Magnetic Properties of the QCD Medium

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Based on C. Bonati, M. D'Elia, MM, F. Negro and F. Sanfilippo, PRD **89**, 054506 (2014)
and on a work in progress with M.Mesiti

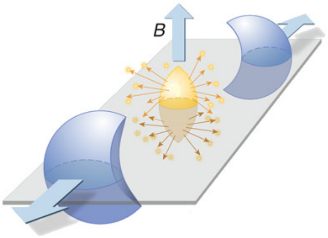
QCD with external B fields

QCD with B fields at the **strong scale**. Found in many phenomenolocical contexts:

- Neutron stars and compact astrophysical objects, $\mathbf{B} \sim \mathbf{10^{10}}$ **T** [Duncan and Thompson, 1992]
- First phase of off-central heavy ion collisions, $\mathbf{B} \sim \mathbf{10^{15}}$ **T** [Skokov et al., 2009]
- Early universe, $\mathbf{B} \sim \mathbf{10^{16}}$ **T** [Vachaspati, 1991]

We consider the heavy-ion collision scenario:

- Off-central collisions: ions generate magnetic fields, almost homogeneus and ortogonal to the reaction plane. Strength controlled by $\sqrt{s_{NN}}$ and the impact parameter.
- At LHC, B fields expected up to $eB \sim 15 m_\pi^2$



These magnetic fields can lead to relevant modifications of the strong dynamics.

QCD with external B fields

Electromagnetic background interacts only with quarks, but loop effects can modify also the gluon dynamics.

- Non perturbative effects lead to non trivial bahavior:
  - ▷ QCD phase diagram (location of the deconfinament cross over, ...)
  - ▷ QCD vacuum structure (chiral symmetry breaking, ...)
  - ▷ QCD equation of state (effect on the free energy of the QCD medium)

  We discuss non perturbative magnetic effects on the **QCD equation of state**.

  $\rightarrow$ Relevant for the description of QGP evolution in non-central HIC.

- We show that the QCD medium reacts as a **paramagnet** to B fields
- Compute magnetic susceptibility and relative magnetic contribution to the pressure.
- Preliminary results for higher order terms.

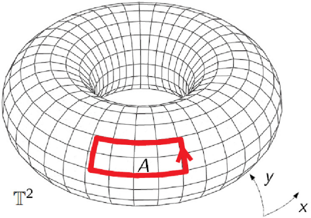
Magnetic fields on the Lattice

- Add proper $U(1)$ phases to $SU(3)$ links:

$$U_\mu(n) \to U_\mu(n) u_\mu(n) \qquad u_\mu = \exp\left(iqa_\mu(n)\right)$$

- Periodic boundary conditions to reduce finite size effects $\to$ **Quantization condition**:

$$e^{iqBA} = e^{iqB(A - L_x L_y a^2)} \to qB = \frac{2\pi b}{L_x L_y a^2}\ , \quad b \in \mathbb{Z}$$

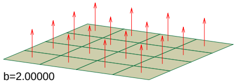


- $\vec{B} = B\hat{z} \to$ gauge fixing $a_y = Bx$, then:

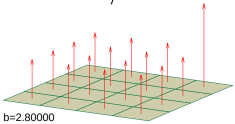
$$u_y^{(q)}(n) = e^{ia^2 qBn_x} \qquad u_x^{(q)}(n)|_{n_x = L_x} = e^{-i\ a^2 qL_x Bn_y}$$

Constant flux $a^2 B$ in all x-y plaquettes, exluded one plaquette at the corner, which has an additional flux $(1 - L_x L_y) a^2 B \to$ Dirac string. Not seen if $b \in \mathbb{Z}$

$b \in \mathbb{Z}$



$b \notin \mathbb{Z}$



- For $b \notin \mathbb{Z}$ string become visible. Non-uniform B $\Longrightarrow$

Our method

- For "small" magnetic fields: $f(T,B) = f(T,0) - \frac{1}{2}\chi_2(T)B^2 + \mathcal{O}(B^3)$
  Then $\chi_2 = -\left.\frac{\partial^2 f(T,B)}{\partial B^2}\right|_{B=0}$... **But** $\frac{\partial}{\partial B}$ not defined on the lattice!

Our method

- To extract χ_2 one can compute **finite free energy differences**:
  $\Delta f(T,b) = f(T,b) - f(T,0)$
- Find an appropriate path that connects two points in parameter space $A = (T;b) \to B = (T;0)$.
  Then $\Delta f(T,b) = -\frac{T}{V}\int_A^B \frac{\partial \log Z}{\partial \vec{p}} d\vec{p}$
- We choose to go straight in b. We interpolate between physical points in parameter space introducing a real valued magnetic field $\to$ we can evaluate $\frac{\partial \log Z}{\partial b} \to$
  $\Delta f(T,b) = -\frac{T}{V}\int_0^b \frac{\partial \log Z}{\partial \tilde{b}} d\tilde{b}$
- For $eB = \frac{2\pi b}{L_x L_y a^2}$ with $b \notin \mathbb{Z}$ intermediate points does not corresponde to the uniform field case
- $\frac{\partial \log Z}{\partial b}$ is not the physical magnetization.

Our method

Renormalization

- $\Delta f(T,b)$ has $(eB)^2$-dependent divergencies. To renormalize, we performe:

$$\Delta f_r(T,b) = \Delta f(T,b) - \Delta f(0,b) \ .$$

$\rightarrow$ Gives us divergences-free magnetic properties of the **thermal medium**.

QED- quenching

- Linear regime:

$$\mathbf{M} = \tilde{\chi}_2 \frac{\mathbf{B}}{\mu_0} = \chi_2 \mathbf{H}$$

where $\mathbf{B}$ total field, $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ external field, and $\chi_2 = \frac{\tilde{\chi}_2}{1-\tilde{\chi}_2}$.
- In the linear regime we can use:

$$\Delta f = \int \mathbf{H} d\mathbf{B} \ \rightarrow \Delta f_r = -\int \mathbf{M} d\mathbf{B} \approx -\frac{\tilde{\chi}_2}{\mu_0} \int \mathbf{B} d\mathbf{B} = -\frac{\tilde{\chi}_2}{2\mu_0} \mathbf{B}^2$$

- Our simulations are QED quenched, no backreaction from the medium $\rightarrow$ $\mathbf{B}$ coincides with the external field added to the Dirac operator. The backreaction of the medium increase Δf_R by a factor $1/(1-\tilde{\chi}_2)^2$ $\rightarrow$ Irrelevant a posteriori.

Numerical Setup

- State of art discretization
  - ▷ Gauge: Tree level improved Symanzik action
    [Weisz, Nucl Phys B 83; Curci, Menotti and Paffuti, Phys Lett B 83]
  - ▷ Fermions: Rooted staggered fermions with stout improvement
    [Morningstar and Peardon, PRD 04]

- Bare parameters taken from [Borsanyi, Endrodi, Fodor et al., JHEP 10] → Line of constant physics.

We performed simulations at 3 values of lattice spacing:

$a[fm]$	N_s	N_t
0.2173	24	4, 6, 8, 10
0.1535	32	4, 6, 8, 10, 12
0.1249	40	4, 6, 8, 12, 16

Physical size ≈ fixed at $L = 5$ fm

Lattice observale

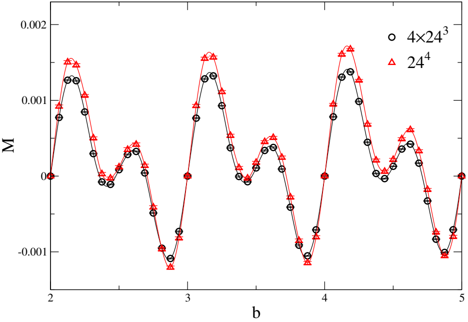
To get $\Delta f(T,b)$ we measured:

$$\mathcal{M} = \frac{\partial \log Z}{\partial b} = -\left\langle \mathrm{Tr}\left(M^{-1}\frac{\partial M}{\partial b}\right)\right\rangle_b$$

We divide the each quantum interval into 16 parts .

Example: $\mathcal{M}$ for $a = 0.2173$ fm set
$N_t = 24 \to T \approx 40$ MeV
$N_t = 4 \to T \approx 227$ MeV

- B no more quantized $\to$ Oscillations due to Dirac string.
- Numerical integration over $\mathcal{M}$ spline interpolations $\to \Delta f$
- Non vanishing T-dependet integrals from b to $b+1$
- We checked our procedure does not depend on the used interpolation.



Magnetic susceptibility

- $\Delta f(B,T) \approx \frac{1}{2} c_2(T) B^2$. To minimize error propagation in the integration we fit:

$$f(b,T) - f(b-1,T) = \int_{b-1}^{b} \mathcal{M}(B,T)dB \quad \text{using} \quad \Delta f(b) = \frac{1}{2} c_2(T)(2b-1)$$

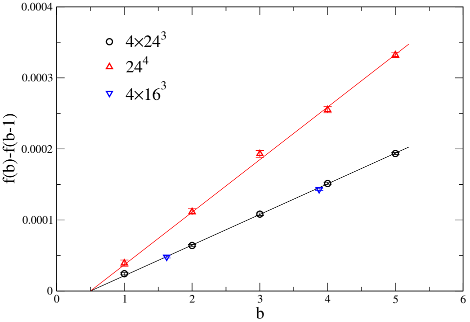
- We don't need to start at $b = 0$ to obtain $c_2(T)$

- $c_2^r(T) = c_2(T) - c_2(0)$ from linear fits. Then:

$$\tilde{\chi}_2(T) = -\frac{e^2 \mu_0 c}{18\hbar\pi^2} L^4 c_2^r(T) \quad \textbf{SI units}$$

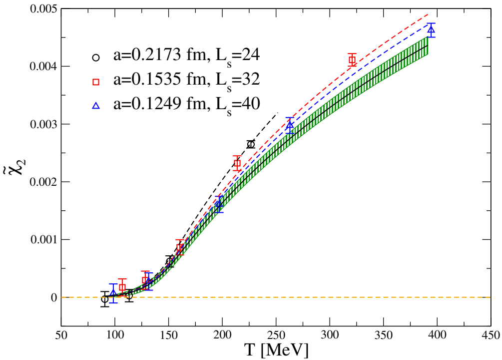
$$\hat{\chi}_2(T) = -\frac{L^4}{18\pi^2} c_2^r(T) \quad \textbf{natural units}$$

- Blue points to check finite size effects$\rightarrow$ Good.



Results

- The QCD medium is a **paramegnet** in all the explored temperature.
- Sharp increase of $\tilde{\chi}_2$ above $T_C \sim 150 - 160$ MeV.
- We observed a linear response up to $eB \approx 0.2$ GeV^2.



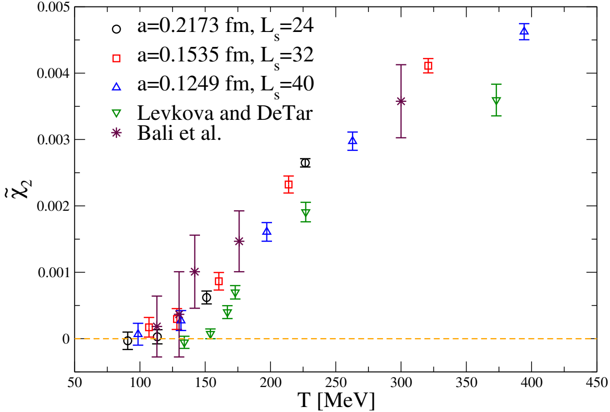
Fit function for the continuum limit:

$$\tilde{\chi}_2(t) = \begin{cases} A \exp\left(-\frac{M}{T}\right), & \text{for } T \leq \tilde{T} \text{ inspired by HRG} \\ a' \log\left(\frac{T}{M'}\right), & \text{for } T > \tilde{T} \text{ inspired by perturbative limit} \end{cases}$$

▷ Impose $\tilde{\chi}_2(T) \in \mathcal{C}^1$, matching at $\tilde{T} \rightarrow$ (5-2)=3 parameters.
▷ Perform the continuum limit by letting $A = A_0 + a^2 A_2$ or $M = M_0 + a^2 M_2$
▷ We found $\tilde{T} \approx 160(10)$ MeV.

Comparison

We can compare our results with other works:

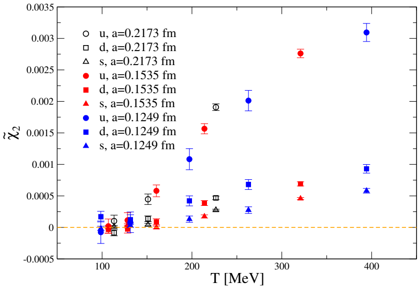


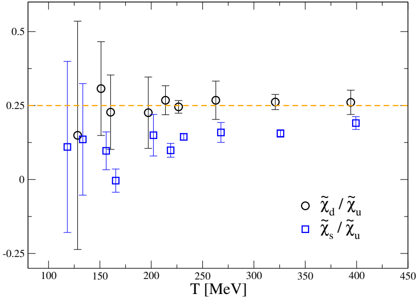
- Taylor expantion method [Levkova and DeTar]
- Anisotropy method [Bali, Bruckmann, Endrodi et. al.]
- Generalized integral method [Bali, Bruckmann, Endrodi et. al.]

Recently, new determination with good agreement [Bali, Bruckmann, Endrodi et. al., arXiv:1406.0269]

Results

We separate different quark contributions

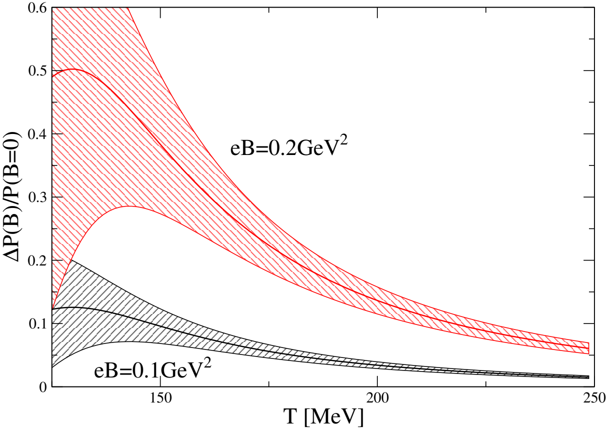




- $\frac{\tilde{\chi}_d}{\tilde{\chi}_u} \approx \left(\frac{q_d}{q_u}\right)^2$ as one would expect
- s-contribution slightly suppressed $\rightarrow$ mass effect
- c quark contribution: $q_c = q_u$, no charge suppression. But $m_c \gg m_u$ $\rightarrow$ Strong mass suppression. We can expect $\tilde{\chi}_c \approx \tilde{\chi}_s$

Pressure contribution

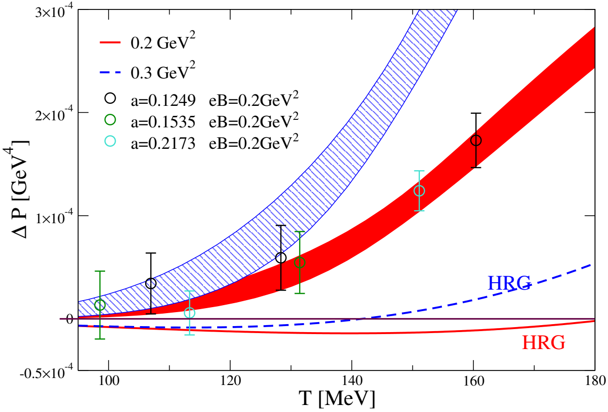
Magnetic contribution to the pressure: $\Delta P(B) = -\Delta f = \frac{1}{2}\tilde{\chi}_2(eB)^2$.



We take $P(B=0)$ from [S. Borsanyi it et al, arXiv:1309.5258].
Relative magnetic contribution of the order 10% for 0.1 GeV^2, 50% for 0.2 GeV^2.

HRG

Low T: check with the hadron resonance model predictions [Endrődi, 2013]



HRG predicts **diamagnetic** behavior at low-T, as one expects:

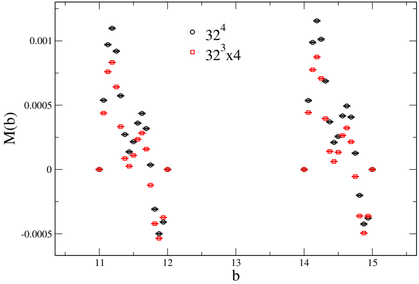
$\rightarrow$ Dominant contributions from pions at low T.

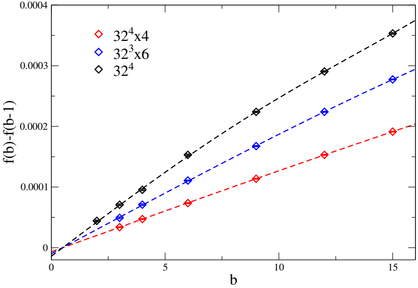
- No evidence with present statistics for susch behavior.
- Recent lattice indications of a possible diamagnetic regime up to $T \approx 120$ MeV.

[Bali, Bruckmann, Endrodi et. al., arXiV:1406.0269]

Higher order terms

- Preliminary results for $a = 0.1535$ fm, for $L = 32$ and $N_t = 32, 6, 4$





- We fit our data with $\Delta f(T, b)$, where $f(T, b) = \frac{c_2}{2} b^2 + \frac{c_4}{4!} b^4 + \frac{c_6}{6!} b^6 + O(b^8)$
  Then $\hat{\chi}_{2n}(T) \cdot (T^2)^{2n-2} = (\frac{L^2}{6\pi N_t})^{2n} \cdot N_t^2 c_{2n}$ in **natural units**

- To determine **thermal medium properties**, subtract all vacuum contribution:
  $\rightarrow$ $\chi_{2n}^r(T) = \chi_{2n}(T) - \chi_{2n}(0)$ for each n

- One can choose to subtract only the divergent $(eB)^2$-term in the free energy difference. We report preliminary results for $\hat{\chi}_4$.

N_t	$\hat{\chi}_4 \cdot T^4$
4	$2(3) \cdot 10^{-4}$
6	$2.7(9) \cdot 10^{-4}$
32	$1.4(2) \cdot 10^{-6}$

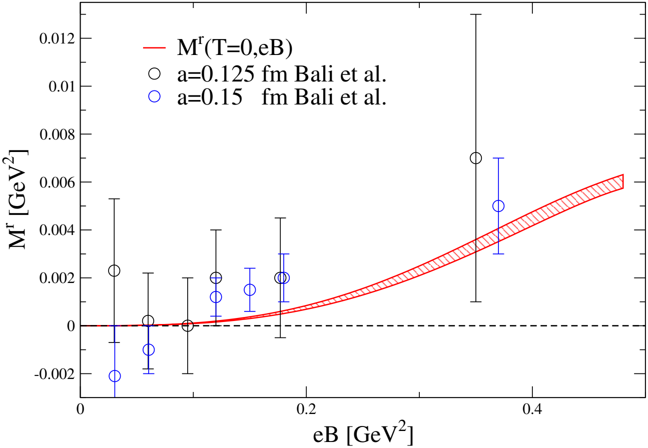
Higher order terms

- We compare $T = 0$ magnetization with previous results [Bali, Bruckmann et al., JHEP 1304 (2013) 130]:

$$M = -\hat{\chi}_2 eB + \frac{\hat{\chi}_4}{3!}(eB)^3 + \frac{\hat{\chi}_6}{5!}(eB)^5 + O\left((eB)^7\right)$$

where $\hat{\chi}_6 \cdot T^8 = -1.5(5) \cdot 10^{-4}$

- We subtract $(eB)^2$-dependent divergence $\rightarrow M^r = M - (-\chi_2 eB)$

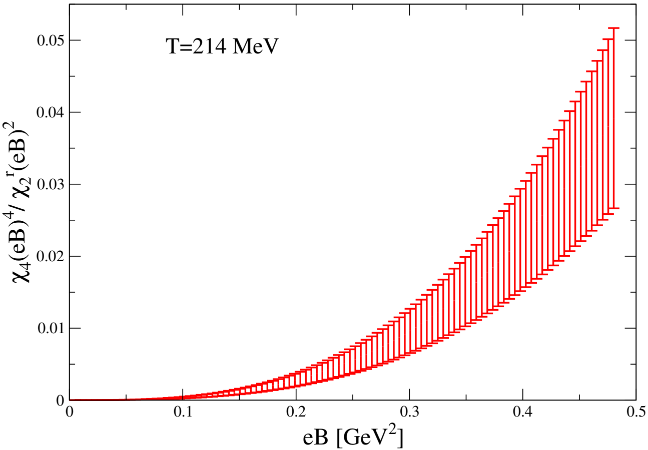


Higher order terms

We compute the relative contribution of the quartic term :

$$\frac{\hat{\chi}_4 (eB)^4/4!}{\hat{\chi}_2^r (eB)^2/2}$$

→ The quartic order term brings relevant contributions for fields $eB \geq 0.2$ GeV^2



Conclusions

- The QCD medium behaves as a paramagnet in all the explored temperatures.
- Weak magnetic activity in the confined phase. Magnetic susceptibility increases sharply across $T_c \approx 160(10)$ MeV.
- The QCD medium has linear response up to $eB \approx 0.2$ GeV^2.
- The magnetic contribution to the preassure is $(10 - 50)\%$ in the range of fields expected at LHC, $(0.1 - 0.2)$ GeV^2.
- Preliminary results for higher order terms

Future studies:

- Measure c quark contributions, which can be relevant at higher temperatures.
- Extend our investigation of higher order terms to finer lattice spacing