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The curvature of the QCD critical line from analytic continuation

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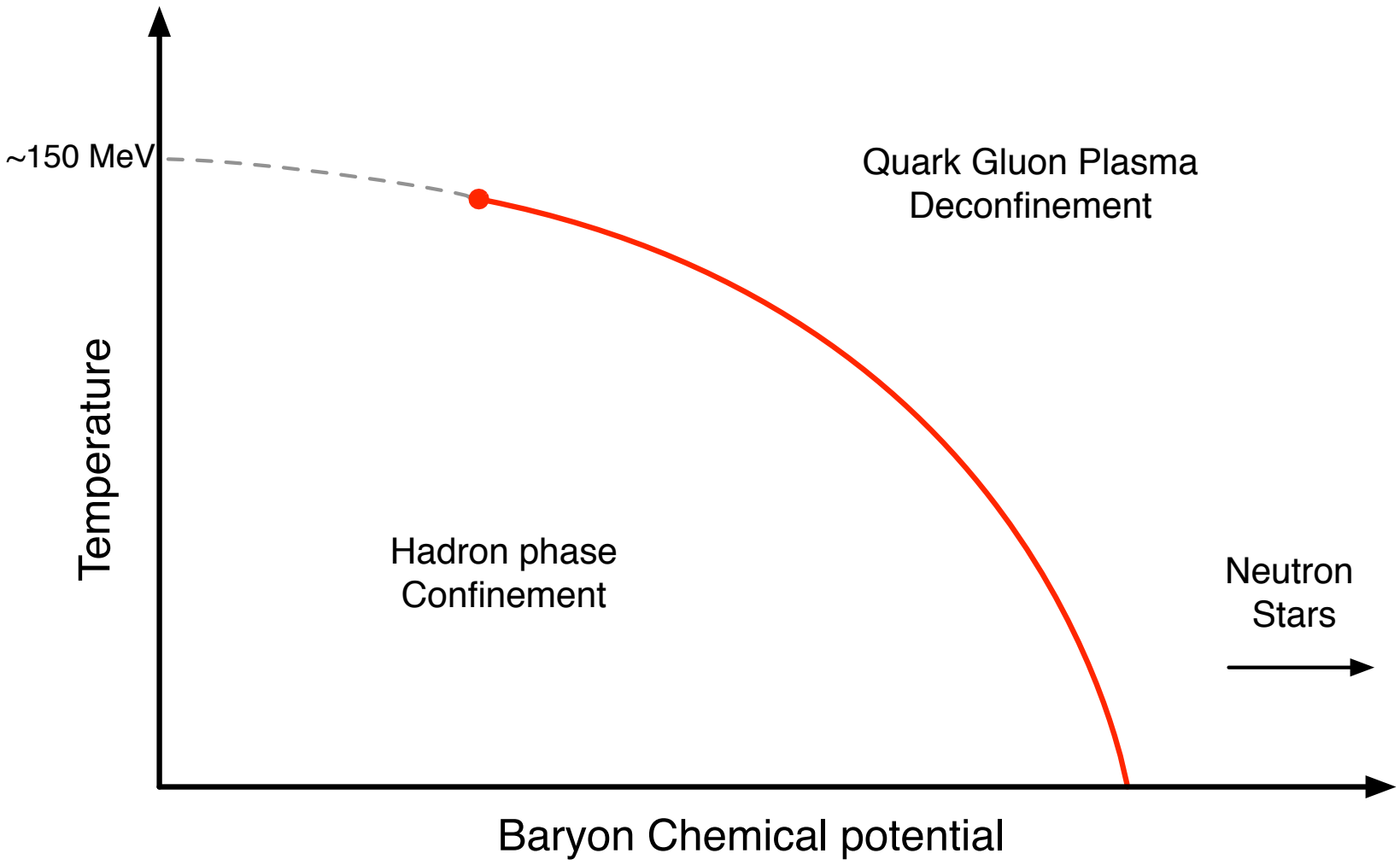
Outline

- the critical line QCD
- the method of analytic continuation
- Observables
- Numerical setup
- (Preliminary) Numerical results
- Conclusions

QCD at nonzero μ_B



The pesudocritical line and analytic continuation

At lowest order in μ, the pseudocritical line can be parametrized as:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 \tag{1}$$

The sign problem and analytic continuation

For purely imaginary μ, the fermion determinant is real positive, and the sign problem is non existent.

With the transformation $\mu \to i\mu$, the pseudocritical line parametrization is modified as:

$$\frac{T_c(\mu_B)}{T_c} = 1 + \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 \tag{2}$$

Observables

Chiral condensate

$$\langle\bar{\psi}\psi\rangle = \frac{1}{V_4}\frac{\partial \log Z}{\partial m} = \frac{N_f}{4V_4}\langle \mathrm{Tr} M^{-1}\rangle \tag{3}$$

Its renormalization [M. Cheng et al, 08]:

$$\Delta^r_{l,s}(T) \equiv \frac{\langle\bar{\psi}\psi\rangle_l(T) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(T)}{\langle\bar{\psi}\psi\rangle_l(0) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(0)} \tag{4}$$

the additive and multiplicative divergencies are thus removed.

Chiral susceptibility

$$\chi_{\bar{\psi}\psi} \equiv \frac{\partial^2 \log Z}{\partial m^2} = \frac{1}{V_4}\left(\frac{N_f}{4}\right)^2 \langle (\mathrm{Tr}\, M^{-1})^2 \rangle - \frac{1}{V_4}\left(\frac{N_f}{4}\right)^2 \langle \mathrm{Tr}\, M^{-1} \rangle^2 + - \frac{1}{V_4}\frac{N_f}{4}\left\langle \mathrm{Tr}\, M^{-2} \right\rangle \tag{5}$$

Its renormalization [Y.Aoki *et al.* 06] :

$$\Delta^r \chi_{\bar{\psi}\psi}(T) \equiv m_l^2 \left(\chi_{\bar{\psi}\psi}(T) - \chi_{\bar{\psi}\psi}(0)\right) \tag{6}$$

the term m_l^2 removes the multiplicative divergencies.
We use the dimensionless quantity $\Delta^r \chi_{\bar{\psi}\psi}(T)/T^4$

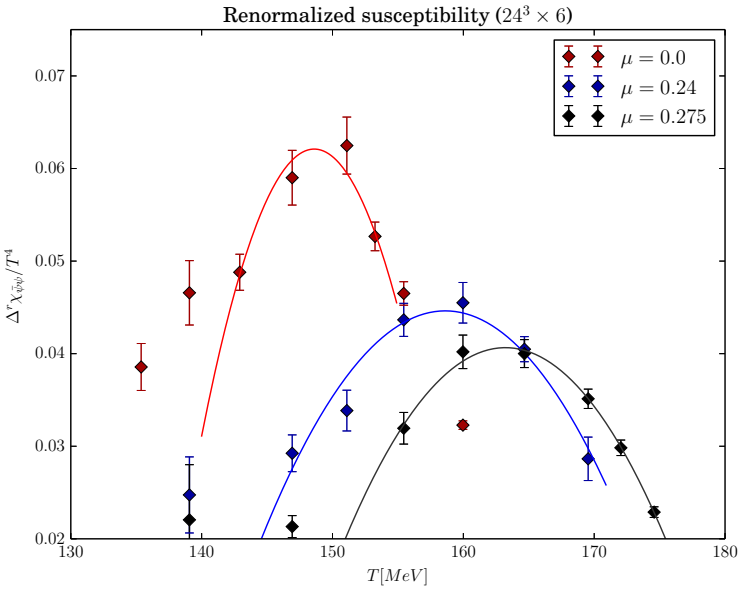
Numerical setup

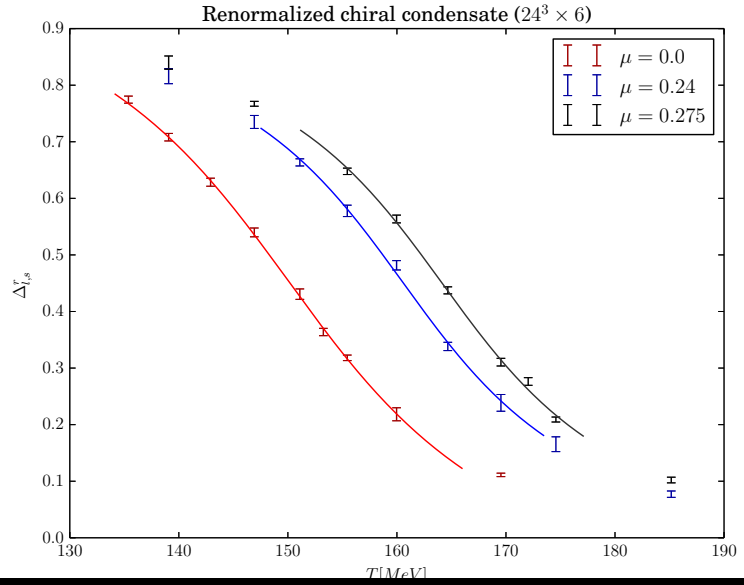
- Study of the $\mu_l \neq 0$, $\mu_s = 0$ case.
- Tree level symanzik improved action with $N_f = 2 + 1$ flavours of stout staggered fermions
- We are at the physical point, on a line of constant physics, taken from [Aoki *et al.*, 09] , around the critical temperature, for different lattices
- Observables evaluated with noisy estimators, with 8 random vectors per quark
- Simulations run on BG-Q machine at CINECA

Lattice	$24^3 \times 6$	$32^3 \times 8$
$i\mu/(\pi T)$	0.00, 0.24, 0.275	0.00, 0.20, 0.24, 0.275
N_{tr}	$2000 \sim 5000$	$2000 \sim 5000$

$24^3 \times 6$ Lattice

Preliminary results





Fit at the peak for the renormalized susceptibility:

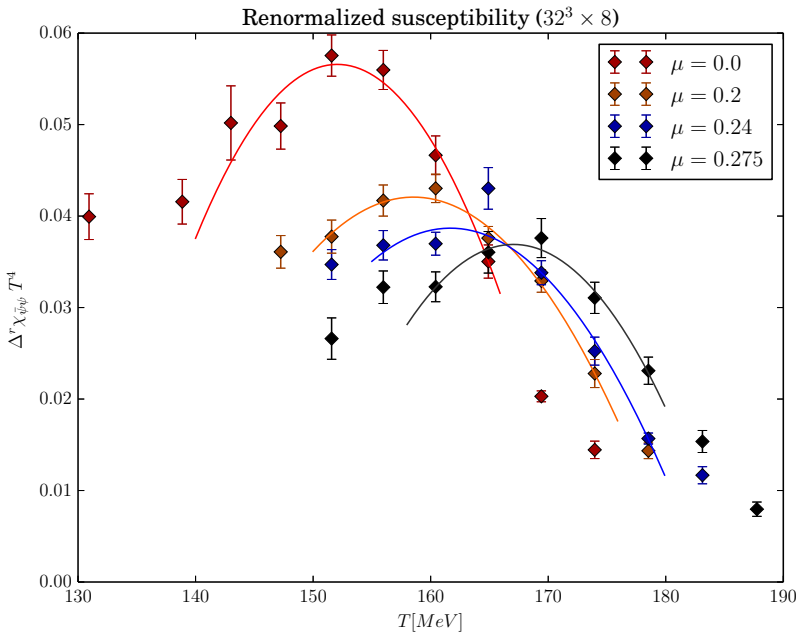
$$\Delta^r \chi_{\bar{\psi}\psi}(T) = \frac{a_s}{\frac{(T-T_{pc})^2}{b_s} + 1}$$

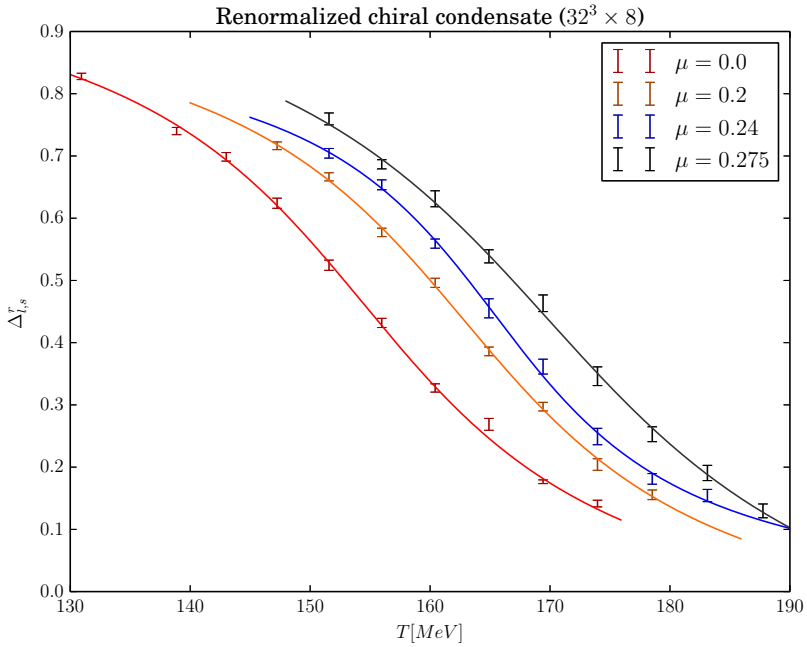
Fit for the chiral condensate:

$$\Delta^r_{l,s}(T) = a_c \operatorname{atan}\left[b_c\left(T - T_{pc}\right)\right] + c_c$$

$32^3 \times 8$ Lattice

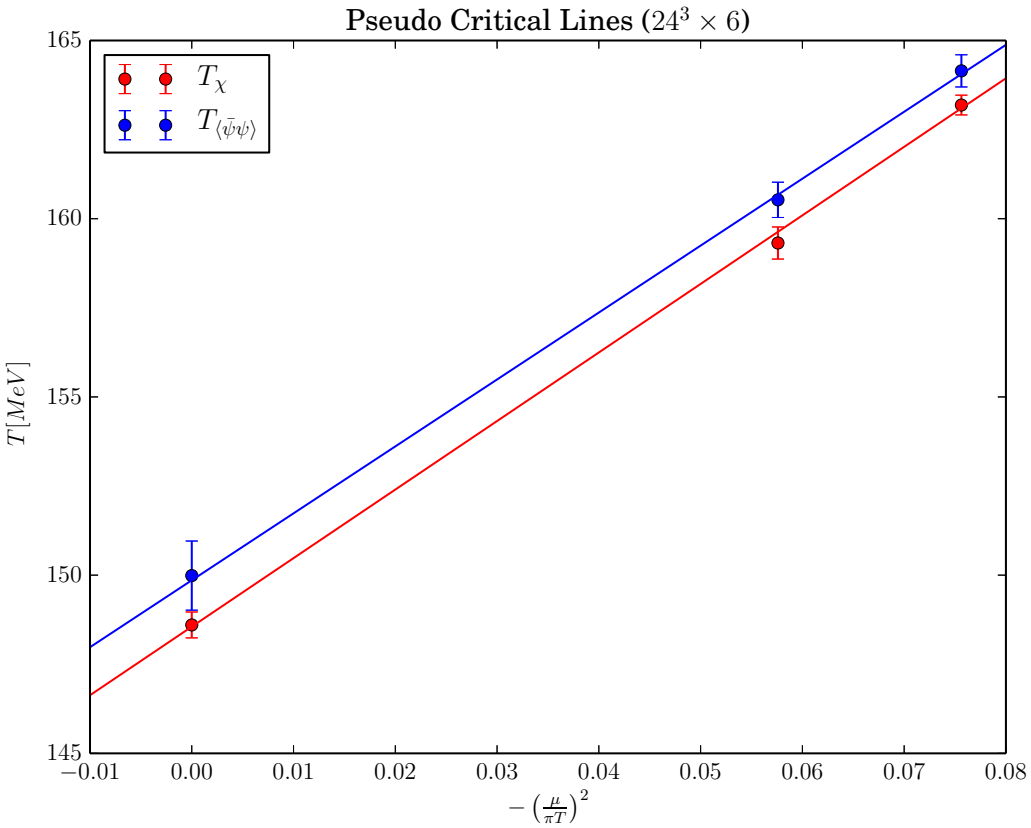
Preliminary results





Critical line ($24^3 \times 6$)

Preliminary results



From $\Delta^r \chi_{\bar\psi\psi}$:

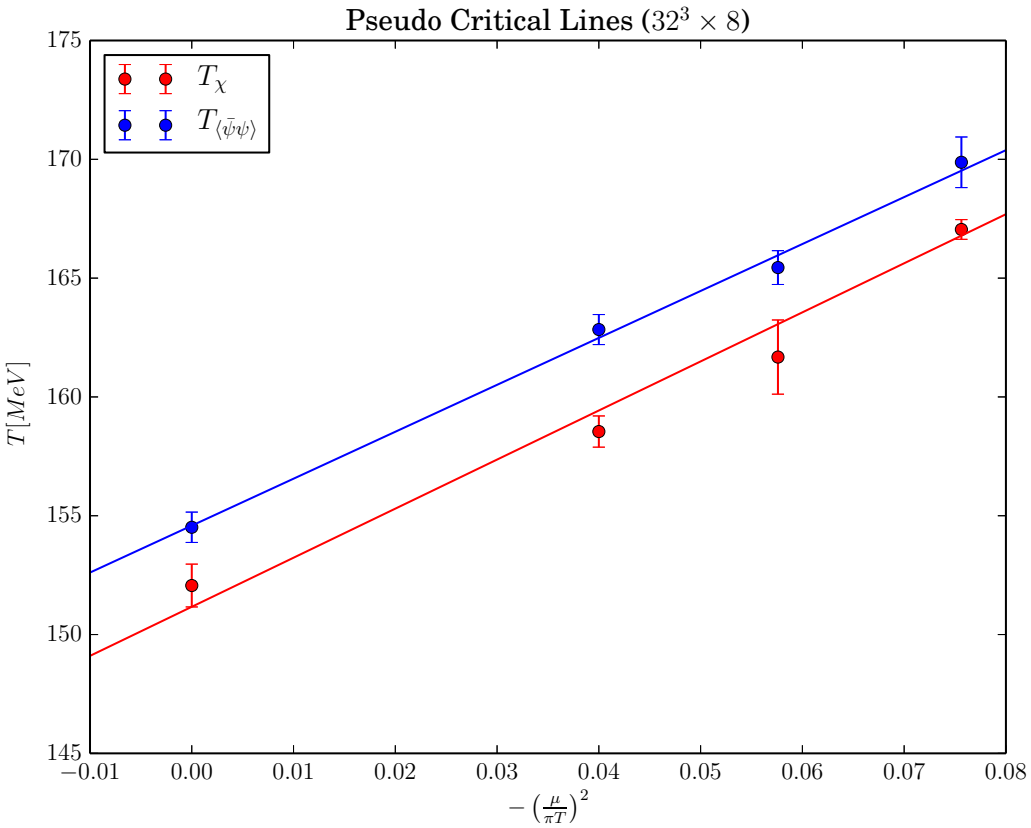
$$\kappa = 0.0146(4)$$

From $\Delta^r_{l,s}$:

$$\kappa = 0.0141(4)$$

Critical line ($32^3 \times 8$)

Preliminary results



From $\Delta^r \chi_{\bar\psi\psi}$:

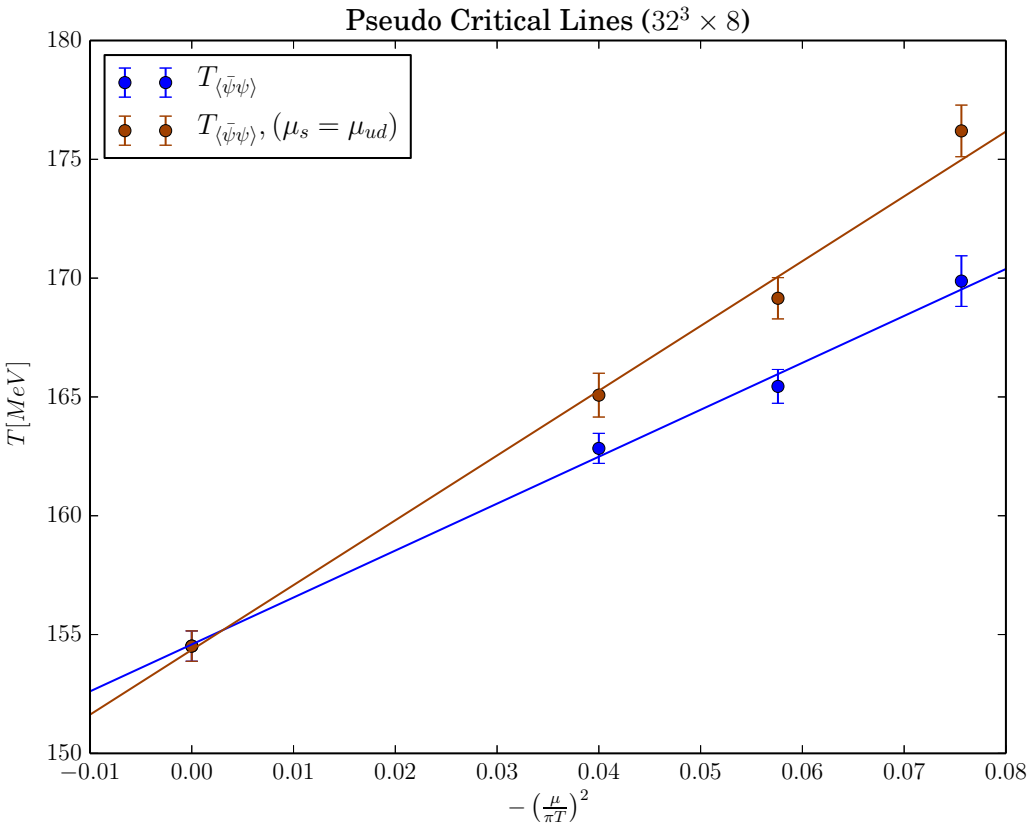
$$\kappa = 0.0153(14)$$

From $\langle\bar\psi\psi\rangle^r_{l,s}$:

$$\kappa = 0.0144(7)$$

Critical line with $\mu_s = \mu_{ud}$ $(32^3 \times 8)$

Preliminary results



From $\langle\bar{\psi}\psi\rangle^r_{l,s}$ (with $\mu_s = \mu_l$)

$$\kappa' = 0.0198(12)$$

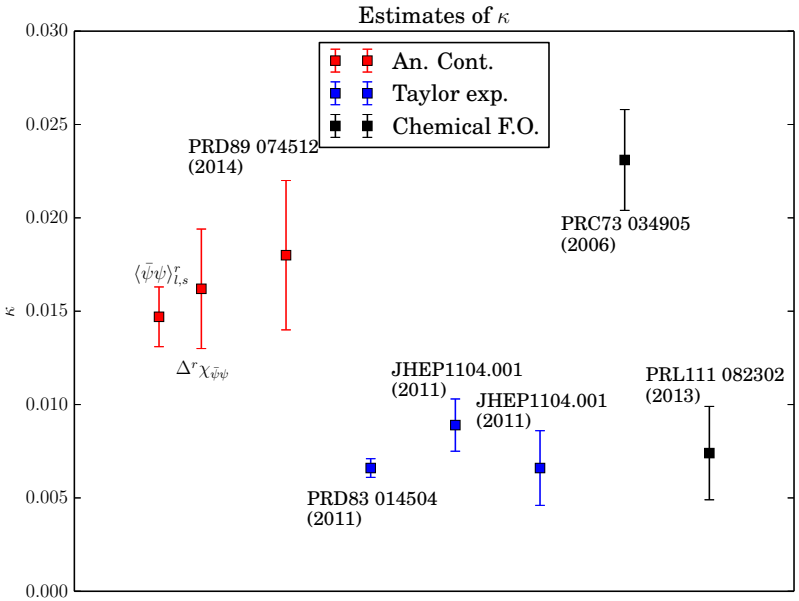
From $\langle\bar{\psi}\psi\rangle^r_{l,s}$ (with $\mu_s = 0$)

$$\kappa = 0.0141(4)$$

$$\kappa'/\kappa \simeq 1.4$$

Estimates of the curvature of the critical line

Comparison with results from the literature



(From left to right)

1. This study, chiral condensate (An Cont)(TlSym)
2. This study, chiral susceptibility (An Cont)(TlSym)
3. chiral susceptibility (An.Cont)(HISQ) [Cea *et al*, 14]
4. chiral condensate (Taylor Exp)(p4) [Kaczmarek *et al*, 10]
5. s-quark number susceptibility (Taylor Exp)(TlSym) [Endrodi *et al*, 11]
6. chiral condensate (Taylor Exp)(TlSym) [Endrodi *et al*, 11]
7. Chemical Freeze out [Cleymans *et al*, 06]
8. Chemical Freeze out [Becattini *et al*, 13]

Conclusions
and necessary developments

- We obtained an estimate for the curvature of the pseudocritical line at the physical point in $N_f = 2 + 1$ QCD, in the case of $\mu_s = 0$
- We started looking at contributions from a strange quark chemical potential: effects are non negligible

We plan to:

- Study the $N_t = 10$ lattice for the continuum extrapolation
- Study other observables, like the quark number susceptibilities