Gauge-invariant signatures of gauge-symmetry breaking in the Hosotani mechanism

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Recall: the Hosotani mechanism

- Compact extra dimension $L_5 \to$ Polyakov loop P_5, gauge potential A_5
  Vacuum $A_5 = 0 \;\Leftrightarrow\; \mathrm{Tr}P_5 = 1$
- Dimensional reduction $\to \bar{A}_5(x) = \frac{1}{L_5}\int_0^{L_5} dx_5 A_5(x, x_5)$, adjoint Higgs field
  with $V_{\mathrm{eff}}(\bar{A}_5)$
- Hosotani, 1983:
  Depending on matter (fermion) fields,
  $V_{\mathrm{eff}}(\bar{A}_5)$ may have non-trivial minimum
- $\langle \bar{A}_5 \rangle \neq 0$, cf. $\langle \phi_{\mathrm{adj}} \rangle \neq 0$: gauge-symmetry breaking by Higgs vev
- Example: $SU(3) \to SU(2) \times U(1) \to U(1) \times U(1)$

Higgs particle may be 5th-dimensional gauge field

Gauge-Higgs unification

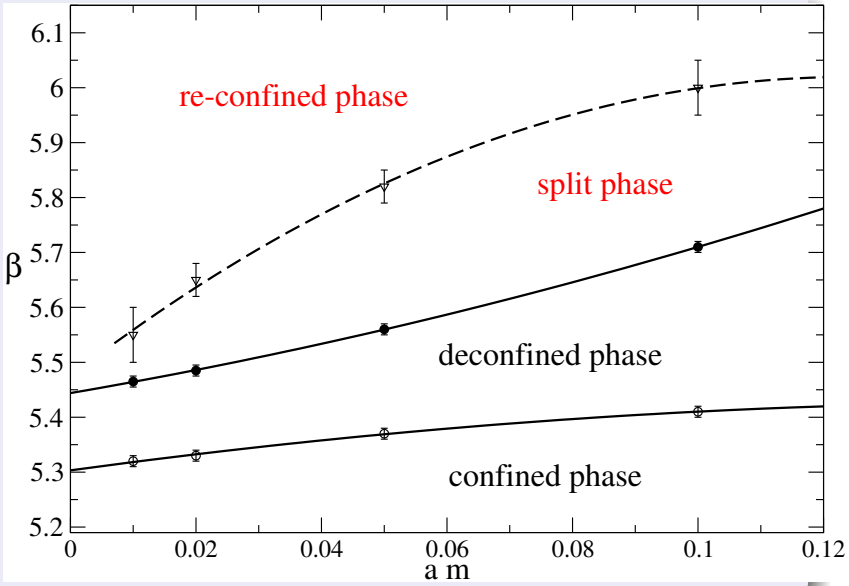
Observing gauge-symmetry breaking I

- $\langle \mathrm{Tr} P_5 \rangle \neq 1$ Cossu & D'Elia, Misumi et al, Cossu et al,..

$SU(3)$ with adjoint fermions, periodic b.c. $\rightarrow \Delta V_{\mathrm{eff}}(\bar{A}_5)$ *opposite* to gluons

2 exotic phases: "split", "reconfined"

Cossu & D'Elia, 0904.1353



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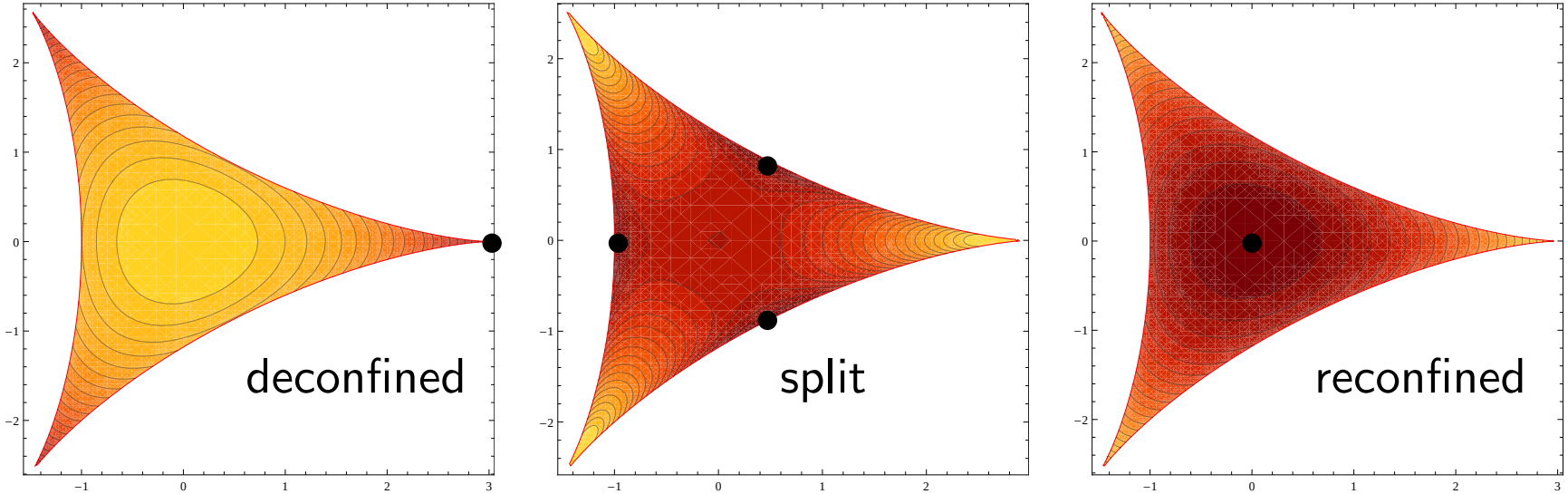


- "split": $\langle P_5 \rangle \approx \left(\begin{array}{cc|c} -1 & 0 & 0 \\ 0 & -1 & 0 \\ \hline 0 & 0 & +1 \end{array}\right)$ + permutations,

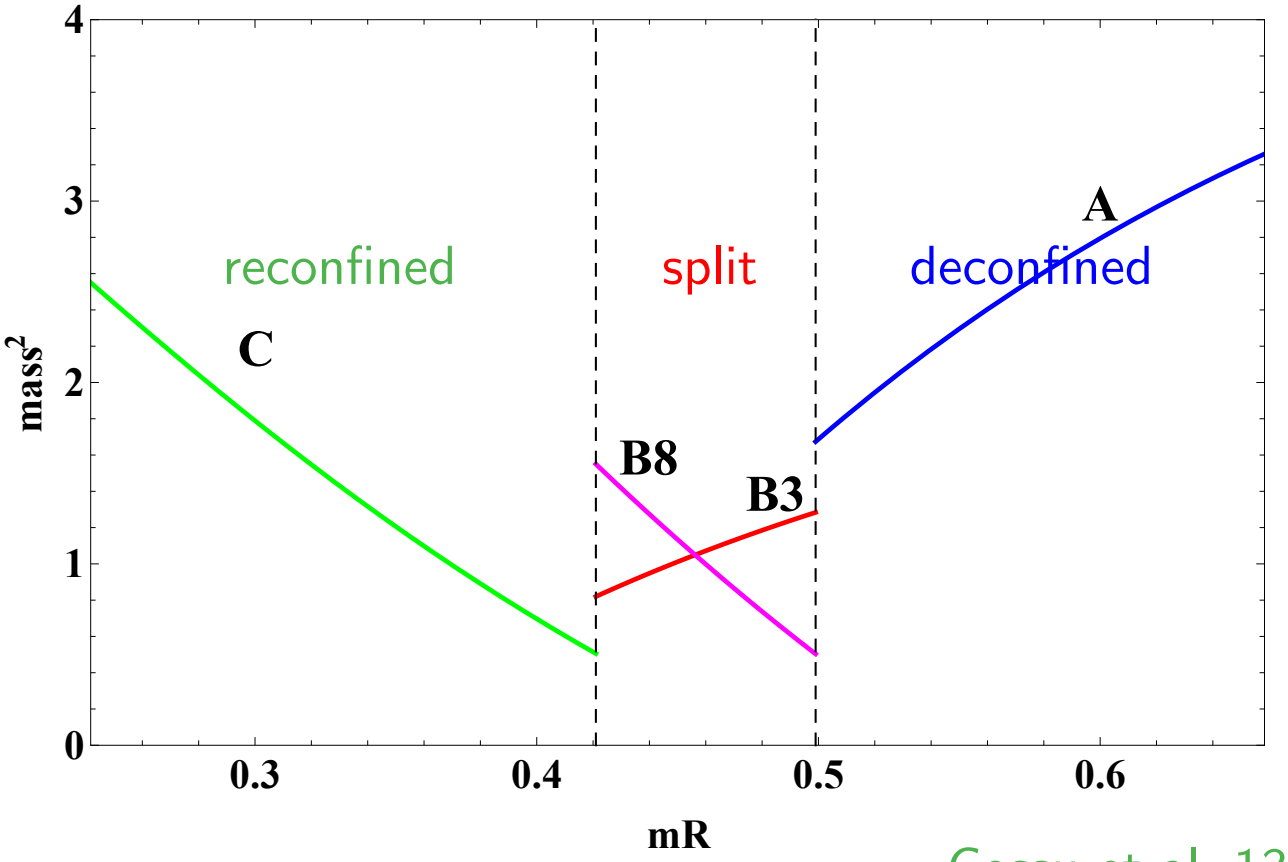
  invariant under $P_5 \rightarrow \Omega^\dagger P_5 \Omega$, $\Omega \in SU(2) \times U(1)$

- "reconfined": $\langle P_5 \rangle \approx \left(\begin{array}{cc|c} e^{+i\frac{2\pi}{3}} & 0 & 0 \\ 0 & e^{-i\frac{2\pi}{3}} & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$ + permutations,

  invariant under $P_5 \rightarrow \Omega^\dagger P_5 \Omega$, $\Omega = \exp(i\theta_3\lambda_3 + i\theta_8\lambda_8)$, ie. $U(1) \times U(1)$

Observing gauge-symmetry breaking II

- Higgs "mass": $P_5 = e^{igL_5 \bar{A}_5{}^k \lambda_k}$, fluctuations $m_k^{-2} \equiv \langle (\bar{A}_5{}^k - \langle \bar{A}_5{}^k \rangle)^2 \rangle$ k-dependent ?



$\longleftarrow$ T increases

Cossu et al, 1309.4198

- deconfined (trivial vacuum, $SU(3)$) & reconfined ($U(1) \times U(1)$): m_k k-independent
- split phase ($SU(2) \times U(1)$): $V_{\text{eff}}(A_5)$ is elliptical $\rightarrow$ $m_3 \neq m_8$

Observing gauge-symmetry breaking III

$\langle \mathrm{Tr} P_5 \rangle$ and m_k are *local* quantities

- Gauge-symmetry breaking has dramatic effect on IR physics:
  esp. $U(1)$ has *massless* photons
- BUT $SU(2) \times U(1)$ Wilson loop shows area law like $SU(3)$:

$$\mathrm{Tr} W = \mathrm{Tr} \prod (U_{SU(2)} \times U_{U(1)}) = \mathrm{Tr} \underbrace{(\prod U_{SU(2)})}_{\text{area law}} \times \underbrace{(\prod U_{U(1)})}_{\text{perimeter law}} \rightarrow \text{area law!}$$

- To observe IR breaking of gauge symmetry: fix gauge ??? Hetrick

Our answer: look for gauge-invariant topological excitations:

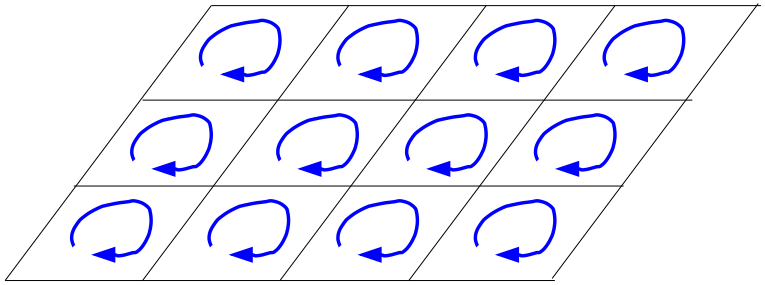
$\Pi_1(U(1)) = \mathcal{Z}$ whereas $\Pi_1(SU(3)) = \Pi_1(SU(2)) = \mathbf{1}$

Abelian fluxes and Abelian monopoles

Stable if $U(1)$ gauge symmetry, otherwise unwinds in $SU(N)$

Recall: Abelian flux in $U(1)$

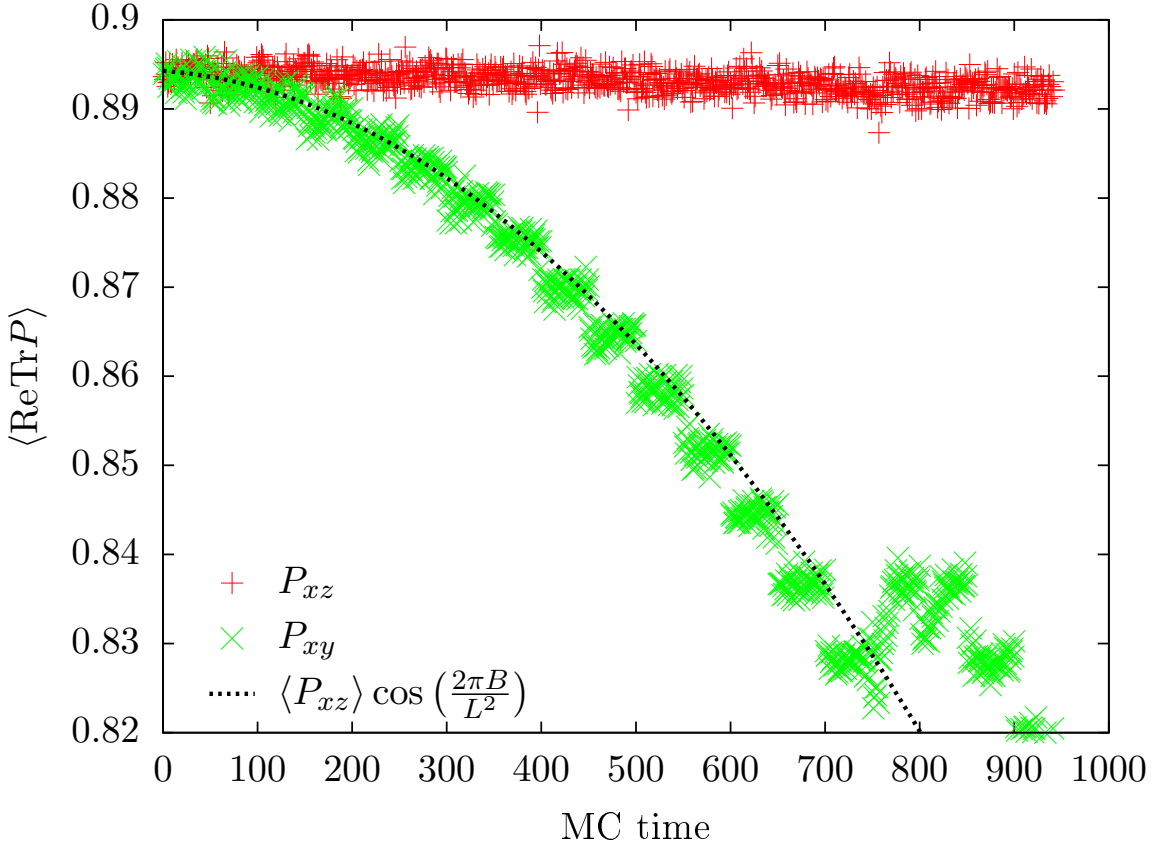
- Start from a cold $U(1)$ configuration: $U_\mu(x) = \mathbf{1}\ \forall x, \mu$
- In each xy plane, prepare flux state: $U_{P_{xy}} = \exp(i\frac{2\pi}{L_x L_y})$



- Perform Monte-Carlo
- Monitor gauge-invariant flux action: $\boxed{\Delta \equiv \langle \mathrm{Tr}\ \underbrace{U_{P_{xz}}}_{\text{no flux}} \rangle - \langle \mathrm{Tr}\ \underbrace{U_{P_{xy}}}_{\text{flux } 2\pi} \rangle}$
- Classical $U(1)$ field: $\mathrm{Tr} U_{P_{xz}} = 1, \quad \mathrm{Tr} U_{P_{xy}} = \cos(\frac{2\pi}{L_x L_y}) \to \Delta \approx \frac{2\pi^2}{L_x L_y}$

Abelian flux in $U(1)$

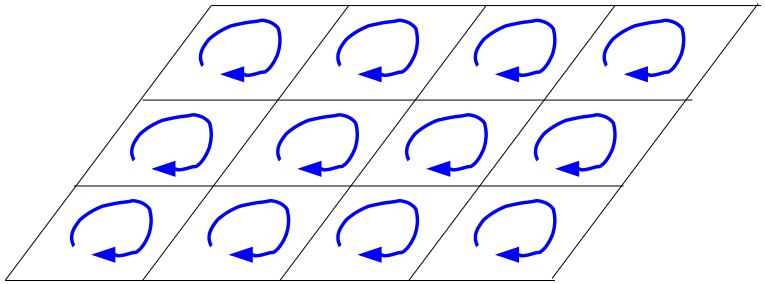
- Flux is absolutely stable in continuum limit (topological sectors)
- Can impose B units of flux, ie. $2B\pi$: $\Delta \approx 1 - \cos \frac{2\pi}{L_x L_y} B$



- When B is large enough, flux can decay:
  in each xy plane, one plaquette angle goes through π

Recipe for Abelian flux in $U(1) \subset SU(3)$: almost the same

- Start from a cold $SU(3)$ configuration: $U_\mu(x) = \mathbf{1}\ \forall x, \mu$
- In each xy plane, prepare flux state in $U(1)$ subgroup: $\theta|_{U(1)} = \frac{2\pi}{L_x L_y}$



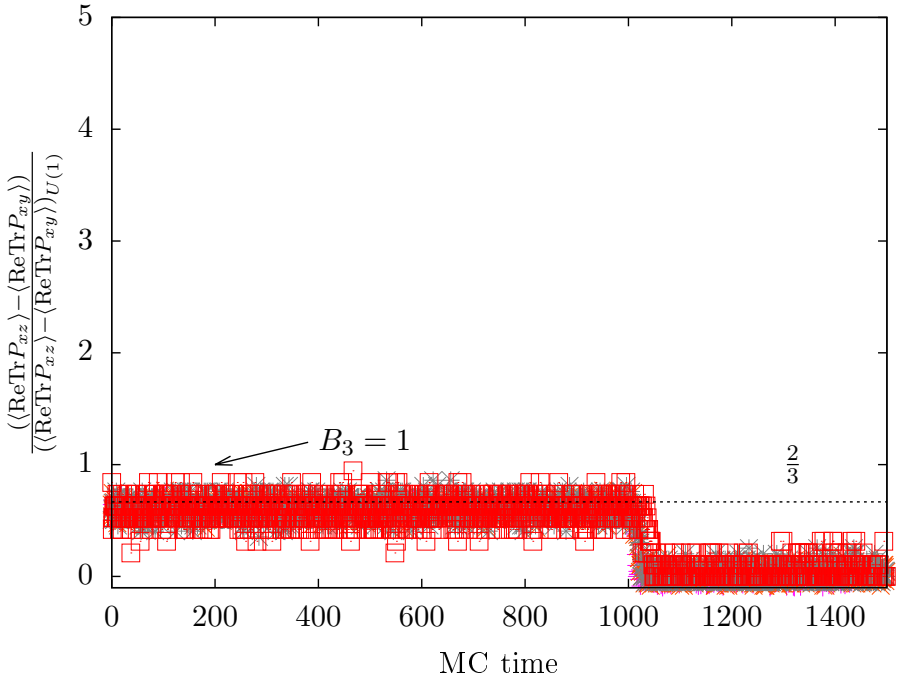
- Couple external field: $h_F \mathrm{Re}\mathrm{Tr} P_5 + h_A |\mathrm{Tr} P_5|^2$ to P_5 to achieve desired phase (simpler than simulating fermionic matter), eg. $SU(3) \to SU(2) \times U(1)$
- Perform Monte-Carlo: $SU(2), U(1)$ subgroups get *scrambled locally*
- Monitor gauge-invariant flux action: $\boxed{\Delta \equiv \langle \mathrm{Tr}\ \underbrace{U_{P_{xz}}}_{\text{no flux}} \rangle - \langle \mathrm{Tr}\ \underbrace{U_{P_{xy}}}_{\text{flux } 2\pi} \rangle}$
- Classical $U(1)$ field: $\Delta = 1 - \cos\theta \approx \frac{1}{2}\theta^2, \quad \theta = \frac{2\pi}{L_x L_y}$

Embed Abelian flux in $U(1)$ subgroup of $SU(3)$

- External field coupled to $P_5 \to$ reconfined phase ($U(1) \times U(1)$) turned off after 1000 MC sweeps $\to$ return to full $SU(3)$

Action of flux depends on $U(1)$ subgroup: λ_3 vs λ_8

$$1-\mathrm{Tr}(e^{i\theta}) \approx \frac{1}{2}\theta^2; \quad 1-\frac{1}{3}\mathrm{Tr}\begin{pmatrix} e^{i\theta} & & \\ & e^{-i\theta} & \\ & & 1 \end{pmatrix} \approx \frac{1}{3}\theta^2; \quad 1-\frac{1}{3}\mathrm{Tr}\begin{pmatrix} e^{i\theta} & & \\ & e^{i\theta} & \\ & & e^{-2i\theta} \end{pmatrix} \approx 1\theta^2$$



$U(1)$ flux unwinds *immediately* when full $SU(3)$ gauge-symmetry is restored

$\downarrow$

gauge-invariant signal of gauge-symmetry breaking/restoration

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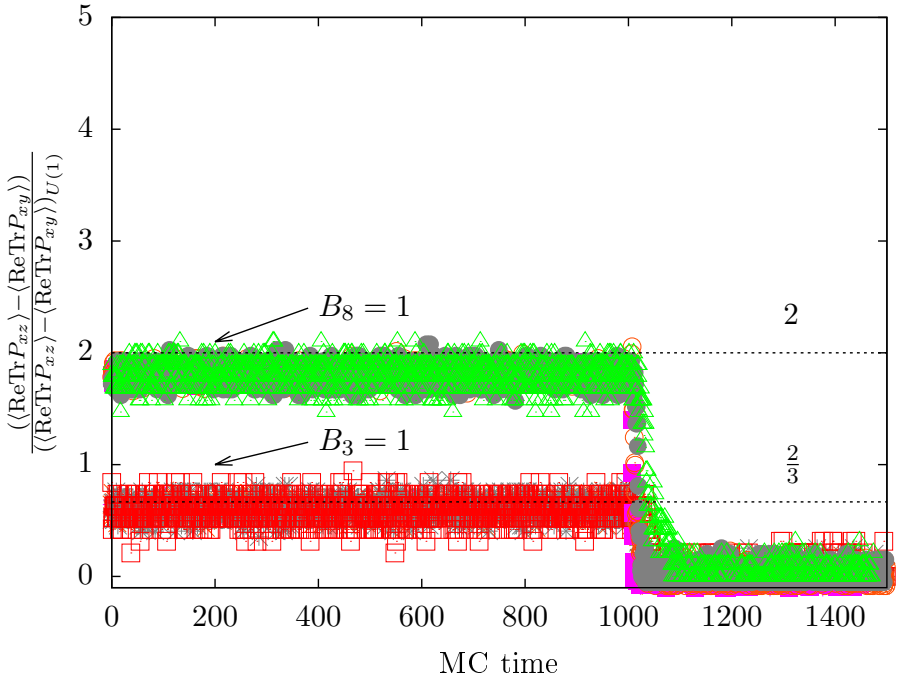

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gauge-invariant signal of gauge-symmetry breaking

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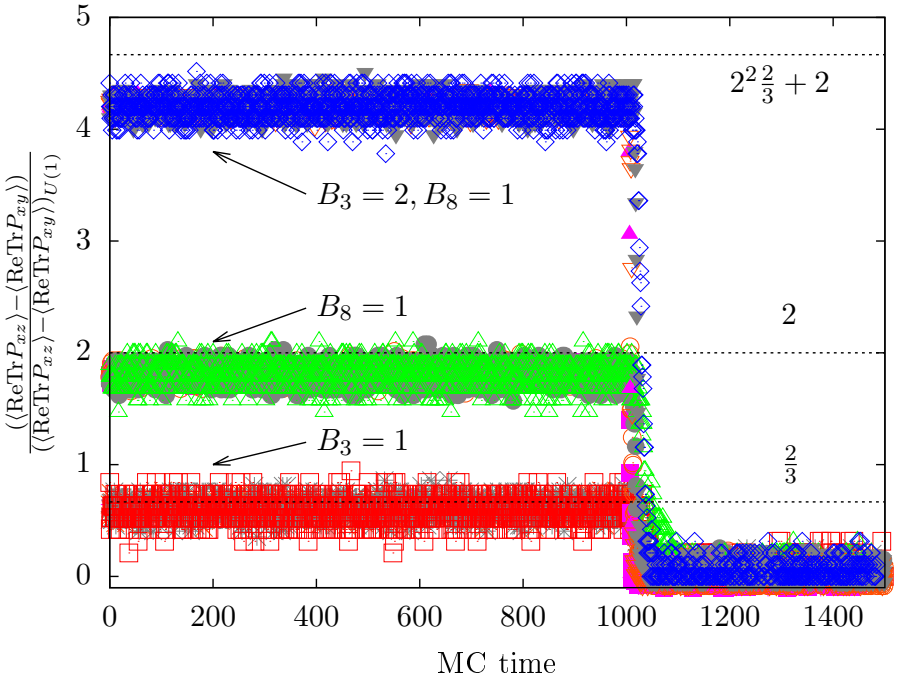

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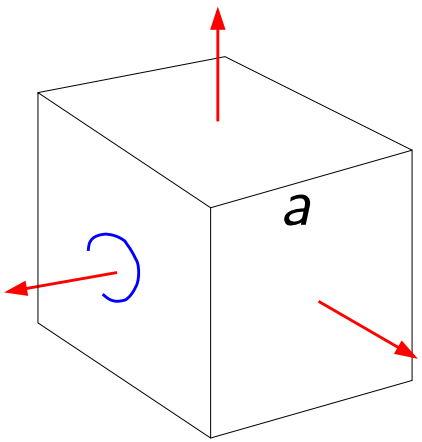
$\downarrow$

gauge-invariant signal of gauge-symmetry breaking

From $U(1)$ flux to $U(1)$ monopole

- $U(1)$ [Dirac magnetic] monopole is gauge-invariant $3d$ object

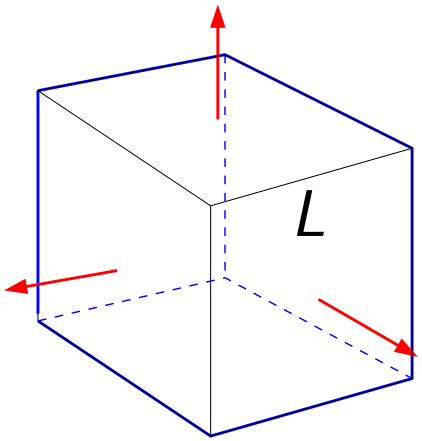
DeGrand & Toussaint 1981



$$\sum_{6\ \mathrm{plaq}} \theta_{\mathrm{plaq}} = 2\pi$$

- To create a classical $U(1)$ monopole, minimize $3d$ action subject to:
  - Charge-conjugated boundary conditions: $U_\mu(x+L) = U_\mu(x)^*$
  - Flux of π through one (or all 3) planes

PdF & Vettorazzo, hep-lat/0311006



$$\sum_{3\ \mathrm{planes}} \theta_{\mathrm{plane}} = \pi$$

$U(1)$ monopole mass and finite-size effects

- Classical, continuum: $S = \int d^3x \, \frac{1}{2}(\cancel{E^2} + B^2) = 2\pi \int dr \; r^2 B(r)^2$

  - Gauss law: $Q_M = 4\pi r^2 B(r) \quad \rightarrow \quad S = \frac{Q_M^2}{8\pi} \int dr \frac{1}{r^2}$

  - UV divergent

  - IR: $S(\text{infinite space}) - S(\text{sphere of diameter } L) = \frac{Q_M^2}{4\pi}\frac{1}{L} \underbrace{=}_{eQ_M=2\pi} \frac{1}{e^2}\frac{\pi}{L}$

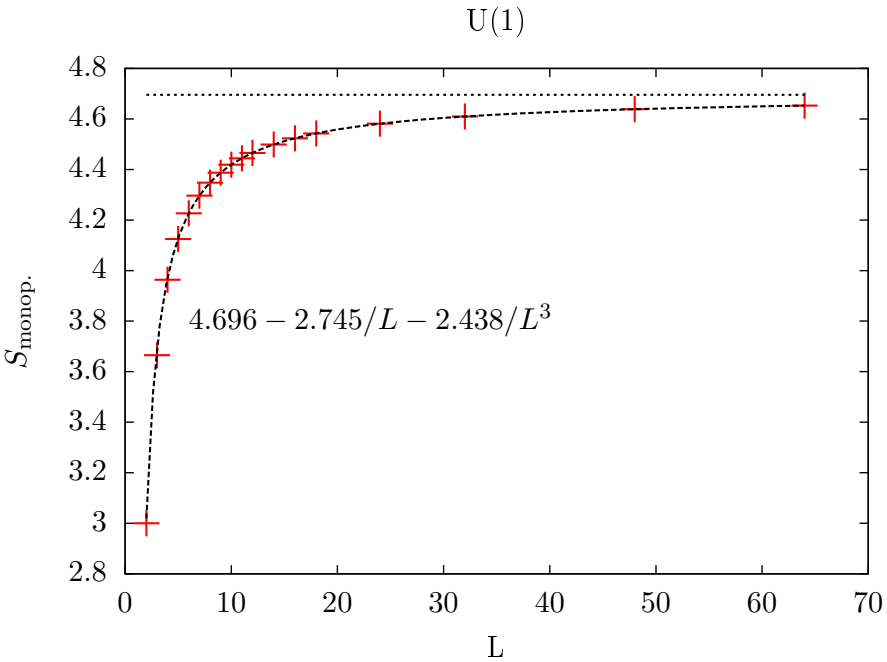
- Charge-conjugated b.c.:
  cubic array of periodic images *with alternating signs* — cf. $Na^+ Cl^-$ crystal

  - Factor $\alpha_3 \equiv \sum'_{ijk} \frac{(-1)^{i+j+k}}{\sqrt{i^2+j^2+k^2}} = -1.74756\ldots$, *Madelung constant*

  - $S(\text{infinite space}) - S(\text{box of size } L) = \frac{1}{e^2}\frac{\alpha_3}{2}\frac{\pi}{L} = \frac{2.745..}{L}$

Monopole mass (UV) action-dependent $\leftrightarrow$ $\frac{1}{L}$ corrections (IR) *universal*

Measure $U(1)$ monopole mass then embed into $U(1)$ subgroup of $SU(3)$



- $\frac{1}{L}$ coefficient as predicted
- tiny $\frac{1}{L^3}$ corrections:

$(\frac{a}{L})^2$ corrections to $\frac{1}{L}$ Coulomb potential

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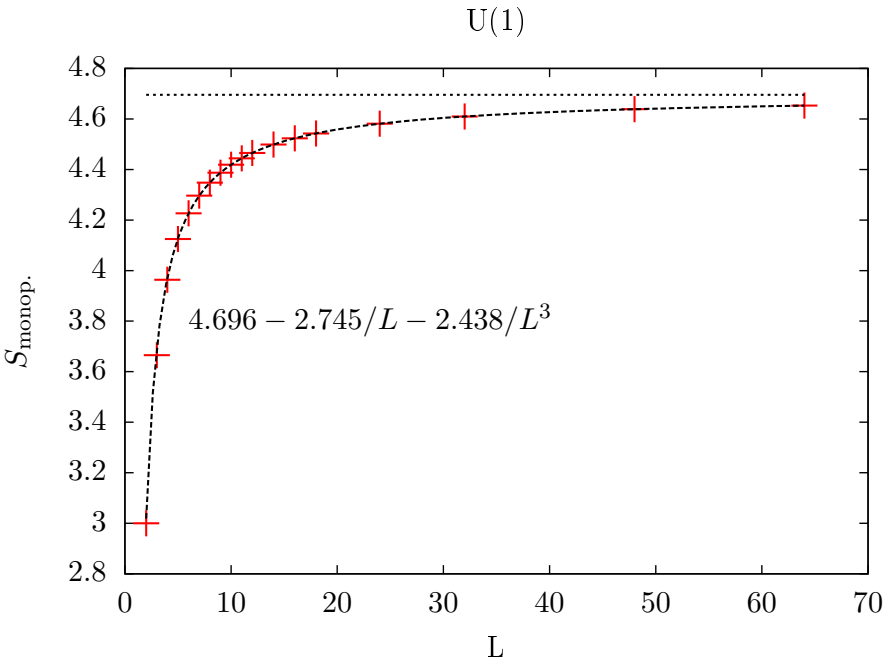
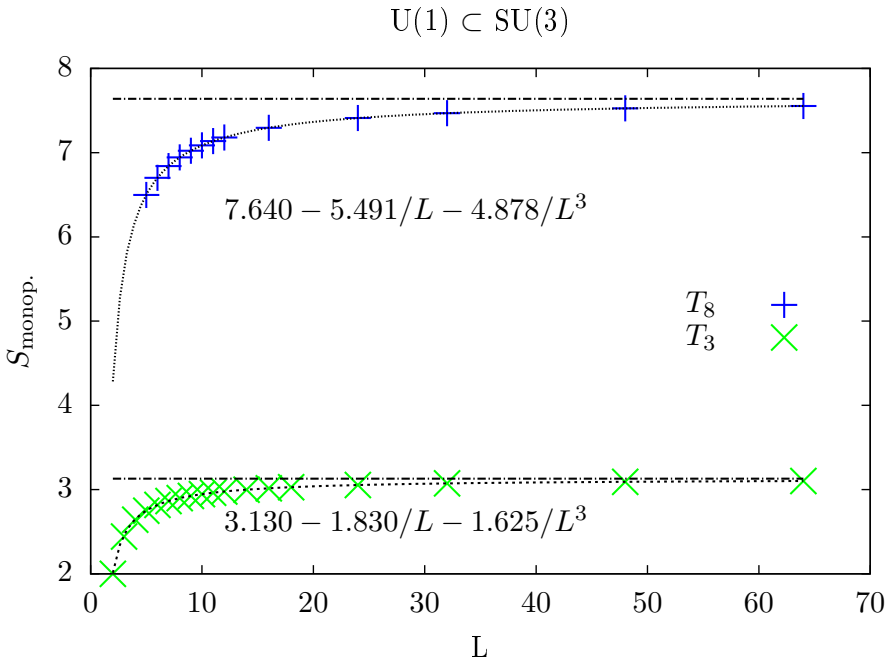


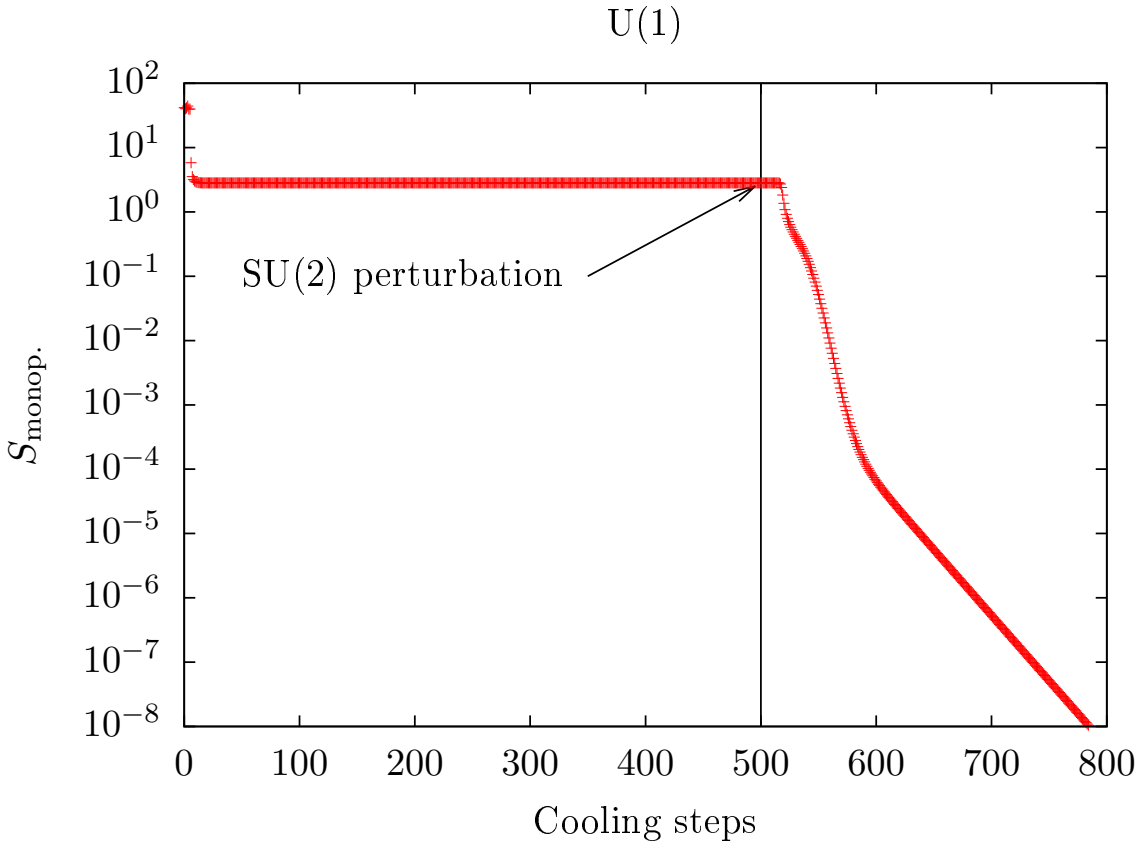


- $\frac{1}{L}$ and $\frac{1}{L^3}$ corrections multiplied by $\frac{2}{3}$ (T_3) and by 2 (T_8) as for fluxes
- Mass ($L \to \infty$) depends on subgroup Cea & Cosmai, hep-lat/0006007

Stability of $U(1)$ monopoles in gauge-symmetry broken $SU(2)$ or $SU(3)$?

- In progress
- So far: prepare monopole in $U(1) \subset SU(2)$ subgroup

Turn on full $SU(2)$ perturbation: monopole decays



Conclusions

Topological excitations can signal gauge-symmetry breaking in *gauge-invariant* way

- Flux states & monopoles are stable in $U(1)$, but decay in $SU(N)$
- Old friends embedded in new environment