UNIVERSITY OF Southampton

NPR for PDAs

Nonperturbative renormalisation for low moments of light-meson distribution amplitudes

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Introduction

Parton distribution amplitudes (PDAs) relevant for exclusive QCD processes at large momentum transfer, distances near the light cone

Give process-independent nonperturbative information on bound-state structure of hadrons

- Momentum-fraction distribution of partons in a particular Fock state of a hadron

Calculated in three main approaches

- Extract from experimental form factor data
- QCD sum rules
- Lattice QCD

Introduction

Low moments of PDAs can be computed from local matrix elements

Example for pseudoscalar meson

$$\langle 0|\mathcal{S}\bar{q}_a\gamma_\mu\gamma_5\overleftrightarrow{D}_\nu q_b|P(p)\rangle = \langle\xi^1\rangle_P f_P\,\mathcal{S}p_\mu p_\nu$$
$$\langle 0|\mathcal{S}\bar{q}_a\gamma_\mu\gamma_5\overleftrightarrow{D}_\nu\overleftrightarrow{D}_\rho q_b|P(p)\rangle = \langle\xi^2\rangle_P f_P\,\mathcal{S}p_\mu p_\nu p_\rho$$

$\mathcal{S}$ means symmetrised and traceless in Lorentz indices

Bare lattice operators need renormalisation and matching to a continuum scheme like $\overline{\text{MS}}$

Introduction

- Moments from non-forward matrix elements with momentum transferred at the operator insertion
- For second moment get mixing of double-covariant derivative operator with double total-derivative operator

$$\mathcal{S}\bar{q}_a\gamma_\mu\gamma_5\overleftrightarrow{D}_\nu\overleftrightarrow{D}_\sigma q_b \qquad \text{and} \qquad \mathcal{S}\partial_\mu\partial_\nu(\bar{q}_a\gamma_\sigma\gamma_5 q_b)$$

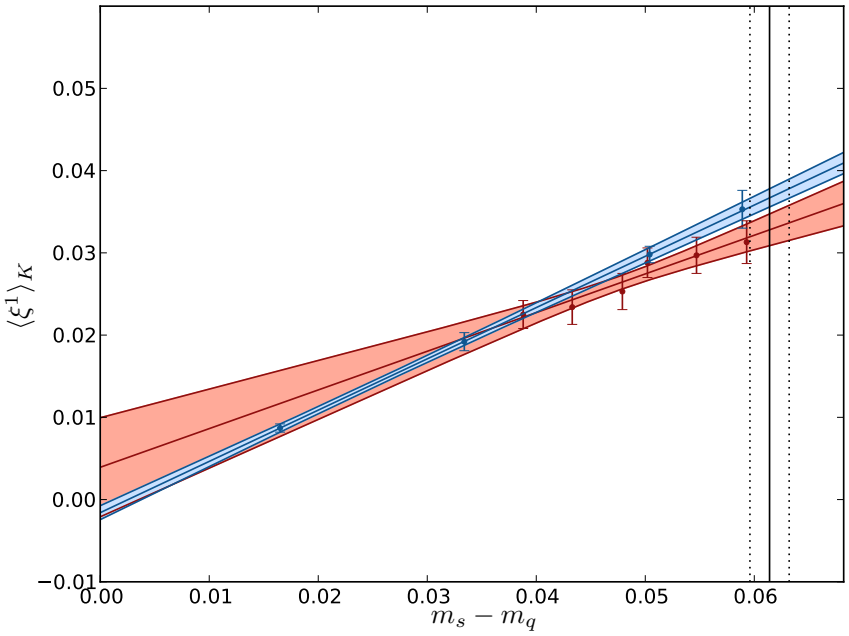
- Evaluate

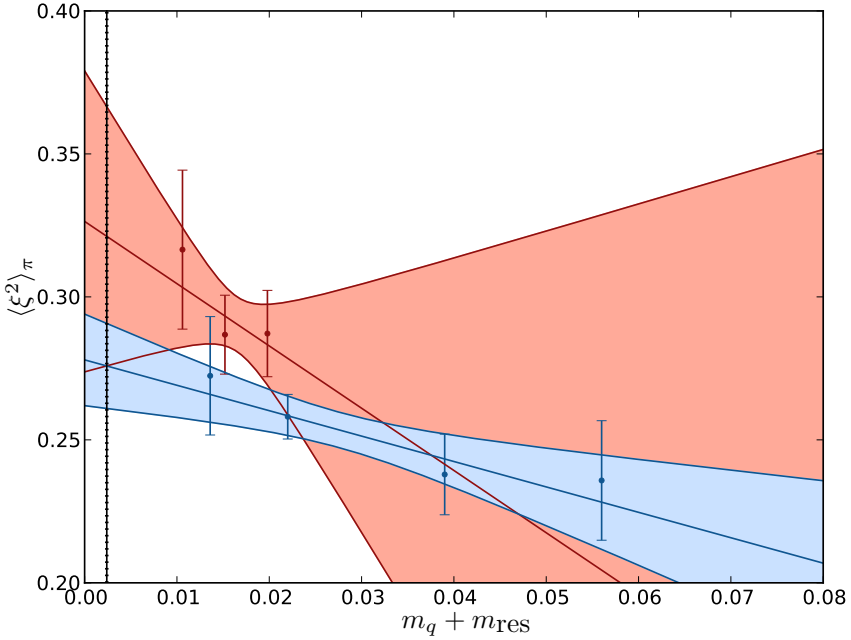
$$\langle\xi^1\rangle^{\overline{\text{MS}}} = \frac{Z_{D,D}}{Z_A}\langle\xi^1\rangle^{\text{bare}}$$

$$\langle\xi^2\rangle^{\overline{\text{MS}}} = \frac{Z_{DD,DD}}{Z_A}\langle\xi^2\rangle^{\text{bare}} + \frac{Z_{DD,\partial\partial}}{Z_A}$$

Previous RBC/UKQCD calculation

Chiral extrapolations: $a^{-1} = 1.73\,\text{GeV}$ blue, $2.28\,\text{GeV}$ red





Kaon 1st moment

Small lattice-spacing dependence, NPR

Pion 2nd moment

More a-dependence, but total-derivative operator perturbatively renormalised

prd83,074505 2011 and in preparation

$$\langle \xi^1 \rangle^{\overline{\mathrm{MS}}} = \frac{Z_{D,D}}{Z_A} \langle \xi^1 \rangle^{\mathrm{bare}}$$

$$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}} = \frac{Z_{DD,DD}}{Z_A} \langle \xi^2 \rangle^{\mathrm{bare}} + \frac{Z_{DD,\partial\partial}}{Z_A}$$

Previously used RI′/MOM scheme

- No momentum transfer at insertion ⇒ could not access mixing with total derivative operator for 2nd moment
- Where had both, PT and NPT renormalisation constants differed

$a^{-1} = 2.28\,\mathrm{GeV}$ lattice with $\overline{\mathrm{MS}}$ scale $\mu = 2\,\mathrm{GeV}$

	$Z_{D,D}/Z_A$	$Z_{DD,DD}/Z_A$	$Z_{DD,\partial\partial}/Z_A$
nonperturbative	1.50(2)	1.97(5)	—
mean-field imp PT	1.28(4)	1.51(6)	0.015(4)

Nonperturbative renormalisation (NPR)

- Use Rome–Southampton regularisation independent (RI) momentum-subtraction scheme (MOM)
- For operator O

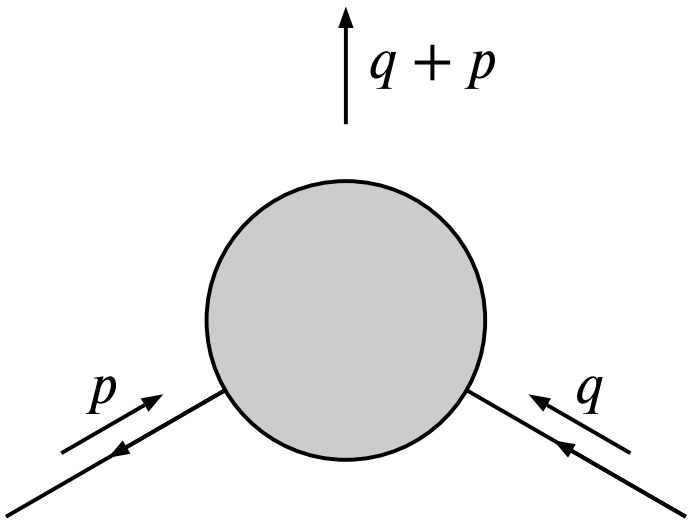
$$\Lambda_R^O = \frac{1}{Z_q} Z_O \Lambda_B^O$$

  - Λ^O is amputated quark two-point function with insertion of O (Lorentz indices implicit)
  - Z_O is operator renormalisation constant (can be matrix) and Z_q is quark wavefunction renormalisation
- Impose renormalisation condition(s) at a particular momentum configuration, with associated scale μ

$$\Lambda_{\mathrm{QCD}} \ll \mu \ll 1/a$$

RI MOM renormalisation

Evaluate amputated two-point quark Green functions at a specified Euclidean momentum configuration



- RI′/MOM *exceptional momentum*

$$q + p = 0, \qquad q^2 = p^2 = \mu^2$$

- SMOM *symmetric momentum*

$$q^2 = p^2 = (q+p)^2 = \mu^2, \qquad q \cdot p = -\mu^2/2$$

- Use SMOM scheme
  - nonexceptional momenta with momentum transfer at operator insertion
  - suppresses contamination from IR effects
  - allows mixing with total derivative operators (*needed* for our application)
  - better-behaved (or at least not worse) perturbative series in conversion to $\overline{\text{MS}}$
- Evaluate conversion functions from SMOM to $\overline{\text{MS}}$
- Relevant continuum calculations done to 3 loops in $\overline{\text{MS}}$ and 2 loops in SMOM by J Gracey [epjc 71,1567 2011; jhep03,109 2011; prd84,016002 2011]

Operators

First moment

$$X_2 = \mathcal{S}\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi$$
$$\partial X_2 = \mathcal{S}\partial_\mu(\bar{\psi}\gamma_\nu\psi)$$

Second moment

$$X_3 = \mathcal{S}\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\sigma \psi$$
$$\partial\partial X_3 = \mathcal{S}\partial_\mu\partial_\nu(\bar{\psi}\gamma_\sigma\psi)$$
$$\partial X_3 = \mathcal{S}\partial_\mu(\bar{\psi}\gamma_\nu \overleftrightarrow{D}_\sigma \psi)$$

- Work with C-eigenstate operators
  - X_2, ∂X_2, ∂X_3 all multiplicatively renormalised
  - X_3, $\partial\partial X_3$ mix
- Single total-derivative operators ($\partial X_{2,3}$) included for some checks
  - ∂X_2 and $\partial\partial X_3$ are total derivatives of vector current
  - ∂X_3 is total derivative of X_2

Scalar coefficients

Expand Green functions as Lorentz tensors with scalar coefficients. Eg for first moment

$$\Lambda^{\mu\nu}(p,q)_{\text{sym}} = \sum_{i=1}^{10} P^{\mu\nu}_{(i)}(p,q)\Sigma_i(\mu^2)$$

$$\Sigma_i(\mu^2) = \frac{1}{\mu^2}\,\text{Tr}\left[M_{ij}P^{\mu\nu}_{(j)}(p,q)\Lambda_{\mu\nu}(p,q)_{\text{sym}}\right]$$

where $P^{\mu\nu}_{(i)}(p,q)$ are 10 Lorentz tensor structures with

$$N_{ij} = \frac{1}{\mu^2}\,\text{Tr}\left[P^{\mu\nu}_{(i)}P_{(j)\mu\nu}\right]_{\text{sym}} \qquad M = N^{-1}$$

Similar decompositions for bilinear and second moment operators

Checking charge-conservation properties

- Gracey used a different basis of operators
- Changing to C-conserving basis gives relations between Gracey's 3-loop $\overline{\text{MS}}$ anomalous dimensions which are all satisfied (and determine one of them to one higher power in g^2)
- For amputated Green functions, charge-conjugation implies relations between scalar coefficients in our basis
  - Satisfied by the Gracey continuum calculations after the change of basis
  - Exactly satisfied by lattice data for a unit gauge field
  - Well-satisfied in our lattice data

SMOM renormalisation conditions

After tracing with some ‘projector’ P, the renormalised amputated Green function should give the tree-level result

$$\frac{1}{Z_q}\,\mathrm{Tr}(Z_O \Lambda^O_{B,\mathrm{sym}} P) = \mathrm{Tr}(\Lambda^O_{\mathrm{tree,sym}} P)$$

- Choose P’s with charge-conjugation properties in mind
- Choose P’s for operators which are total derivatives of vector current to maintain Ward identity

Show examples. . .

Vector current and derivatives

- SMOM renormalisation condition for vector current

$$\frac{1}{12\mu^2}\frac{Z_V}{Z_q}\mathrm{Tr}(k_\mu \Lambda^\mu_{V,B}\not{k}) = 1 \qquad \text{where} \qquad k = q + p$$

  maintains WID $k_\mu \Lambda^\mu_{V,R} = S_R^{-1}(-p) - S_R^{-1}(q)$
- Choose renormalisation conditions for total derivatives of the vector current

$$\frac{Z_{\partial X_2}}{Z_q}\mathrm{Tr}\left[\left(\mathcal{S}k_\mu k_\nu\right)\not{k}\Lambda^{\mu\nu}_{\partial X_2,B}\right] = 9i(\mu^2)^2$$

$$\frac{Z_{\partial\partial X_3}}{Z_q}\mathrm{Tr}\left[\left(\mathcal{S}k_\mu k_\nu k_\rho\right)\not{k}\Lambda^{\mu\nu\rho}_{\partial\partial X_3,B}\right] = -6(\mu^2)^3$$

- Conversion functions, SMOM to $\overline{\mathrm{MS}}$, for all three are then 1

2nd moment operator

$X_3 = \mathcal{S}\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\sigma \psi$ mixes with $\partial\partial X_3 = \mathcal{S}\partial_\mu\partial_\nu(\bar{\psi}\gamma_\sigma\psi)$ and

$$\Lambda^{\overleftrightarrow{D}\overleftrightarrow{D}}_{\mu\nu\sigma}(p,q)_{\text{tree}} = -\mathcal{S}(q_\mu - p_\mu)(q_\nu - p_\nu)\gamma_\sigma$$
$$= \frac{\mu^2}{3}(P_{3,\mu\nu\sigma} + P_{1,\mu\nu\sigma} - P_{2,\mu\nu\sigma})$$

Impose renormalisation condition

$$\frac{1}{Z_q}\,\mathrm{Tr}\Big[\big((MP)_3 + (MP)_1 - (MP)_2\big)(Z_{DD,DD}\Lambda_B^{\overleftrightarrow{D}\overleftrightarrow{D}} + Z_{DD,\partial\partial}\Lambda_B^{\partial\partial})\Big]$$
$$= \mathrm{Tr}\big[\big((MP)_3 + (MP)_1 - (MP)_2\big)\Lambda^{\overleftrightarrow{D}\overleftrightarrow{D}}_{\text{tree}}\big] = \mu^2$$

2nd moment operator

Need another condition

$$\frac{1}{Z_q}\mathrm{Tr}\Big[\big((MP)_3+(MP)_1+(MP)_2\big)\big(Z_{DD,DD}\Lambda_B^{\overleftrightarrow{D}\overleftrightarrow{D}}+Z_{DD,\partial\partial}\Lambda_B^{\partial\partial}\big)\Big]$$

$$=\mathrm{Tr}\big[\big((MP)_3+(MP)_1+(MP)_2\big)\Lambda_{\mathrm{tree}}^{\overleftrightarrow{D}\overleftrightarrow{D}}\big]=\frac{\mu^2}{3}$$

These fix $Z_{DD,DD}$ and $Z_{DD,\partial\partial}$ to get from bare lattice to SMOM

Conversion functions: SMOM to $\overline{\text{MS}}$

$$O_{\overline{\text{MS}}} = CO_R \qquad \Rightarrow \qquad \Lambda_{\overline{\text{MS}}} = \frac{1}{C_q} C\Lambda_R \quad \text{where} \quad C_q \equiv \frac{Z_{q,\overline{\text{MS}}}}{Z_q}$$

Expand Green function Λ_a for operator O_a in terms of tensors P_i with scalar coefficients Σ_{ai}

$$\Lambda_a = \sum_i \Sigma_{ai} P_i \qquad \Sigma_{ai} = \text{Tr}\left[(MP)_i \Lambda_a\right]$$

Renormalisation: tracing Λ_{Ra} with some 'projector' P_A gives tree-level (or other chosen) result, T_A

$$\text{Tr}\left(\Lambda_{Ra} P_A\right) = T_{aA}$$

May need several P_A if operators mix

Conversion functions

Let $N^P_{iA} \equiv \mathrm{Tr}(P_i P_A)$ and use $\Lambda_R = C_q C^{-1} \Lambda_{\overline{\mathrm{MS}}}$

$$C_q C^{-1}_{ab} \Sigma^{\overline{\mathrm{MS}}}_{bi} N^P_{iA} = T_{aA}$$

- $\Sigma^{\overline{\mathrm{MS}}}$, N^P and T (and C_q) known; impose enough conditions to solve for elements of C
- Combine C with Z from SMOM renormalisation conditions to get from lattice to $\overline{\mathrm{MS}}$ at scale μ
- Use $\overline{\mathrm{MS}}$ anomalous dimensions to scale to a common value, say 2 GeV
- Can also compute the SMOM anomalous dimensions

$$\gamma_{\mathrm{SMOM}} = C^{-1} \gamma_{\overline{\mathrm{MS}}} C - \mu \frac{dC^{-1}}{d\mu} C$$

First moment

$$
\begin{aligned}
C_{11} =& 1 - (1.63903\alpha + 5.12484)a \\
& - (3.8244\alpha^2 + 6.37866\alpha - 12.1458N_f + 106.359)a^2 \\
C_{22} =& 1
\end{aligned}
$$

Second moment

$$
\begin{aligned}
C_{11} =& 1 - (2.18537\alpha + 8.24516)a \\
& - (5.18357\alpha^2 + 2.38666\alpha - 19.8008N_f + 156.444)a^2 \\
C_{12} =& (0.138749\alpha + 1.15755)a \\
& + (0.419338\alpha^2 + 1.95065\alpha - 2.31945N_f + 20.0837)a^2 \\
C_{22} =& 1 \\
C_{33} =& 1 - (1.63903\alpha + 5.12484)a \\
& - (3.8244\alpha^2 + 6.37866\alpha - 12.1458N_f + 106.359)a^2
\end{aligned}
$$

Outlook

- Aiming for 1st and 2nd moments of PDAs with fully nonperturbative renormalisation
- Continuum calculations exist (Gracey) to allow the needed conversion from SMOM to $\overline{\text{MS}}$ once renormalistion conditions imposed
- Will need care with renormalisation conditions for lattice Green functions when considering decomposition into hypercubic representations