Exploring confinement in SU(N) gauge theories with double-trace Polyakov loop deformations

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In Memoriam

Pierre van Baal
1955-2013

Gerry Guralnik
1936-2014

Resurgence and QCD

- The behavior of observables are described by a trans-series

$$\langle O \rangle = \sum_{n=0}^{\infty} p_{0,n} \lambda^n + \sum_c e^{-S_c/\lambda} \sum_{n=0}^{\infty} p_{c,n} \lambda^n$$

- Sum includes contributions which are not topologically stable, such as an instanton-anti-instanton contribution.

- Applies even in theories without topologically stable objects.

- Deep connection between perturbative and non-perturbative sector.

- QCD: Argyres and Unsal, 2012.
- CP^{N-1}: Dunne and Unsal, 2012.
- Quantum Mechanics, Dunne and Unsal, 2013.
- Principal Chiral Model: Cherman *et al.*, 2013, 2014.
- Self-dual model: Basar *et al.*, 2013.

O(3) model in d=2: the XY limit

- Asymptotically free (like QCD)

$$S = \int d^2x \frac{1}{2g^2} (\nabla\vec{\sigma})^2$$

- instantons (like QCD):

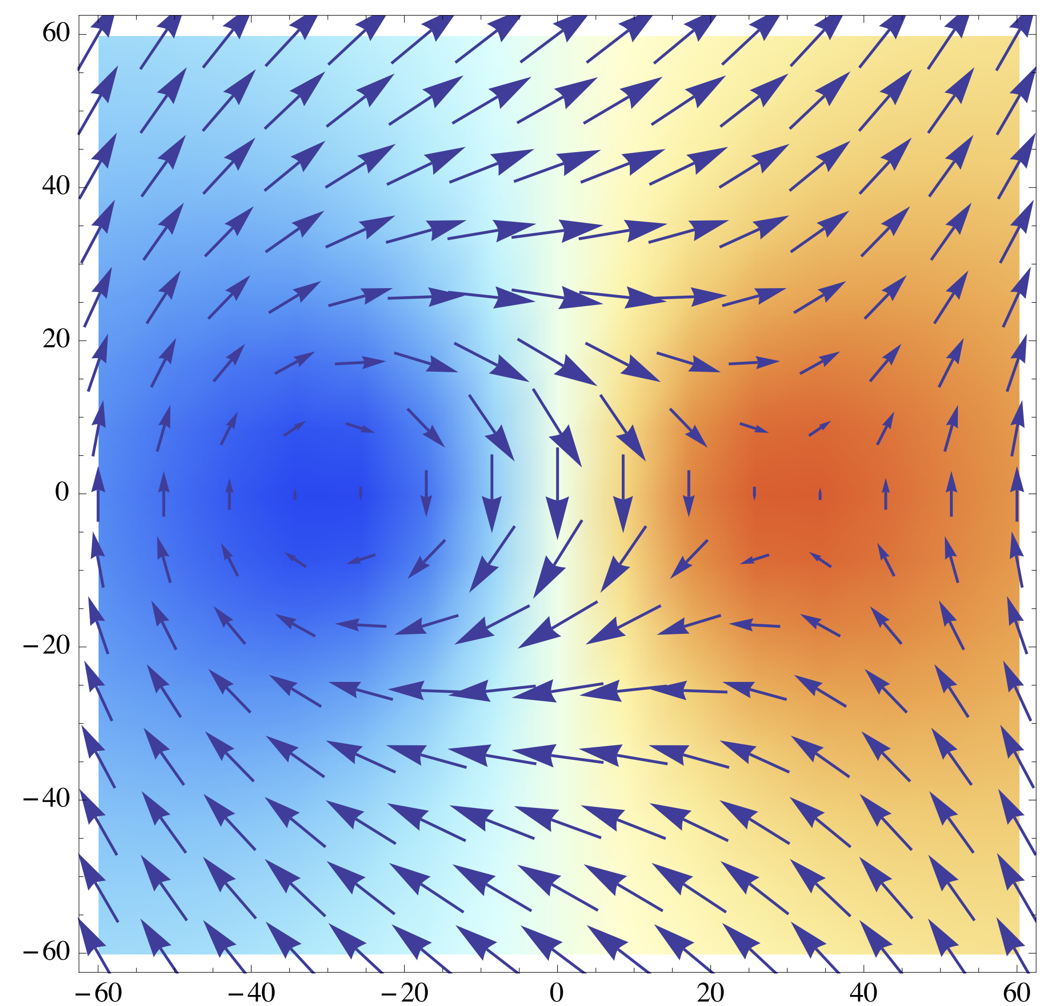
$$\pi_2\left(S^2\right) = Z \qquad w = \frac{\sigma_1 + i\sigma_2}{1-\sigma_3} = c\frac{z-z_1}{z-z_2}$$

$$\vec{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 1$$

- XY model vortices emerge as constituents of instantons (like monopoles in QCD)
- Can deform O(3) model into an XY model with h negative G[2] and mco, 1981; Affleck, 1986.

$$S \to S - \int d^2x \, \frac{1}{2} h\sigma_3^2$$

- Vortices responsible for KT critical behavior and for mass gap

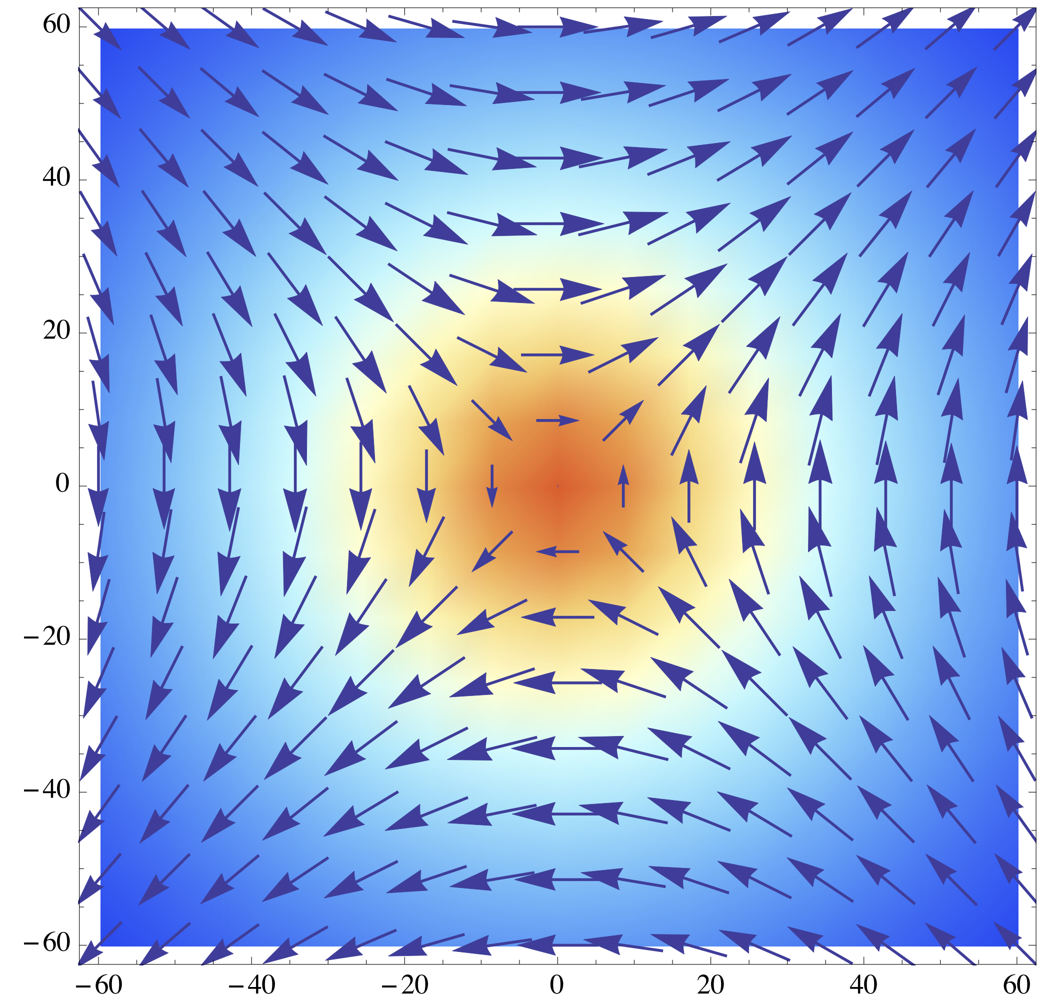


O(3) model in d=2: the Z(2) limit

- h positive changes the boundary conditions to $\sigma_3=\pm 1$ at infinity.
- h breaks the classical scale invariance of the O(3) model and determines instanton size.
- The interpretation of an instanton as a vortex-antivortex pair is lost.
- instantons look like flipped spin in Ising-model low-T expansion.

$$S = \int d^2x \frac{1}{2g^2} \left(\nabla\vec{\sigma}\right)^2$$

$$\vec{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 1$$



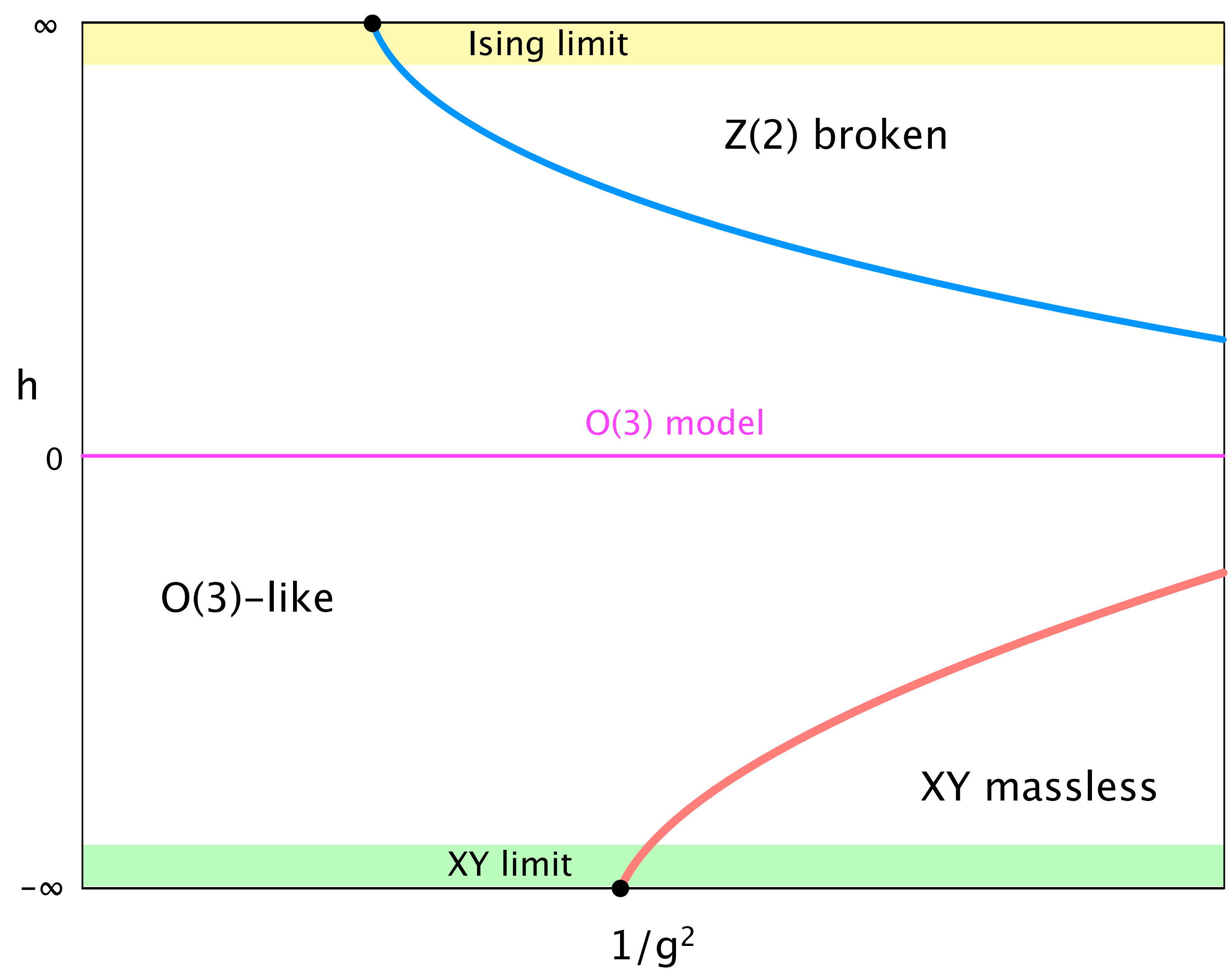
$$S \to S - \int d^2x\, \frac{1}{2} h \sigma_3^2$$

mco and Guralnik, 1981

Phase diagram of deformed O(3) model

- Adiabatic continuity: the unbroken, high-T phase of the Ising model is continuously connected to the high-T phase of the XY model via deformation of O(3).
- σ_3^2 tells us where we are in the phase diagram.
- O(N) models with N>3 do not have stable instantons, but have XY and Ising deformations!
- Smallest charge vortices suppressed at θ=π, and higher-charge excitations must be included (Affleck, 1986, 1991).

O(3) phase diagram $$S \to S - \int d^2x \, \frac{1}{2} h \sigma_3^2$$



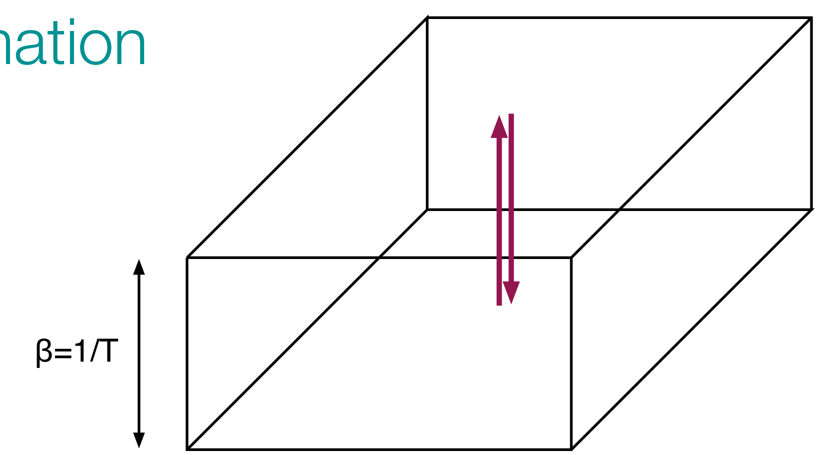
High-T confinement on R^3 x S^1: restoring Z(N)

- Coupling gets weak as T gets large

$$g^2(T) \to 0$$

- Modify action to restore Z(N) symmetry **and** force the theory to be Abelian at large distances

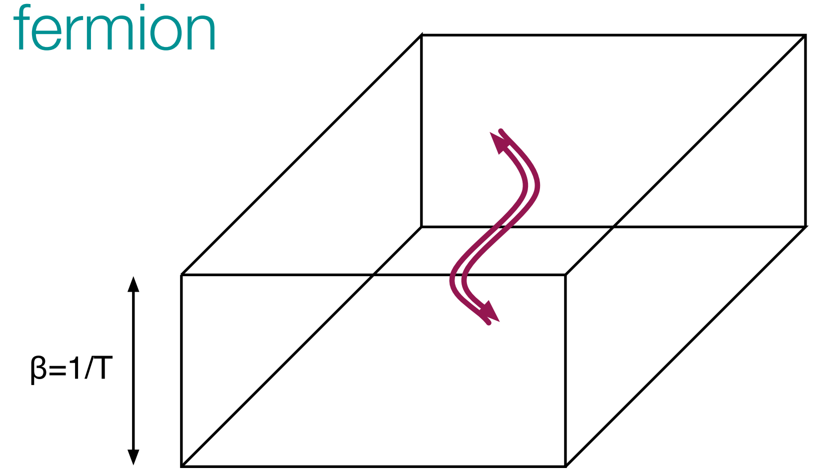
deformation



  - double trace deformation (Meyers and mco, 2008)

$$S \to S - \int d^3x \, H_A \, |Tr_F P|^2$$

adjoint fermion



  - adjoint fermions (Unsal, 2008)

- A_4 behaves as a 3d scalar with a center-symmetric expectation value; Euclidean monopoles solutions!

High-T confinement on R^3 x S^1: topology

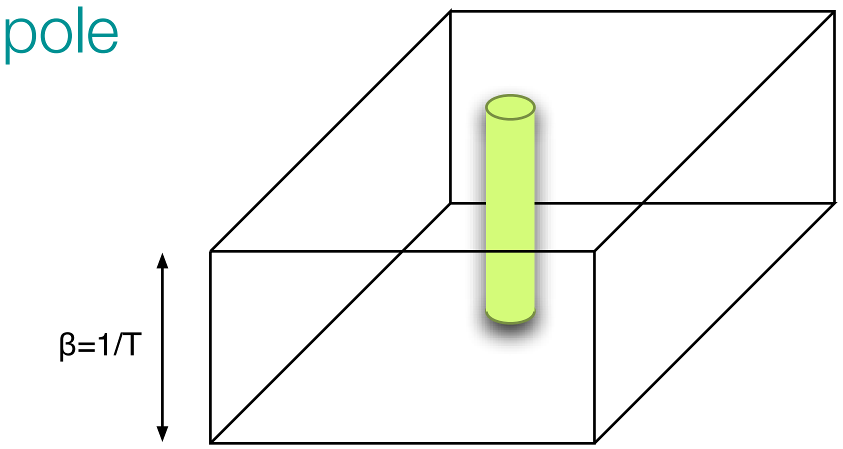
- Euclidean monopoles are constituents of instantons (Lee & Yi, 1997, Kraan & van Baal 1998) and confine (Unsal 2008; Unsal and Yaffe, 2008).
- Dimensional reduction yields confinement as in 3d Georgi-Glashow model (Polyakov 1976) by monopole gas
- Monopole gas is represented by a sine-Gordon model for SU(2):

$$S_{eff} = \int d^3x \left[\frac{g^2(T)T}{32\pi^2} \left(\partial_j \sigma\right)^2 - 4y\cos(\sigma) \right]$$

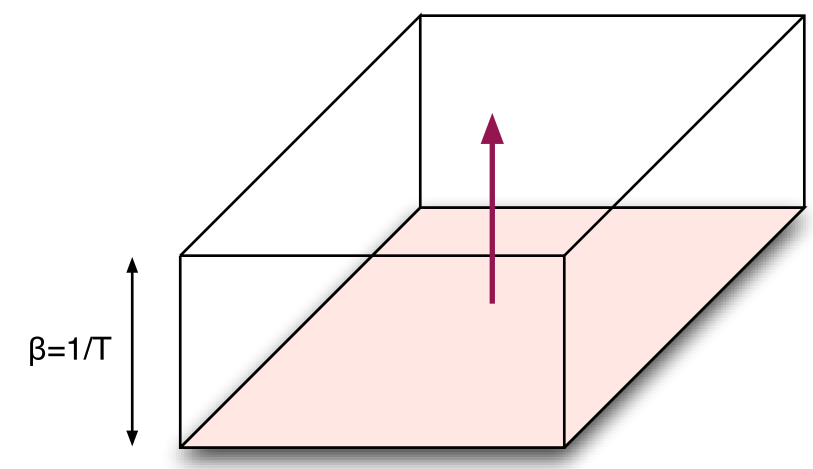
$$y \propto T^3 \left(\Lambda/T\right)^{11/3}$$

- Program works on lattice as well mco, 2012 and in press.

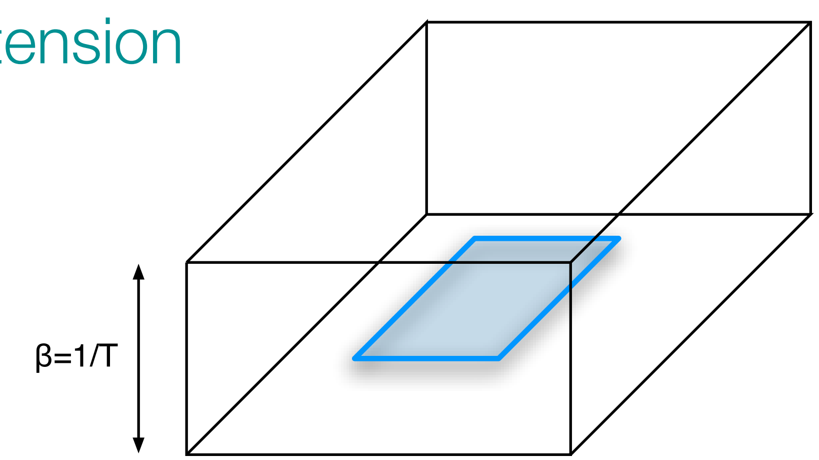
monopole



dimensional reduction



string tension



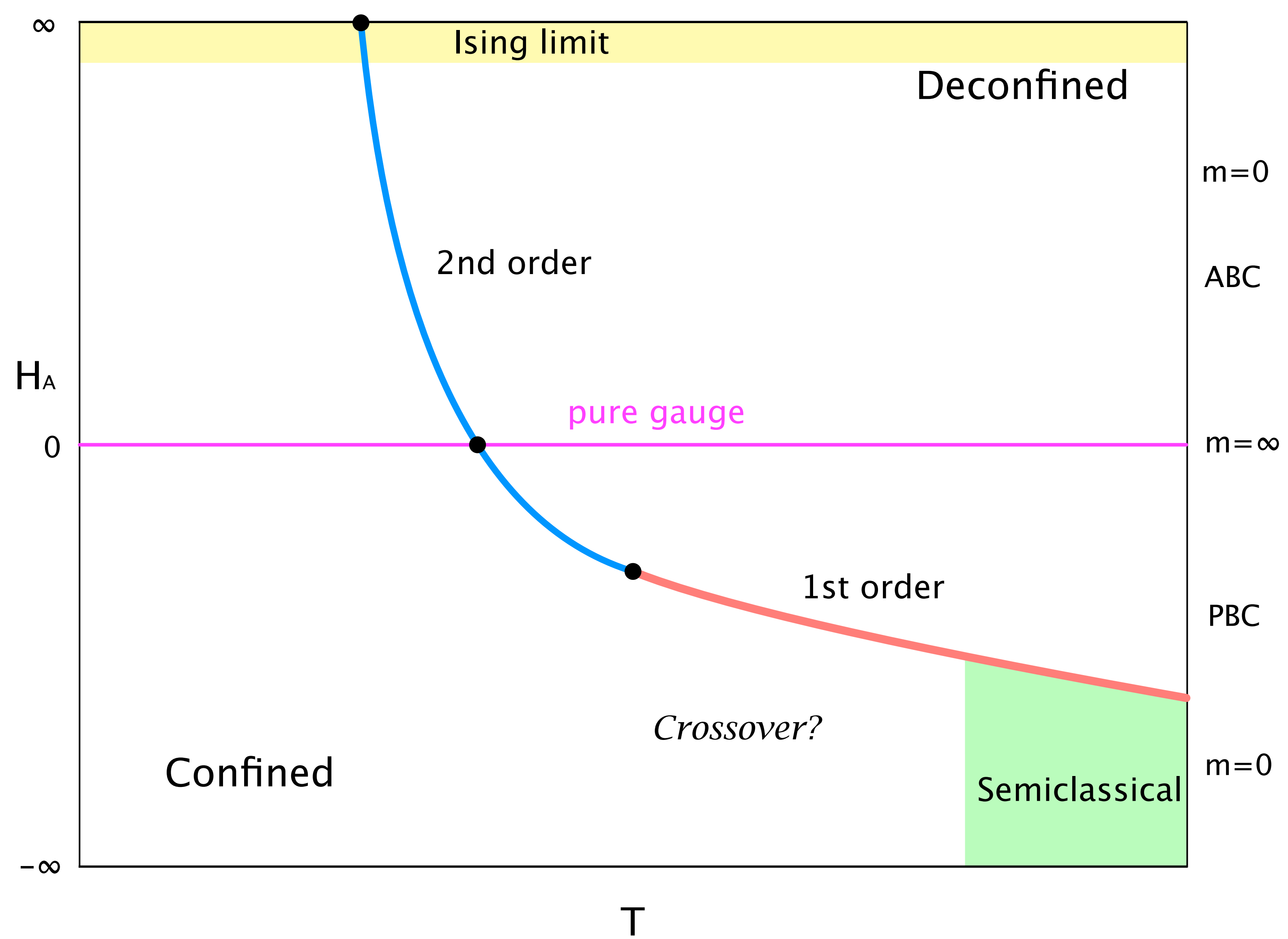
High-T confinement on R^3 x S^1: phase diagram

- Positive H_A promotes Z(2) breaking and decreases the deconfinement temperature
- Negative H_A increases the deconfinement temperature
- Deconfinement transition changes from 2nd-order to 1st at tricritical point (location and existence are non-universal- H. Nishimura & mco, 2012
- Reach region of high-T semiclassical confinement

$$T \gg \Lambda$$

SU(2) phase diagram

$$S \to S - \int d^3x\, H_A\, |Tr_F P|^2$$

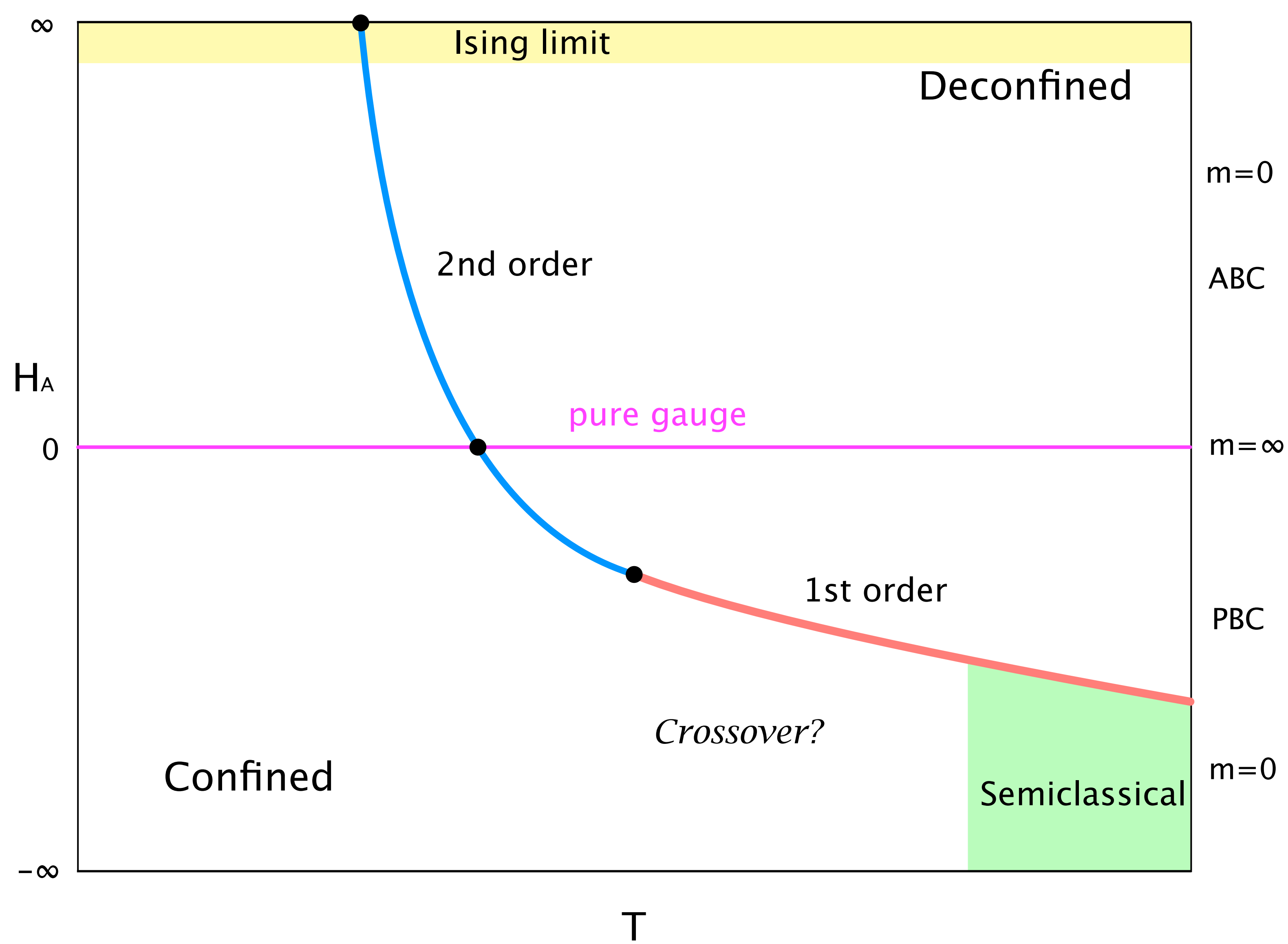


More about the phase diagram

- $Tr_A P$ determines where we are in phase diagram.
    - Ising limit: $Tr_A P=2$
    - pure gauge: $Tr_A P \sim 0$
    - U(1) limit: $Tr_A P=-1$
- The SU(2) phase diagram has the form of a Blume-Emery-Griffiths model; the tricritical point has non-Ising critical indices.

SU(2) phase diagram

$$S \to S - \int d^3x \, H_A \, |Tr_F P|^2$$



String tensions

- In semiclassical region of SU(N), inclusion of only the lightest monopole states gives a generalized sine-Gordon model based on the affine roots of SU(N) Unsal and Yaffe, 2008.

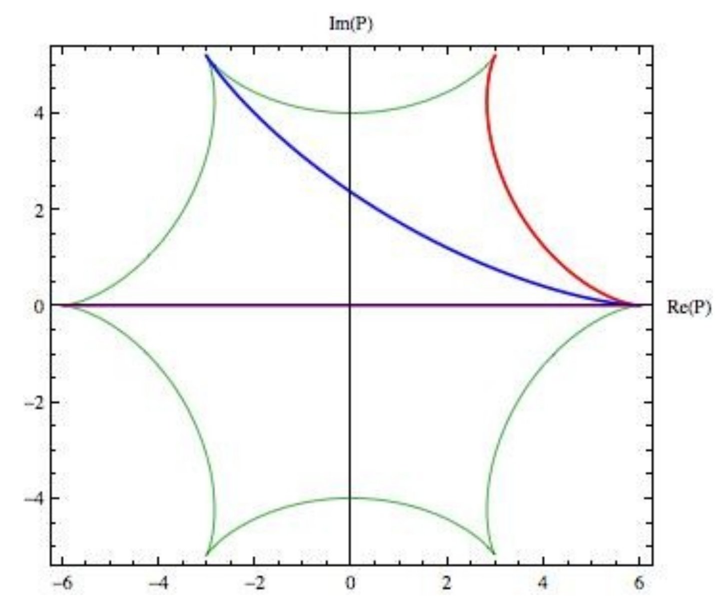
$$S_{mag} = \int d^3x \left[\frac{T}{2} (\partial \rho)^2 - 2\xi \sum_{j=1}^{N} \cos\left(\frac{2\pi}{g} \alpha_j \cdot \rho \right) \right]$$

- This in turn leads to unsatisfying results for string tension Meisinger and mco, 2010.

$$\sigma_k^{(s)} \leq \frac{8}{\pi} \left[\frac{g^2 T \xi}{N} k (N-k) \right]^{1/2}$$

- Inclusion of all roots with equal with equal ξ is known to lead to string tension Casimir scaling Giovannangeli and C.P. Korthals Altes, 2001.

$$\sigma_k = \sigma_1 \frac{k(N-k)}{N-1}$$



Lattice U(1)^N models

- Start from a Villain U(1)^N system:

$$S_1 = \frac{1}{2g^2} \sum_{a=1}^{N} \sum_{p} Tr \left(\partial_\mu \phi_\nu^a - \partial_\nu \phi_\mu^a - 2\pi n_{\mu\nu}^a \right)^2$$

- Define a set of monopole currents:

$$m_\mu^a (X) = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial_\nu n_{\rho\sigma}^a (x)$$

- The remaining degrees of freedom can be integrated out, giving a Coulomb gas representation:

$$S_{dual} = \frac{2\pi^2}{g^2} \sum_{R,R'} m_\mu^a (R)\, G (R - R')\, m_\mu^a (R')$$

Lattice U(1)$^{N-1}$ models

- We can restrict U(1)^N to U(1)$^{N-1}$ using a periodic delta function:

$$S_2 = -i \sum_{x,\mu} p_\mu(x) \left[\sum_a \phi_\mu^a(x) \right]$$

- This gives rise to an electric interaction in the Coulomb gas representation:

$$S_{dual} = \frac{2\pi^2}{g^2} \sum_{R,R'} m_\mu^a(R)\, G(R-R')\, m_\mu^a(R') + \frac{g^2}{2} \sum_{r,r'} p_\mu^a(r)\, G(r-r')\, p_\mu^a(r') - i \sum_{r,R} \left(\sum_a m_\mu^a(R) \right) \Theta_{\mu\nu}(R-r)$$

- Similarly, we can add a potential term that favors or disfavors the Z(N) center subgroup of SU(N).

- On $R^3 \times S^1$, the dominant terms will be short monopole world lines with $m_4^a=+1$ and $m_4^b=-1$ for some $a \neq b$. This leads naturally to Casimir scaling when center symmetry is unbroken.

Conclusions

- Double trace deformations allow us to interpolate between the $U(1)^{N-1}$ instanton gas picture of confinement and a Z(N) gauge theory, with the pure gauge theory in the middle.
- $Tr_A P$ indicates where we are in the phase diagram, with the system behaving as a generalization of the BEG model.
- Calorons change their role in moving between regions, but are important throughout.
- Casimir scaling is associated with the inclusion of monopoles on a democratic basis, and appears naturally in a $U(1)^{N-1}$ lattice model.