A study of the anomalous magnetic moment of the muon computed from the Adler function

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Outline

Adler function

The Adler function is defined as[1]

$$\frac{D(q^2)}{q^2} = \frac{3\pi}{\alpha}\frac{d}{dq^2}\Delta\alpha_{had}(q^2),$$

and it can be measured in $e^+e^- \to hadrons$. The Adler function is related to the vacuum polarization by

$$D(q^2) = 12\pi^2 q^2 \frac{d\Pi(q^2)}{d(q^2)},$$

and if we use $D(q^2)$ to determine $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$ we avoid the extrapolation to $\Pi(q^2 = 0)$.
Steps of the analysis:

1. determine $\Pi(q^2)$ on the lattice
2. take the numerical derivative of $\Pi(q^2) \to D(q^2)$
3. in order to describe the q^2-dependence apply a fit to $D(q^2)$
4. determine a_μ^{HLO} from this fit

[1] Adler, Phys. Rev. **D** 10, 3714, 1974

CLS-Ensembles

In our study we use $O(a)-$improved Wilson-fermions with $N_f = 2$ with partially twisted boundary conditions. The strange and charm quarks are partially quenched.

Label	V	β	a[fm]*	m_π[MeV]	$m_\pi L$	N_{cnfg}	N_{meas}
A3	64×32^3	5.20	0.079	473	6.0	251	1004
A4	64×32^3	5.20	0.079	363	4.7	400	1600
A5	64×32^3	5.20	0.079	312	4.0	251	1004
B6	96×48^3	5.20	0.079	267	5.1	306	1224
E5	64×32^3	5.30	0.063	456	4.7	1000	4000
F6	96×48^3	5.30	0.063	325	5.0	300	1200
F7	96×48^3	5.30	0.063	277	4.2	250	1000
G8	128×64^3	5.30	0.063	193	4.0	205	820
N5	96×48^3	5.50	0.050	430	5.2	347	1392
N6	96×48^3	5.50	0.050	340	4.1	559	2236
O7	128×64^3	5.50	0.050	261	4.4	138	552

* [arXiv:1110.6365]

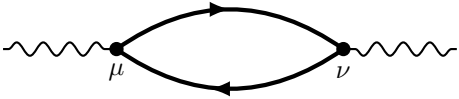
Step 1: Determination of $\Pi(q^2)$

The vacuum polarization tensor can be computed by

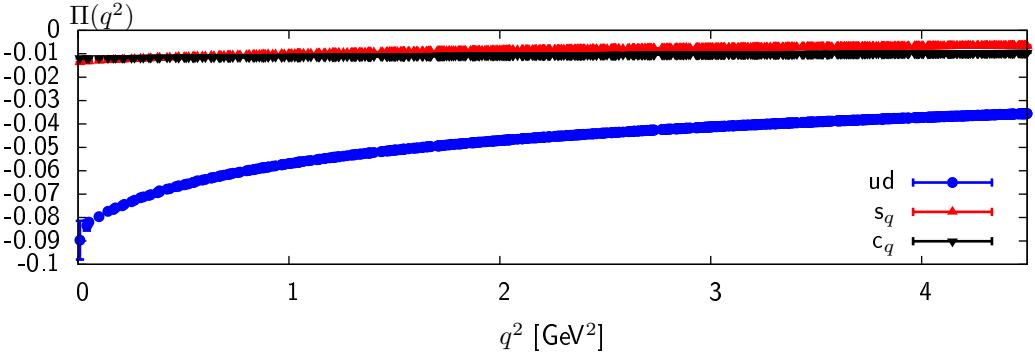
$$\Pi_{\mu\nu}(q^2) = \int d^4x e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle \,.$$

From Euclidean invariance and current conservation one finds

$$\Pi_{\mu\nu}(q^2) = \left(g_{\mu\nu} q^2 - q_\mu q_\nu\right) \Pi(q^2).$$



$\Pi(q^2)$ on F7, $m_\pi = 277$MeV, $a = 0.063$fm



Step 2: Methods to compute the Adler function

Fit to $\Pi(q^2)$

Fit an ansatz to $\Pi(q^2)$, and compute the derivative of the fit function. We use the Padé ansatz

$$\Pi_{fit}(q^2) = \Pi(0) - q^2 \left(\frac{a_1}{q^2 + b_1} + \frac{a_2}{q^2 + b_2} \right),$$

$$\frac{d}{dq^2}\Pi_{fit}(q^2) = -\frac{a_1 b_1}{(b_1 + q^2)^2} - \frac{a_2 b_2}{(b_2 + q^2)^2}.$$

[arXiv:1112.2894, arXiv:1205.3695]

Numerical derivative of $\Pi(q^2)$

We use linear fits with varying ranges to approximate the derivative of $\Pi(q^2)$.

Step 2: Procedures for the numerical derivative 1

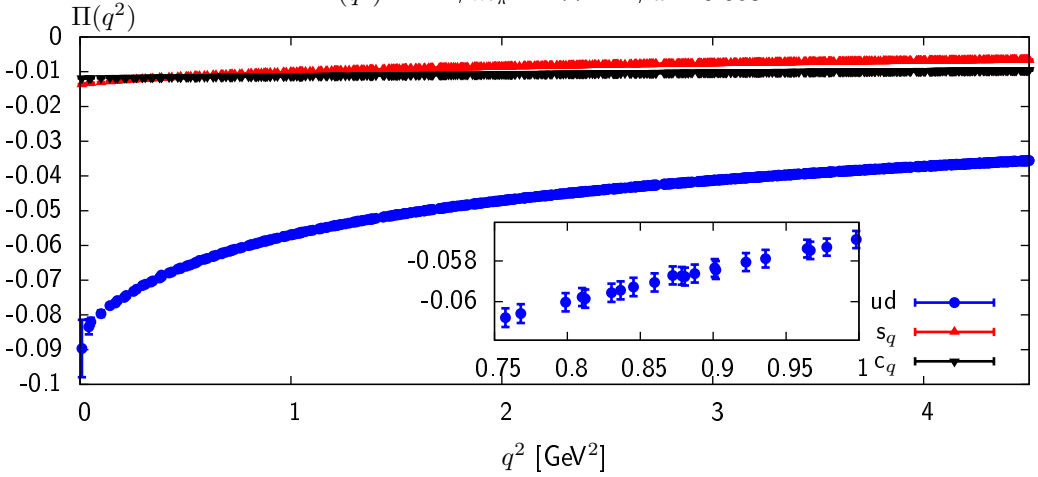
Procedure I

- at each q^2 perform a linear fit

$$\Pi^{[l]}_{fit}(q^2) = a_l + b_l q^2,$$

- repeat these fits for several fit ranges $\epsilon \in [0.1, 1.0]\text{GeV}^2$,
- search for a region in ϵ where variations in b_l are small.

$\Pi(q^2)$ on F7, $m_\pi = 277\text{MeV}$, $a = 0.063\text{fm}$



Step 2: Procedures for the numerical derivative 2

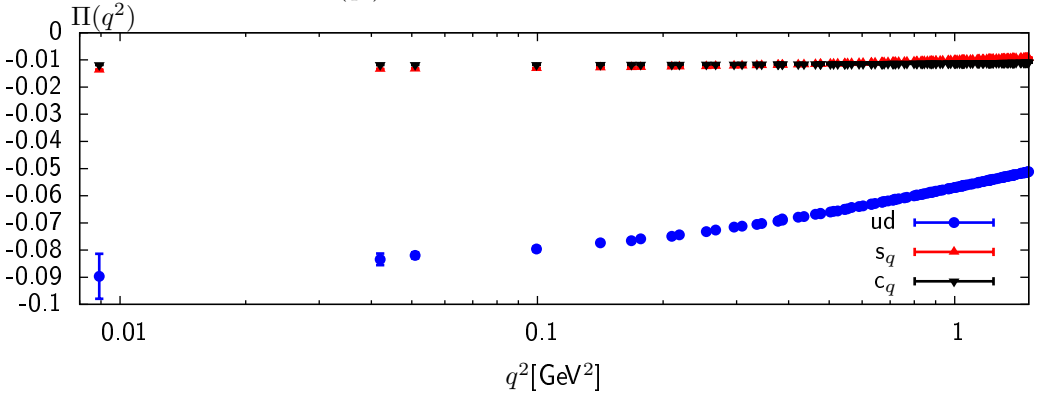
Procedure II

- at each q^2 we fit the two functions

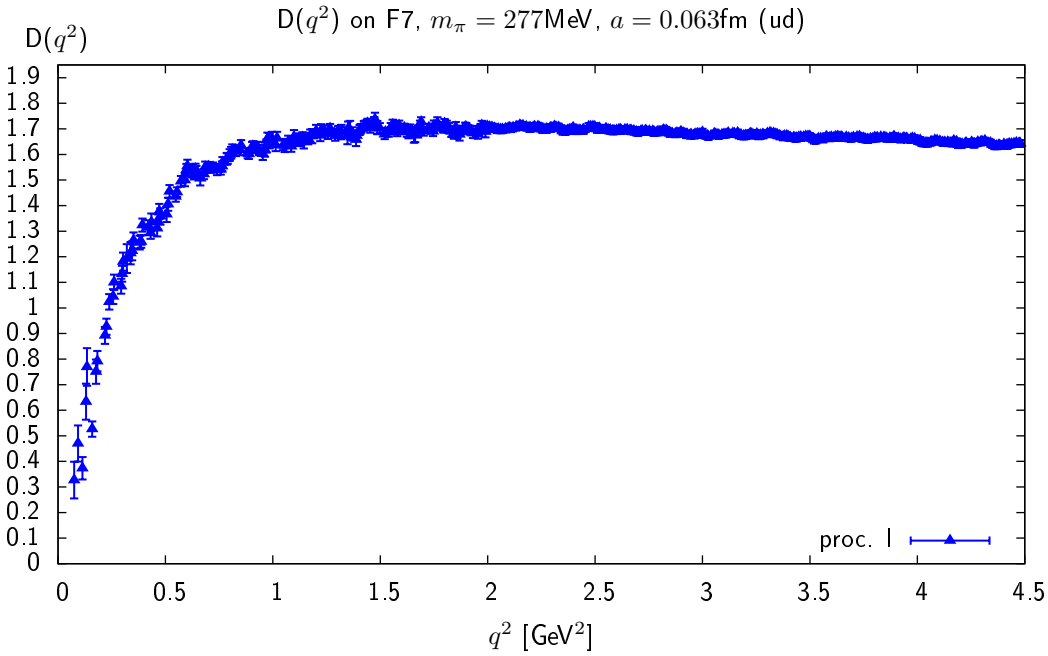
$$\Pi^{[l]}_{fit}(q^2) = a_l + b_l \ln\left(q^2\right),$$
$$\Pi^{[q]}_{fit}(q^2) = a_q + b_q \ln\left(q^2\right) + c_q \left(\ln\left(q^2\right)\right)^2,$$

- repeat these fits for several fit ranges $\epsilon \in [0.1, 1.0]\text{GeV}^2$,
- apply cuts to the fits, such as removing fits with a large curvature c_q,
- from the fits that survive pick the result, where the coefficients b_l and b_q are similar.

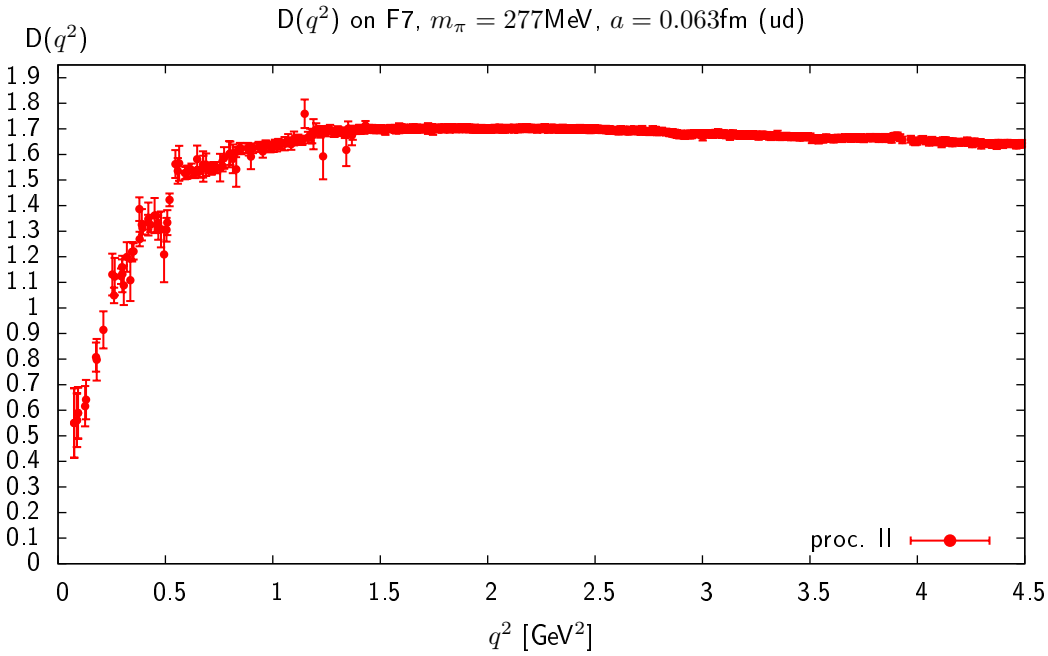
$\Pi(q^2)$ on F7, $m_\pi = 277\text{MeV}$, $a = 0.063\text{fm}$



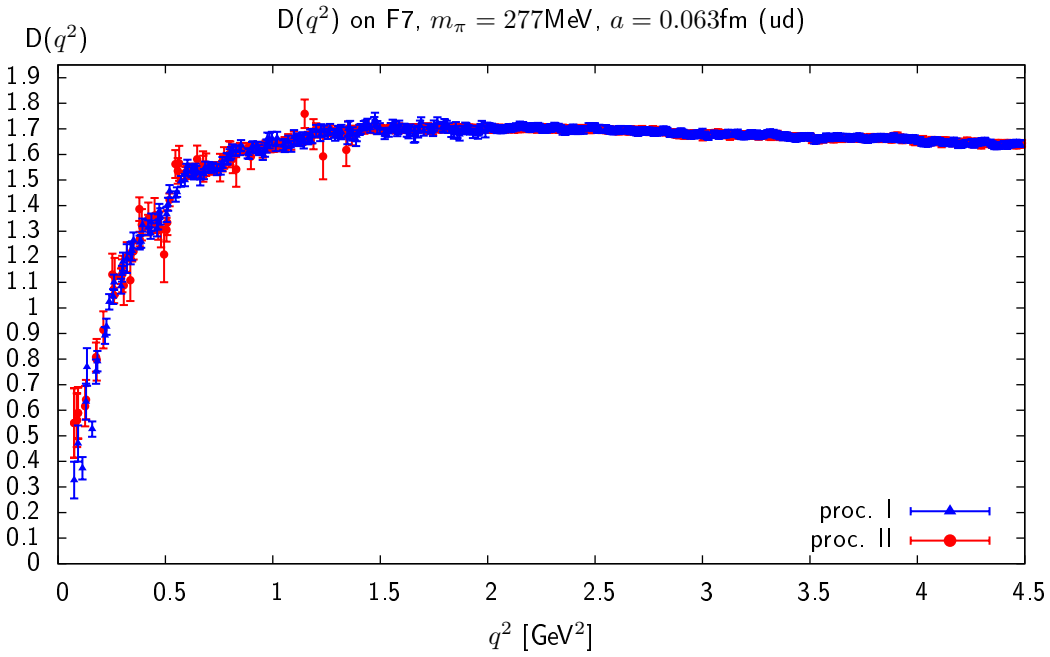
Results for the Adler function on F7: Procedure I



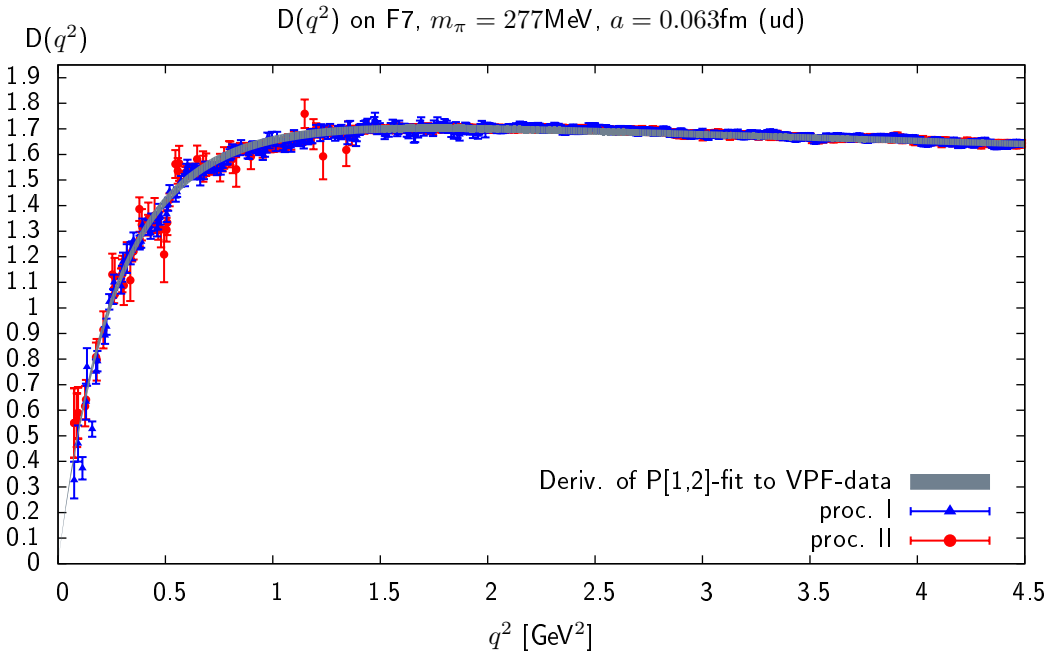
Results for the Adler function on F7: Procedure II



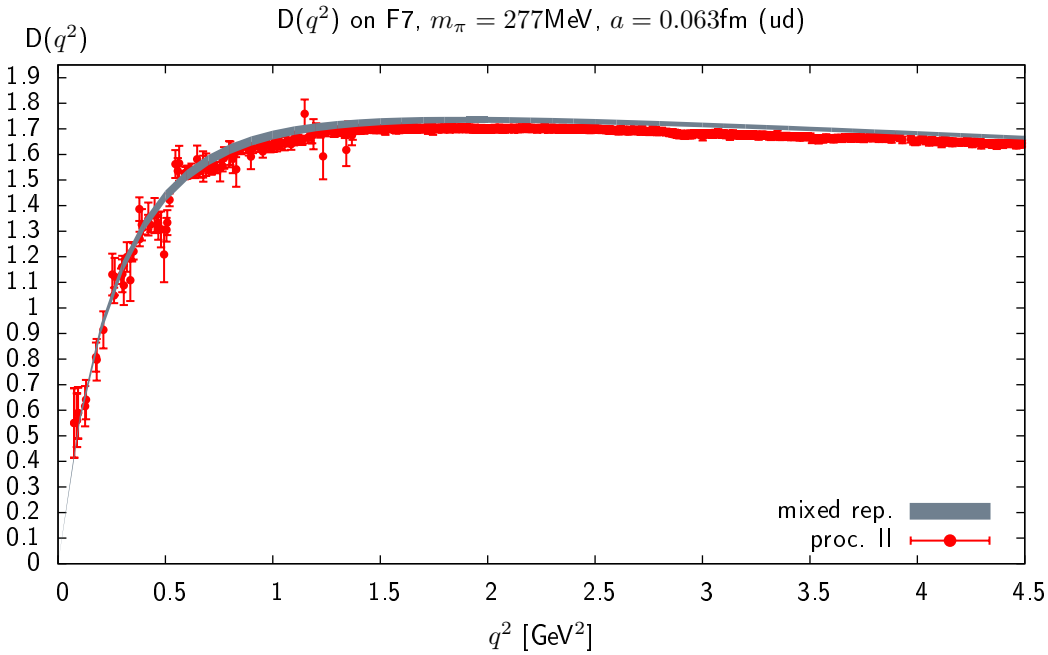
Comparison of the different methods on F7



Comparison of the different methods on F7



Comparison with the mixed representation method



For the mixed representation method see [arXiv:1306.2532], and the talk by Anthony Francis.

Step 3: Combined fits

To determine the Adler function at the physical point and in the continuum we apply combined fits of the type:

$$D(q^2) = A(q^2)(1 + B(a,q) + C(m_\pi, q^2))$$

- The momentum dependence is described by the derivative of Padé-approximants

$$A_{[12]}(q^2) = q^2 \left(\frac{a_1 b_1}{(b_1 + q^2)^2} + \frac{a_2 b_2}{(b_2 + q^2)^2} \right),$$

$$A_{[22]}(q^2) = q^2 \left(\frac{a_1 b_1}{(b_1 + q^2)^2} + \frac{a_2 b_2}{(b_2 + q^2)^2} + a_0 \right).$$

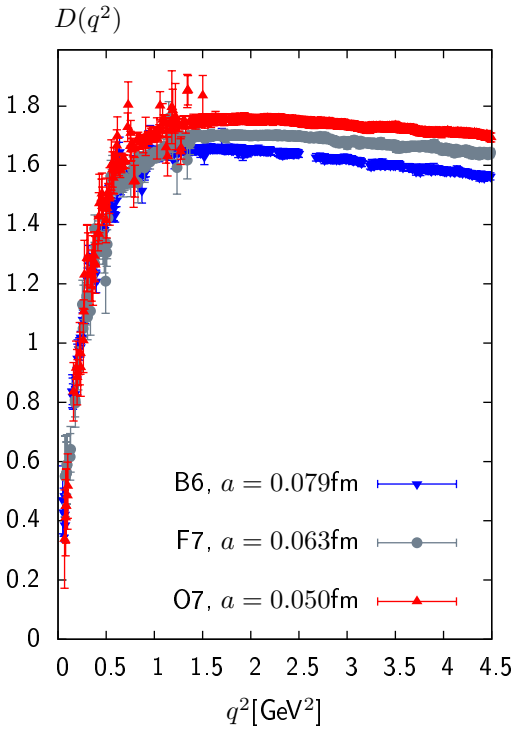
- The lattice spacing dependence is given by

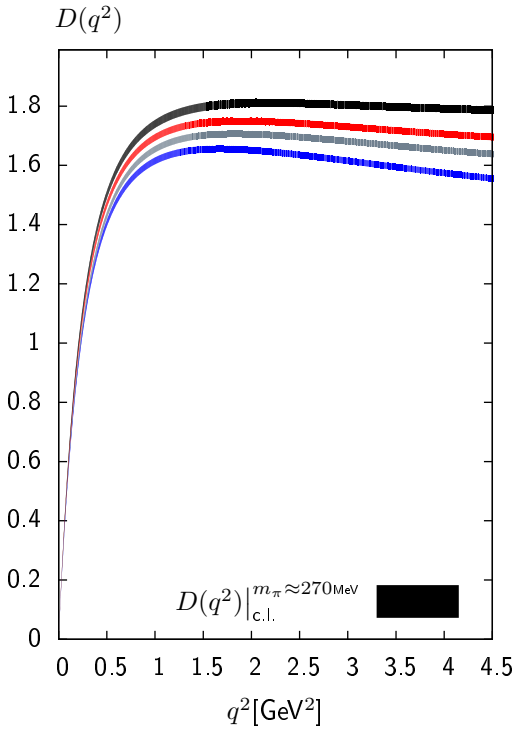
$$B_1(a,q) = c_1(aq) + c_2(4\pi f_\mathrm{K} a), \quad B_2(a,q) = c_1\,(aq)^2 + c_2\,(4\pi f_\mathrm{K} a)^2\,.$$

- The light quark mass dependence is

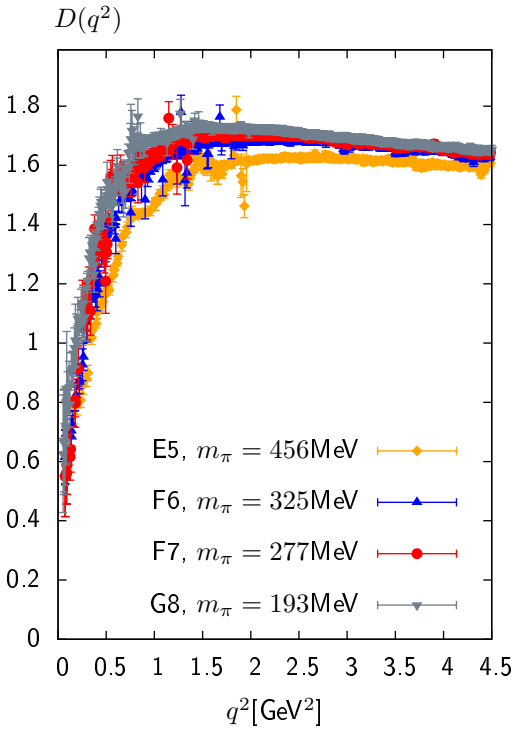
$$C(m_\pi, q^2) = d_1 \frac{m_\pi^2 - (m_\pi^{exp})^2}{d_2 + q^2}$$

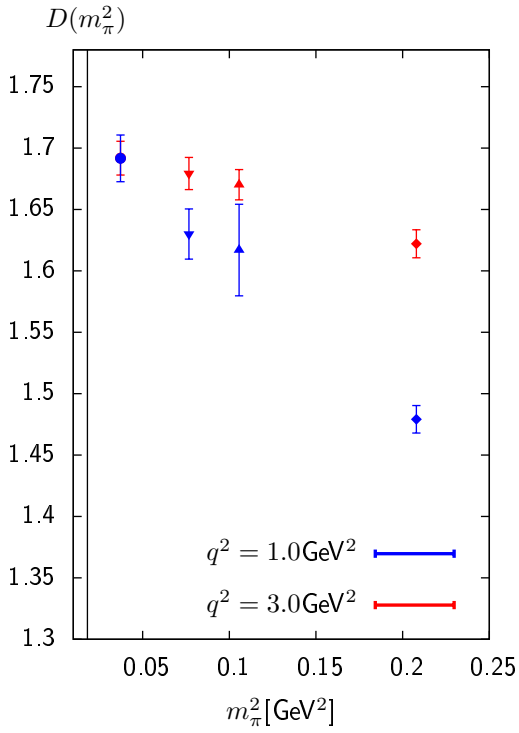
Results for $m_\pi \approx 270$MeV for P[2,2], $O(a^2)$, $N_f = 2$



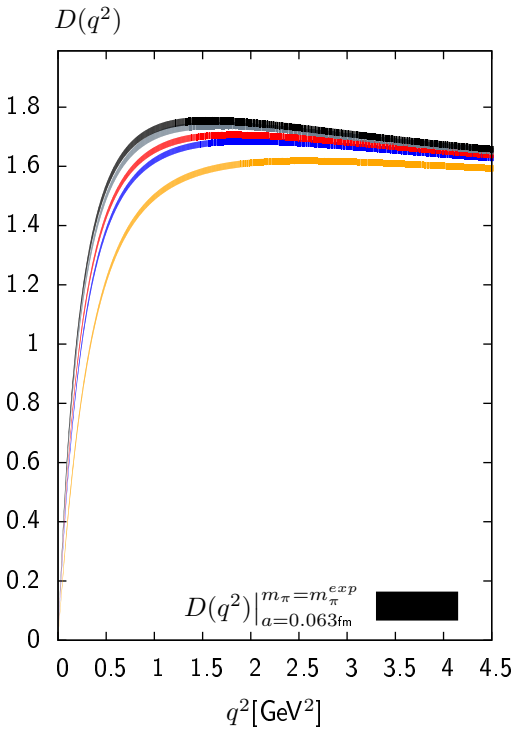


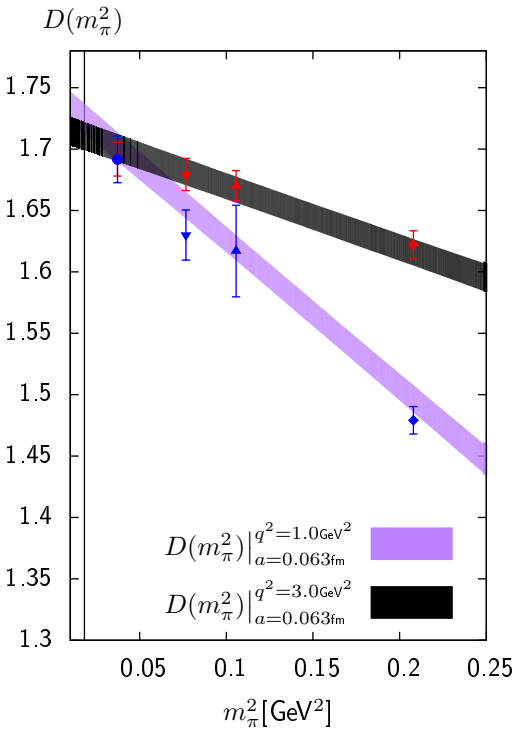
Results for $\beta = 5.30$, $a = 0.063$fm, $N_f = 2$





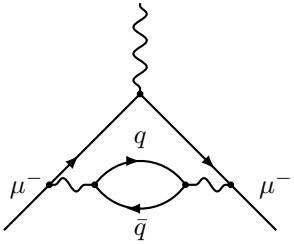
Results for $\beta = 5.30$, $a = 0.063$fm for P[2,2], $O(a^2)$, $N_f = 2$





Step 4: Determination of a_μ^{HLO}

We can use the results from the combined fits to determine a_μ^{HLO}:



$$a_\mu^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 f(q^2) 4\pi^2 \hat{\Pi}(q^2),$$

$$f(q^2) = \frac{m_\mu^2 q^2 Z^3 (1 - q^2 Z)}{1 + q m_\mu^2 q^2 Z^2},$$

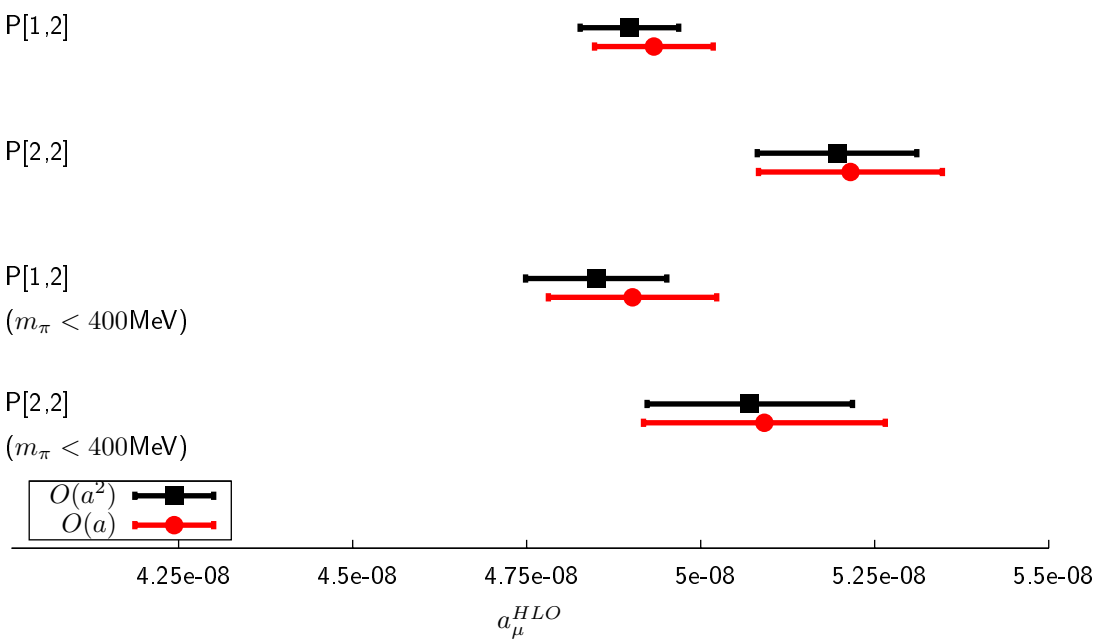
$$Z = -\frac{q^2 - \sqrt{q^4 + 4 m_\mu^2 q^2}}{2 m_\mu^2 q^2},$$

where we insert the coefficients of the combined chiral and continuum fits of the Adler function to determine the q^2-behaviour of $\hat{\Pi}(q^2)$:

$$\hat{\Pi}(q^2) \rightarrow \hat{\Pi}_{12}(q^2) = -q^2 \left(\frac{a_1}{b_1 + q^2} + \frac{a_2}{b_2 + q^2}\right) \text{ or}$$

$$\rightarrow \hat{\Pi}_{22}(q^2) = -q^2 \left(\frac{a_1}{b_1 + q^2} + \frac{a_2}{b_2 + q^2} + a_0\right).$$

Preliminary results for a_μ^{HLO} from combined fits, $N_f = 2$



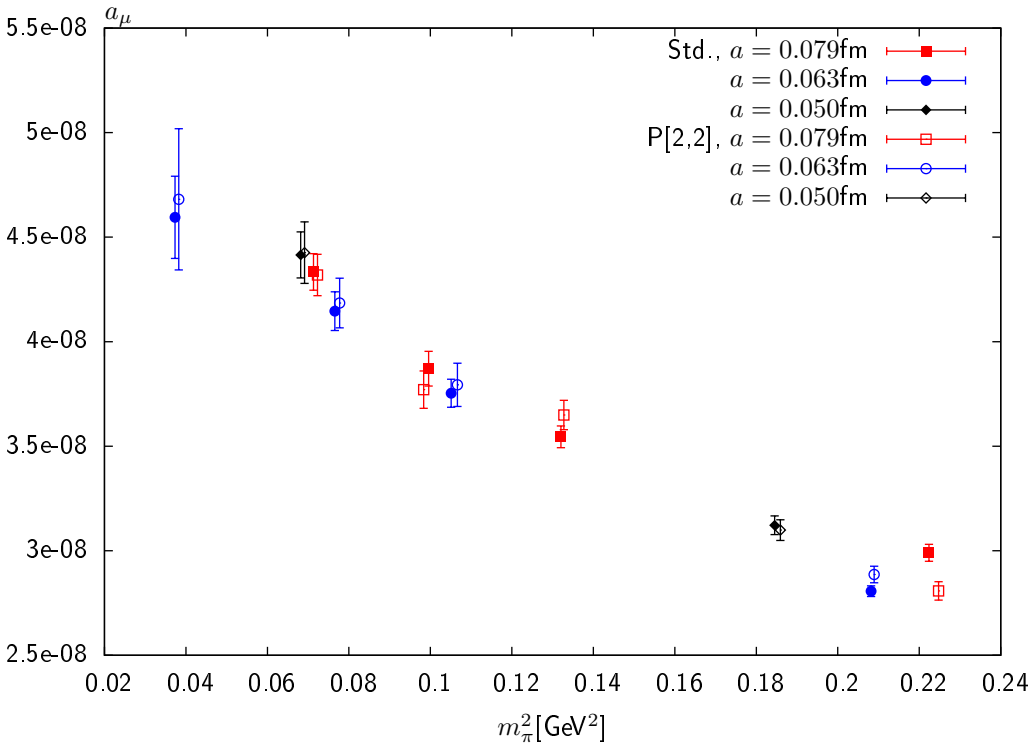
Summary and Outlook

- From the vacuum polarization we can obtain the Adler function with different methods, which agree within errors for a large range of q^2, and are similar to the mixed representation method, cf. talk by Anthony Francis on Monday (1D).
- We presented a method to extract the Adler function in the continuum and at the physical point using a combined fit for the light quark sector.
- From the extrapolated Adler function we can extract the hadronic contribution to the anomalous magnetic moment of the muon.
- We plan to extend the analysis to the already available strange and charm data.
- E. Shintani is currently investigating AMA for the vacuum polarization.
- In the future we will also investigate methods which make use of the moments of $\Pi(q^2)$ to compute a_μ^{HLO} [arXiv:1403.1778, arXiv:1406.4671].

Thank you for your attention.

Backup

Preliminary results for a_μ^{HLO} for P[2,2], $N_f = 2$



Low q^2−region of $\Pi(q^2)$