Gradient Flow Analysis on MILC HISQ Ensembles

Nathan Brown
brownnathan@wustl.edu

Washington University in St. Louis

MILC Collaboration
Co-Authors: A. Bazavov, C. Bernard, C. DeTar, J. Foley, S. Gottlieb, U.M. Heller, J.E. Hetrick, J. Komijani, J. Laiho, L. Levkova, R.L. Sugar, D. Toussaint, R.S. Van de Water

Motivation

- Any dimensionful quantity with a finite continuum value can be used for scale setting.
- Eventually an experimentally determined quantity is needed to calculate in physical units, but relative scale setting can be done without physical units.
- The ideal scale setting routine:
  - easy and fast to compute
  - low statistical errors
  - insensitive to systematic effects
    (valence/sea quark mass, finite volume, etc.)
- Gradient flow is particularly promising for its ease, speed, low statistical error, and small dependence on the lattice spacing and masses.

Gradient Flow and Scales

- Gradient flow is a smoothing of the original gauge fields U towards stationary points of the action S. [Lüscher, JHEP 1008 (2010) 071]
- Successive links $V(t)$ are updated in flowtime according to the diffusion equation,

$$\frac{d}{dt}V(t)_{i,\mu} = -V_{i,\mu}\frac{\partial S(V)}{\partial V_{i,\mu}}, \quad V(0)_{i,\mu} = U_{i,\mu} \quad \left[\frac{dA_\mu}{dt} = D_\nu F_{\nu\mu}\right]$$

- Dimensionless quantities can be defined through the energy density $\langle E(t)\rangle$ and flowtime $t[a^2]$. [Lüscher, JHEP 1008 (2010) 071] and [BMW (S. Borsanyi et al.), JHEP 1209 (2012) 010]

$$T(t) = t^2\langle E(t)\rangle \qquad W(t) = t\frac{d}{dt}T(t)$$

- From which a dimensionful quantity can be determined at the fiducial point

$$T(t_0) = W(w_0^2) = 0.3$$

Measurements of w_0/a and $\sqrt{t_0}/a$ ($m_s' = m_s$)

a(fm)	m_l'/m_s'	volume	N_{run}/N_{bins}	$\sqrt{t_0}/a$	w_0/a[%]
0.15	1/5	$16^3 \times 48$	1021/127	1.1004(05)	1.1221(08)[0.07%]
0.15	1/10	$24^3 \times 48$	1000/125	1.1092(03)	1.1381(05)[0.04%]
0.15	1/27	$32^3 \times 48$	999/124	1.1136(02)	1.1468(04)[0.03%]
0.12	1/5	$24^3 \times 64$	1040/70	1.3124(06)	1.3835(10)[0.07%]
0.12	1/10	$32^3 \times 64$	999/66	1.3228(04)	1.4047(09)[0.06%]
0.12	1/10	$40^3 \times 64$	1001/66	1.3226(03)	1.4041(06)[0.04%]
0.12	1/27	$48^3 \times 64$	34/34	1.3285(05)	1.4168(10)[0.07%]
0.09	1/5	$32^3 \times 96$	102/34	1.7227(08)	1.8957(15)[0.08%]
0.09	1/10	$48^3 \times 96$	119/29	1.7376(05)	1.9299(12)[0.06%]
0.09	1/27	$64^3 \times 96$	67/16	1.7435(05)	1.9470(13)[0.07%]
0.06	1/5	$48^3 \times 144$	127/42	2.5314(13)	2.896(03)[0.11%]
0.06	1/10	$64^3 \times 144$	38/19	2.5510(14)	2.948(03)[0.11%]
0.06	1/27	$96^3 \times 192$	49/16	2.5833(07)	3.0118(19)[0.06%]

- MILC HISQ ensembles with $N_f = 2+1+1$ dynamical quarks, at physical m_s'
- Significantly more configurations can be run for ensembles with $a < 0.12$fm.
- Almost all statistical errors have been reduced below 0.1%.
- $\sqrt{t_0}$ has consistently lower statistical errors than w_0.

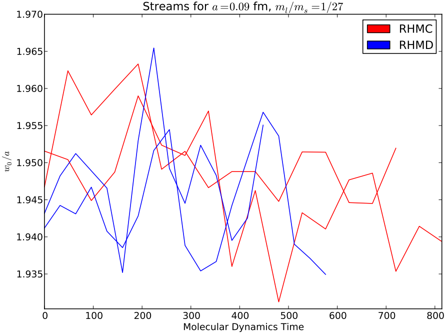
Measurements of w_0/a and $\sqrt{t_0}/a$ ($m_s' < m_s$)

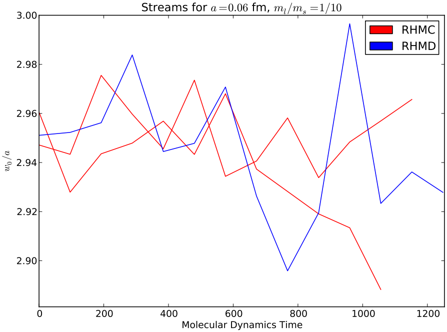
- MILC HISQ ensembles at lighter than physical m_s'.
- m_s' and m_l' are the sea quark masses of the ensemble
- m_s is the physical strange quark mass
- All ensembles are at $a = 0.12$ fm.

m_l'/m_s	m_s'/m_s	volume	N_{run}/N_{bins}	$\sqrt{t_0}/a$	w_0/a[%]
0.10	0.10	$32^3 \times 64$	102/25	1.3596(06)	1.4833(13)[0.08%]
0.10	0.25	$32^3 \times 64$	204/51	1.3528(04)	1.4676(10)[0.07%]
0.10	0.45	$32^3 \times 64$	205/51	1.3438(05)	1.4470(10)[0.07%]
0.10	0.60	$32^3 \times 64$	107/26	1.3384(08)	1.4351(16)[0.11%]
0.175	0.45	$32^3 \times 64$	133/33	1.3385(05)	1.4349(13)[0.09%]
0.20	0.60	$24^3 \times 64$	255/63	1.3297(06)	1.4169(12)[0.08%]
0.25	0.25	$24^3 \times 64$	255/63	1.3374(07)	1.4336(14)[0.10%]

Stream Analysis: RHMC vs RHMD

- There are 3 HISQ ensembles with streams generated on RHMD and RHMC. w_0 was measured for streams of both types on 2 of these ensembles.
- No significant difference was found between w_0/a measured on ensembles generated with RHMD compared to RHMC.
    - For $a = 0.09$fm, $m_l/m_s = 1/27$: $w_0(RHMC)/w_0(RHMD) = 1.0009(12)$
    - For $a = 0.06$fm, $m_l/m_s = 1/10$: $w_0(RHMC)/w_0(RHMD) = 1.0002(26)$





Charm Quark Mistuning

- From decoupling analysis, variations in the charm quark mass can be accounted for through the leading dependence of the QCD scale with three flavors $\Lambda_{QCD}^{(3)}$. If $Q \propto \Lambda_{QCD}^{(3)}$, then

$$\frac{\partial Q}{\partial m_c} = \frac{2}{27}\frac{Q}{m_c}$$

- The meson masses aM_π, aM_K, scales w_0/a, $\sqrt{t_0}/a$, and decay constant aF_{p4s} were adjusted.
  - F_{p4s} is the pseudoscalar decay constant with degenerate valence masses $m_{val} = 0.4 m_s$ and physical sea quark masses
- Because of small errors in the quantities studied and large charm mass mistunings ($\sim 10\%$ in some cases) adjustments were often significant.
  - Meson mass corrections ranged from 0.3 to $7\sigma_{\text{stat}}$
  - Gradient flow scale corrections ranged from 0.5 to $18\sigma_{\text{stat}}$
- Adjustments were largest for the unphysical quark mass, $a = 0.15$ and $a = 0.06$ fm ensembles.

Chiral Expansion

- Including quark mass dependence allows us to include ensembles with $m_s' \neq m_s$ and correct for mistuning errors.
- For the continuum, $N_f = 2+1$ theory the mass dependence of w_0 to NNLO is: [0. Bär and M. Golterman, Phys. Rev. D 89, 034505 (2014)]

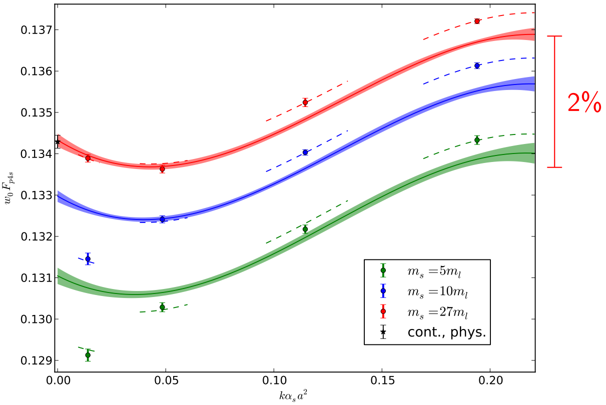
$$\begin{aligned} w_0 &= w_{0,ch}\left(1 + k_1 \frac{2M_K^2 + M_\pi^2}{(4\pi f)^2}\right. \\ &+ \frac{1}{(4\pi f)^2}\left((3k_2 - k_1)M_\pi^2\mu_\pi + 4k_2 M_K^2\mu_K + \frac{k_1}{3}(M_\pi^2 - 4M_K^2)\mu_\eta + k_2 M_\eta^2\mu_\eta\right) \\ &+ \left. k_4\frac{(2M_K^2 + M_\pi^2)^2}{(4\pi f)^4} + k_5\frac{(M_K^2 - M_\pi^2)^2}{(4\pi f)^4}\right), \quad \mu_Q = \frac{M_Q^2}{(4\pi f)^2}\log\frac{M_Q^2}{\mu^2} \end{aligned}$$

- Expansion for $\sqrt{t_0}$ is identical in form, because the meson masses are independent of flowtime.

Combined Continuum Extrapolation

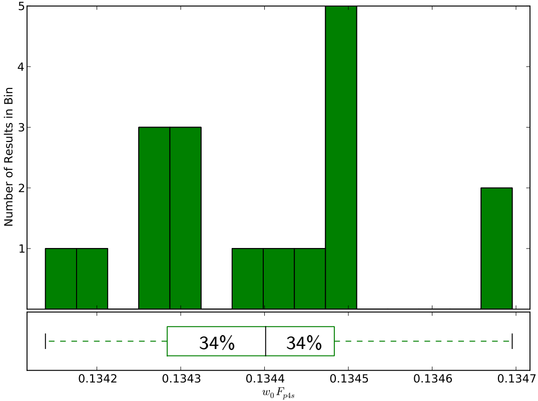
- We simultaneously performed the continuum extrapolation and meson mass interpolation, using F_{p4s} to make all masses and the gradient flow scale dimensionless.
- We considered many different versions of the extrapolation/interpolation:
  - To extrapolate to the continuum, we included $\alpha_s a^2$ and possible higher orders of a^2 (with or without α_s), up to a^6.
  - Both NLO and NNLO chiral expansions were considered.
  - Products between terms in the chiral and continuum expansions were included up to the same order as the highest of other included terms. For this purpose, $(\Lambda_{QCD} a)^2 \sim (M/(4\pi f))^2$.
  - Due to the large range of M_K covered by the full set of ensembles, some fits drop ensembles with low values of M_K.
- Overall, we consider $5_{\text{cont}} \times 2_{\text{chiral}} \times 7_{\text{kaon}} = 70$ versions of the fit.

Central Extrapolation for Physical m_s



- The central fit form is up to $(\alpha_s a^2)^3$, chiral NNLO, and across all values of M_k.
- Only $m_s = m_s^{physical}$ ensembles are plotted, but fit includes all $m_s \leq m_s^{physical}$ ensembles
- Dotted lines are for actual masses run; solid lines are for re-tuned masses per legend
- Curvature typical of highly improved actions: "leading" term reduced, so "higher" terms evident

Histogram of Extrapolated $w_0 F_{p4s}$

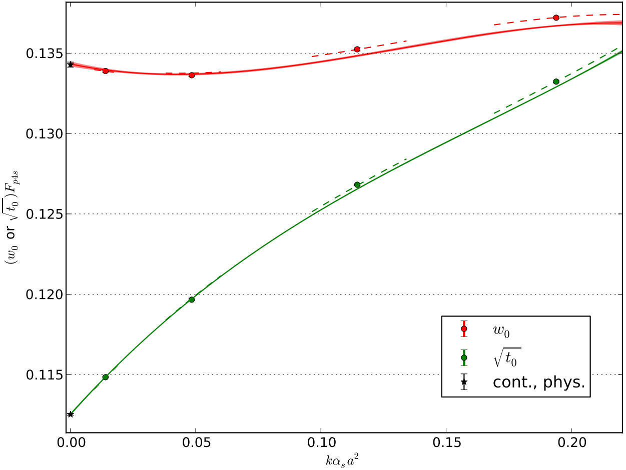


- Histogram only includes fits with p-value> 0.01.
- This only included fits with a^6 or $(\alpha_s a^2)^3$ terms.
- Both NLO and NNLO chiral expansions are represented:
  - For NLO, ensembles with $M_K / F_{p4s} > 0.8$ are included
  - For NNLO, all ensembles can be included

Continuum, Physical Mass Results

- The central fit has $\chi^2/dof = 10.6/10$, $p = 0.39$, and is 0.1σ from the physical 0.06 fm ensemble.
- Half the full width of the histogram is used to conservatively estimate a systematic fit error of 4×10^{-4}
- There is also residual finite volume error in F_{p4s} (coming from residual FV error in f_π, which is estimated using χPT) that cannot be corrected for, adding another systematic error of 2×10^{-4} fm.
- Result: $w_0 = 0.1722(2)_{\text{stat}}(4)_{a^2}(2)_{\text{FV}}(3)_{F_{p4s}}$ fm
  First is the statistical error, then systematic error from the continuum extrapolation, residual finite volume effects, and the value of F_{p4s} in MeV, respectively.

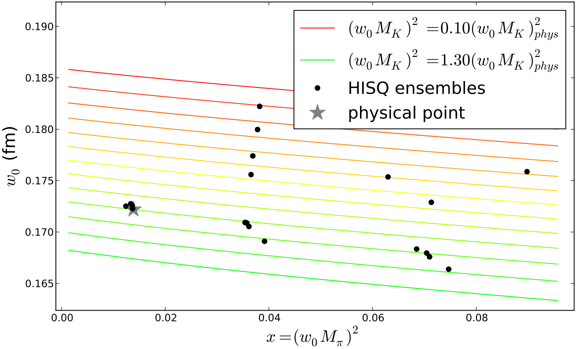
Comparison of w_0 and $\sqrt{t_0}$



- Comparison of $w_0 F_{p4s}$ and $\sqrt{t_0} F_{p4s}$ plotted for physical mass ensembles
- The fits are over the full dataset and for the same central fit form shown before.
- Discretization dependence of $\sqrt{t_0} F_{p4s}$ is larger.

Mass Dependence

- The central fit can be used to construct the continuum mass dependence of w_0 with masses in units of w_0.
- This is useful for scale setting: measure w_0/a, aM_π, and aM_K to construct the independent variables $x = (w_0 M_\pi)^2$ and $y = (w_0 M_K)^2$, then read off $w_0(x, y)$(fm) from the plot (or a corresponding interpolation).



- Lines are for fixed values of $y = (w_0 M_K)^2$ from 0.1 to 1.3 times the physical value.

Summary

- Our preliminary value of
$$w_0 = 0.1722(2)_{\mathrm{stat}}(4)_{a^2}(2)_{\mathrm{FV}}(3)_{F_{p4s}} \text{ fm}$$
agrees with HPQCD within 1σ, but deviates from BMW by 1.7σ (joint) compared to their HEX smeared, Wilson result $w_0 = 0.1755(18)(04)$ fm.
[HPQCD (R. J. Dowdall et al.), arXiv:1303.1670] [BMW (S. Borsanyi et al.), JHEP 1209 (2012) 010]
- Some of this deviation may be due to the difference in N_f. We will be computing w_0 on the asqtad $N_f = 2 + 1$ ensembles.
- Charm mass mistunings can have a significant effect for precise quantities, such as w_0.
- Compared to Lat'13, systematic error from the extrapolation/interpolation is cut in half; this is primarily due to charm mass adjustments and the χPT handle on mass dependence.
- Evidence was provided that the discretization effects of $w_0 F_{p4s}$ are smaller than $\sqrt{t_0} F_{p4s}$, in agreement with what BMW found.