Simulating two dimensional two-color QCD using purely bosonic variables

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Outline

Motivation

Why QCD_2 with two colors?

- confinement, chiral flavor symmetry, fermion boson equivalence, etc.
- we have many theoretical results [G.'t Hooft 1974][A.Ferrando and V.Vento 1991][M. Herrchen 1990]
- Kosterlitz–Thouless transition
- chemical potential

Introduction

ordinary approach our approach gluonic observables

$$\mathcal{L} = -\overline{\psi} i \not{D} \psi + \frac{1}{2} tr F^{\mu\nu} F_{\mu\nu} \longrightarrow \text{Link variables}$$

d.o.f.: $\psi, \overline{\psi}, A_\mu$

$$\mathcal{L} = -\frac{4}{V} \log \left| \frac{\Theta}{\eta} \right| - W + \mathcal{L}_{YM} \longrightarrow \text{Observables}$$

d.o.f.: $\alpha, \overline{\alpha}, \phi, h_0, h_1$

Analysis
Degrees of freedom

introduce complex variables and complexify our gauge fields

$$A_z = -ig^{-1}\left(\partial_z - ic\sigma_3\right)g$$

$$c = \frac{2\pi}{L_0}h_0 - i\frac{2\pi}{L_1}h_1, h_0, h_1 \in \mathbb{R}, \quad g \in SL(2, C)$$

Iwasawa decomposition

$$g = \begin{pmatrix} e^{\phi} & \nu e^{\phi} \\ 0 & e^{-\phi} \end{pmatrix} V \qquad \Bigg| \qquad \begin{array}{c} \phi \in \mathbb{R}, \nu \in \mathbb{C} \\ V \in SU(2) \end{array}$$

$$\alpha = \partial_z \nu + 2\nu\partial_z\phi - 2ic\nu$$

Analysis
Functions

Fermion Determinant

$$det(i\not{D}) = \left|\frac{\Theta}{\eta}\right|^4 e^W$$

$$\Theta = \Theta(h_0, h_1) \quad \eta = \eta(\frac{L_0}{L_1}) \quad W = W(\phi, \alpha)$$

Dirac operator has zero modes only for $(h_0 + \frac{1}{2}, h_1) \in \mathbb{Z}^2$

Yang-Mills action

$$S_{YM}(\phi, \alpha, h_0, h_1)$$

Measure

$$\int \mathcal{D}A_z \mathcal{D}A_{\overline{z}} = \int \mathcal{D}VJ(V)dh_0 dh_1 \mathcal{D}\phi \mathcal{D}\alpha \mathcal{D}\overline{\alpha}$$

only the Jacobian depends on V $\rightarrow$ V drops out for observables

Symmetries

$\phi \to \phi + \Delta\phi$

- $A_z(\phi, \alpha, h_0, h_1) = A_z(\tilde{\phi}, \alpha, h_0, h_1)$
- we set $\int\limits_{\mathcal{T}} dx^2 \phi(x) = 0$

residual gauge symmetry

- $\alpha \to e^{-i4\pi(\frac{n_0 x_0}{L_0} + \frac{n_1 x_1}{L_1})}\alpha$
- $h_0 \to h_0 + n_0 \quad h_1 \to h_1 + n_1$
- $n_0, n_1 \in \mathbb{Z}$
- $A_z(\phi, \alpha, h_0, h_1) = \Omega A_z(\phi, \tilde{\alpha}, \tilde{h}_0, \tilde{h}_1)\Omega^\dagger + i(\partial_z \Omega)\Omega^\dagger \quad \Omega \in SU(2)$
- we restrict h_0 and h_1 to $[0, 1]$

Simulation

Partition function

$$Z = \int_0^1 \int_0^1 dh_0 dh_1 \, |\Theta(h_0, h_1)|^4 \int D\phi D\alpha_1 D\alpha_2 e^{-\beta S_{YM} + W}$$

Simulation:

1. HMC step for $\phi, \alpha, \overline{\alpha}$ with constant h_0, h_1
2. n Metropolis steps for h_0, h_1 with constant $\phi, \alpha, \overline{\alpha}$
3. calculate Link variables $\rightarrow$ calculate gluonic variables
4. calculate fermionic variables

Link variables

Polyakov loop

$P(s) = tr \prod_{i=0}^{N_t} U_0(x_i, s)$

Link variables

$U_\mu(x) \sim e^{iA_\mu(x)}$

we can calculate $A_\mu(\phi, \alpha, \overline{\alpha}, h_0, h_1)$

Problem: $e^{iA_\mu(x)} \to \Omega^\dagger(x) e^{i\tilde{A}_\mu(x) + i(\partial_\mu \Omega(x))\Omega^\dagger} \Omega(x)$

wrong transformation for Link variables

Solution: $U_\mu(x) = e^{\frac{iA_\mu(x)}{2}} e^{i2\pi \frac{h_\mu x_\mu}{L_\mu} \sigma_3} e^{\frac{iA_\mu(x+e_\mu)}{2}}$

right transformation $U_\mu(x) = \Omega(x)^\dagger \tilde{U}_\mu(x) \Omega(x + e_\mu)$

Link variables

Test

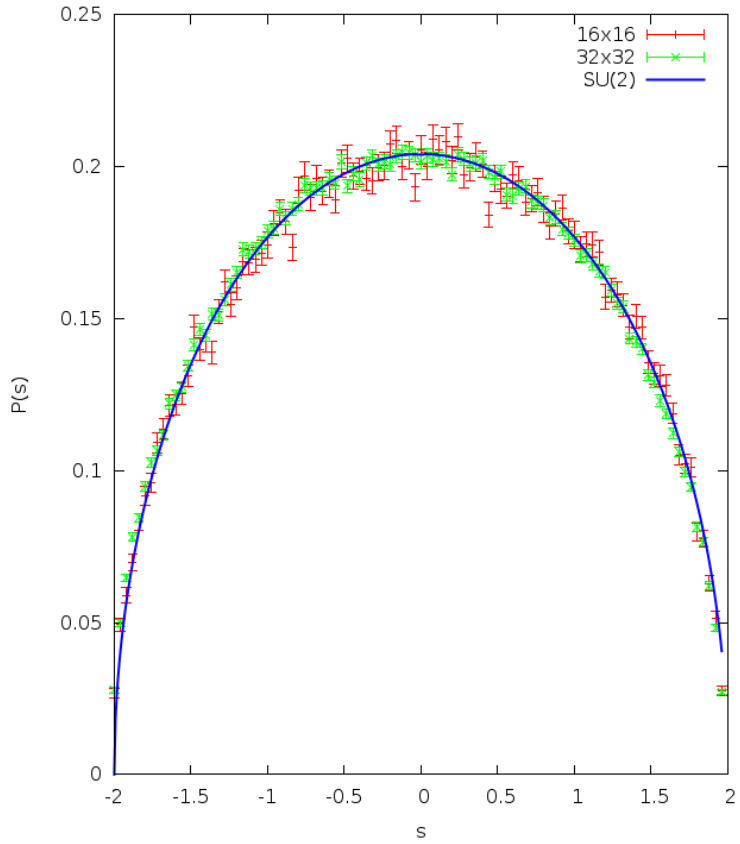

Figure : Polyakov loop Distribution for pure gauge theory

- pure gauge theory
- correct distribution function
- no volume dependance

Results

- We find $\langle P \rangle = 0$ for all simulated β for $T = 0$
- We can not produce enough statistics for Polyakov loop Correlators or Wilson loop Correlators
- We found the correct Plaquette action for no too small β

Introduction

Meson Mass spectrum

Meson	m_0	m_1
Scalar	0	m_{s1}
Pseudoscalar	0	m_{s1}
Vector	m_{v0}	m_{v1}
Axial vector	m_{v0}	m_{v1}

- Scalar $\leftrightarrow$ Pseudoscalar: massless fermions
- Vector $\leftrightarrow$ Axial Vector: $\gamma_\mu = -i\epsilon^{\mu\nu}\gamma_\nu\gamma_5$
- massless Meson: theoretical results

Fermionic correlator

Calculation

Dirac Operator: $\not{D} = G(\not{\partial} - i\not{c}\sigma_3)G^\dagger \quad G = \begin{pmatrix} g^{-1} & 0 \\ 0 & g^\dagger \end{pmatrix}$

SLAC operator: $\not{\partial}(x) = \frac{1}{|\Lambda|}\sum_{p\in\tilde{\Lambda}} \not{p} e^{-ipx}$

axial and vector meson correlator vanish
diquark correlator is the same as the correlator for free fermions $\rightarrow$ massless ground state
scalar meson is the only interesting quantity left

Result

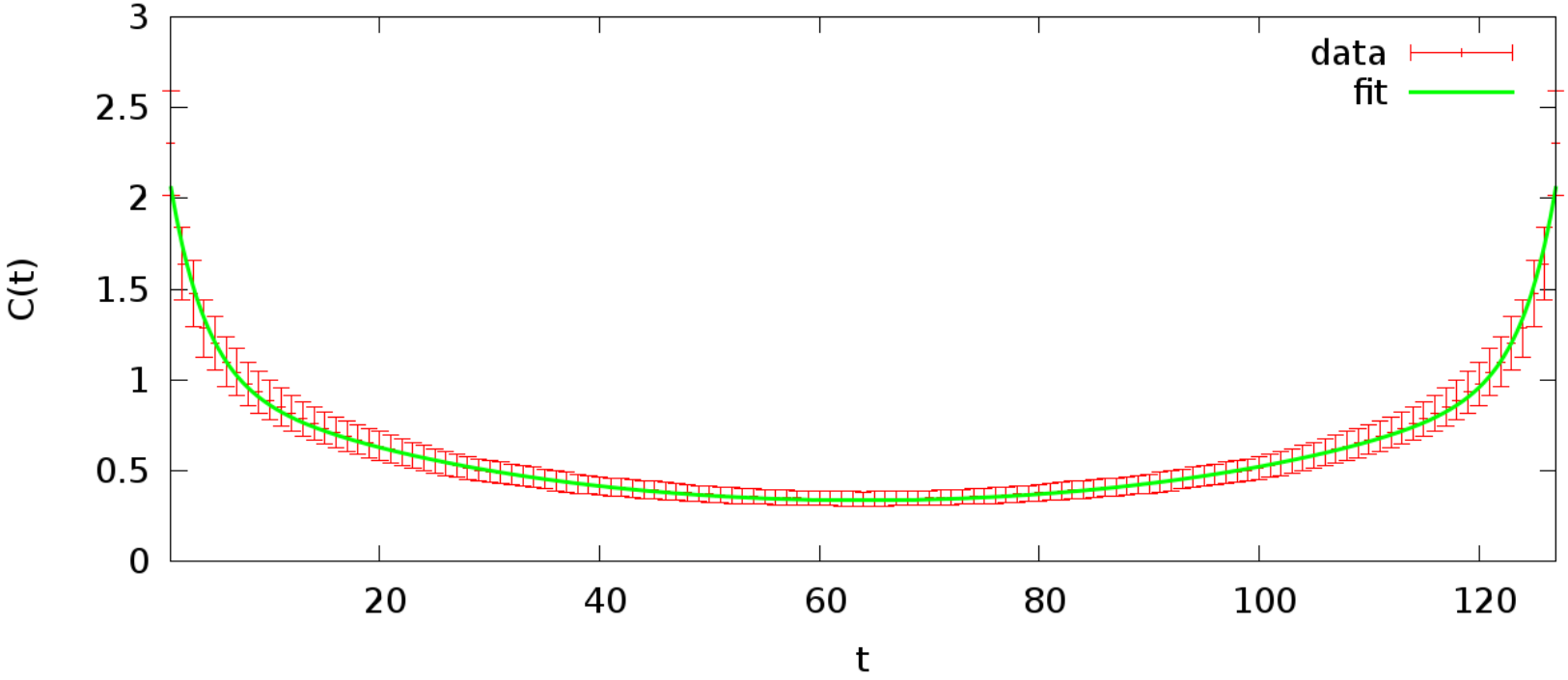

Figure : Correlator for a 128x128 lattice with NF=1 and $\beta = 0.24$
fit function: $C(t) = a\cosh(m_0(x - L/2)) + b\cosh(m_1(x - L/2))$

Mass determination

L	$\frac{\pi}{L}$	m_0	m_1
128	0.02454	0.0277(2)	0.31(1)
64	0.04909	0.059(4)	0.8(2)
32	0.09817	0.092(6)	1.1(2)
16	0.19635	0.19(2)	1.0(4)

the smallest mass is always of order $\frac{\pi}{N_0} \rightarrow$ massless ground state

Summary:

- We reintroduced Link variables for gluonic variables
- We found a massless ground state for the scalar meson

Outlook:

- add chemical potential
- look at non zero temperature $\rightarrow$ Kosterlitz-Thouless transition
- Supersymmetry