Targeting the Conformal Window: Scalars on the Lattice

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LATTICE2014

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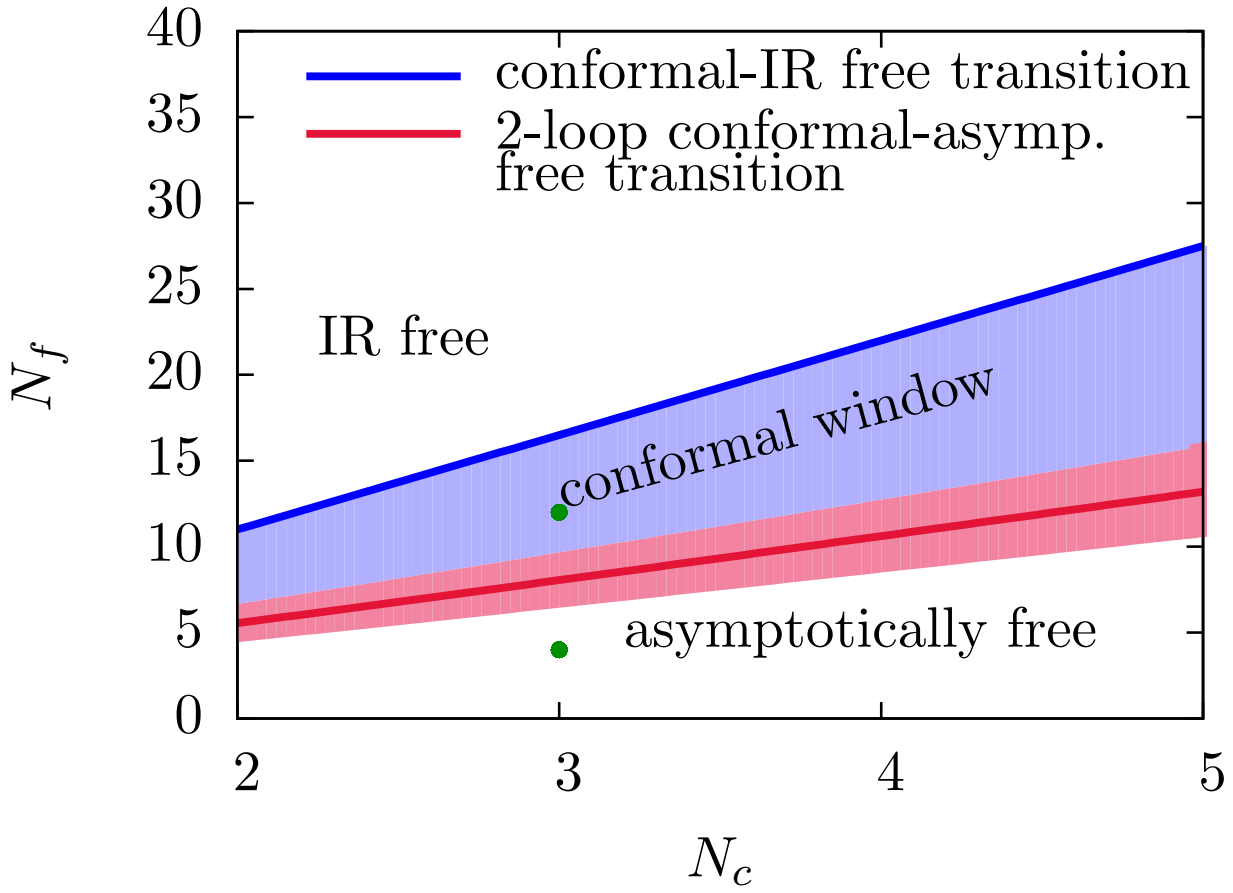
June 23, 2014

Higgs: A Light, Composite Scalar?

- LHC results show the Higgs is a relatively light scalar.
- A composite scalar is a natural mechanism, e.g., in superconductivity.
- Its mass is a function of dynamics: avoids fine-tuning.
- Can composite strong dynamics produce a light scalar relative to the rest of the spectrum?
  - Partially conserved dilatation current (PCDC)?
  - Consequence of near-conformal dynamics?
- Recent lattice studies have indicated a light scalar in possibly near-conformal theories.
  - $SU(3)$ 8 flavor fundamental (Y. Aoki et al., arXiv:1403.5000)
  - $SU(3)$ 2 flavor sextet (Fodor et al. PoS(Lattice 2013)062, arXiv:1401.2176)

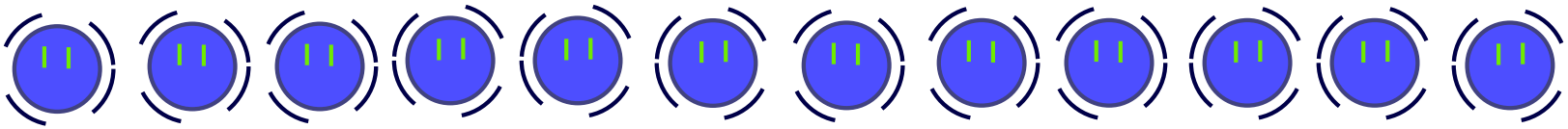
Targeting the Conformal Window: $SU(3)$ fundamental

- It is difficult to target the conformal window.
- $N_f = 4$ has confining, chirally broken behavior. No IRFP.
- $N_f = 12$ has growing evidence for conformal behavior at an IRFP.



Fermion mass parameterization

- What if there was another knob to turn?
- Start with 12 massless fermions.



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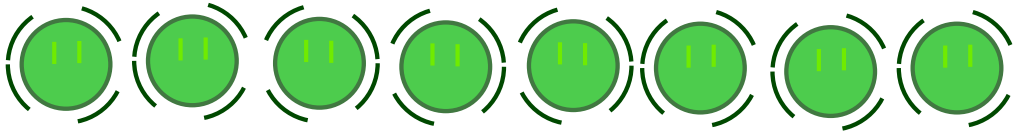
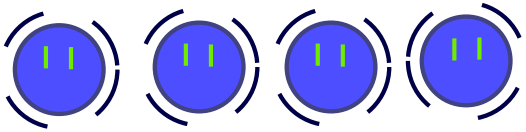
- What if there was another knob to turn?
- Start with 12 massless fermions.
- Give mass to 8 of them.
  - 4 remain massless fermions: $m_\ell = 0$
  - 8 tuneable massive fermions: $m_h \neq 0$





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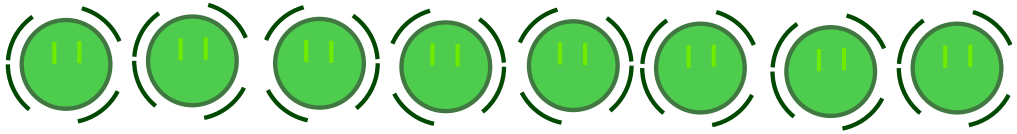
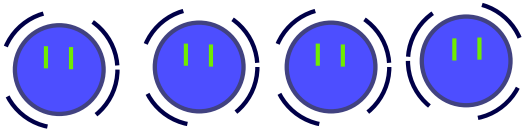
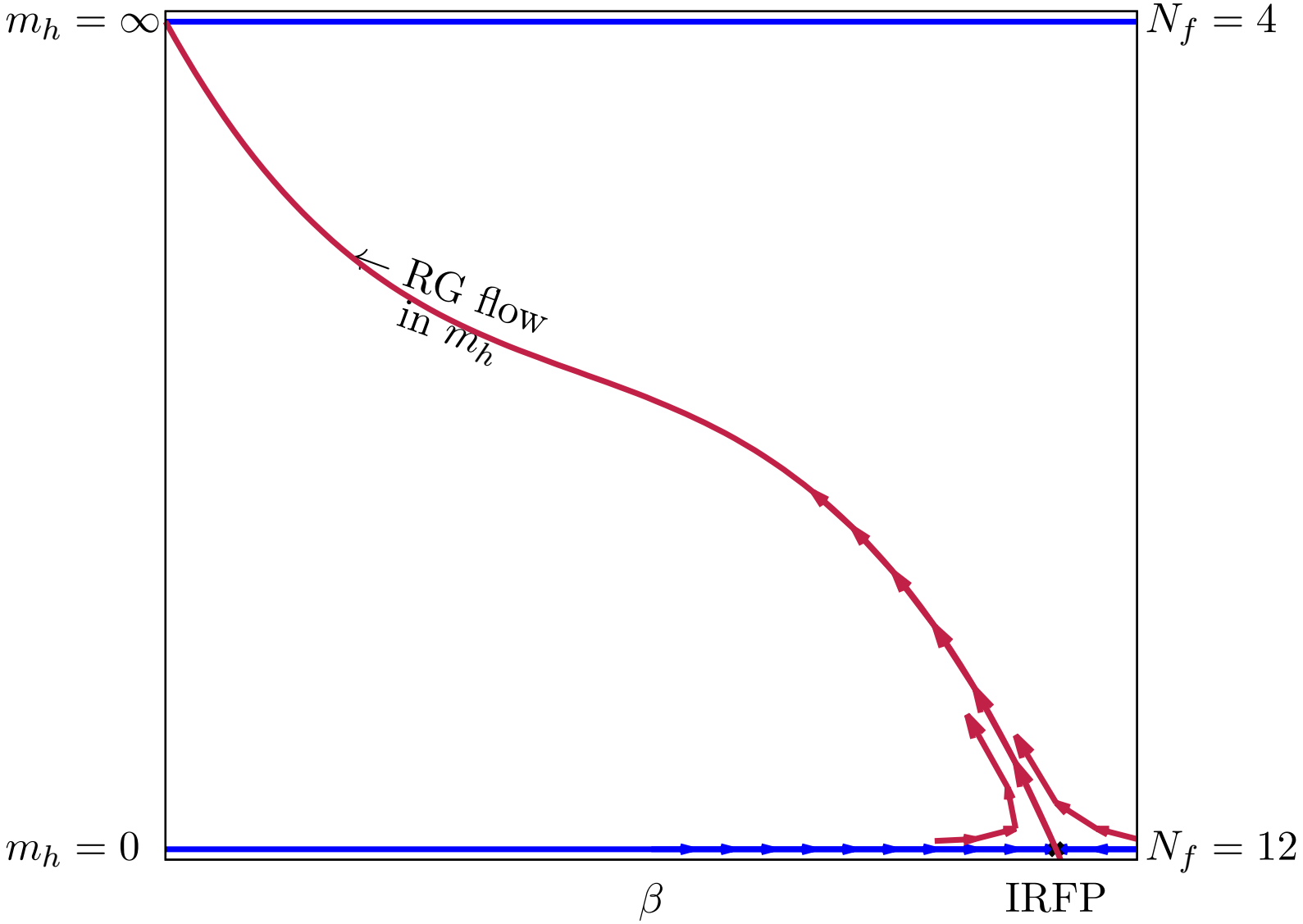


- As $m_h \to 0$, we recover the 12 flavor chirally symmetric theory exactly at $m_h = 0$.
- As $m_h \to \infty$, we get the 4 flavor chirally broken theory due to decoupling.



Flow Diagram

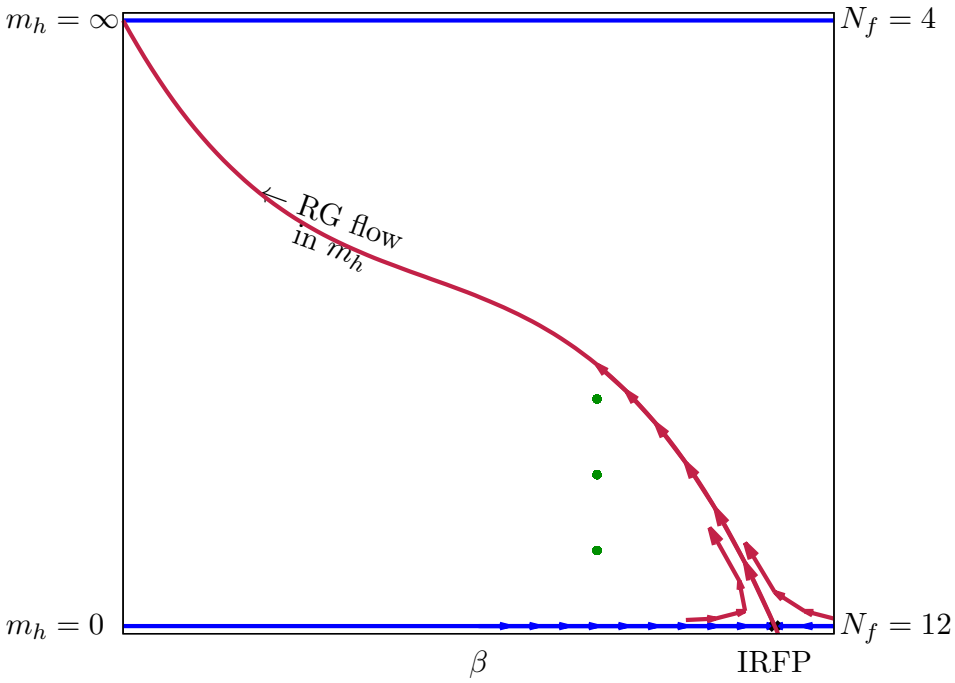


Studying the 4+8 Model

- We can tune ourselves to walking behavior by varying m_h.
- This is a 4+8 flavor model to study walking dynamics.
  - Gauge Action: Mixed Fundamental-Adjoint Action.
  - Fermion Action: nHYP Smeared Staggered
  - FUEL for HMC and Measurements
    - Developed by James Osborn, Algorithms and Machines: Parallel 1F at 3:35 earlier today.
- Explorations of the 4+8 flavor theory:
  1. Running Coupling
  2. Scale-dependent Fermion Anomalous Dimension
  3. **Spectrum**
  4. Finite T Phase Diagram
- Preliminary results on:
  - (1) and (2) are being presented by Oliver Witzel during Tuesday's poster session.

Setting m_h

- Our choice of m_h should set how long our theory walks.
- Need to choose m_h large enough to see chirally broken behavior.
- Plan:
  - Fix $m_\ell \approx 0$
  - Use $\langle \bar{\psi}\psi \rangle_\ell$ as an order parameter for chirally broken phase.
  - Start $m_h = m_\ell$: chirally symmetric.



- Raise m_h until onset of chiral symmetry breaking.

Physics Parameter: $\langle\bar{\psi}\psi\rangle_\ell$

- Chirally symmetric phase: $\langle\bar{\psi}\psi\rangle_\ell \to 0$ as $m_\ell \to 0$.
  - Parametrically small.
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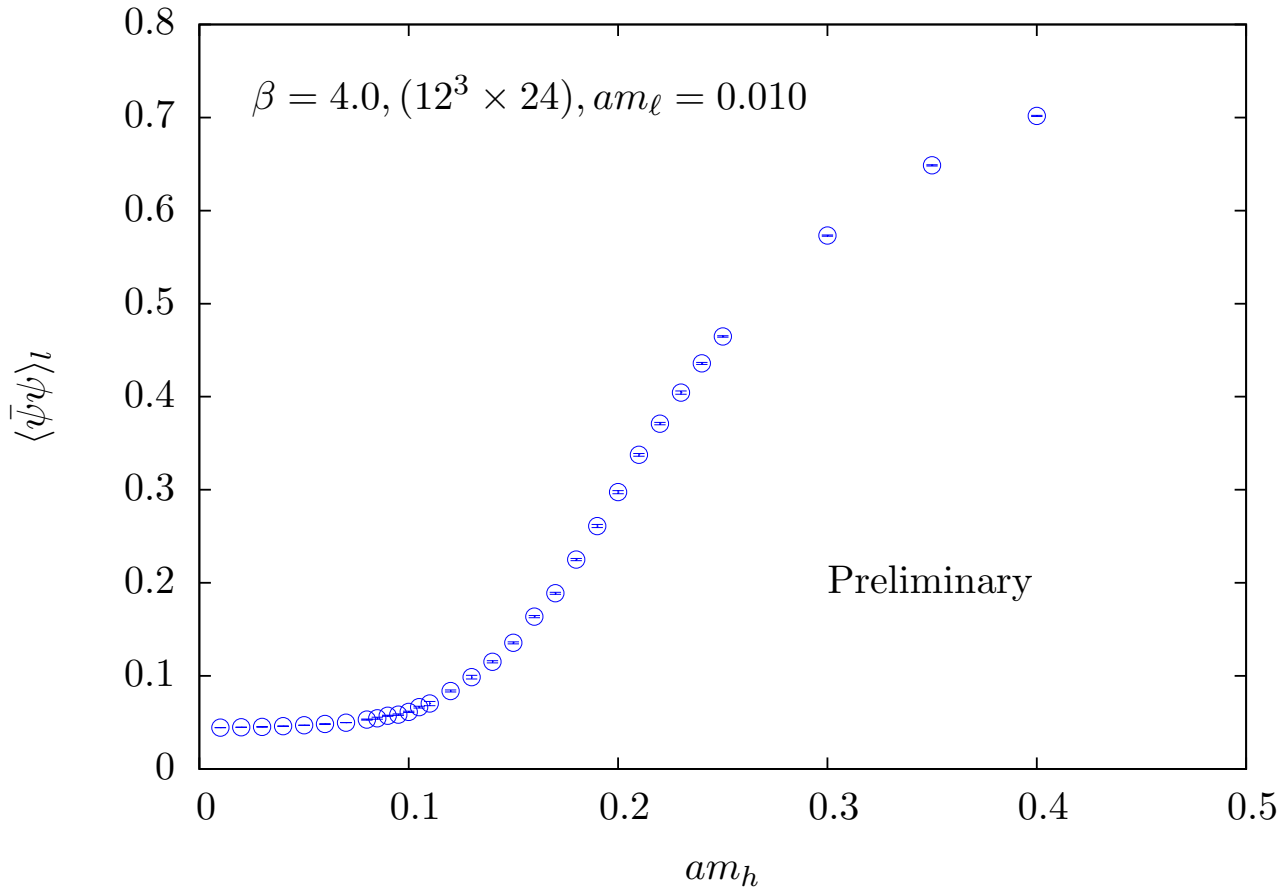

Volume Dependence: Zero m_ℓ Limit

- Recall: $\langle\bar{\psi}\psi\rangle_\ell^{lattice} = \frac{cm_\ell}{a^2} + \langle\bar{\psi}\psi\rangle_\ell^{phys}$
- Define $\langle\bar{\psi}\psi\rangle_\ell\big|_{m_\ell=0.0} \equiv$ linear extrapolation of $\langle\bar{\psi}\psi\rangle_\ell$ to $m_\ell = 0$.
  - Linear extrapolation from $m_\ell = 0.005, 0.010$, errors in quadrature.

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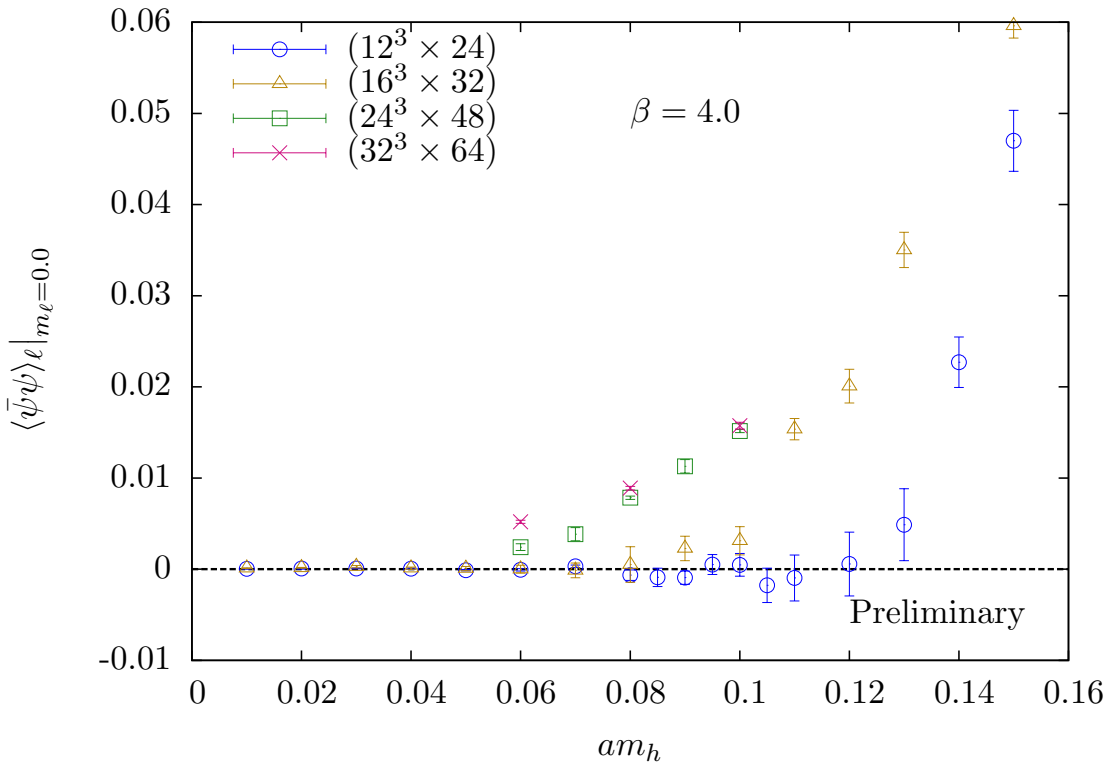

- We find a regime of volume-independent physics for $24^3 \times 48$ volumes.

Looking at the Scale: Wilson Flow

- Wilson Flow can be used to define a lattice scale.

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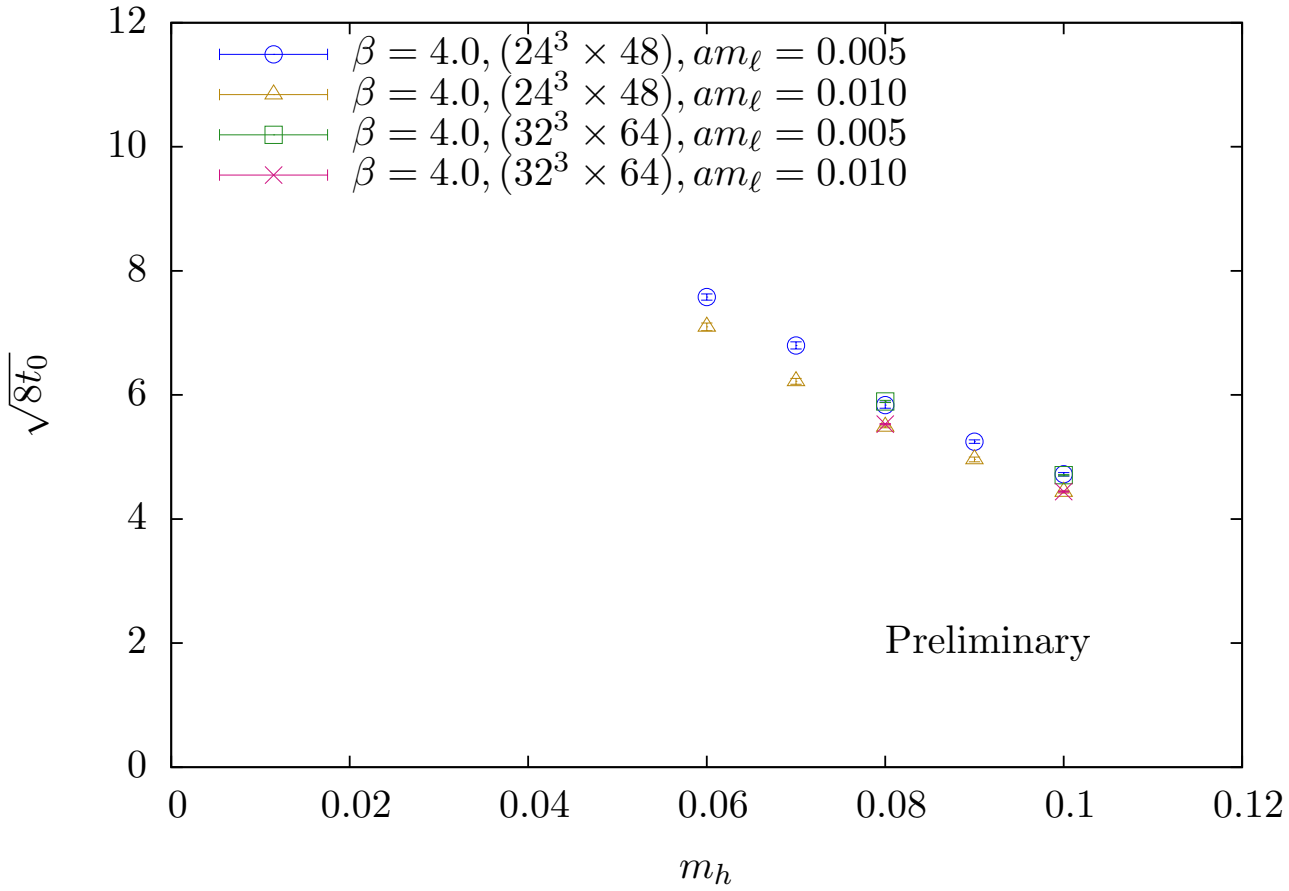

- Scale is independent of volume.

Spectrum finite volume effects: $M_{\pi,GB}$

- Wall sources, point sinks following Gupta et al. Phys.Rev.D(43) 1991.
- Focus on the Goldstone Boson pion mass for finite size effects.

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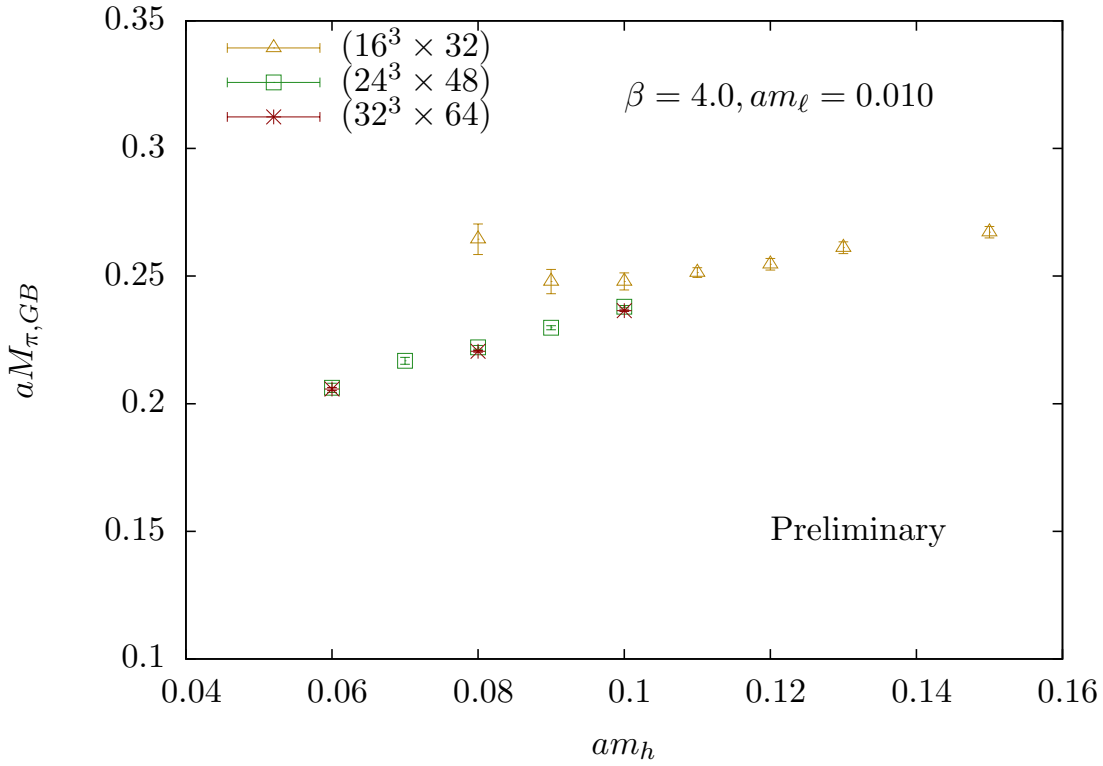

- 16^3 suffer finite size effects, while 24^3 and 32^3 are safe for $m_\ell = 0.010, m_h \geq 0.060$.

Control and Measurements

- We've identified a chirally broken regime independent of volume effects.
- Meaningful results on modest $24^3 \times 48$ lattices.
- 1000 configurations separated by 10 MDTU.
- 9 sets of ensembles:
  - $m_l = 0.005, 0.010, 0.015$ for a light flavor chiral limit.
  - $m_h = 0.060, 0.080, 0.100$ to test theories with different RG flow.
- There is autocorrelation depending on m_ℓ, m_h—understood and controlled with blocking.

Connected Spectrum: $M_{\pi,GB}$

- Performed correlated fit of a single cosh to folded data.

$$C(t) = A_\pi \cosh\left(M_\pi(\frac{T}{2} - t)\right)$$

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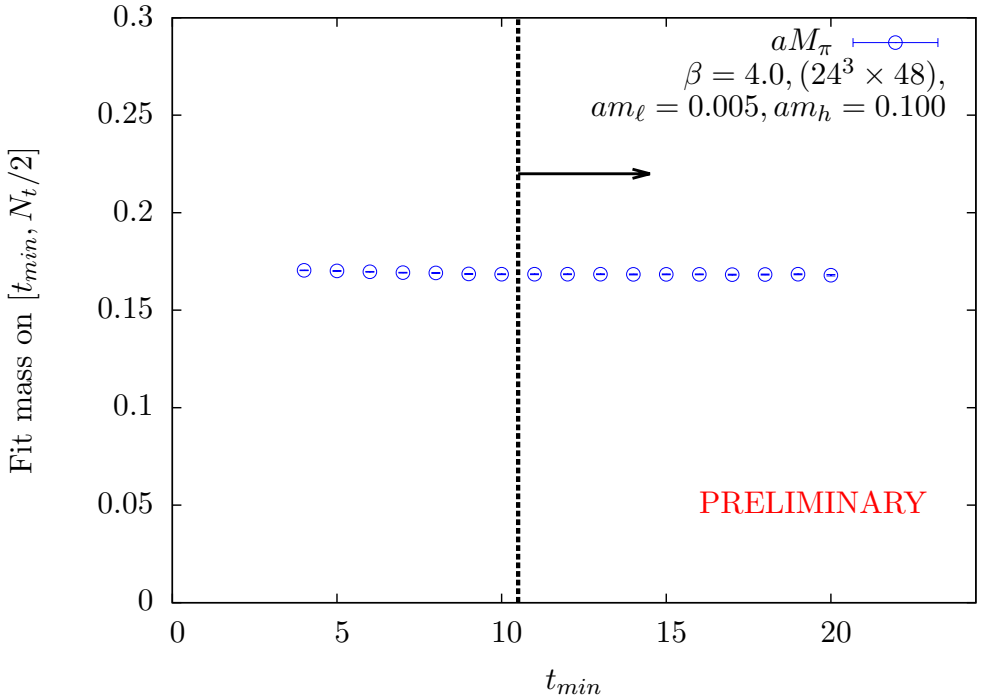
- Fit on $t = 11$ to 24.
- Largest interval with p-value $> 5\%$ confidence.

$$A_\pi = 232(1)$$
$$M_\pi = 0.1684(3)$$



Connected Spectrum: M_{a_0}

- Performed correlated fit of a single cosh plus oscillating term to folded data.

$$C(t) = A_{a_0}\cosh\left(M_{a_0}(\frac{T}{2} - t)\right) + A_{\pi_{sc}}(-1)^t\cosh\left(M_{\pi_{sc}}(\frac{T}{2} - t)\right)$$

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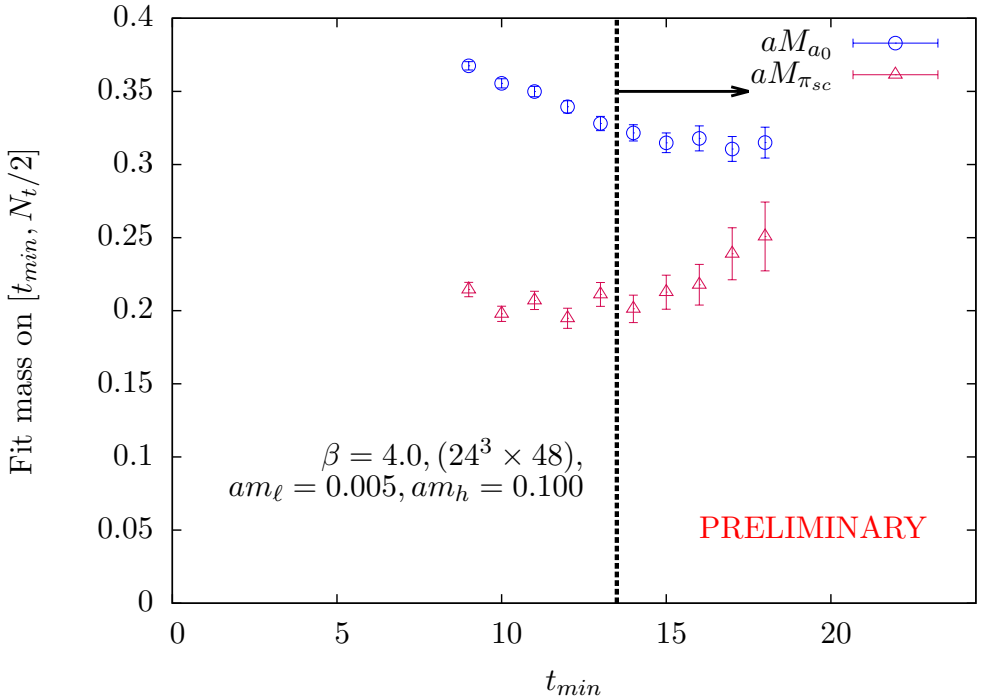
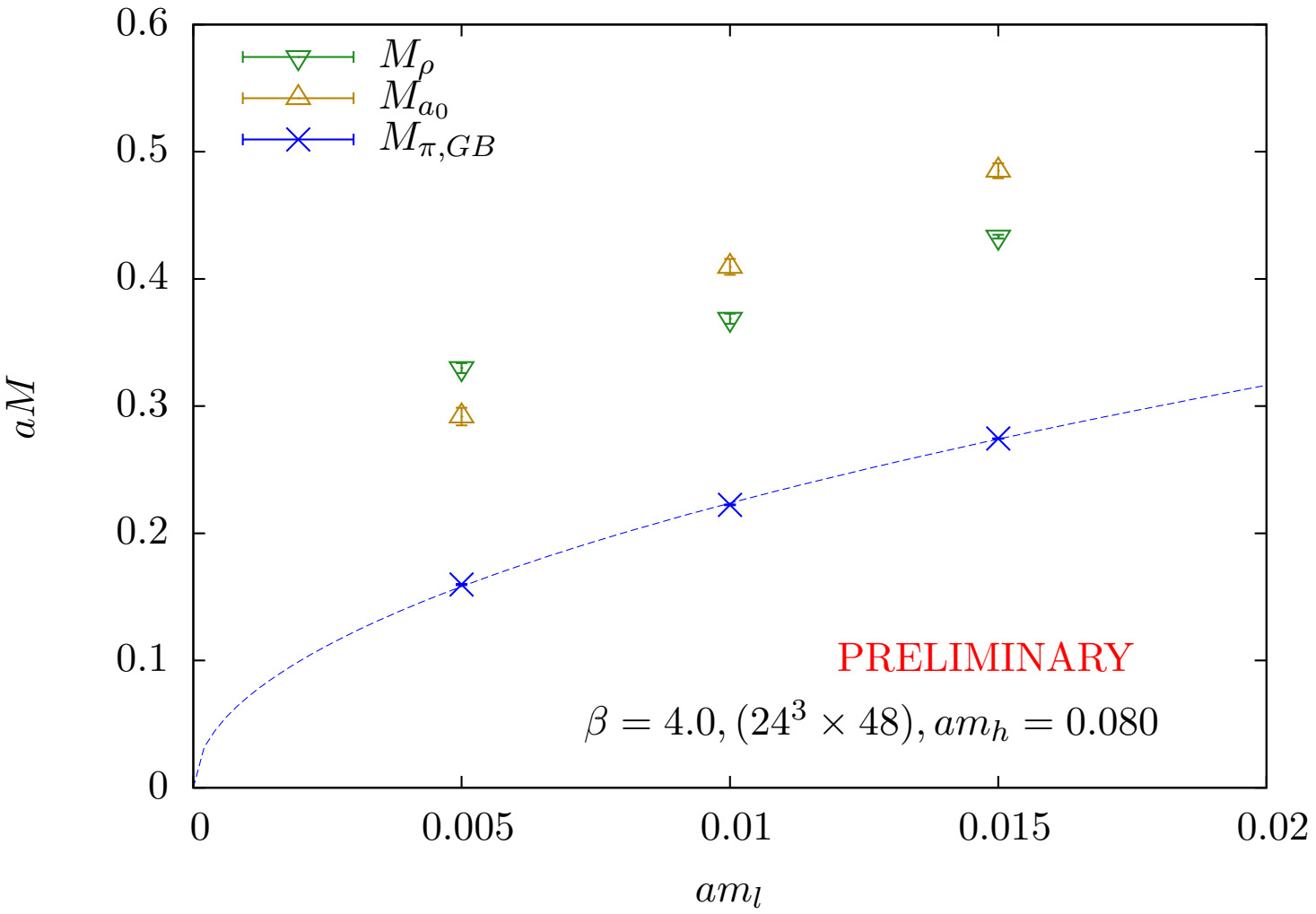
- Fit on $t = 13$ to 24.
- Largest interval with p-value $> 5\%$ confidence.

$$A_{a_0} = -1.73(14) \times 10^{-4}$$
$$M_{a_0} = 3.206(75) \times 10^{-1}$$
$$A_{\pi_{sc}} = 3.12(19) \times 10^{-5}$$
$$M_{\pi_{sc}} = 2.13(14) \times 10^{-1}$$



Connected Spectrum: Overall

- Measured light flavor meson spectrum (π, a_0, ρ).



Disconnected Spectrum: Stochastics and Dilution

- Stochastic sources and dilution to probe the light-flavor 0^{++} meson.
  - 6 $U(1)$ noise sources diluted in time, color, and even/odd spatially.
  - **1728** inversions per $24^3 \times 48$ configuration.
  - Current statistics reflect **250** configurations per ensemble.
- Used improved operators for disconnected measurement.

$$C_{conn}(t) = -A_{a_0} e^{-M_{a_0} t} - (-1)^t \left(A_{\pi_{sc}} e^{-M_{\pi_{sc}} t} \right) + \cdots$$

$$C_{disc}(t) = A_\sigma e^{-M_\sigma t} - A_{a_0} e^{-M_{a_0} t} + (-1)^t \left(A_{\pi_{\bar{s}c}} e^{-M_{\pi_{\bar{s}c}} t} - A_{\pi_{sc}} e^{-M_{\pi_{sc}} t} \right) + \cdots$$

$$\downarrow$$

$$C_\sigma(t) \equiv C_{disc}(t) - C_{conn}(t) = A_\sigma e^{-M_\sigma t} + (-1)^t \left(A_{\pi_{\bar{s}c}} e^{-M_{\pi_{\bar{s}c}} t} \right) + \cdots$$

Practical Considerations

- Vacuum subtraction is noisy in $C_{disc}(t)$.
  - Idea: Fit correlators with an additional constant.
- $C_{conn}(t)$, $C_\sigma(t)$ have large contamination from higher energy states.
  - Idea: Replace $C_{conn}(t)$ with just analytic fit to a_0, π_{sc} state.

$$C'_{conn}(t) \equiv -A_{a_0} e^{-M_{a_0} t} - (-1)^t \left(A_{\pi_{sc}} e^{-M_{\pi_{sc}} t}\right)$$
$$C'_\sigma(t) \equiv C_{disc}(t) - C'_{conn}(t)$$

  - Conclusion: Consistent at large t with using measured $C_{conn}(t)$, gives consistent mass at smaller t.

Disconnected Spectrum: M_σ

- 250 configurations separated by 40 MDTU each.
- Errors are by jackknife analysis, blocksize of 1.
- Data has fit constant subtracted for ease of visualization.

$$C'_\sigma(t) = A_\sigma \cosh\left(M_\sigma(\frac{T}{2} - t)\right) + (-1)^t A_{\pi_{\bar{s}c}} \cosh\left(M_{\pi_{\bar{s}c}}(\frac{T}{2} - t)\right) + V$$

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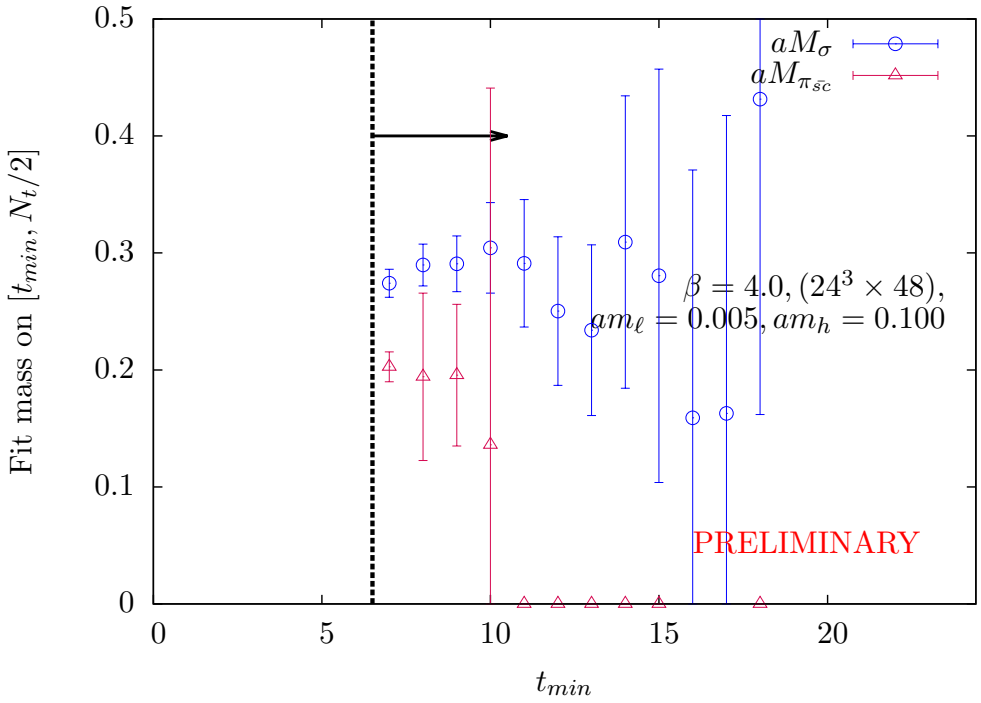
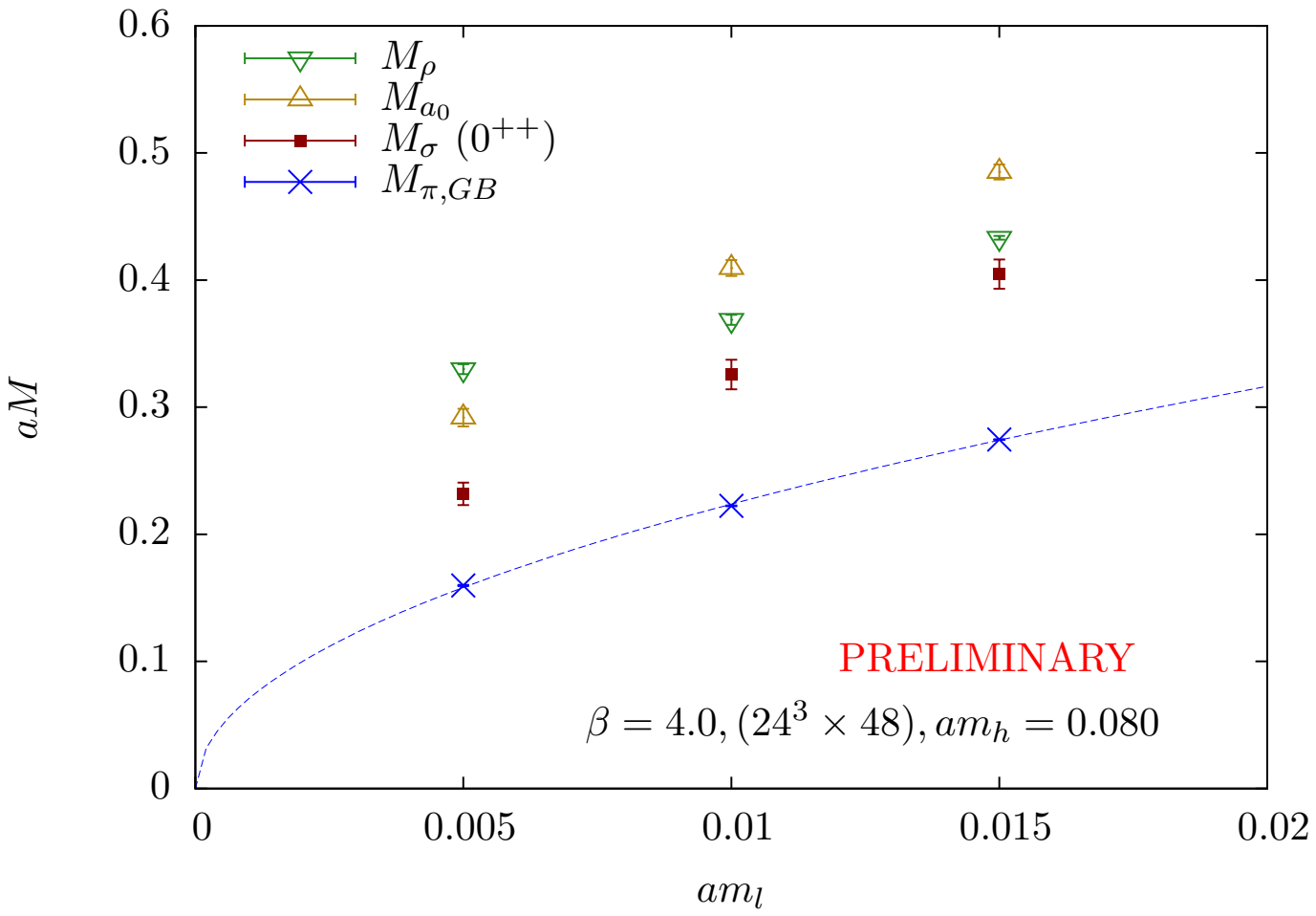
- Fit on $t = 6$ to 24.
- Largest interval with p-value $> 5\%$ confidence.

$$A_\sigma = 9.5(17)\text{e-}4$$
$$M_\sigma = 2.665(83)\text{e-}1$$
$$A_{\pi_{\bar{s}c}} = -1.8(18)\text{e-}4$$
$$M_{\pi_{\bar{s}c}} = 2.02(26)\text{e-}1$$
$$V = -1.87(13)\text{e-}2$$



0^{++} Results

- Results for different m_h.
- Fit lines for $M_{\pi,GB}$ reflect PCAC relation.



Conclusion

- Model to study walking behavior with 4+8 flavors.
- Tune to near-conformal behavior by shifting the mass of the 8 flavors.
  - m_h can be tuned continuously as opposed to discretely for N_f.
- $\langle\bar{\psi}\psi\rangle_\ell$ on finite volume, as a probe of chiral symmetry breaking, shows a transition with m_h.
  - Suggests physically viable parameters where simulations on moderate lattices can be done.
- The spectrum shows a splitting of the 0^{++} scalar state from the connected spectrum.

Thank you!