Identification of Intra-Bunch Dynamics of CERN SPS
LARP / HiLumi Collaboration Meeting

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Outline

- Introduction
- Why Reduced Models and Dynamics Identification
- Estimation of Model Parameters
- Results
- Conclusion

Feedback Control

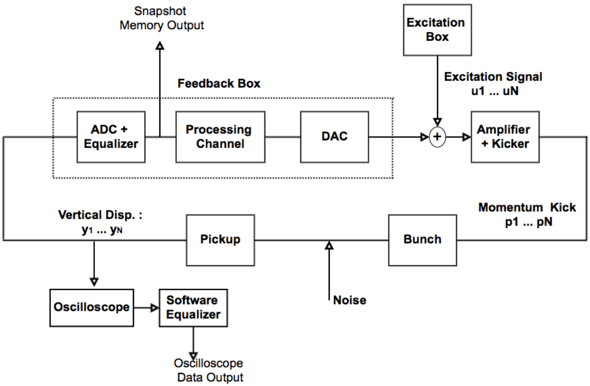


Figure : Closed Loop Operation Block Diagram

Open Loop Operation

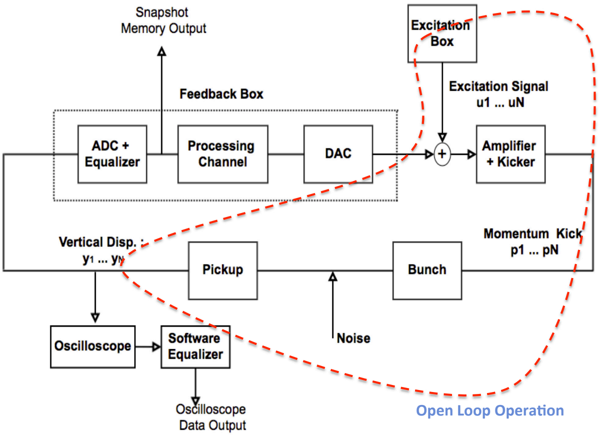


Figure : Open loop operation where $\overrightarrow{U_t}$ is applied and $\overrightarrow{Y_t}$ is measured where $\overrightarrow{U_t} \in R^m$, $\overrightarrow{Y_t} \in R^p$ and $t = 1 : Nturns$

Bunch Dynamics Extraction

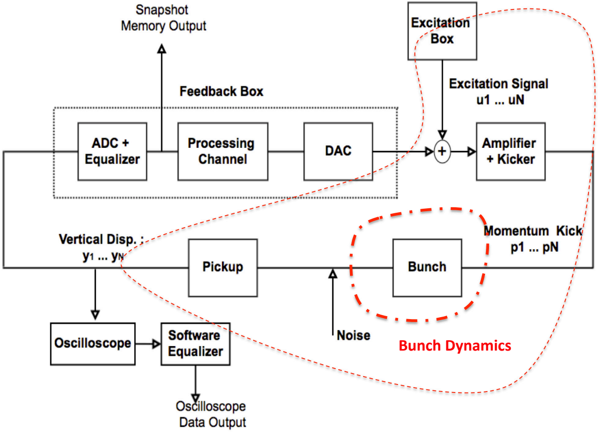


Figure : Objective is for given $\vec{U_t}$ and measured $\vec{Y_t}$, find the reduced order model and estimate its parameters so that model can replicate the dominant bunch dynamics

MDs, Simulations and Reduced Order Models

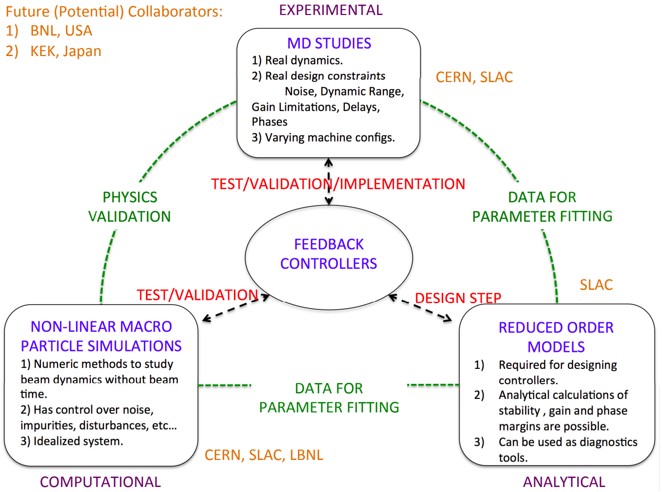


Figure : We use both machine data and non-linear macro-particle simulations codes (HEADTAIL, CMAD) in reduced model parameter estimation

Reduced Order Model Parameter Identification

- The ultimate goal is to stabilize Ecloud and TMCI effects using wide-band intra-bunch feedback system.
- Controlling multiple locations across the bunch requires multi-input multi-output (MIMO) analysis.
  - Important ! For example, you can make higher modes unstable while trying to control dipole mode.
- The goal of identification is to fit parameters of the reduced order MIMO model using MD measurements and macro-particle simulation codes.
- Reduced order model is required to design a control architecture to stabilize effects of disturbances on intra-bunch dynamics under hardware and processing power constraints.

Model and Formalism

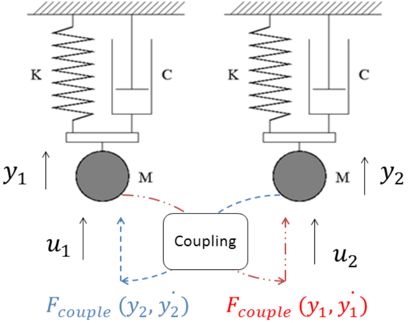


Figure : Reduced Model for Dipole and Head -Tail Modes

$$\mathbf{y}(z) = \hat{\mathbf{D}}(z)^{-1}\hat{\mathbf{N}}(z)\mathbf{u}(z)$$

$$\mathbf{u}(z) = \sum_{i=0}^{\infty} \mathbf{u}_i z^{-i}; \quad \mathbf{y}(z) = \sum_{i=0}^{\infty} \mathbf{y}_i z^{-i}$$

$$\hat{\mathbf{D}}(z) = \sum_{i=0}^{k} \mathbf{D}_i z^{i}; \quad \hat{\mathbf{N}}(z) = \sum_{i=0}^{k} \mathbf{N}_i z^{i}$$

$$\mathbf{D}_k = \mathbf{I}_p$$

$$\mathbf{Q}_o = [\mathbf{C}^T | \mathbf{A}^T\mathbf{C}^T | \cdots | (\mathbf{A}^T)^{k-1}\mathbf{C}^T]^T$$

$$\rho[\mathbf{Q}_o] = n = k * p$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{O} & \cdots & \mathbf{O} & -\mathbf{D}_0 \\ \mathbf{I}_p & \mathbf{O} & \cdots & \mathbf{O} & -\mathbf{D}_1 \\ \mathbf{O} & \mathbf{I}_p & \cdots & \mathbf{O} & -\mathbf{D}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \mathbf{I}_p & -\mathbf{D}_{k-1} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{N}_0 - \mathbf{D}_0\mathbf{N}_k \\ \mathbf{N}_1 - \mathbf{D}_1\mathbf{N}_k \\ \mathbf{N}_2 - \mathbf{D}_2\mathbf{N}_k \\ \vdots \\ \mathbf{N}_{k-1} - \mathbf{D}_{k-1}\mathbf{N}_k \end{bmatrix}$$

$$\mathbf{C} = [\mathbf{O} \quad \mathbf{O} \quad \cdots \quad \mathbf{O} \quad \mathbf{I}_p]; \quad \mathbf{D} = \mathbf{N}_k$$

Figure : Observable Canonical Form for Discrete Time MIMO System

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= CX_k \end{aligned} \tag{1}$$

where control variable (external excitation) $U \in R^p$, vertical displacement measurements $Y \in R^q$, system matrix $A \in R^{n\times n}$, input matrix $B \in R^{n\times p}$, and output matrix $C \in R^{q\times n}$.

Extension of the Reduced Order Model

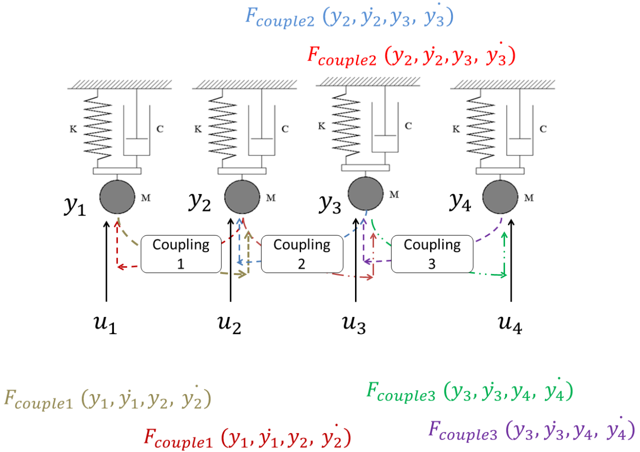


Figure : 4 x 4 MIMO Representation of the Intra-Bunch Dynamics

- Higher order dynamics can be analyzed by extending the model up to N coupled harmonic oscillators.
- For example, the model above can capture up to 4 modes.

Data for Identification

- Experimental data was collected from a single bunch with 1×10^{11} protons at 26 GeV with low chromaticity configuration at CERN SPS.
- Both open loop driven and closed loop feedback measurements were taken. Main focus is on open loop driven measurements for identification in this presentation.
- In analysis, we use data from April, November, December 2012 and January, February 2013 MDs together with CMAD and HEADTAIL simulations.

MD Measurements - 2 x 2 Model

- Drive SPS bunch using frequency chirp excitation signal and record corresponding vertical motion.
- Based on the transfer functions of cable plant, amplifiers and kicker, we calculate the momentum kick that beam goes through.
- Using the momentum kick signal and vertical displacement measurement, estimate the parameters of reduced order model.
- Drive the reduced order model with the same momentum kick signal to get vertical motion that model estimates.

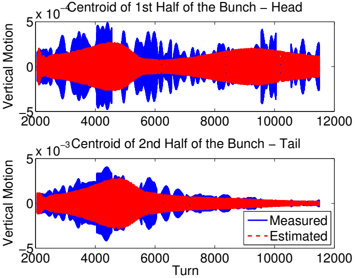


Figure : Comparison between the measured vertical motion of the head and the tail with model's response.

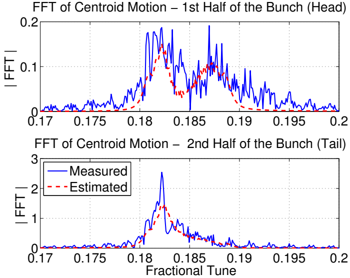
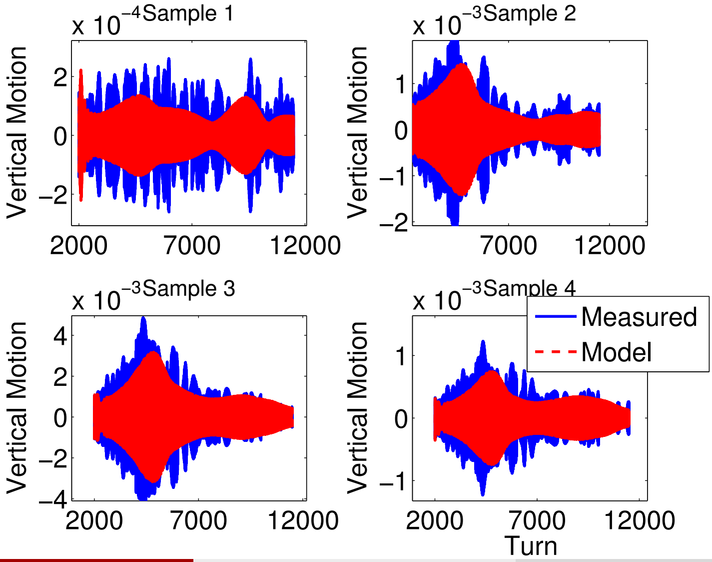
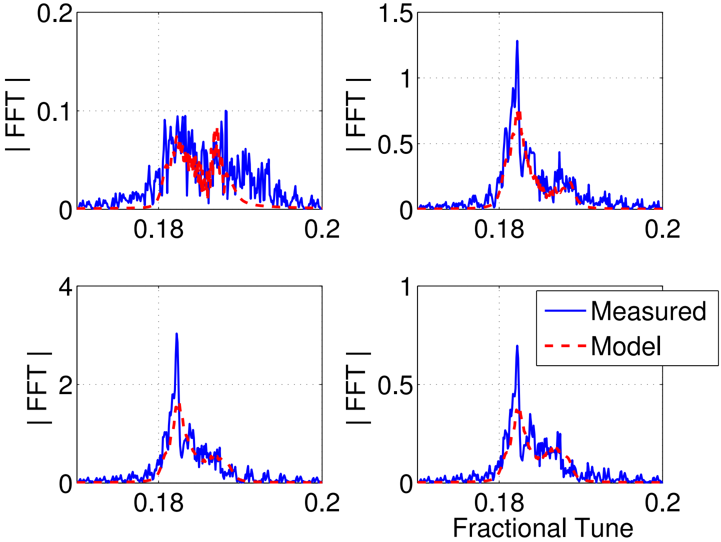


Figure : The FFT's of measured vertical motion of the head and the tail together with model's response.

MD Measurements - 4 x 4 Model, In Time Domain

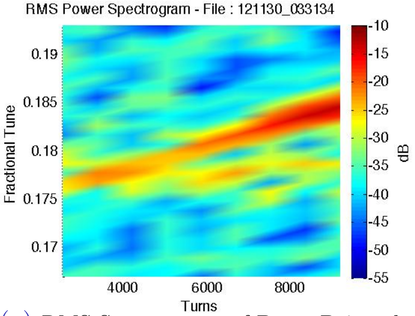


MD Measurements - 4 x 4 Model, In Freq. Domain

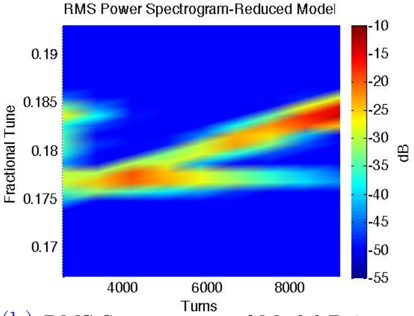


Comparison of Measurements with Reduced Model

- Driven chirp SPS measurement spectrogram (left), reduced model spectrogram (right)
- Chirp tune 0.175 - 0.195 turns 2K - 17K
- Tune 0.177 barycentric mode, tune 0.183 (first upper synchrotron sideband)
- Model and measurement agreement suggests dynamics can be closely estimated.



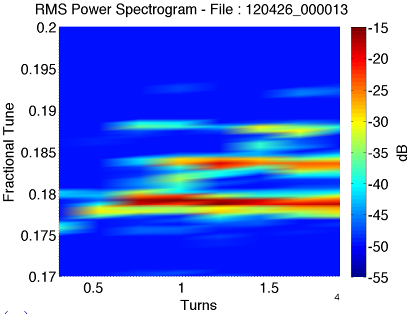
(a) RMS Spectrogram of Beam Driven by 200 MHz Chirp Excitation Sequence



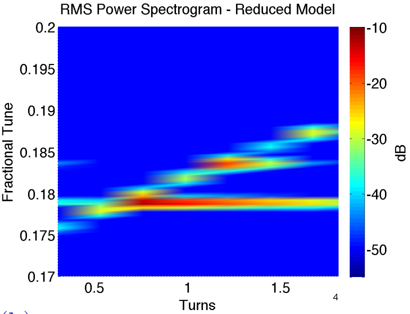
(b) RMS Spectrogram of Model Driven by 200 MHz Chirp Excitation Sequence

Exciting Mode 0, 1^{st} and 2^{nd} Upper Side Bands ?

- A specific machine condition with very low chromaticy configuration.
- Agreement between measurement and model shows that reduced order model can capture dynamics.
- Robustness of the identification algorithm has to be analyzed for such machine conditions.



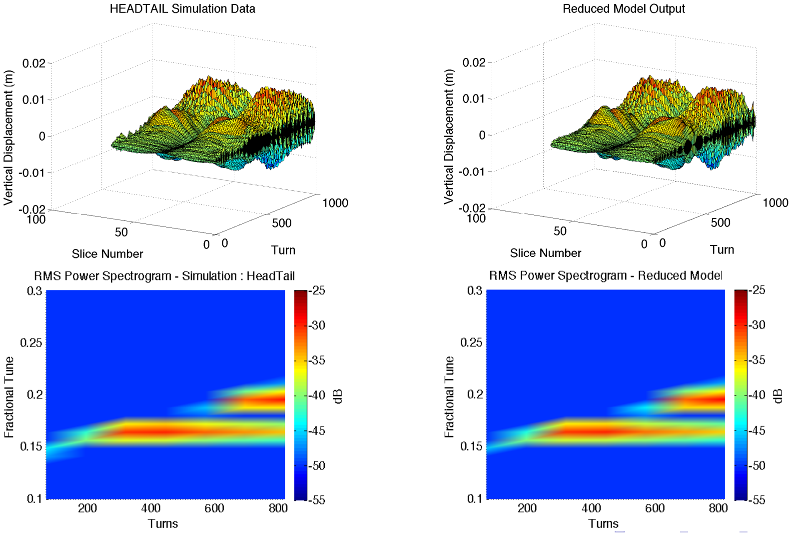
(a) RMS Spectrogram of Bunch Driven by 200 MHz Chirp Excitation



(b) RMS Spectrogram of Model Driven by 200 MHz Chirp Excitation

Comparison of HEADTAIL with Reduced Model

- Figures on top show vertical motion of bunch, driven by 200 MHZ, 0.144 - 0.22 Chirp, 1000 Turns. Bottom figures are corresponding spectrograms.



HEADTAIL Dominant Dynamics / Model Reduction

- If we look at the Henkel Singular Value analysis, we can realize that 8 or 14 states (4 or 7 modes) out of >128 states are main contributors to the dynamics. Therefore we should be able to fit an 8th / 14th order model to capture these dynamics. Rest should be redundant.

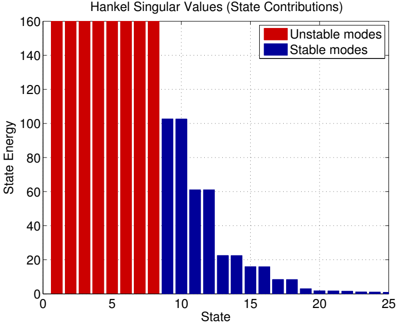
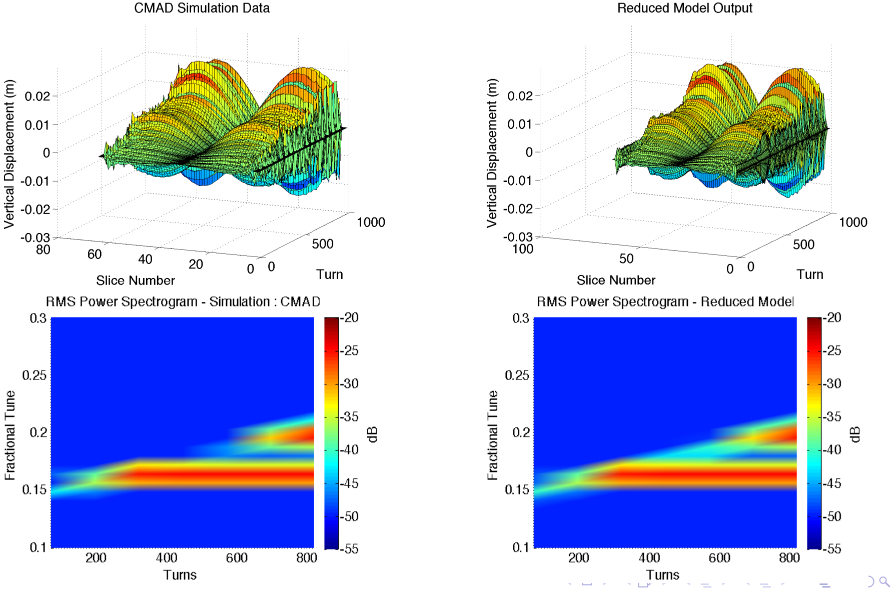


Figure : Henkel Singular Values Analysis - 4 Dominant Modes

Table : Dominant Modes, Synchrotron Tune 0.017

Mode	Eigenvalue
1	$\pm 0.1800i$
2	$\pm 0.1632i$
3	$\pm 0.1959i$

Comparison of CMAD with Reduced Model



CMAD Dominant Dynamics / Model Reduction

- If we look at the Henkel Singular Value analysis, we can realize that 6 states (3 modes) out of >128 states are main contributors to the dynamics. Therefore we should be able to fit a 6th order model to capture these dynamics. Rest should be redundant. Notice the small differences between CMAD and HeadTail eigenvalues.

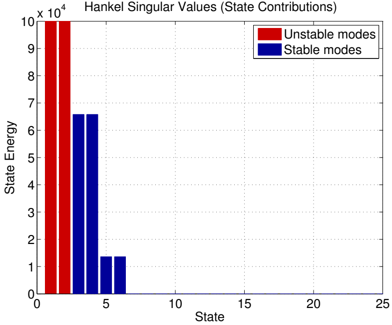


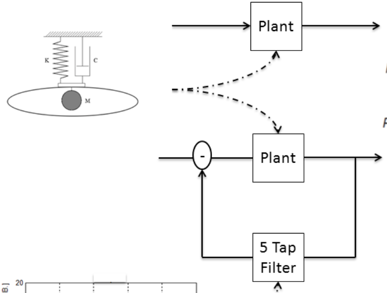
Figure : Henkel Singular Values Analysis - 3 Dominant Modes

Table : Dominant Modes, Synchrotron Tune 0.017

Mode	Eigenvalue
1	$\pm 0.180i$
2	$\pm 0.163i$
3	$\pm 0.197i$

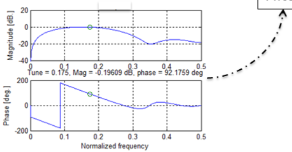
Identification in Simulation Studies

- Let's look at a simple example in CMAD and think about where identification studies can lead to.
- Bunch is represented by a centroid. Using the exact same filter we used in MDs, close the loop on bunch while driving it with an external frequency chirp excitation.



$$P(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \zeta \approx 0, \omega_n = 0.185, \lambda = \pm 1.1624i$$

$$P(z) = \frac{0.4386z}{z^2 - 0.7943z + 1}, \lambda_d = 0.3971 \pm 0.9177i$$



$$F_z = a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}$$

With one turn delay :

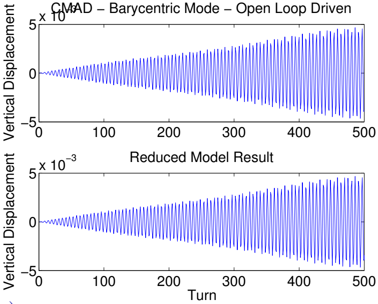
$$F_z = a_0 z^{-1} + a_1 z^{-2} + a_2 z^{-3} + a_3 z^{-4} + a_4 z^{-5}$$

Figure : Open and closed loop driven simulation for mode 0 dynamics analysis.

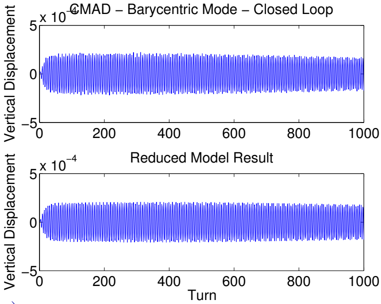
System Identification Results

- 5 tap filter $\rightarrow$ 7th order closed loop system. Reduced order model and identification techniques identify the dominant dynamics.

$$\begin{aligned} \lambda_{closedLoop} &= -0.0692 \pm 1.1456i, -0.755 \pm 0.97i, ... \\ \lambda_{estimated} &= -0.0683 \pm 1.1459i \end{aligned} \tag{2}$$



(a) Vertical displacement of the driven system in open loop.



(b) Vertical displacement of the closed loop driven system.

Online MD Analysis Tool

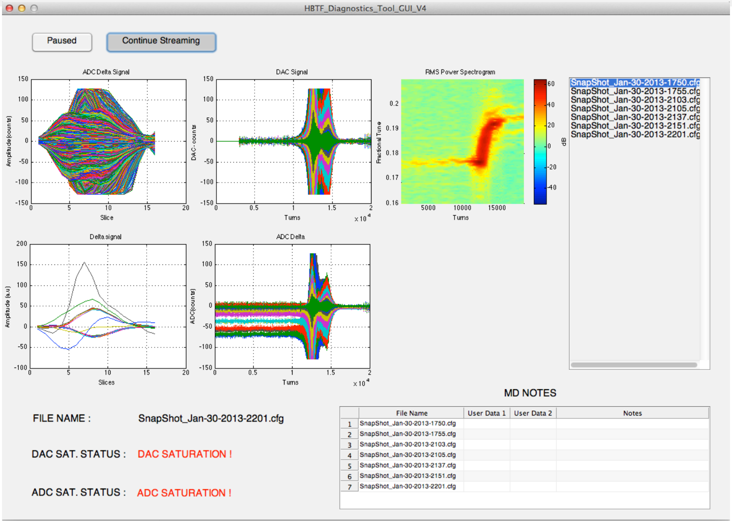


Figure : We are designing MD Tools to be able to have quick online data analysis while we are taking data

Why System Identification and Reduced Order Model

- Beam Diagnostics Tool
  - Beam parameters can be measured during MD such as modes, tunes,etc. . .
- Required for Controller Design
  - Enables us to use control design techniques.
  - Powerful solution to study different kinds of control techniques, i.e. optimal control, robust control, without using too much machine time.
  - Applicable to machine measurements together with simulation data (HeadTail/CMAD).
  - Validation tool for simulations using machine measurements or vice-versa.
  - Allows us to predict the future behavior of the system exploring wide parameter space as opposed to running simulations for each unique condition. For example, predicting the minimum gain to stabilize as opposed to running simulation for different gains until finding a stabilizing gain.

Thank you for your attention !

- Any questions ? ...