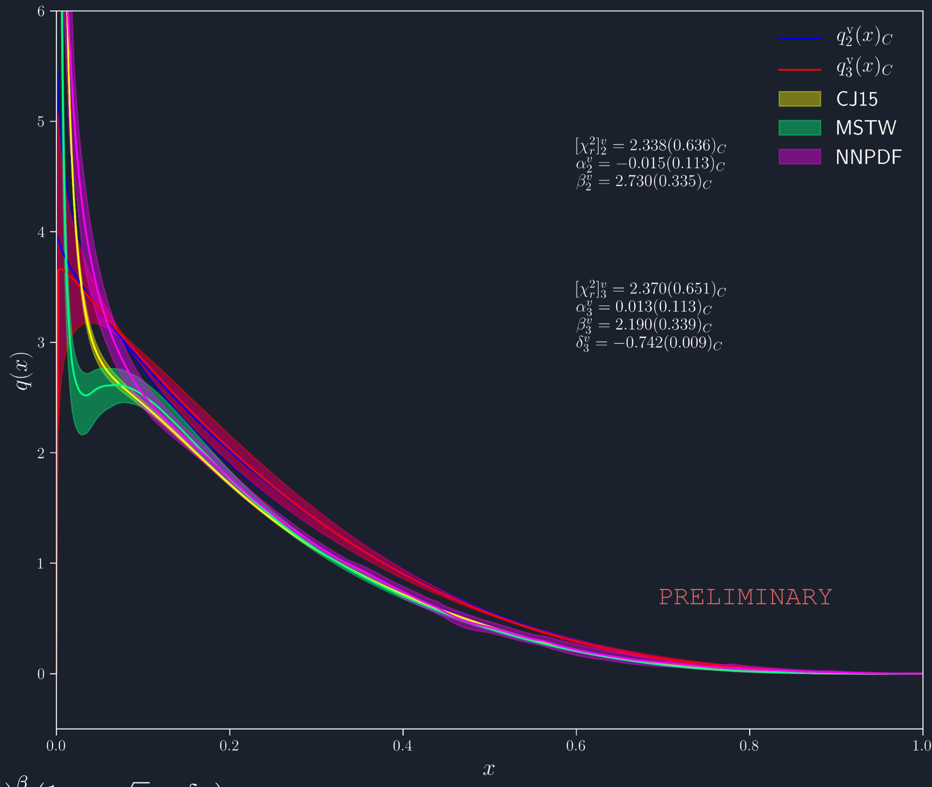




$$q_{\rm V}(x,\mu) = \frac{x^\alpha (1-x)^\beta \left(1+\gamma\sqrt{x}+\delta x\right)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+3/2,\beta+1)+\delta B(\alpha+2,\beta+1)}$$

PDFs and Phenomenological Comparison



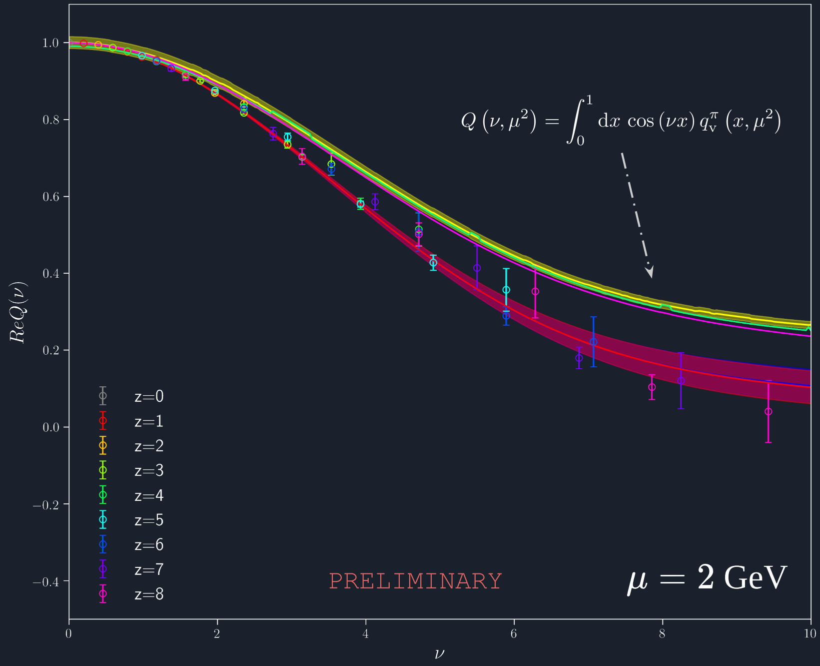


$$q_{\mathrm{V}}(x,\mu) = \frac{x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\delta x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+3/2,\beta+1)+\delta B(\alpha+2,\beta+1)}$$

Phenomenological ITDs

➔ Model/parametrization - independent determination

- isolate ITDs of phenomenological PDFs



➔ discrepancy not a surprise

➔ role of [as yet] unquantified systematics

$$\frac{\mathcal{M}_\mathrm{p}(\nu, z^2; p_z)/\mathcal{M}_\mathrm{p}(\nu, 0; p_z)}{\mathcal{M}_\mathrm{p}(\nu, z^2; p'_z \neq 0)/\mathcal{M}_\mathrm{p}(\nu, 0; p'_z \neq 0)}$$

X. Gao et al., arXiv: 2007.06590 [hep-lat]

➔ nuisance terms

$$\mathfrak{M}(\nu, z^2) = \mathfrak{M}_\mathrm{cont}(\nu, z^2) + \sum_n (a|p_z|)^n \mathfrak{G}_n(\nu) + (a/|z|)^n \mathfrak{H}_n(\nu)$$

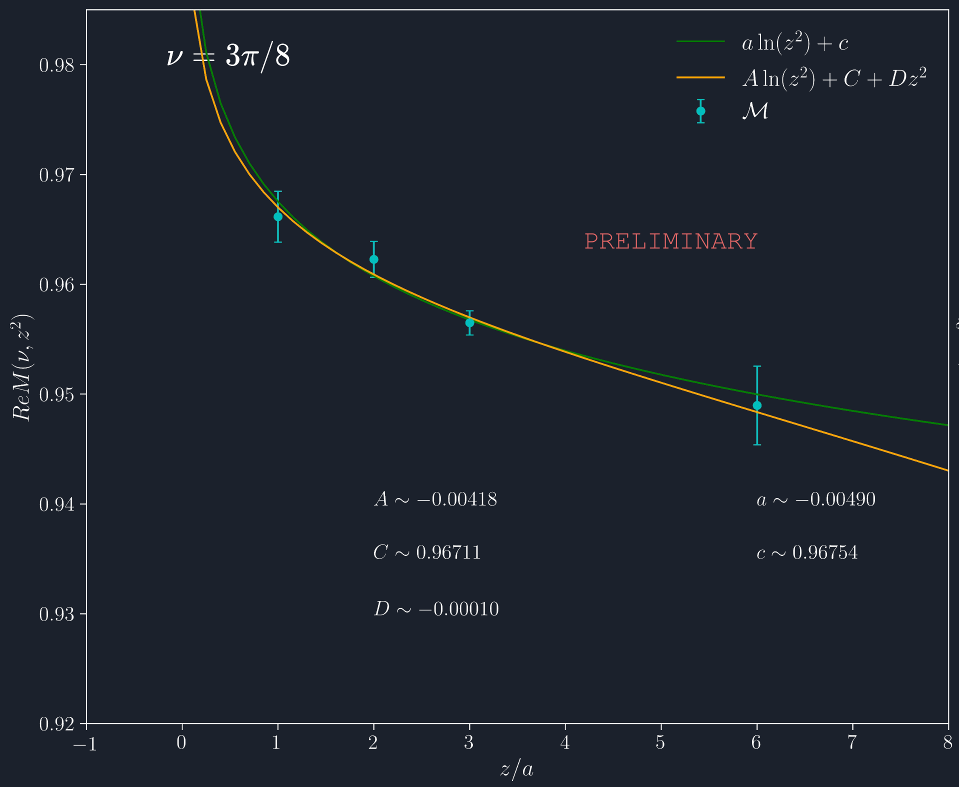
...more ensembles...

Summary

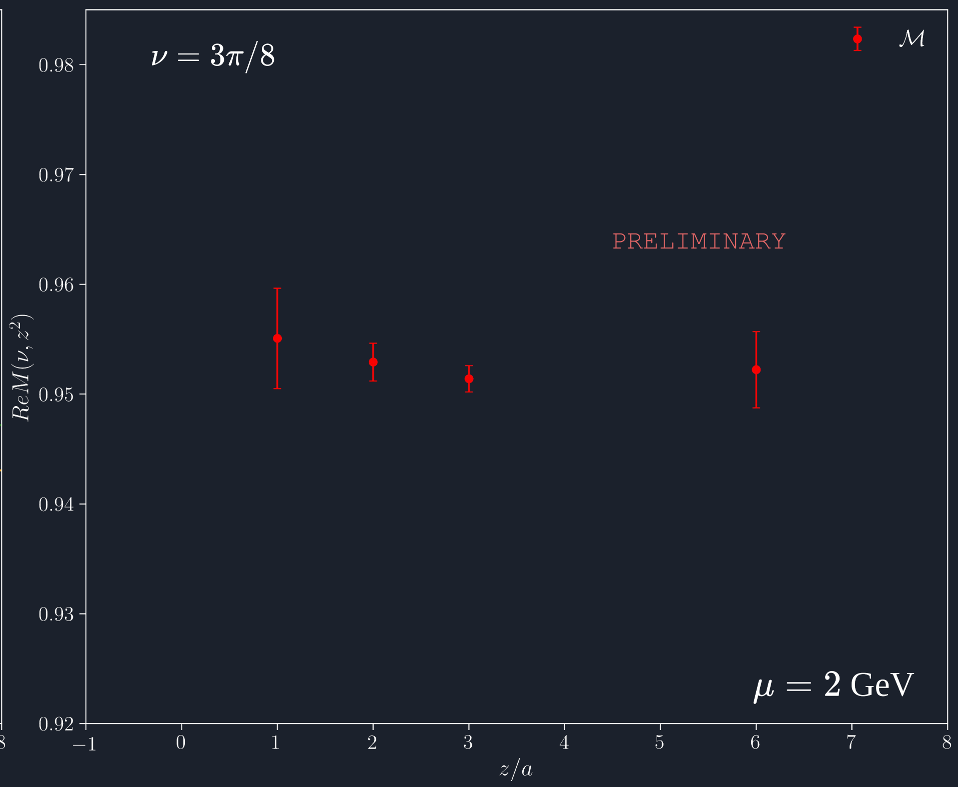
- Light-cone physics inaccessible directly in LQCD
  - factorizable matrix elements
    - Ioffe-time pseudo-distributions (pITDs)
- Distillation - first use in [partonic] structure calculations
  - improved statistical precision [evidence of dglap]
  - attenuation of excited-states
  - reusability - e.g. hadronic PDFs (polarized)/GPDs/DAs
- PDF parametrizations [pITD -> PDF   vs.   ITD -> PDF]
  - pheno. ITDs highlight need to quantify systematics
- Excited to share results in coming months...

THANK YOU!

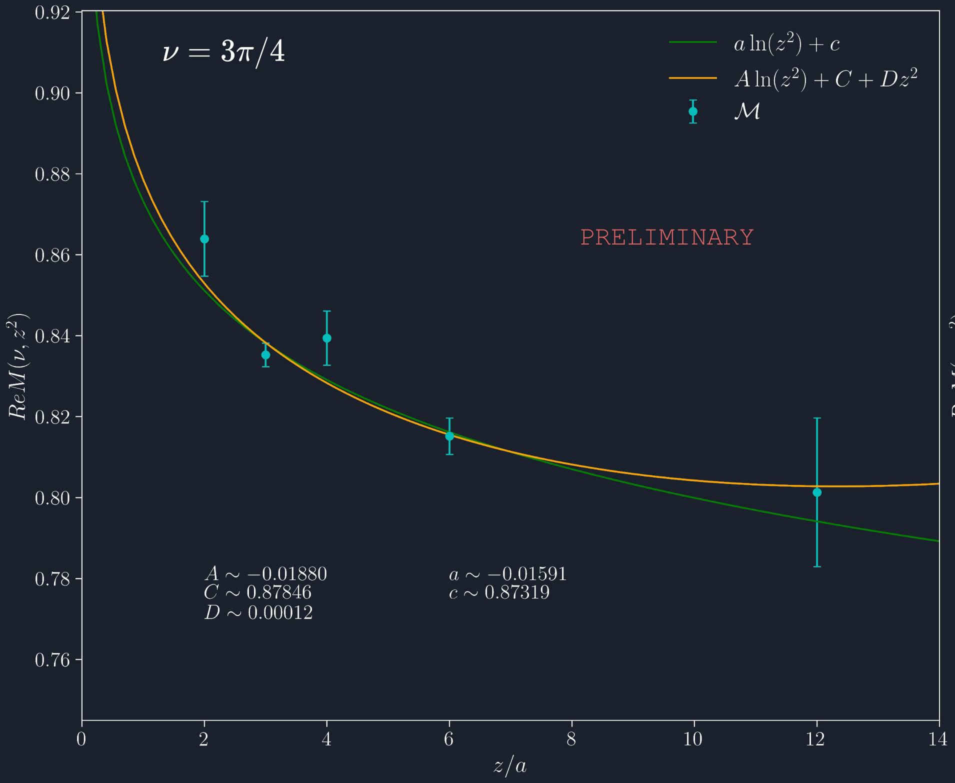
Backups...



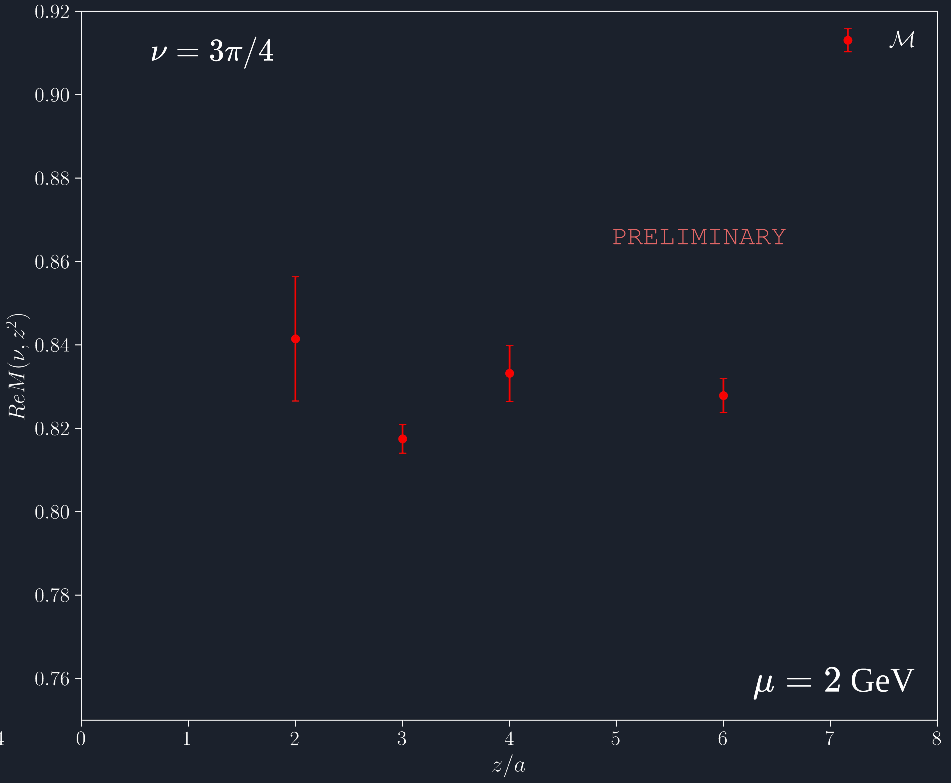
$\nu = 3\pi/8$
$a\ln(z^2) + c$
$A\ln(z^2) + C + Dz^2$
$\mathcal{M}$
PRELIMINARY
$A \sim -0.00418$
$C \sim 0.96711$
$D \sim -0.00010$
$a \sim -0.00490$
$c \sim 0.96754$
$ReM(\nu, z^2)$
z/a



$\nu = 3\pi/8$
$\mathcal{M}$
PRELIMINARY
$\mu = 2$ GeV
$ReM(\nu, z^2)$
z/a



$\nu = 3\pi/4$
$a\ln(z^2) + c$
$A\ln(z^2) + C + Dz^2$
$\mathcal{M}$
PRELIMINARY
$A \sim -0.01880$
$C \sim 0.87846$
$D \sim 0.00012$
$a \sim -0.01591$
$c \sim 0.87319$
$ReM(\nu, z^2)$
z/a



$\nu = 3\pi/4$
$\mathcal{M}$
PRELIMINARY
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$ReM(\nu, z^2)$
z/a