NPR step-scaling across the charm threshold

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Outline

Motivations

The story so far

- LQCD has made huge progresses, especially with chiral extrapolation
- NPR allows us to get Z factors with high precision for many operators
- Perturbative matching introduces the dominant error in B_K

What more can we do?

- Claim PT is not our job?
- Increase the scale !
  If we get PT to higher order the effect of this increase will be even stronger.
- Then we should treat the charm quark accordingly

General strategy

Take $0.8\mathrm{GeV} \sim \mu_0 < \mu_1 \ldots < m_c^{SMOM} < \ldots \mu_n \sim 5\mathrm{GeV}$
Define threshold step scaling functions:

$$\sigma(\mu_n, \mu_{n+1}, m_c) = \lim_{a \to 0} \left[\Lambda^{2+1+1}(a, \mu_{n+1}, m_c)\right]^{-1} \Lambda^{2+1+1}(a, \mu_n, m_c)$$

Then

$$\langle \mathcal{O}(\mu_1, m_c)\rangle_{ren}^{2+1+1} = \Pi_n \sigma(n, n+1) \langle \mathcal{O}(\mu_0)\rangle_{ren}^{2+1}$$

- Choose scale from W_0 at suff. IR Wilson flow time that we match the IR limit of 2+1+1 flav theory to the 2+1f theory.
- For $\mu_0 >> m_s, m_u, m_d$ this is equivalent to matching massless mu,d,s.
- Fix m_c to its physical value, defined by NPR in a small volume by taking hierarchy of scales:

$$\mu_{d/s} < \mu_0 < m_c < \mu_n$$

- Run from off-shell amplitudes in approx massless 3f theory to off shell amplitudes in approx massless 4f theory.
- Treat charm threshold effects treated non-perturbatively, and the charm at its physical mass at all stages.
- Mass independence of Z_m in RI schemes is satisfied if

$$p, a^{-1} \gg \Lambda, m_q$$

- Do *not* need $m_q \to 0$

Ensembles

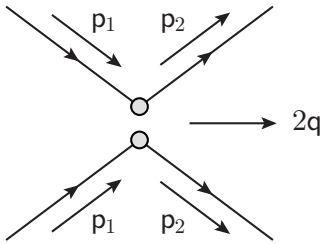
$N_f = 2 + 1$ Ensembles

B_K has been computed on a wide set of (M)DWF ensembles, including two ensembles at the physical quark masses, and lattice spacing going up to 3 GeV.

$N_f = 2 + 1 + 1$ Ensembles

β	$L^3 \times T \times L_5$	m_l	m_c	a^{-1}
5.70	$32^3 \times 64 \times 12$	0.0047	0.243	3.0 GeV
5.70	$32^3 \times 64 \times 12$	0.002	0.243	3.0 GeV
5.70	$32^3 \times 64 \times 12$	0.0047	0.01	3.0 GeV
5.77	$32^3 \times 64 \times 12$	0.0044	0.213	3.6 GeV
5.84	$32^3 \times 64 \times 12$	0.0041	0.183	4.3 GeV
5.84	$32^3 \times 64 \times 12$	0.002	0.183	4.3 GeV

RI-SMOM scheme



Kinematics

- Non-exceptional schemes avoid π pole
- $p_1^2 = p_2^2 = (p_1 - p_2)^2$
- no $\sum p_i$ combination cancels out
- many orientations satisfy this condition but cont. limit is universal

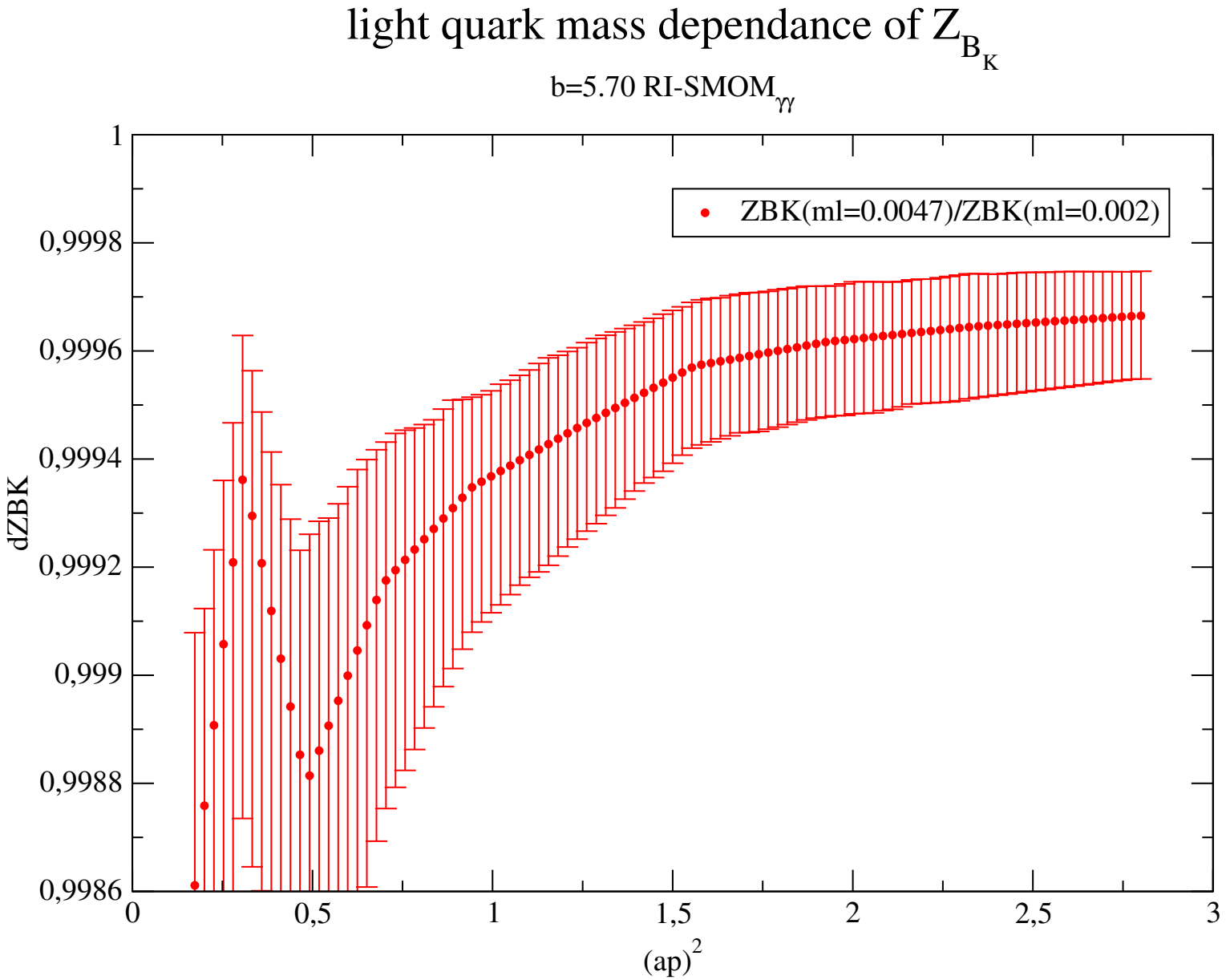
Renormalisation condition

- $Z\,\mathrm{Tr}\left[P_{ijkl}G_{ijkl}\right] = \mathrm{Tr}\left[P_{ijkl}G_{ijkl}\right]\big|_{\mathrm{tree}}$
- $P_{ijkl} = \gamma_i\delta_{ij}\gamma_k\delta_{kl}$ or $P_{ijkl} = \not{q}_{ij}\not{q}_{kl}$
  different schemes allow us to evaluate the truncation error
- Very versatile method, with many knobs to turn
- With five 4-volume factors plus HDCG it is very cheap

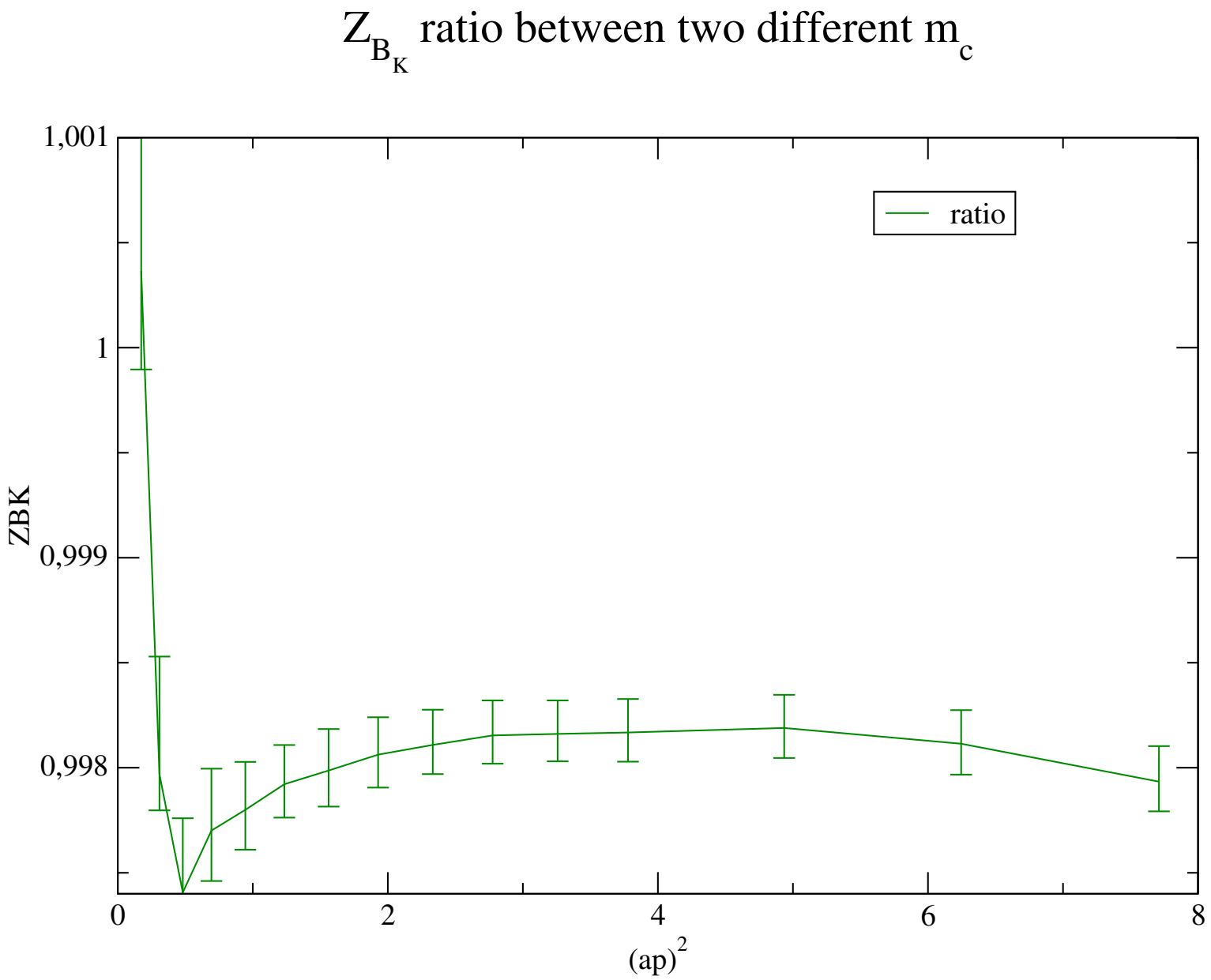
Step-scaling and ratios

- $Z_{\mathrm{lat}\to\mathrm{RI/SMOM}}(p^2)/Z_{\mathrm{lat}\to\mathrm{RI/SMOM}}(p_0^2)$ has an universal cont. limit
- Even if you use Wilson, Twisted, Staggered or anything, you can use our result
- As a corollary we can form other interesting ratios:
  $Z_{dir_1}(p^2)/Z_{dir_2}(p^2)$ is 1 up to discr. effects
  $Z_{ens_1}(p^2)/Z_{ens_2}(p^2)$ is constant up to discr. effects
- Those ratios have several advantages:
  - No dependance on p_0 nor $(ap_0)^2$ contamination
  - Correlated through a^{-1} (often main src of error)
  - Allow an easy study of p^2 dependance of discr. effects, instead of working slice-by-slice and throwing away a lot of information

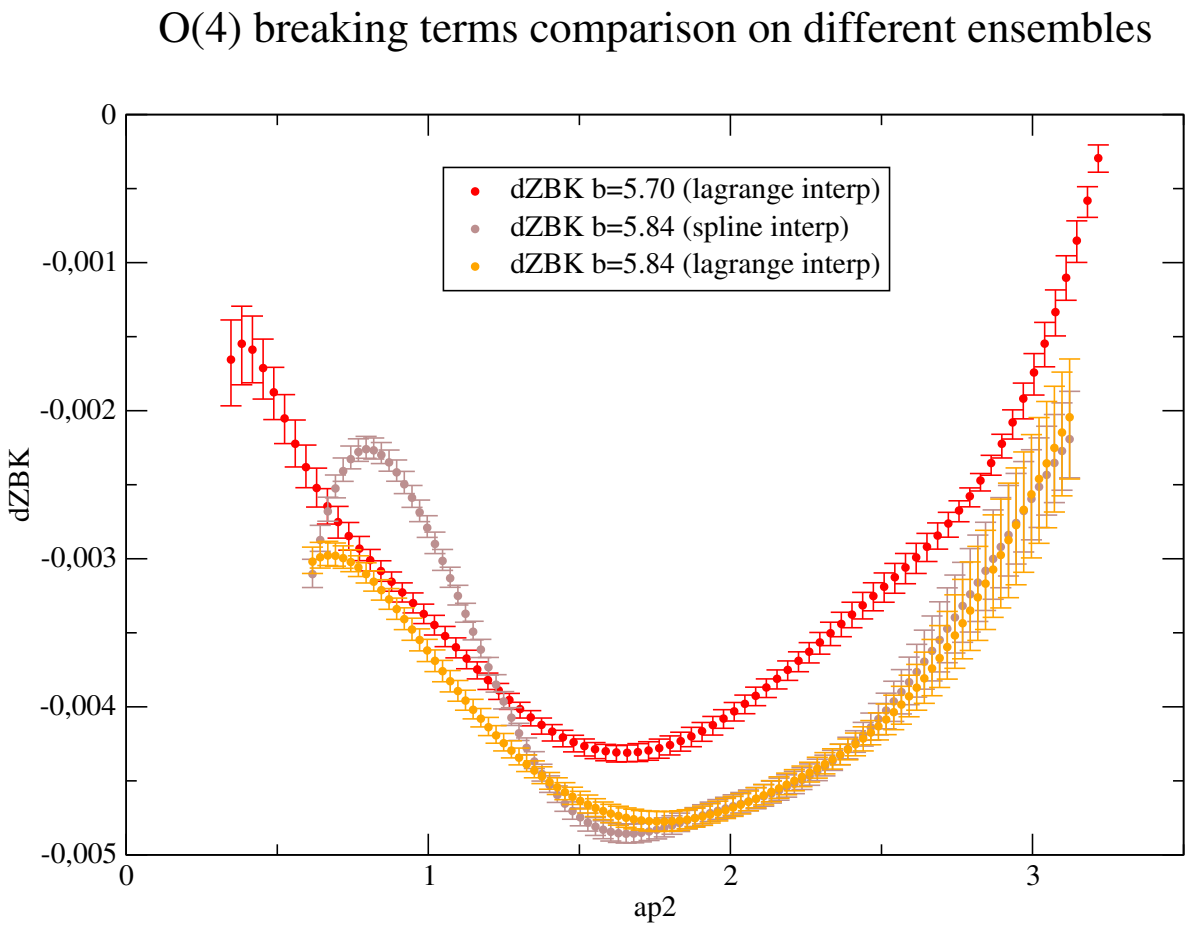
Chiral extrapolation systematics



Charm effects

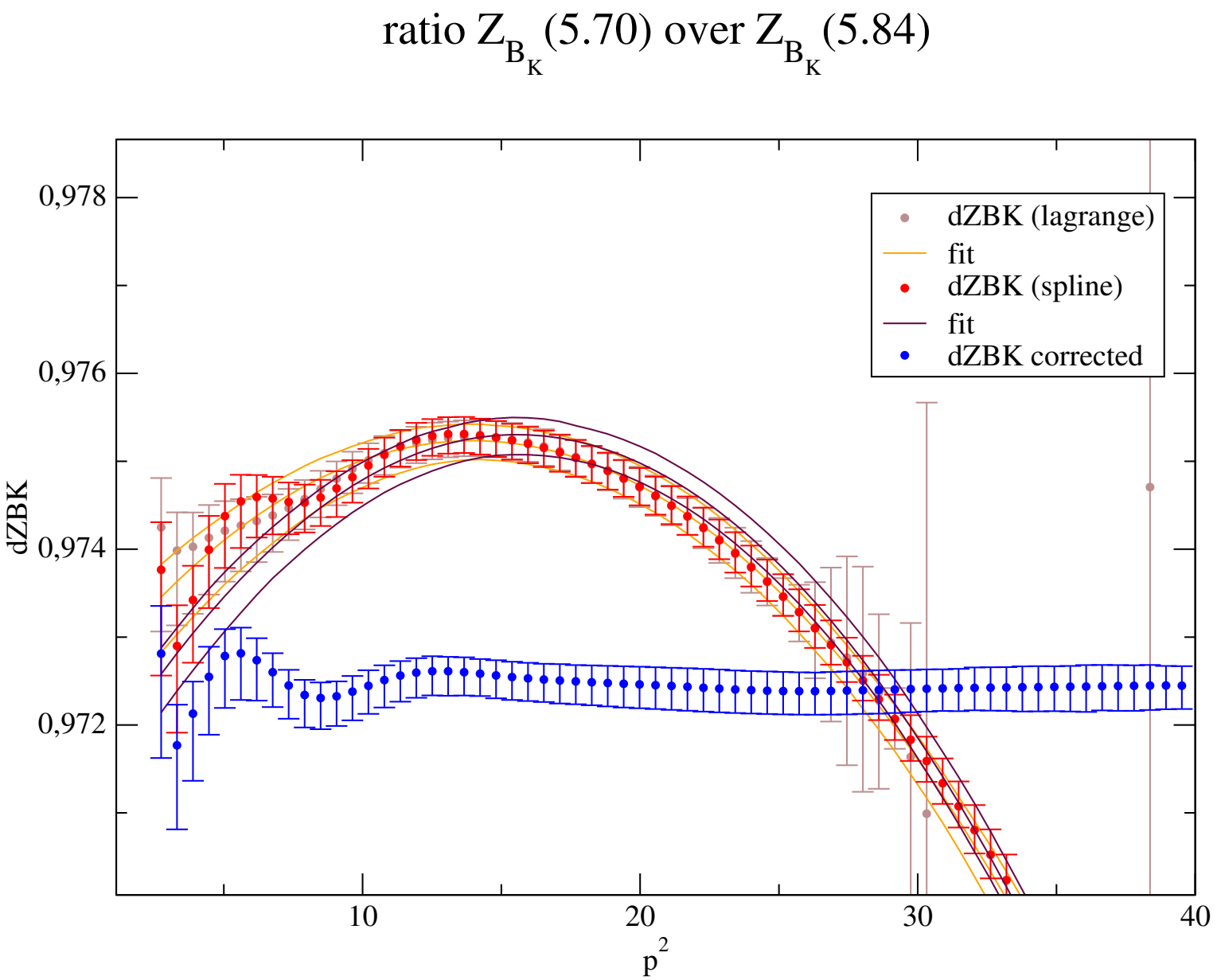


Looking at different orientations

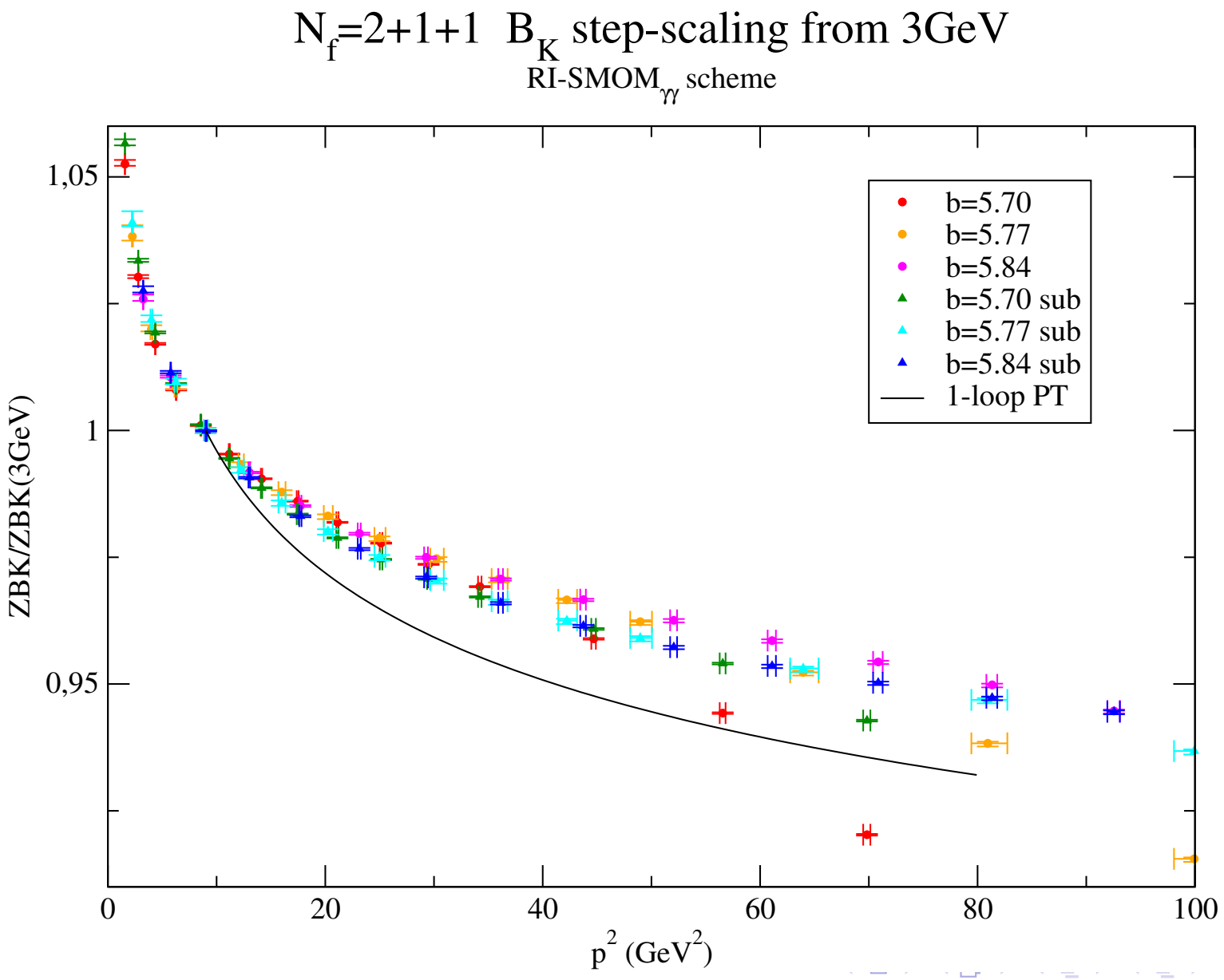


In principle $Z_a(p^2) = Z_0 s(p^2, p_0^2)(1 + \alpha(p)(ap)^2 + \beta(p)(ap)^4)$, but p dependence small after Λ_{QCD}

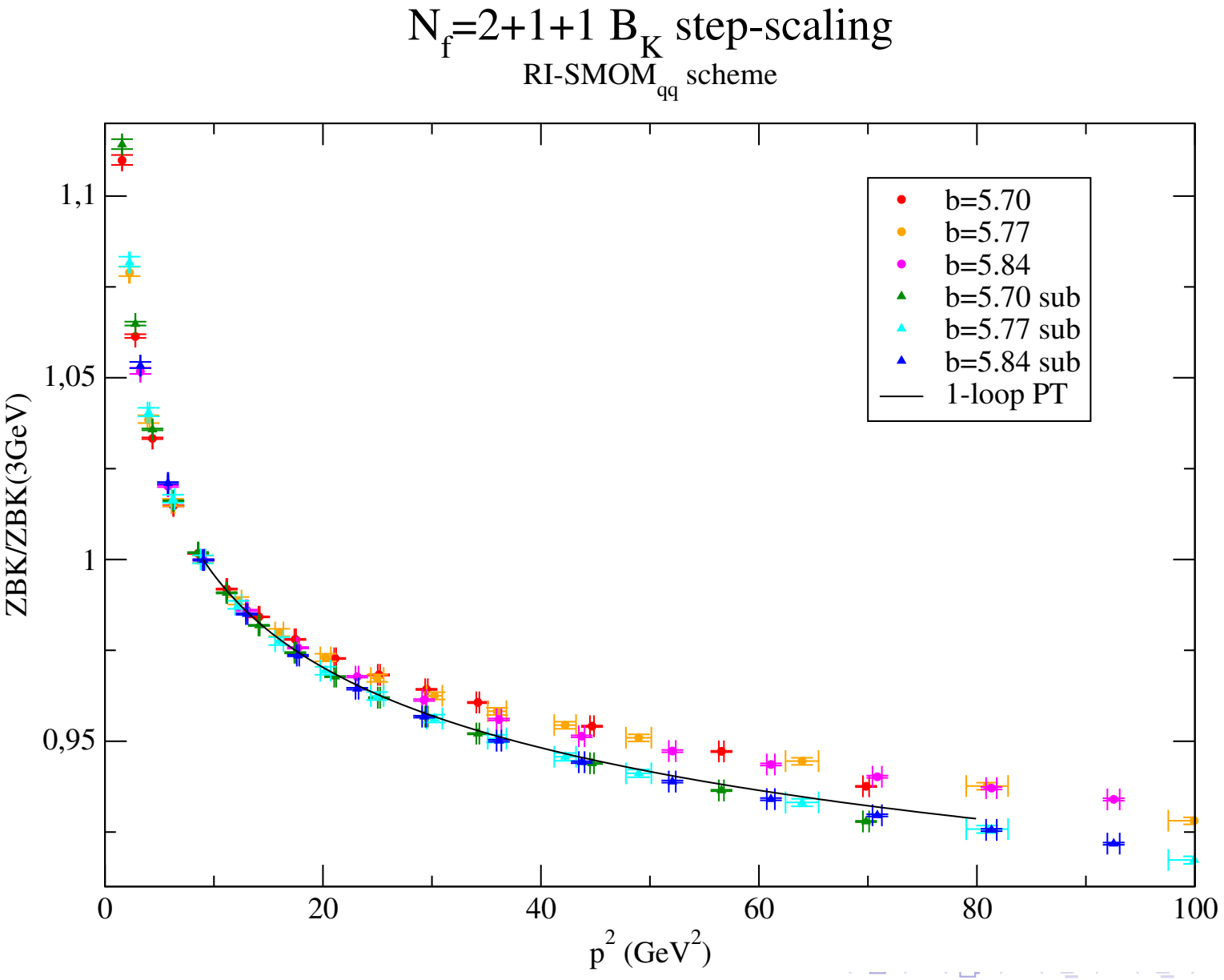
Discretisation effects substraction



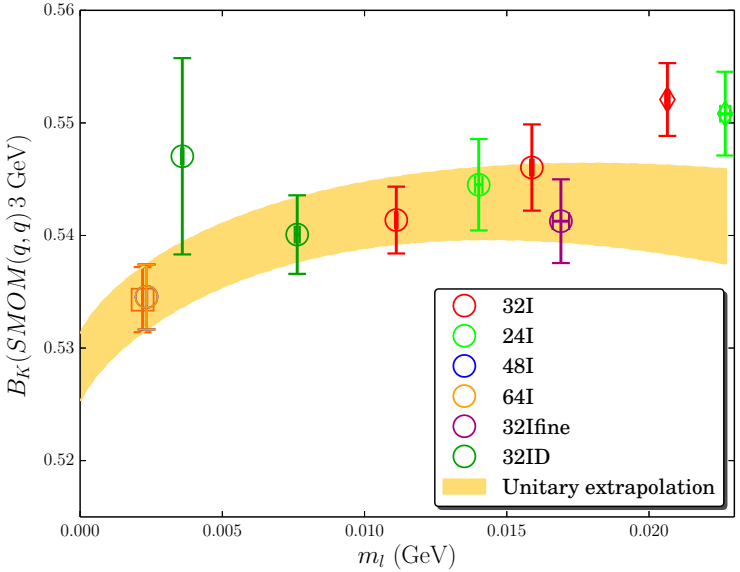
Step-scaling results ($\gamma\gamma$)

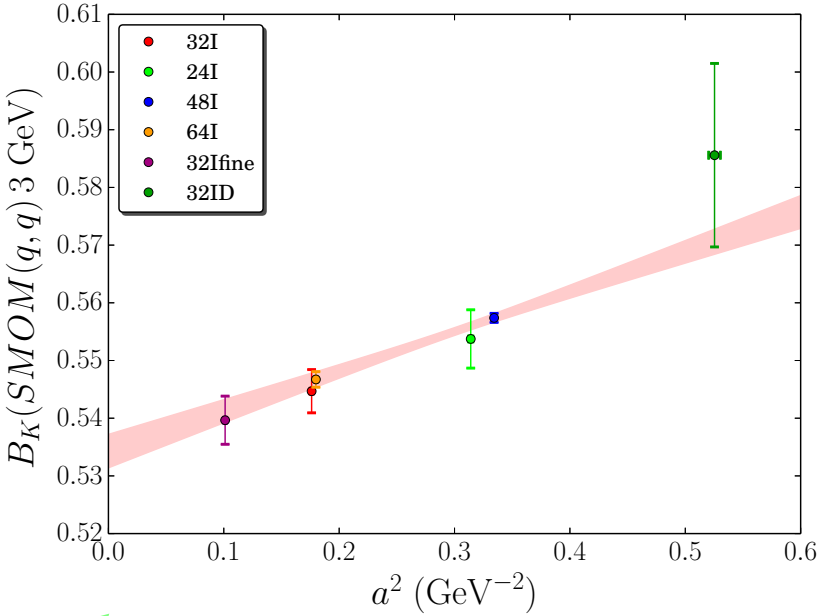


Step-scaling results ($\not{q}\not{q}$)



The 3 GeV starting point

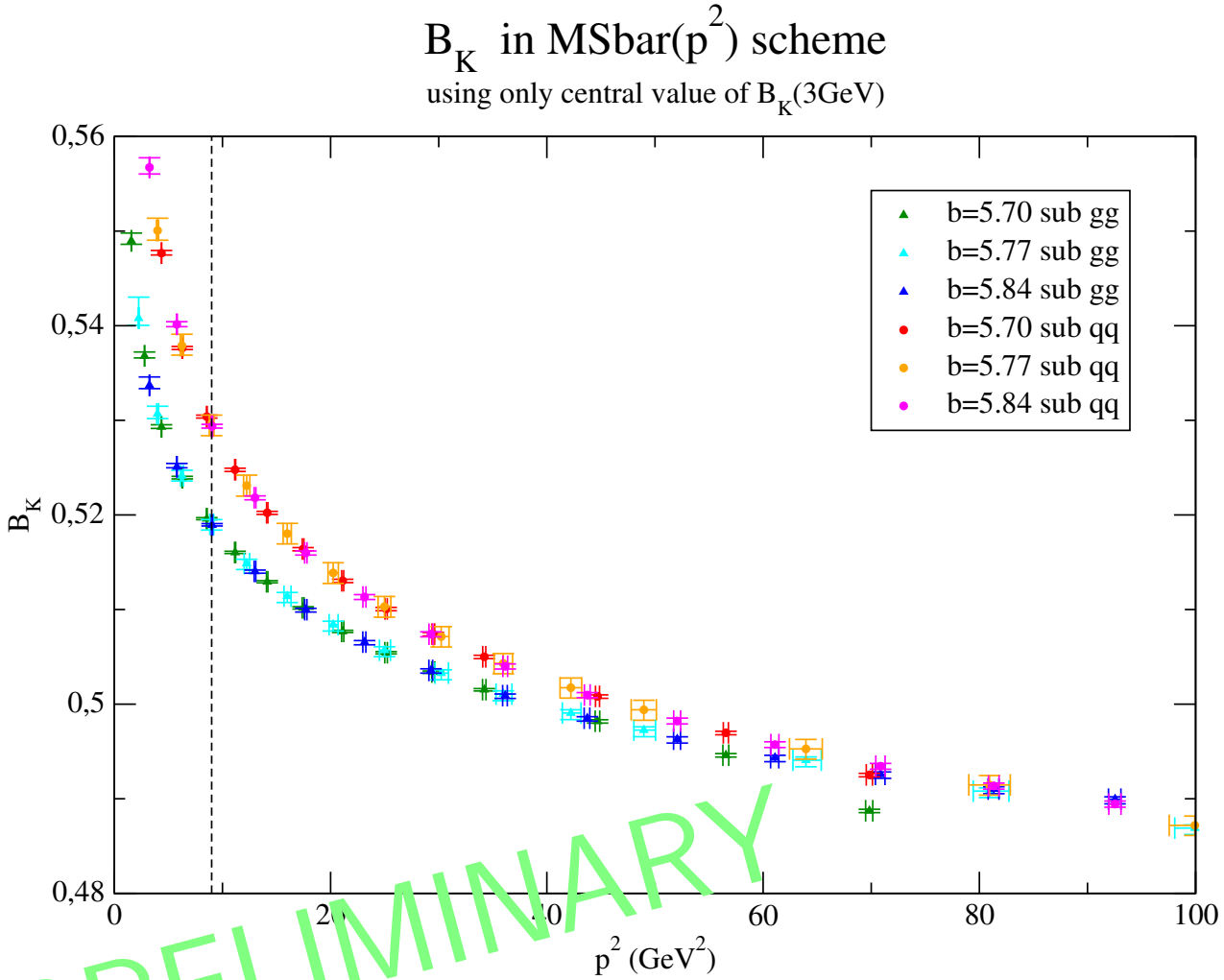




PRELIMINARY

$$B_K(\not{q}\not{q}, 3\text{ GeV}) = 0.5343(29)$$
$$B_K(\gamma\gamma, 3\text{ GeV}) = 0.5168(28)$$
$$\Rightarrow \quad B_K(\overline{\text{MS}}, 3\text{ GeV}) = 0.5296(29)_{\text{stat}}(20)_{\text{FV}}(2)_{\chi}(107)_{\text{NPR}}$$

Running to 5 GeV and higher



$$B_K(\overline{\mathrm{MS}}, 5\ \mathrm{GeV}) = 0.5103(28)_{\mathrm{stat}}(20)_{\mathrm{FV}}(2)_{\chi}(45)_{\mathrm{NPR}}$$
$$B_K(\overline{\mathrm{MS}}, 9\ \mathrm{GeV}) = 0.4913(28)_{\mathrm{stat}}(20)_{\mathrm{FV}}(2)_{\chi}(3)_{\mathrm{NPR}}\ ??$$

- The discr. errors, which are the main challenge for increasing the scale, are well under control
- This is also a strong evidence that, more generally, our action is well-behaved
- Our strategy of getting discr errors from the p^2 dependence seems payful
- We have presented a very promising preliminary result at 5 GeV, and more than halfed the error bar
- Our strategy seems to be valid up to 9 GeV, however one has to be careful about the systematics we've presented, in particular charm effects
- A FV study would be necessary to complete those results. For the moment we can only extrapolate our previous experience
- Our results confirm quite impressively something we have always observed: the convergence is much faster in $\mathrm{RI/SMOM}_{\not{q}}$
- Generalisation to Z_m, B_K BSM, $K \to \pi\pi$ $\Delta I = 1/2$ or $3/2$, ...

Thanks for your attention!