Exploring confinement in SU(N) gauge theories with double-trace Polyakov loop deformations

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Lattice 2014
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In Memoriam

Pierre van Baal
1955-2013

Gerry Guralnik
1936-2014
Resurgence and QCD

• The behavior of observables are described by a trans-series

\[ \langle O \rangle = \sum_{n=0}^{\infty} p_{0,n} \lambda^n + \sum_{c} e^{-S_c/\lambda} \sum_{n=0}^{\infty} p_{c,n} \lambda^n \]

• Sum includes contributions which are not topologically stable, such as an instanton-anti-instanton contribution.

• Applies even in theories without topologically stable objects.

• Deep connection between perturbative and non-perturbative sector.

• QCD: Argyres and Unsal, 2012.

• CP^{N-1}: Dunne and Unsal, 2012.

• Quantum Mechanics, Dunne and Unsal, 2013.

• Principal Chiral Model: Cherman et al., 2013, 2014.

• Self-dual model: Basar et al., 2013.
O(3) model in $d=2$: the XY limit

- Asymptotically free (like QCD)

- Instantons (like QCD):
  $$\pi_2(S^2) = Z \quad w = \frac{\sigma_1 + i\sigma_2}{1 - \sigma_3} = c \frac{z - z_1}{z - z_2}$$

- XY model vortices emerge as constituents of instantons (like monopoles in QCD)

- Can deform O(3) model into an XY model with $h$ negative $G^2$ and mco, 1981; Affleck, 1986.

  $$S \rightarrow S - \int d^2x \frac{1}{2g^2} (\nabla \bar{\sigma})^2$$

  $$\bar{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 1$$

- Vortices responsible for KT critical behavior and for mass gap
O(3) model in d=2: the Z(2) limit

- h positive changes the boundary conditions to $\sigma_3 = \pm 1$ at infinity.

- h breaks the classical scale invariance of the O(3) model and determines instanton size.

- The interpretation of an instanton as a vortex-antivortex pair is lost.

- Instantons look like flipped spin in Ising-model low-T expansion.

\[ S = \int d^2x \frac{1}{2g^2} (\nabla \vec{\sigma})^2 \]

\[ \vec{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 1 \]

\[ S \rightarrow S - \int d^2x \frac{1}{2} h\sigma_3^2 \quad \text{mco and Guralnik, 1981} \]
Phase diagram of deformed O(3) model

- Adiabatic continuity: the unbroken, high-T phase of the Ising model is continuously connected to the high-T phase of the XY model via deformation of O(3).

- $\sigma_3^2$ tells us where we are in the phase diagram.

- O(N) models with N>3 do not have stable instantons, but have XY and Ising deformations!

- Smallest charge vortices suppressed at $\theta=\pi$, and higher-charge excitations must be included (Affleck, 1986, 1991).
High-T confinement on $R^3 \times S^1$: restoring $Z(N)$

- Coupling gets weak as $T$ gets large
  \[ g^2 (T) \to 0 \]
- Modify action to restore $Z(N)$ symmetry and force the theory to be Abelian at large distances

  - double trace deformation
    (Meyers and mco, 2008)
    \[ S \to S - \int d^3x \ H_A \ |Tr_F P| ^2 \]

  - adjoint fermions
    (Unsal, 2008)

- $A_4$ behaves as a 3d scalar with a center-symmetric expectation value; Euclidean monopoles solutions!
Euclidean monopoles are constituents of instantons (Lee & Yi, 1997, Kraan & van Baal 1998) and confine (Unsal 2008; Unsal and Yaffe, 2008).

Dimensional reduction yields confinement as in 3d Georgi-Glashow model (Polyakov 1976) by monopole gas.

Monopole gas is represented by a sine-Gordon model for SU(2):

\[ S_{eff} = \int d^3x \left[ \frac{g^2(T)T}{32\pi^2} (\partial_j \sigma)^2 - 4y \cos(\sigma) \right] \]

\[ y \propto T^3 (\Lambda/T)^{11/3} \]

Program works on lattice as well mco, 2012 and in press.

\[ \beta = 1/T \]

\[ \text{monopole} \]

\[ \text{dimensional reduction} \]

\[ \text{string tension} \]
High-T confinement on \( R^3 \times S^1 \): phase diagram

- Positive \( H_A \) promotes \( Z(2) \) breaking and decreases the deconfinement temperature

- Negative \( H_A \) increases the deconfinement temperature

- Deconfinement transition changes from 2nd-order to 1st at tricritical point (location and existence are non-universal—H. Nishimura & mco, 2012)

- Reach region of high-T semiclassical confinement

\[
T \gg \Lambda
\]
More about the phase diagram

• $\text{Tr}_A P$ determines where we are in phase diagram.
  - Ising limit: $\text{Tr}_A P = 2$
  - Pure gauge: $\text{Tr}_A P \sim 0$
  - U(1) limit: $\text{Tr}_A P = -1$

• The SU(2) phase diagram has the form of a Blume-Emery-Griffiths model; the tricritical point has non-Ising critical indices.

SU(2) phase diagram

$$S \rightarrow S - \int d^3 x H_A \left| \text{Tr}_F P \right|^2$$
String tensions

- In semiclassical region of SU(N), inclusion of only the lightest monopole states gives a generalized sine-Gordon model based on the affine roots of SU(N) Unsal and Yaffe, 2008.
  \[ S_{mag} = \int d^3x \left[ \frac{T}{2} (\partial \rho)^2 - 2\xi \sum_{j=1}^{N} \cos \left( \frac{2\pi}{g} \alpha_j \cdot \rho \right) \right] \]

- This in turn leads to unsatisfying results for string tension Meisinger and mco, 2010.
  \[ \sigma_k^{(s)} \leq \frac{8}{\pi} \left[ \frac{g^2T\xi}{N} k (N - k) \right]^{1/2} \]

- Inclusion of all roots with equal with equal $\xi$ is known to lead to string tension Casimir scaling Giovannangeli and C.P. Korthals Altes, 2001.
  \[ \sigma_k = \sigma_1 \frac{k (N - k)}{N - 1} \]
Lattice U(1)\(^N\) models

- Start from a Villain U(1)\(^N\) system:

\[
S_1 = \frac{1}{2g^2} \sum_{a=1}^{N} \sum_{p} Tr \left( \partial_{\mu} \phi_{\nu}^a - \partial_{\nu} \phi_{\mu}^a - 2\pi n_{\mu\nu}^a \right)^2
\]

- Define a set of monopole currents:

\[
m_{\mu}^a (X) = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial_{\nu} n_{\rho\sigma}^a (x)
\]

- The remaining degrees of freedom can be integrated out, giving a Coulomb gas representation:

\[
S_{\text{dual}} = \frac{2\pi^2}{g^2} \sum_{R, R'} m_{\mu}^a (R) G (R - R') m_{\mu}^a (R')
\]
Lattice U(1)\(^{N-1}\) models

- We can restrict U(1)\(^N\) to U(1)\(^{N-1}\) using a periodic delta function:
  \[
  S_2 = -i \sum_{x, \mu} p_\mu(x) \left[ \sum_a \phi^a_\mu(x) \right]
  \]

- This gives rise to an electric interaction in the Coulomb gas representation:
  \[
  S_{\text{dual}} = \frac{2\pi^2}{g^2} \sum_{R, R'} m_{\mu}^a(R) G(R - R') m_{\mu}^a(R') + \frac{g^2}{2} \sum_{r, r'} p_\mu^a(r) G(r - r') p_\mu^a(r') - i \sum_{r, R} \left( \sum_a m_{\mu}^a(R) \right) \Theta_{\mu\nu}(R - r)
  \]

- Similarly, we can add a potential term that favors or disfavors the Z(N) center subgroup of SU(N).

- On \(R^3 \times S^1\), the dominant terms will be short monopole world lines with 
  \(m_4^a = +1\) and \(m_4^b = -1\) for some \(a \neq b\). This leads naturally to Casimir scaling when center symmetry is unbroken.
Conclusions

• Double trace deformations allow us to interpolate between the U(1)$^{N-1}$ instanton gas picture of confinement and a Z(N) gauge theory, with the pure gauge theory in the middle.

• Tr$_{AP}$ indicates where we are in the phase diagram, with the system behaving as a generalization of the BEG model.

• Calorons change their role in moving between regions, but are important throughout.

• Casimir scaling is associated with the inclusion of monopoles on a democratic basis, and appears naturally in a U(1)$^{N-1}$ lattice model.