Testing volume independence of large N gauge theories on the lattice

A. González-Arroyo* and Masanori Okawa

Lattice 2014

* Instituto de Física Teórica UAM/CSIC
Departamento de Física Teórica, UAM

June 24, 2014

1

1

Large N and the lattice

- Two powerful non-perturbative methods which can be used in combination.
- The concept of **volume independence** or **reduction** emerged from this confluence
- large N SU(N) gauge theories are interesting in their own right.

In this work we restrict ourselves to pure gauge on a symmetric box L^4 and Wilson action

$$S_W = -bN \sum_P \mathrm{Tr}(U(P))$$

where $b = \beta/(2N^2) = 1/\lambda_L$
We also restrict the observables O to small Wilson loops $W(R, T)$ (simple and precise).

The problem

A lattice expectation value: $O(b, N, L)$
The large N quantity is given by

$$O_\infty(b) = \lim_{N \longrightarrow \infty} \quad \lim_{L \longrightarrow \infty} O(b, N, L)$$

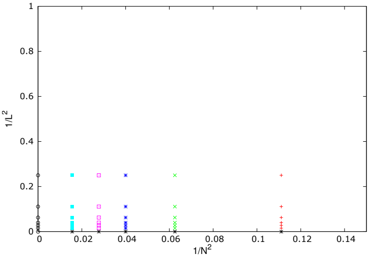
$$\longleftarrow O(b, N) \longrightarrow$$

The problem

A lattice expectation value: $O(b, N, L)$
The large N quantity is given by

$$O_\infty(b) = \lim_{N\longrightarrow\infty} \quad \lim_{L\longrightarrow\infty} O(b, N, L)$$

$$\longleftarrow O(b, N) \longrightarrow$$

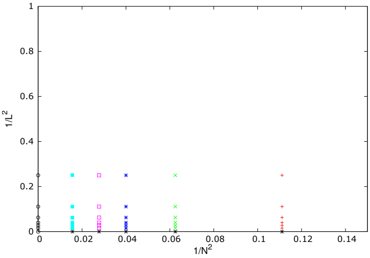


The problem

A lattice expectation value: $O(b, N, L)$
The large N quantity is given by

$$O_\infty(b) = \lim_{N \longrightarrow \infty} \quad \lim_{L \longrightarrow \infty} O(b, N, L)$$

$$\longleftarrow O(b, N) \longrightarrow$$



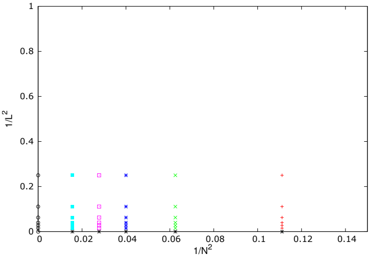
- Do the limits commute?

The problem

A lattice expectation value: $O(b, N, L)$
The large N quantity is given by

$$O_\infty(b) = \lim_{N \longrightarrow \infty} \quad \lim_{L \longrightarrow \infty} O(b, N, L)$$

$$\longleftarrow O(b, N) \longrightarrow$$



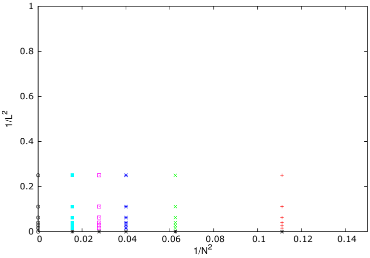
- Do the limits commute?
- Are there phase transition points on the x or y axis?

The problem

A lattice expectation value: $O(b, N, L)$
The large N quantity is given by

$$O_\infty(b) = \lim_{N \longrightarrow \infty} \quad \lim_{L \longrightarrow \infty} O(b, N, L)$$

$$\longleftarrow O(b, N) \longrightarrow$$



- Do the limits commute?
- Are there phase transition points on the x or y axis?

$\lim_{N \longrightarrow \infty} O(b, N, L) = O_\infty(b)$ **VOLUME INDEPENDENCE**

Important ingredients/results

- **The statement can be b-dependent**
  True at strong coupling, wrong at weak coupling (BHN)?
- **The statement can depend on the boundary conditions**
  Twisted Boundary conditions (GAO), discrete Z_N fluxes through each plane $n_{\mu\nu}$:

$$S_W = -bN \sum_P Z(P)\mathrm{Tr}(U(P))$$

  All that matters is $Z(\mathrm{Plane}) = \exp\{2\pi i \; n(\mathrm{Plane})/N\}$
- **There might be phase transitions at fixed $L_c(b)$**
  Partial reduction (NN)
- **Might depend on the lattice action (UY)-...**

Weak coupling results (large b)

The theory can be studied by standard weak-coupling/perturbative methods.

$$O(b,N,L) = \hat{O}_0 - \sum_{j=1} \hat{O}_j(N,L)\frac{1}{b^j}$$

- Leading order dominated by flat connections: $\hat{O}_0 = \hat{W}_0 = 1$

For periodic B.C. :**TORONS**

For T.B.C: orthogonal twists
$+$ isotropy $\Rightarrow$ $N = \hat{L}^2$ and $n(\mathrm{Plane}) = k\hat{L}$
SYMMETRIC TWIST k coprime with $\hat{L}$

- $\hat{W}_1^{\mathrm{PBC}}(N,L) = \hat{W}_1(N,\infty) - \frac{R^2T^2}{24L^4}(1-\frac{1}{N^2}) + \dots$ $\cancel{\mathrm{VI}}$
  $\hat{W}_1^{\mathrm{TBC}}(N,L) = \hat{W}_1^{\mathrm{PBC}}(\infty, L\hat{L}) - \frac{1}{N^2}\hat{W}_1^{\mathrm{PBC}}(\infty,L) \sim$
  $\hat{W}_1(N,\infty) + \mathcal{O}(\frac{1}{L^6N^2})$
- General arguments suggest this is true to all orders (GAO)
  (now estimating size of $\hat{W}_2^{\mathrm{TBC}}(N,L,k)$ with M. Garcia Perez)

Results in Non-perturbative weak-coupling region

Can we recover volume independence by simply setting $k \neq 0$?
Results by several authors (IO-TV) show one cannot keep k fixed

A surviving candidate (GAO 2010):

$$\lim_{\hat{L} \longrightarrow \infty} O(b, N = \hat{L}^2, L, k_{\hat{L}}) = O_\infty(b)$$

THIS WORK

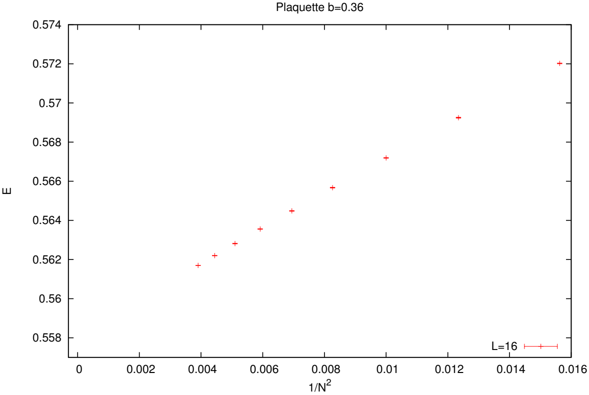
Make a **DIRECT** test of the proposal by a **QUANTITATIVELY PRECISE** measurement of $W(R, T; b, N, L, k)$
Our study includes ordinary LGT with PBC ($k = 0$), the EK model ($L = 1$ $k = 0$) and TEK model ($L = 1$ $k \neq 0$)
Volume independence $\Leftrightarrow$ $W(R, T; b, \infty, L, k) = W_{N=\infty}(R, T; b)$

N-dependence of plaquette e.v.

We studied the plaquette ($R = T = 1$) at $b = 0.36$ at various L and N. This is a very precise quantity (errors 10^{-5}).

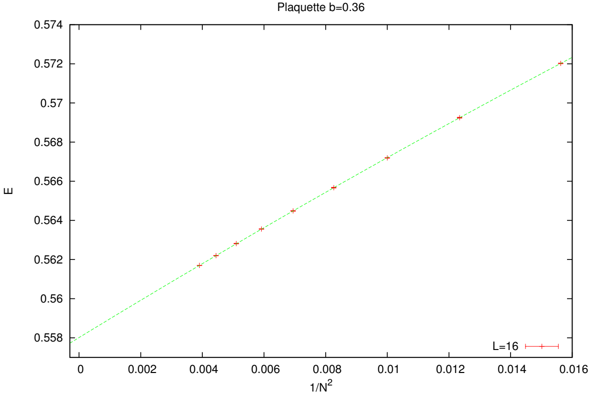
N-dependence of plaquette e.v.

We studied the plaquette ($R = T = 1$) at $b = 0.36$ at various L and N. This is a very precise quantity (errors 10^{-5}).
For periodic boundary conditions ($k = 0$)



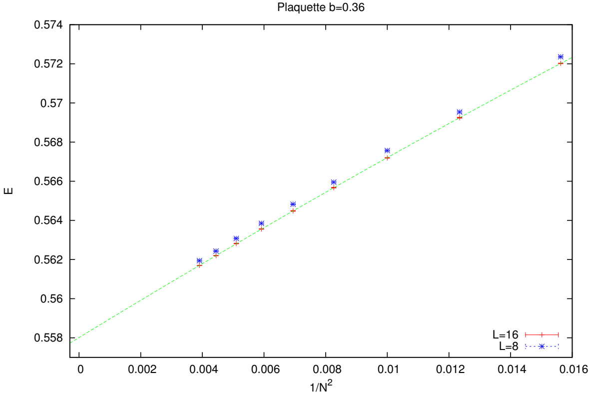
N-dependence of plaquette e.v.

We studied the plaquette ($R = T = 1$) at $b = 0.36$ at various L and N. This is a very precise quantity (errors 10^{-5}).
For periodic boundary conditions ($k = 0$)



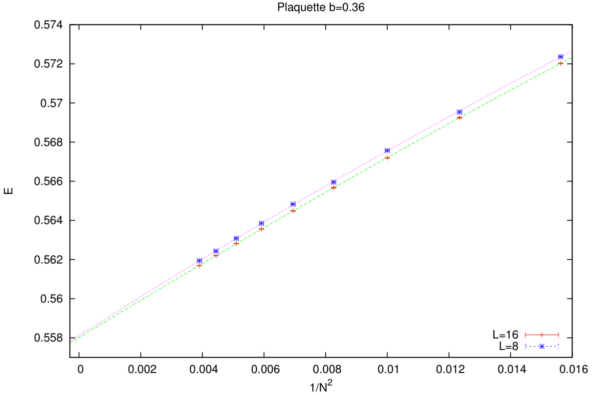
N-dependence of plaquette e.v.

We studied the plaquette ($R = T = 1$) at $b = 0.36$ at various L and N. This is a very precise quantity (errors 10^{-5}).
For periodic boundary conditions ($k = 0$)



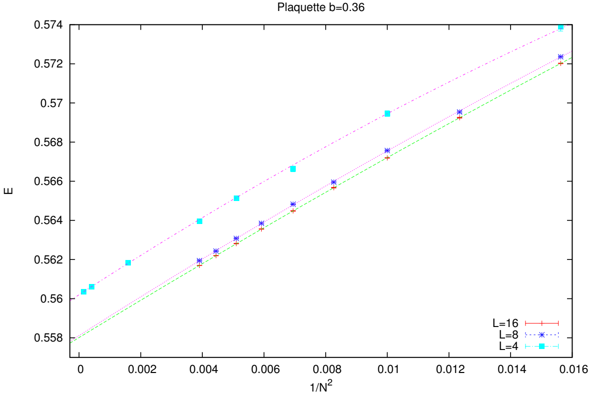
N-dependence of plaquette e.v.

We studied the plaquette ($R = T = 1$) at $b = 0.36$ at various L and N. This is a very precise quantity (errors 10^{-5}).
For periodic boundary conditions ($k = 0$)



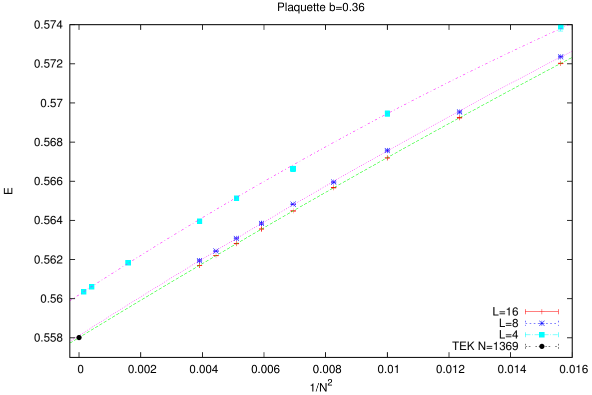
N-dependence of plaquette e.v.

We studied the plaquette ($R = T = 1$) at $b = 0.36$ at various L and N. This is a very precise quantity (errors 10^{-5}).
For periodic boundary conditions ($k = 0$)



N-dependence of plaquette e.v.

We studied the plaquette ($R = T = 1$) at $b = 0.36$ at various L and N. This is a very precise quantity (errors 10^{-5}).
And twisted boundary conditions $k \neq 0$ (TEK $L = 1$)

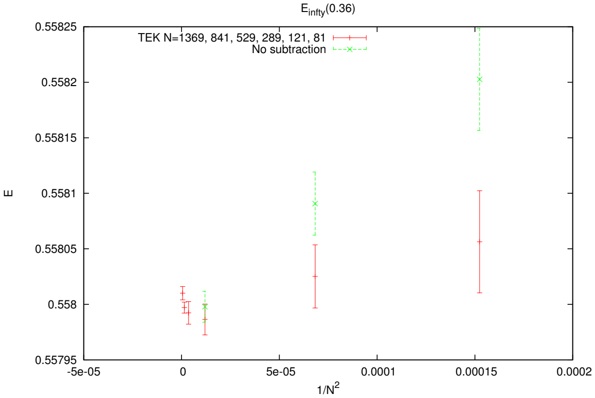


N-dependence of TEK

We tried various values of $N = \hat{L}^2$

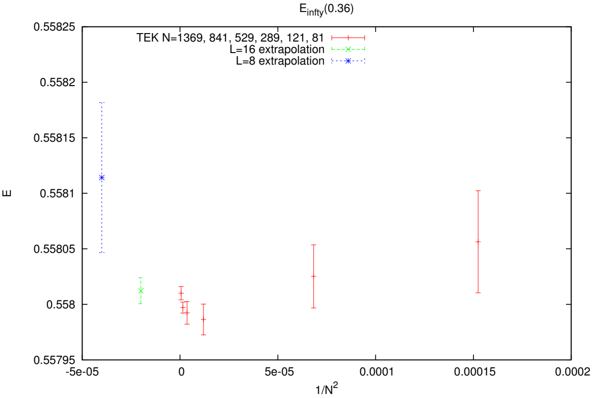
N-dependence of TEK

We tried various values of $N = \hat{L}^2$



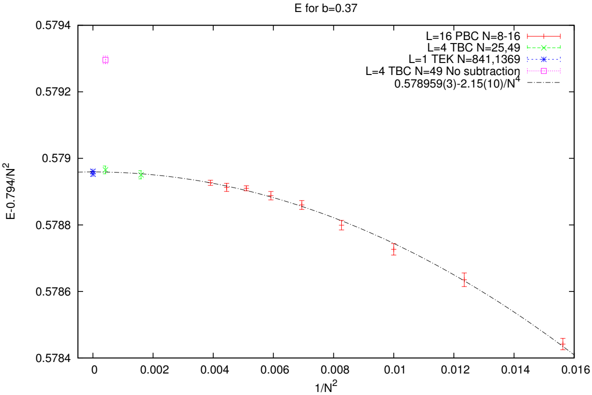
N-dependence of TEK

We tried various values of $N = \hat{L}^2$



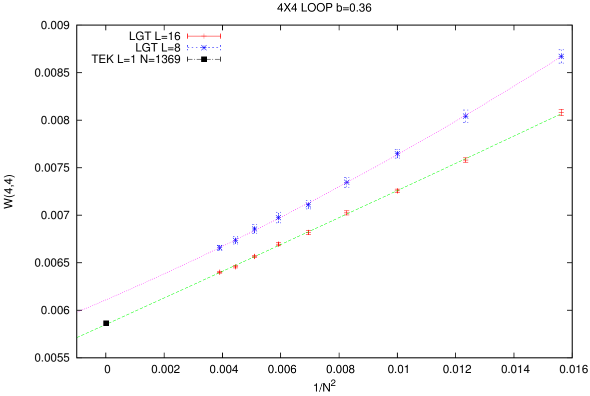
Other values of b and R

The result extends to other values of b:
Example $b = 0.37$: The $1/N^2$ is approximately universal



Other values of b and R

The same is true about other Wilson loops $R = 2, 3, 4$



Global description for all N

All the data points obtained over the last 3 years can be fitted to a unified picture:

$$\bar{E}(b,N) = \frac{1+\sum_{n=1}^{3}\sum_{m=0}^{\min(2,n)} a_{nm}x^n y^m}{1+\sum_{n=1}^{3}\sum_{m=0}^{\min(2,n)} a'_{nm}x^n y^m}$$

with $x = 1/b_I = 1/(bE(b))$ and $y = 1/N^2$.
a'_{nm} adjusted to match perturbative expansion to 3 loops

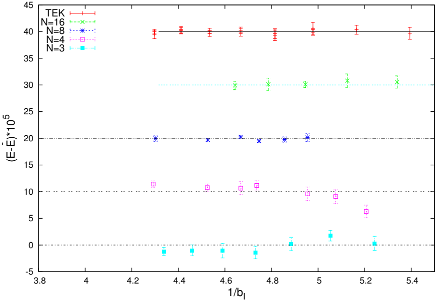
Global description for all N

All the data points obtained over the last 3 years can be fitted to a unified picture:

$$\bar{E}(b,N) = \frac{1+\sum_{n=1}^{3}\sum_{m=0}^{\min(2,n)} a_{nm}x^n y^m}{1+\sum_{n=1}^{3}\sum_{m=0}^{\min(2,n)} a'_{nm}x^n y^m}$$

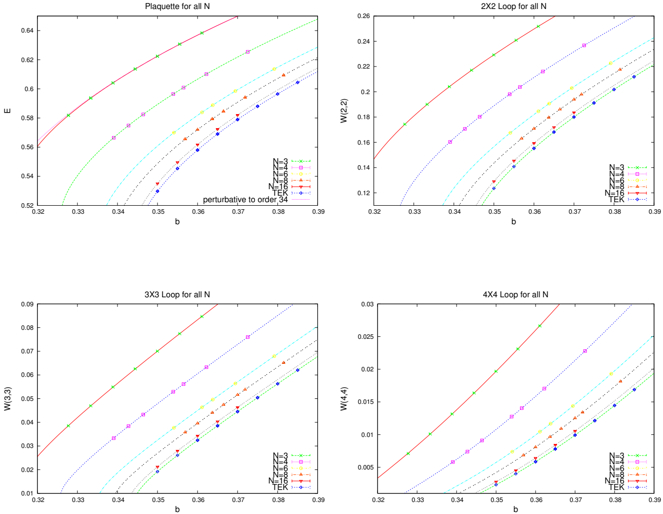
with $x = 1/b_I = 1/(bE(b))$ and $y = 1/N^2$.

a'_{nm} adjusted to match perturbative expansion to 3 loops



- The formula is invertible
- matches the data within $\mathcal{O}(10^{-5})$ errors
- The $N=\infty$ parameters are determined with TEK only
- Similar results can be obtained for $W(R,R)$

A final view


Plaquette for all N
E
b
N=3
N=4
N=6
N=8
N=16
TEK
perturbative to order 34
2X2 Loop for all N
W(2,2)
3X3 Loop for all N
W(3,3)
4X4 Loop for all N
W(4,4)

Conclusions and outlook

Volume independence holds up to a very high level of precision if the appropriate boundary conditions are used.

The data is consistent with VI[2] being valid at all values of b.

Finite N corrections are small and allow the reduced version to be a practical tool to access the large N world.

- Complete the study in perturbation theory (with M. Garcia Perez)
- Extend the study to other observables and asymmetric boxes (finite temperature?)

Large N WORKSHOP at IFT(Madrid)
18 May- June 5 2015

[2] VI stands for Volume Independence not the middle name of Felipe VI