Four-fermi anomalous dimension with adjoint fermions

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Hierarchies and scaling dimensions

Linearized RG flow in a neighbourhood of a fixed point

$$\mu \frac{d}{d\mu} \hat{g}_k = -y_k \hat{g}_k \,, \qquad \hat{g}_k(\mu) = \left(\frac{\mu}{\Lambda_{\rm UV}} \right)^{-y_k} \hat{g}_0$$

$$\boxed{y_k = d_k + \gamma_k}$$

Associated IR scale:

$$\Lambda_{\rm IR} \sim \hat{g}_0^{1/y_k} \, \Lambda_{\rm UV} \,, \qquad y_k \ll 1 \quad \Longrightarrow \quad \text{natural hierarchy}$$

Global-singlet **relevant** operators (GSRO) require fine-tuning.

Stable hierarchies generated by *weakly* relevant operators. [Strassler 03, Sannino 04, Luty&Okui 04]

YM theory at the GFP is a limiting case:

$$\Lambda_{\rm IR} \sim \Lambda_{\rm UV} \exp\{-\frac{1}{\beta_0 g^2}\}$$

Hierarchies and the flavor sector

In the **SM** + elementary Higgs:

$$\boxed{\dim(H^\dagger H) \simeq 2}$$ dimension = 2

$$\mathcal{L}_Y = Y^u\, H\, \bar{L} u_R + Y^d H^\dagger \bar{L} d_R$$ dimension = 1+3 = 4

For **FCNC**:

$$\frac{f}{\Lambda_{\rm UV}^2}\, \bar{q} q \bar{q} q$$ dimension = 6

In **DEWSB**: scalar is composite [Dimopoulos et al 79, Eichten et al 1980]

$$\mathcal{L}_Y = \frac{Y^q}{\Lambda_{\rm UV}^2}\, \bar{Q} Q\, \bar{q} q$$ dimension = 3+3 = 6

Four-fermi interactions & walking scenarios

Alleviate the problem if we have **smaller** dimension of the composite scalar

Theory at the EW scale is **near** a non-trivial fixed point

Scaling dimension of the fermion bilinear is smaller

$$\dim\, H = \dim\, \bar{Q}Q = 3 - \gamma_m$$ [Holdom,Yamawaki, Appelquist, Eichten, Lane]

Scaling dimension of the Yukawa term < scaling dimension of FCNC terms

However, large anomalous dimension could generate a relevant four-fermi interaction

$$\dim\, QQQQ = 6 - \gamma_{4f} \approx 6 - 2\gamma_m$$

[Sannino 04, Luty 04, Rattazzi et al 08]

Operator mixing in the adjoint representation

$$\left(\bar{\psi}_1\Gamma_1\psi_2\right)\left(\bar{\psi}_3\Gamma_2\psi_4\right) \qquad \left(\bar{\psi}_1\Gamma_1 T^A\psi_2\right)\left(\bar{\psi}_3\Gamma_2 T^A\psi_4\right)$$

Color trace identity in the adjoint:

$$\left(T^A\right)_{\alpha\beta}\left(T^A\right)_{\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\delta}$$

Fierzing the fermionic fields:

$$\left(\bar{\psi}_1\Gamma_1\psi_4\right)\left(\bar{\psi}_3\Gamma_2\psi_2\right) \qquad \left(\bar{\psi}_1\Gamma_1 C\bar{\psi}_3^T\right)\left(\psi_4 C\Gamma_2\psi_2\right)$$

Complete basis of operators

$$O^{(1)}_{\Gamma_1\Gamma_2} = (\bar{\psi}_1\Gamma_1\psi_2)(\bar{\psi}_3\Gamma_2\psi_4)\,,$$
$$O^{(2)}_{\Gamma_1\Gamma_2} = (\bar{\psi}_1\Gamma_1\psi_4)(\bar{\psi}_3\Gamma_2\psi_2)\,,$$
$$O^{(3)}_{\Gamma_1\Gamma_2} = (\bar{\psi}_1\Gamma_1 C\bar{\psi}_3^T)(\psi_2^T C\Gamma_2\psi_4)\,,$$
$$O^{(4)}_{\Gamma_1\Gamma_2} = (\bar{\psi}_1\Gamma_1 C\bar{\psi}_3^T)(\psi_4^T C\Gamma_2\psi_2)\,,$$

parity-even	parity-odd
$\gamma_\mu \otimes \gamma_\mu$	$\gamma_\mu \otimes \gamma_\mu\gamma_5$
$\gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5$	$\gamma_\mu\gamma_5 \otimes \gamma_\mu$
$\mathbf{1} \otimes \mathbf{1}$	$\mathbf{1} \otimes \gamma_5$
$\gamma_5 \otimes \gamma_5$	$\gamma_5 \otimes \mathbf{1}$
$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$	$\sigma_{\mu\nu} \otimes \tilde{\sigma}_{\mu\nu}$

Operator mixing in the adjoint representation

Classify the operators according to their transformation properties under:

$$C : \left\{ \begin{array}{lcl} \psi_k(x) & \longrightarrow & C\bar{\psi}_k(x)^T \,, \\ \bar{\psi}_k(x) & \longrightarrow & \psi_k(x)^T C \,, \end{array} \right.$$

$$P : \left\{ \begin{array}{lcl} \psi_k(x) & \longrightarrow & \gamma_0 \psi_k(\tilde{x}) \,, \\ \bar{\psi}_k(x) & \longrightarrow & \bar{\psi}_k(\tilde{x})\gamma_0 \,, \end{array} \right.$$

$$S_{kl} : \left\{ \begin{array}{lcl} \psi_k(x) & \longleftrightarrow & \psi_l(x) \,, \\ \bar{\psi}_k(x) & \longleftrightarrow & \bar{\psi}_l(x) \,, \end{array} \right.$$

$$\mathcal{C}_{1234} = C S_{12} S_{34}$$
$$\mathcal{C}_{1423} = C S_{14} S_{23}$$

$$\boxed{C\,\Gamma\,C = \eta\,\Gamma^T}$$

Parity-odd sector & discrete symmetries

$$Q^{(k)}[\Gamma_1\Gamma_2 \pm \Gamma_2\Gamma_1] = O^{(k)}[\Gamma_1, \Gamma_2] \pm \eta_1\eta_2 O^{(k)}[\Gamma_2, \Gamma_1]$$

Concentrate on the sector $\mathcal{C}_{1234} = \mathcal{C}_{1423} = -1$

$$Q^{(1)}[VA - AV], Q^{(2)}[VA - AV], Q^{(3)}[SP - PS], Q^{(4)}[SP - PS]\,.$$

The operators above can be rearranged:

$$Q_1^{\pm} = Q^{(1)}[VA - AV] \pm Q^{(2)}[VA - AV]$$
$$A_1^{\pm} = Q^{(3)}[SP - PS] \pm Q^{(4)}[SP - PS]\,.$$

These are eigenstates of S_{14}

Mixing matrix

$$\begin{pmatrix} Q_1 \\ A_1 \end{pmatrix}^{\pm}_R = \begin{pmatrix} Z_{Q_1Q_1} & Z_{Q_1A_1} \\ Z_{A_1Q_1} & Z_{A_1A_1} \end{pmatrix}^{\pm} \begin{pmatrix} Q_1 \\ A_1 \end{pmatrix}^{\pm},$$

In this sector:

$$Q^{(3)}[SP - PS] = Q^{(4)}[SP - PS]$$

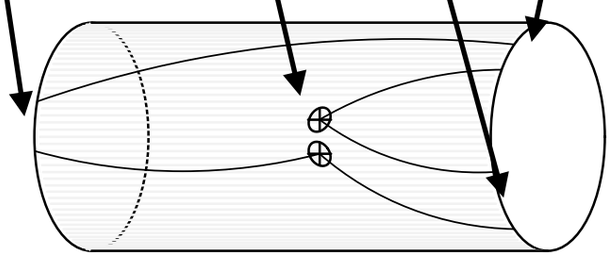
$$A_1^- = 0$$

Hence multiplicative renormalization:

$$\left(Q^{(1)}[VA - AV] - Q^{(2)}[VA - AV]\right)_R = Z_{Q_1Q_1}\left(Q^{(1)}[VA - AV] - Q^{(2)}[VA - AV]\right)$$

SF - four-fermi anomalous dimension

$$F^{\pm}_{i;A,B,C} = \frac{1}{L^3}\,\langle \mathcal{O}'_{53}[\Gamma_C]\,\mathcal{Q}^{\pm}_i\,\mathcal{O}_{21}[\Gamma_A]\,\mathcal{O}_{45}[\Gamma_B]\rangle$$



$$\mathcal{O}'_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}'_{f_1}(\mathbf{y})\Gamma\zeta'_{f_2}(\mathbf{z})$$

$$\mathcal{O}_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}_{f_1}(\mathbf{y})\Gamma\zeta_{f_2}(\mathbf{z})$$

SF - four-fermi anomalous dimension

$$f_1 = -\frac{1}{2L^6}\,\langle \mathcal{O}'_{12}[\gamma_5]\,\mathcal{O}_{21}[\gamma_5]\rangle$$

$$k_1 = -\frac{1}{6L^6}\sum_{k=1,2,3}\langle \mathcal{O}'_{12}[\gamma_k]\,\mathcal{O}_{21}[\gamma_k]\rangle$$

$$\boxed{h^{\pm}_{i;A,B,C}(x_0) = \frac{F^{\pm}_{i;A,B,C}(x_0)}{f_1^{\eta}\,k_1^{3/2-\eta}}}$$

$$\boxed{Z^{-}(g_0, a\mu)\,h^{-}_{1;A,B,C}(L/2) = h^{-}_{1;A,B,C}(L/2)\Big|_{g_0=0}}$$

SF - four-fermi anomalous dimension

Step-scaling functions:

$$\Sigma^{\pm}(s;u,L/a) = Z^{\pm}(g_0, sL/a)\, Z^{\pm}(g_0, L/a)^{-1}\big|_{\bar{g}(L)^2=u}$$

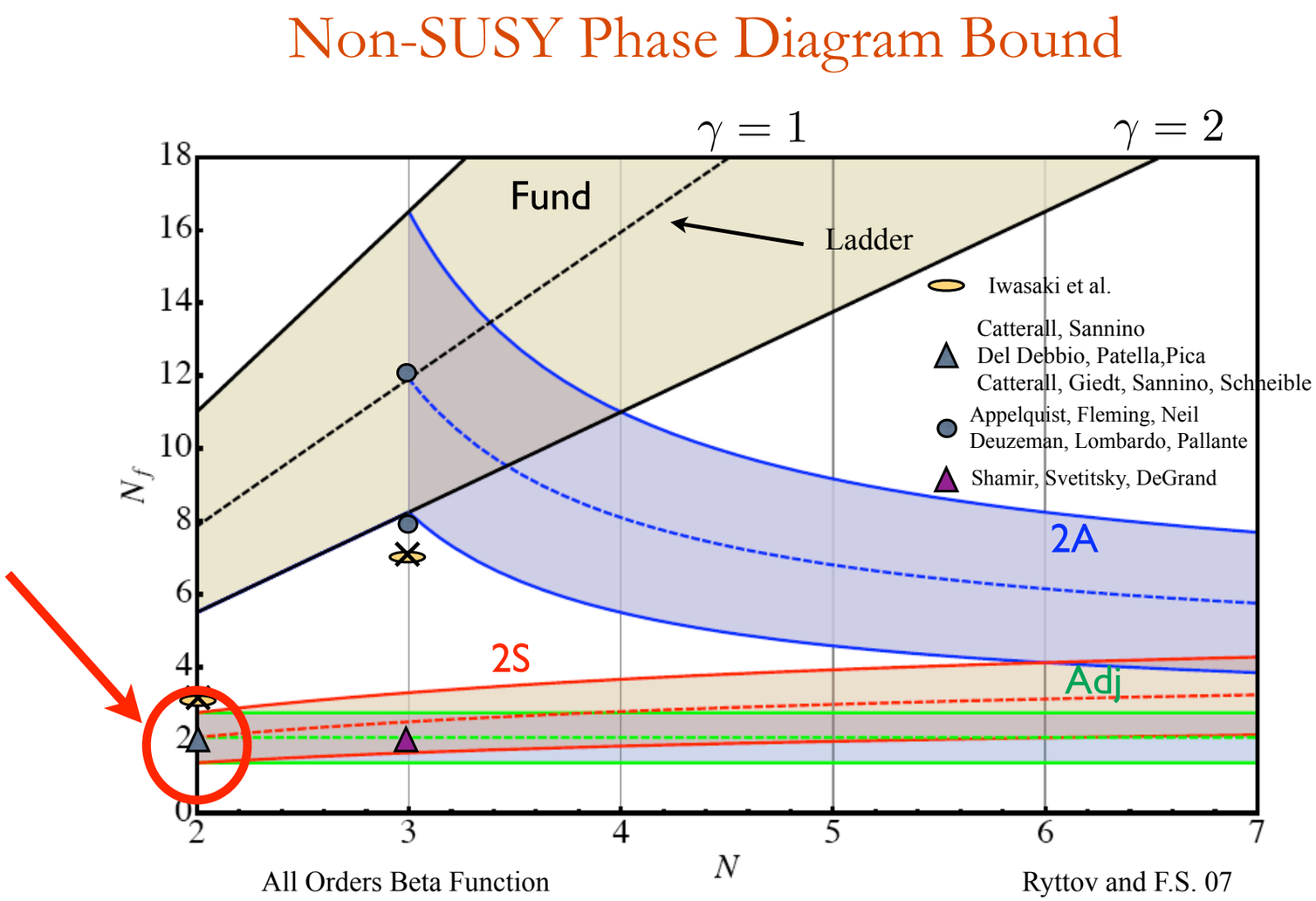
$$\sigma^{\pm}(s;u) = \lim_{a\to 0} \Sigma^{\pm}(s;u,L/a) = \mathrm{T}\,\exp\left\{\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg\, \frac{\gamma^{\pm}(g)}{\beta(g)}\right\}$$

In a neighbourhood of a fixed point:

$$\boxed{\gamma^{\pm}(u) = \frac{\log \sigma^{\pm}(s;u)}{\log s}}$$

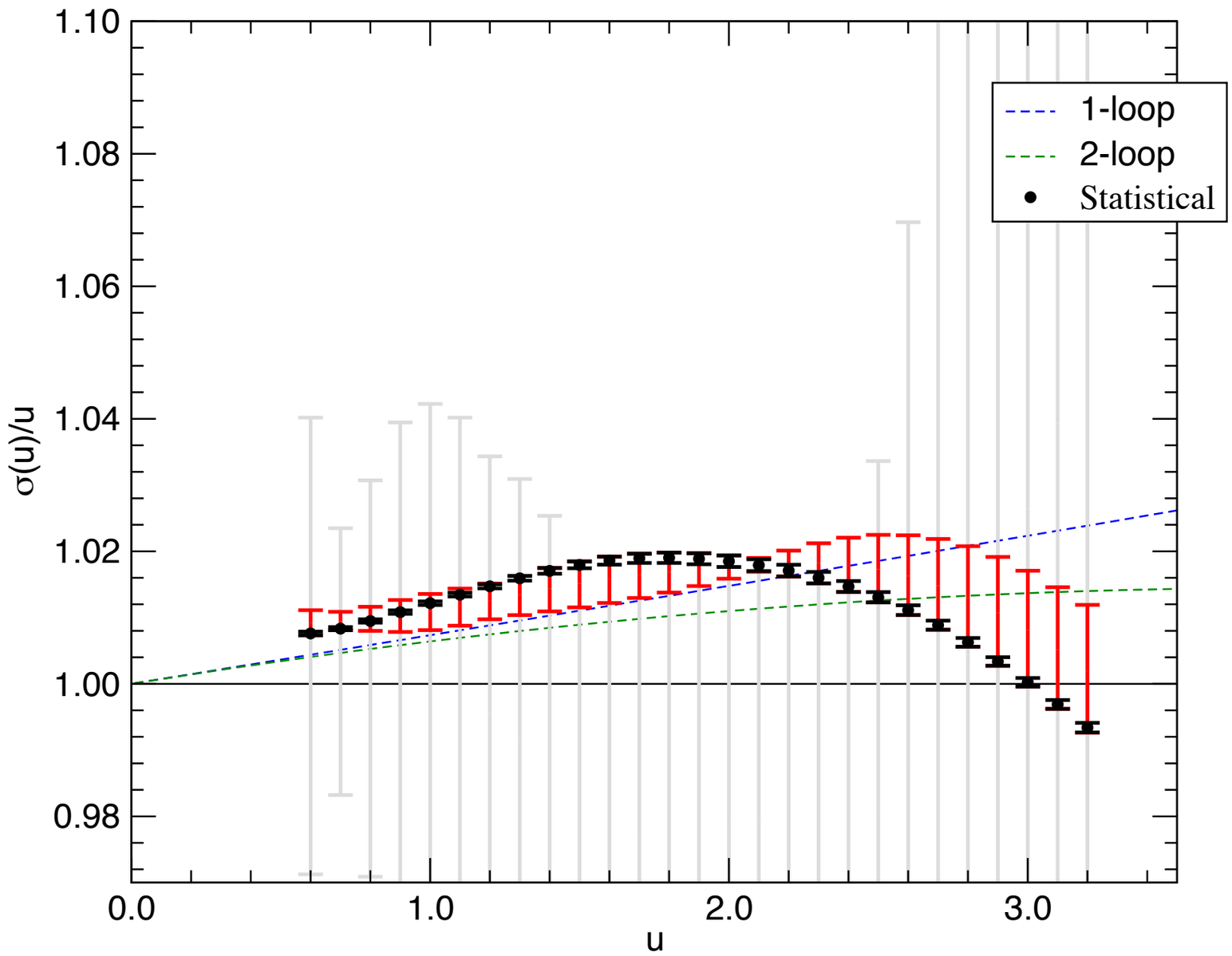
Phase diagram of SU(N) gauge theories

Use lattice tools to **search** for IRFPs in 4D SU(N) gauge theories



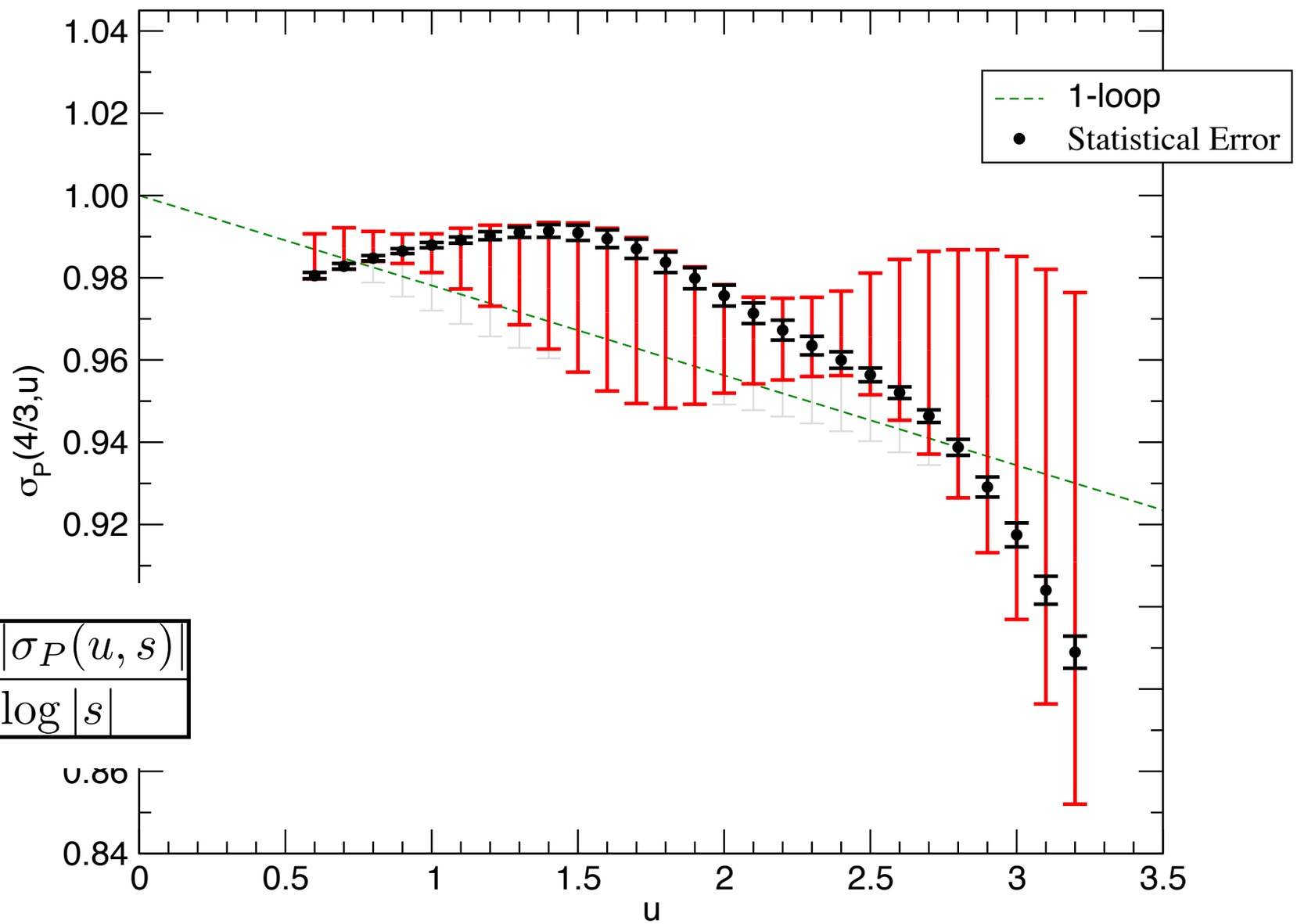
[Yamawaki, Appelquist, Miransky, Schrock, Nunez, Piai, Hong, Braun, Gies]

Running coupling



[Bursa et al 09]

Running of the mass

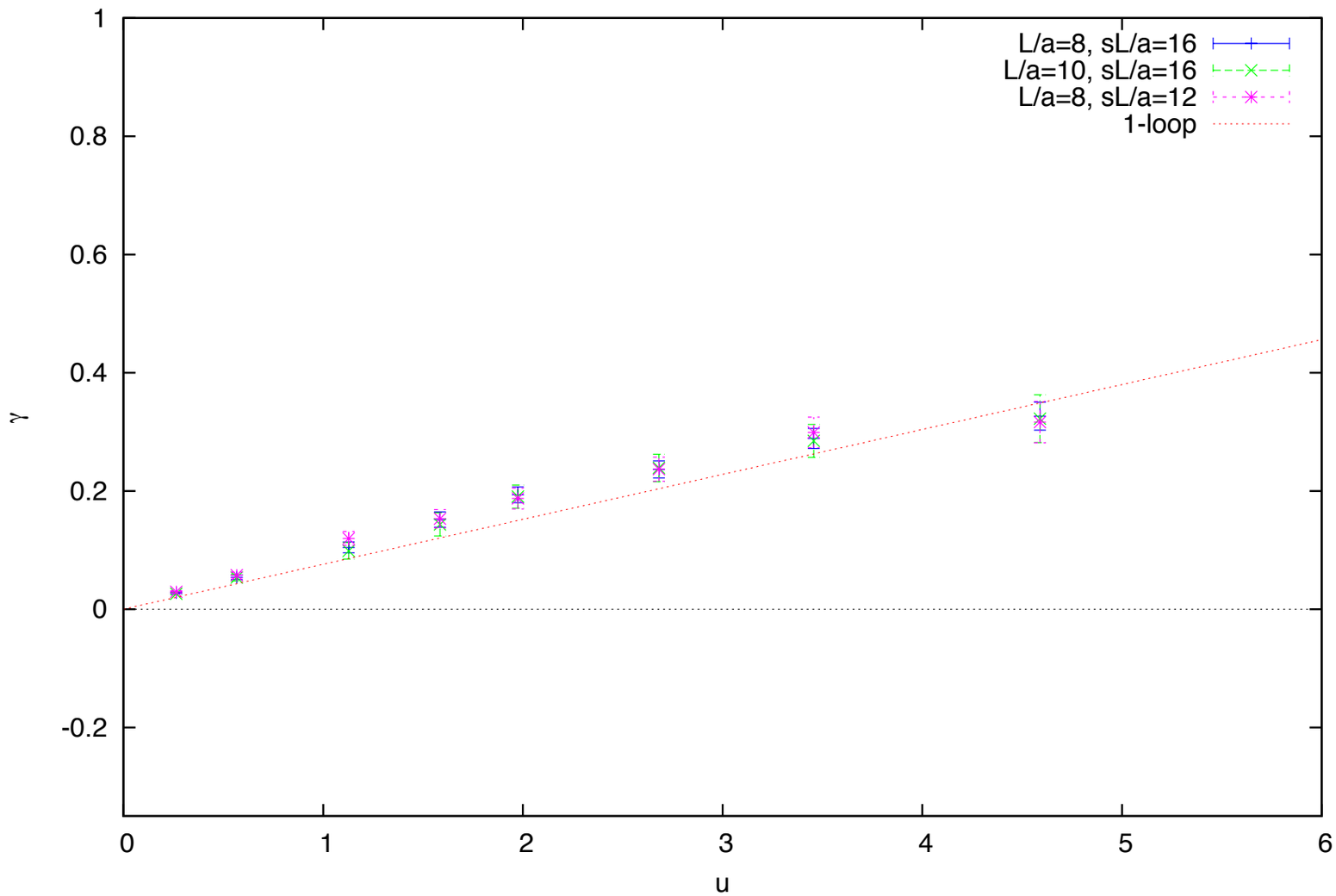


$$\hat{\gamma}(u) = -\frac{\log|\sigma_P(u,s)|}{\log|s|}$$

$\gamma < 0.6$

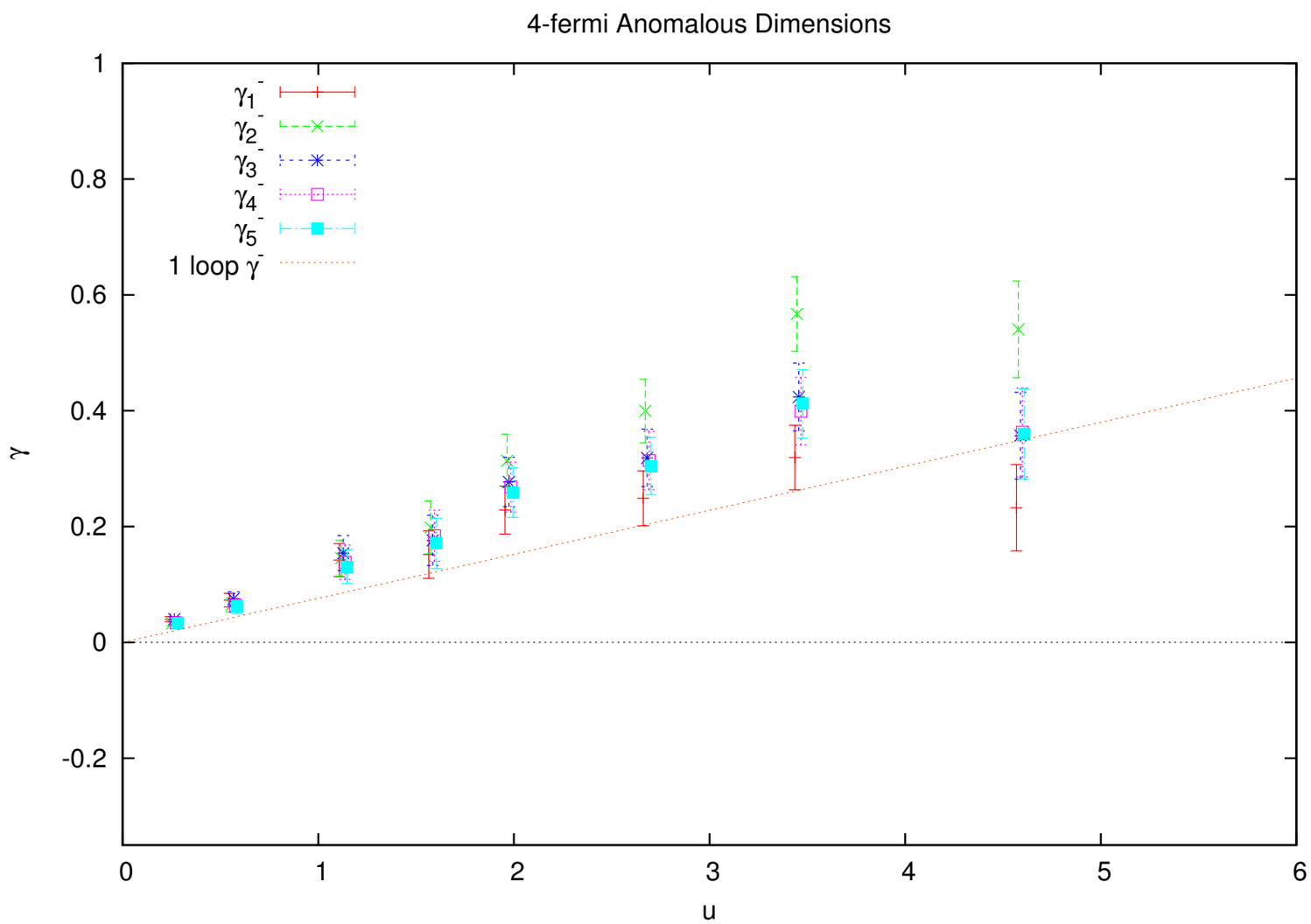
[Bursa et al 09]

Running of the mass - revisited



Four-fermi anomalous dimension

Multiplicative renormalization in this channel

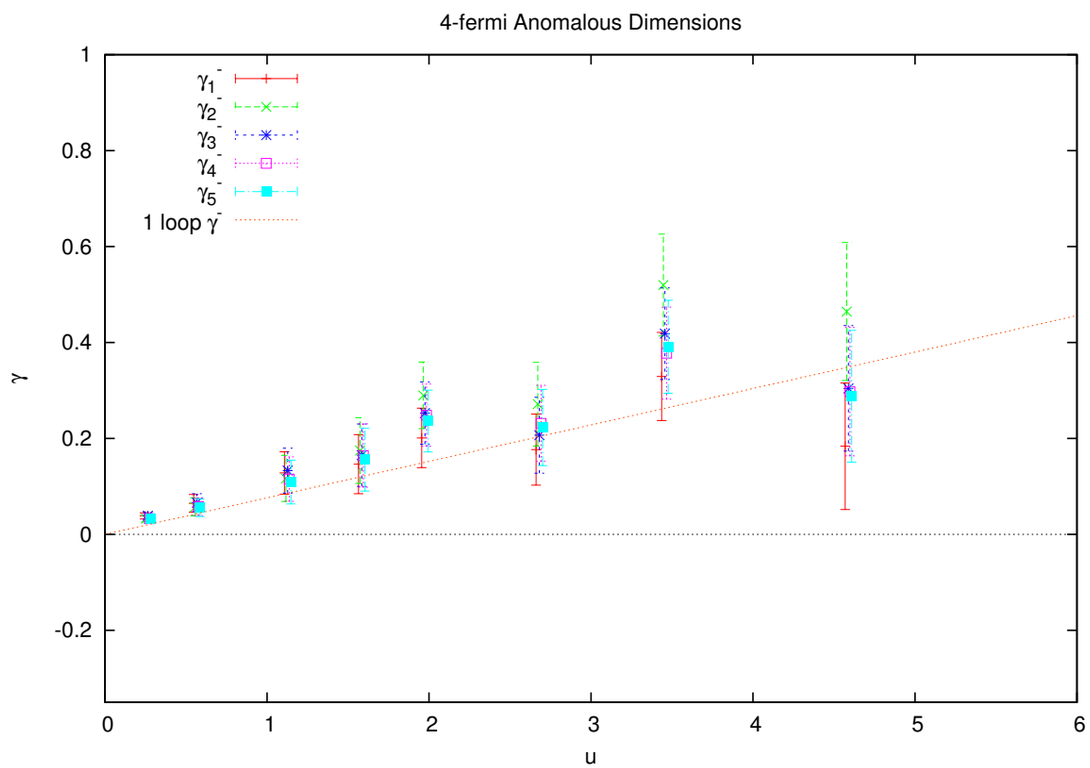


$$L/a = 8 \rightarrow L/a = 16$$

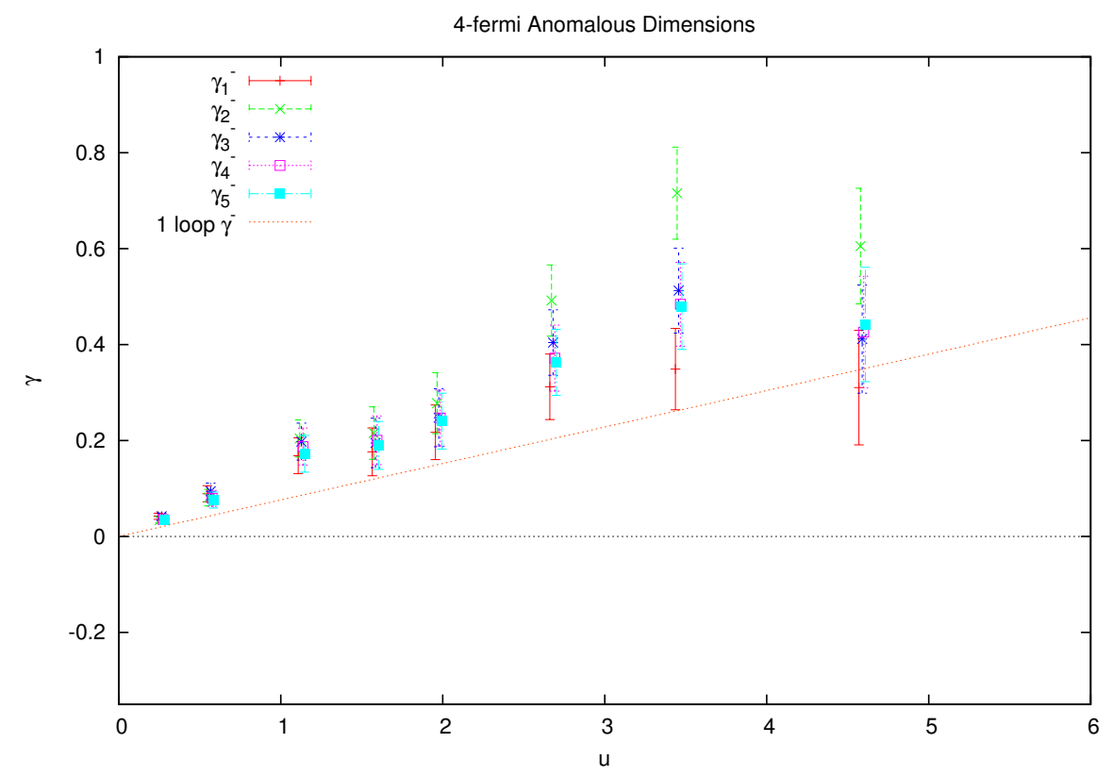
Scheme-independence: system in a neighbourhood of a fixed point?

Four-fermi anomalous dimension

Checks with different scaling steps



$L/a = 10 \rightarrow L/a = 16$



$L/a = 8 \rightarrow L/a = 12$

Results are consistent

Conclusions & perspectives

Goal: characterize the IR fixed point in SU(2) adj

Determination of the critical exponents at the IRFP using SF

computed mass anomalous dimension & 4fermi anomalous dimensions

gradient flow to define the coupling, chirally rotated [Ramos 13, Sint 10]

Comparison with other methods:

scaling of spectral quantities

energy-momentum tensor

RG flows

$$S[\phi; g, \mu] = \int d^D x \left[\frac{1}{2} \left(\partial_\mu \phi \right)^2 + \sum_k \mu^{d_i} \hat{g}_k O_k(x) \right]$$

$$O_k = \partial^{p_k} \phi^{n_k} \,, \quad D - d_k = n_k \frac{D-2}{2} + p_k$$

Integrate UV modes

$$\boxed{\mathcal{O}(\hat{g}; \mu) = \mathcal{O}(\hat{g}'; \mu') \,, \quad \mu' = \mu / b < \mu}$$

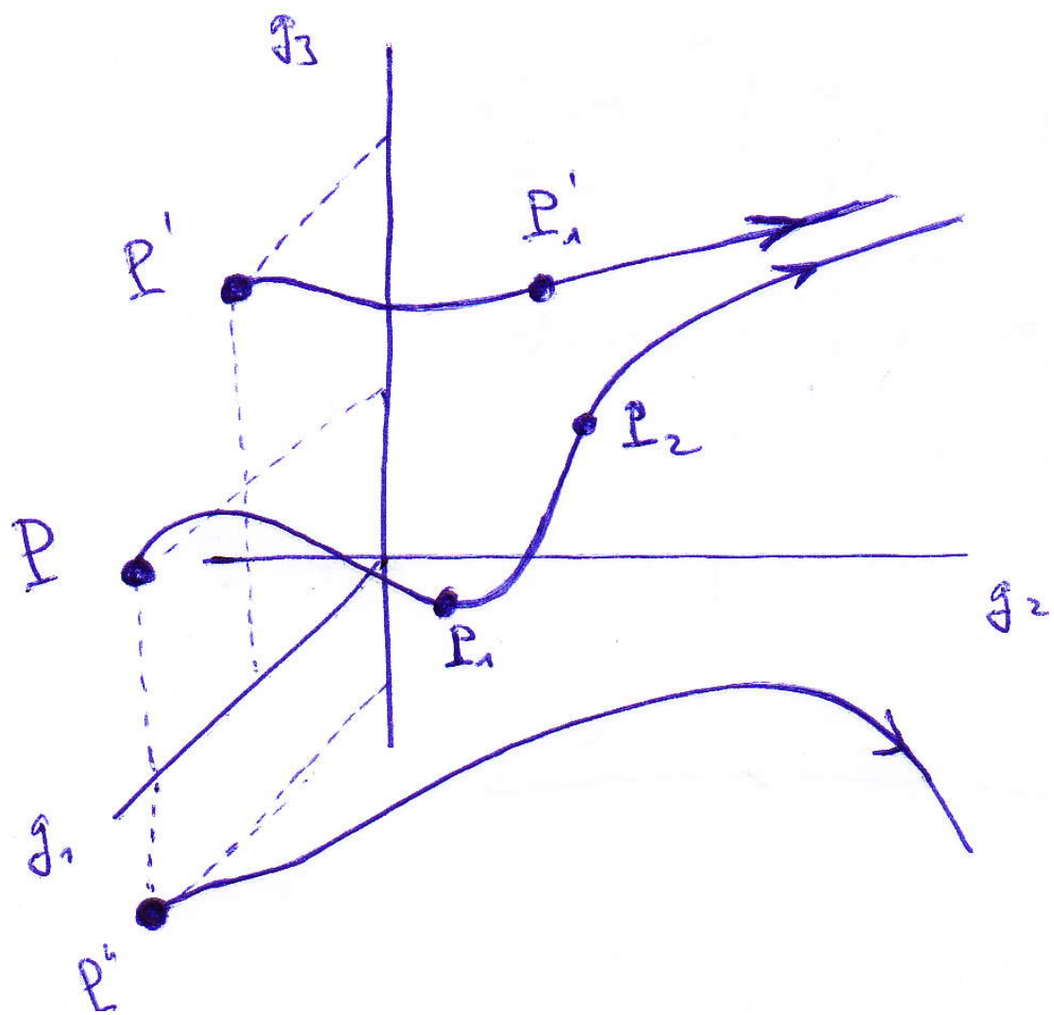
This introduces a dependence of the dimensionless couplings on the cut-off.

The effect of integrating out the high-energy modes is compensated by the change in the couplings (and rescaling of the fields). Scheme-dependence.

Integrating out the UV degrees of freedom generates all the interactions that are compatible with the symmetries of the system.

RG flows

RG transformations generate a **flow** in the space of couplings.



$$\mu \frac{d}{d\mu} \hat{g}_k = \hat{\beta}(\hat{g})$$

$$\boxed{\hat{\beta}_k(\hat{g}^*) = 0}$$