JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

TECHNISCHE UNIVERSITÄT DARMSTADT

Lattice simulations of G_2-QCD at finite density I

32nd International Symposium on Lattice Field Theory

Lattice 2014

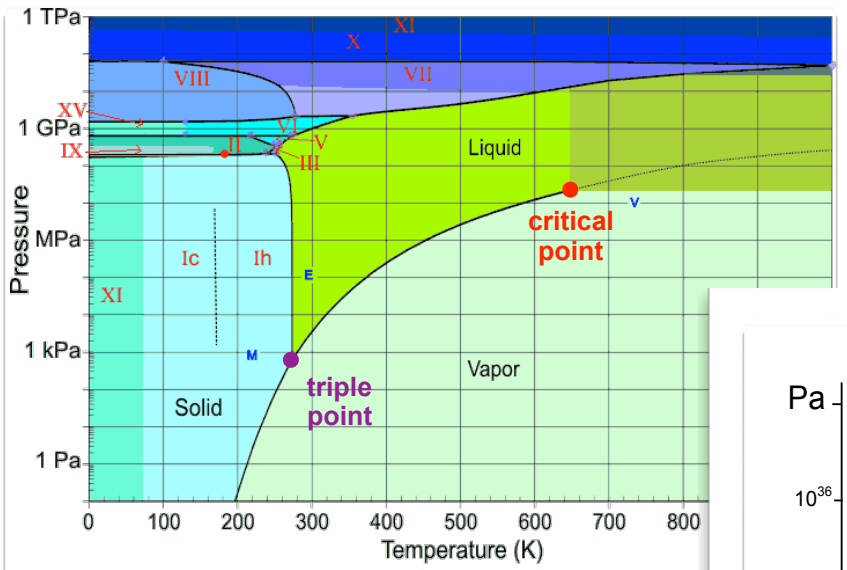
Columbia University, NY, 23 June 2014

Lorenz von Smekal

HIC for FAIR
Helmholtz International Center

Contents

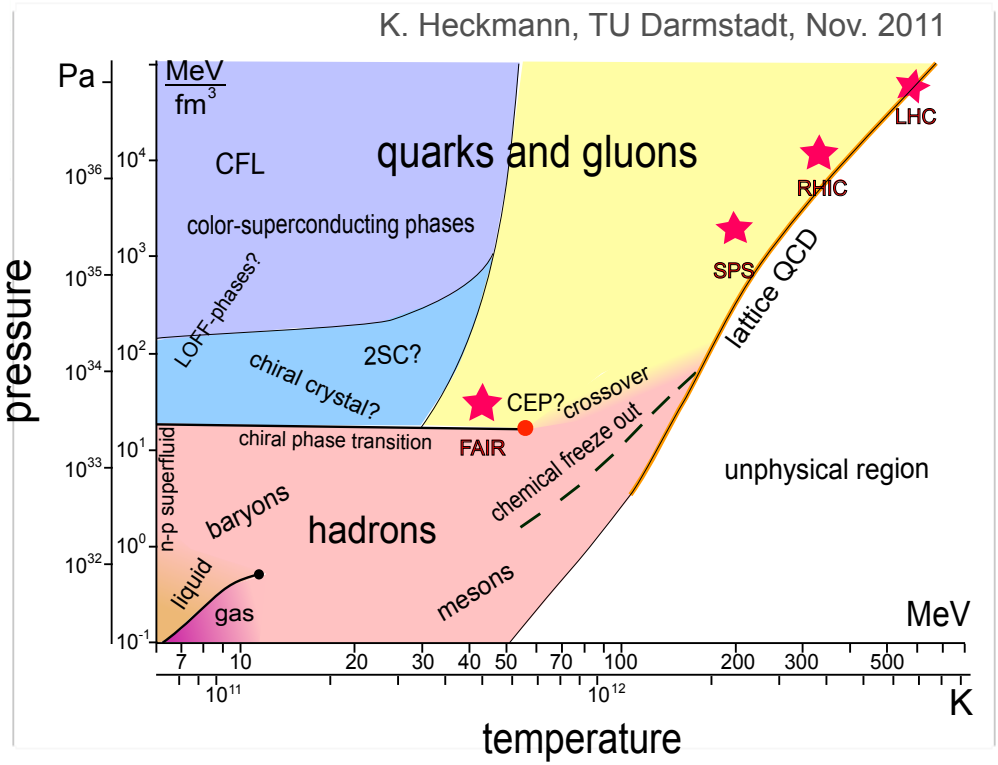
Phase Diagram



http://www.lsbu.ac.uk/water/phase.html

Water

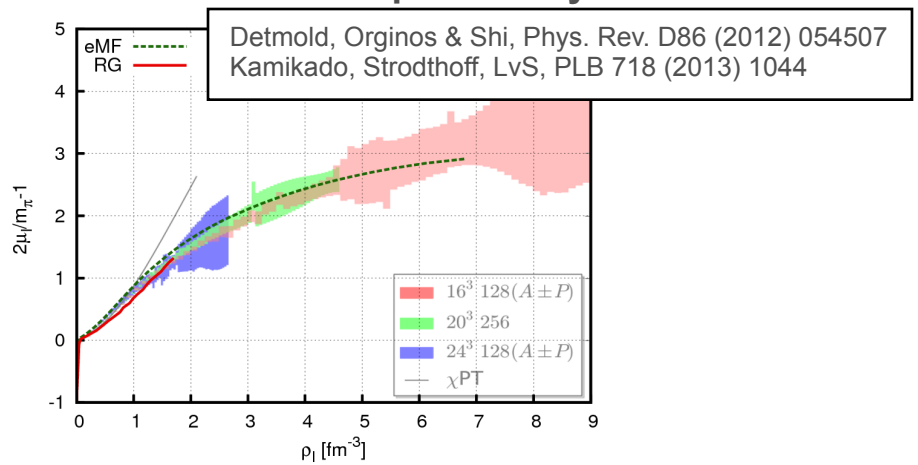
QCD



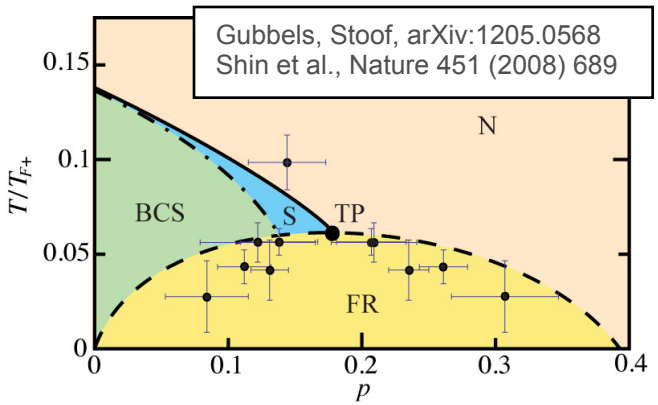
QCD-like Theories

- **compare lattice simulations with functional methods and effective models**
  where there's no sign problem
- **apply to ultracold fermi gases**
  exploit analogies and
  more experimental data

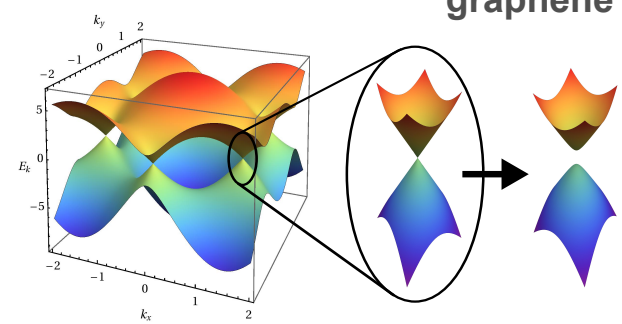
QCD at finite isospin density



polarised fermi gas at unitarity



graphene



- **strongly correlated fermions in 2+1 dimensions**
  electronic properties of graphene

[see Dominik Smith's talk on Thu, Applications beyond QCD]

Fermion-Sign Problem

sign problem:

$$\left(\mathrm{Det}\, D(\mu_f)\right)^* = \mathrm{Det}\, D(-\mu_f)$$

- **except if:**

(a) two degenerate flavors with isospin chemical potential

Dyson index:

fermion determinant $\rightsquigarrow \mathrm{Det}\big(D(\mu_I)D(-\mu_I)\big)$ $\beta = 2$

QCD at finite isospin density

(b) anti-unitary symmetry $TD(\mu)T^{-1} = D(\mu)^*$ $T^2 = \pm 1$

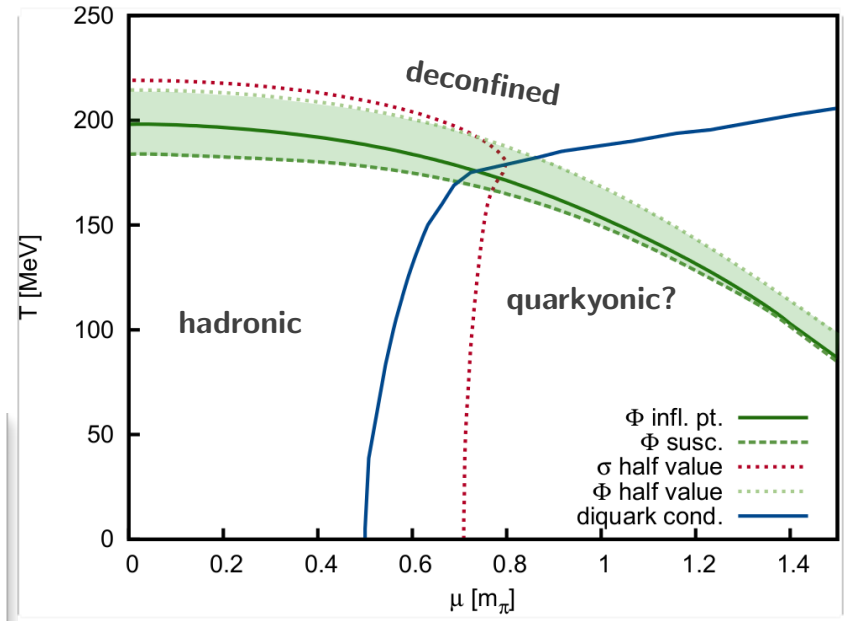
fermion color representation:

(i) pseudo-real $T^2 = 1$ **two-color QCD** $\beta = 1$

(ii) real $T^2 = -1$ **adjoint QCD, or G_2-QCD** $\beta = 4$

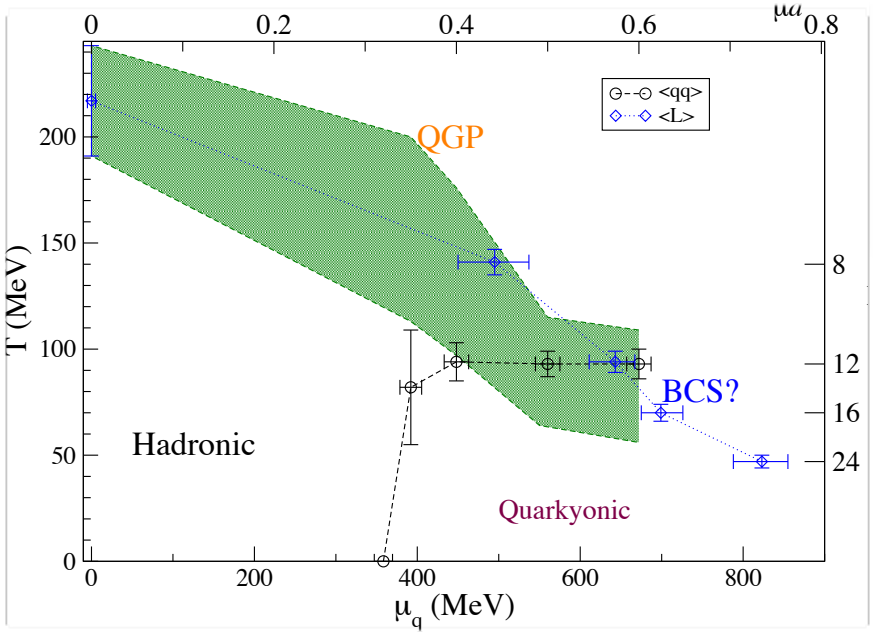
Two-Color QCD

- **Polyakov-Quark-Meson-Diquark model phase diagram:**



Strodthoff & L.v.S., PLB 731 (2014) 350

- **Lattice simulations:**



Can we describe the two-color world with the 3d effective lattice theory for heavy quarks? [cf. Philipp Scior's talk]

Cotter, Giudice, Hands & Skullerud, PRD 87 (2013) 034507

G_2 Gauge Theory

- **smallest exceptional Lie group**
  subgroup of SO(7)
- **rank = 2 (as SU(3)), dimension = 14**
  7 colors, 14 gluons
  fund. reps.: $\{7\} = (1,0)$, $\{14\} = (0,1)$ (= adjoint)

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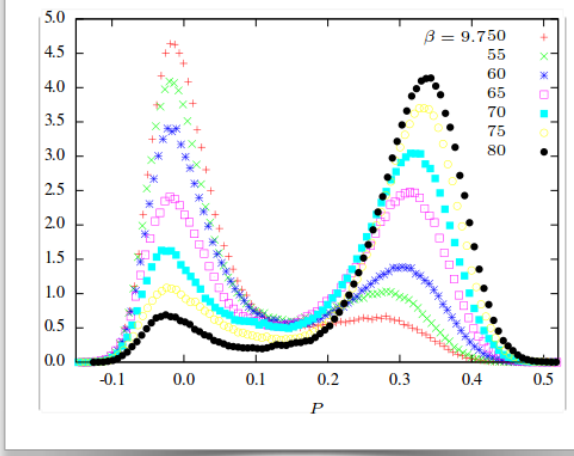
  yet (as SU(3)), 1st order deconfinement finite *T* phase transition in pure gauge theory (also in chiral condensate)



Holland, Minkowski, Pepe & Wiese, Nucl. Phys. B 668 (2003) 207

Pepe & Wiese, NPB 768 (2007) 21

Danzer, Gattringer, Maas, JHEP 01 (2009) 024

Wellegehausen, Wipf & Wozar, PRD 83 (2011) 114502

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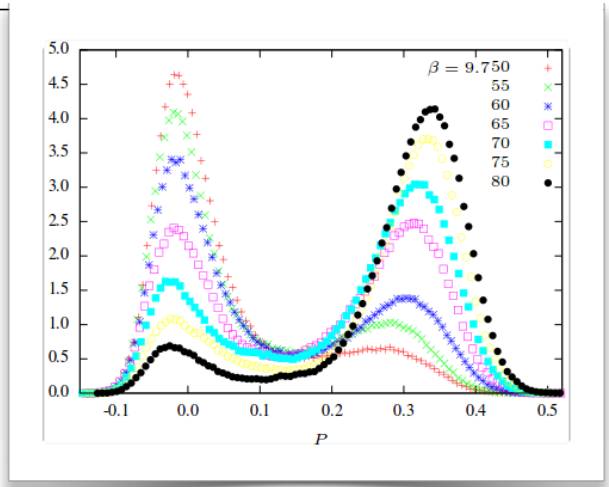
- **simple & simply connected, no center**
  yet (as SU(3)), 1st order deconfinement finite *T* phase transition in pure gauge theory (also in chiral condensate)
- **all reps. real**
  Dirac operator $\mathrm{D}(\mu)$ has antiunitary symmetry S, with $\mathrm{S}^2 = -1$ (symplectic, $\beta = 4$) and extended $\mathrm{SU}(2\mathrm{N_f})$ flavor symmerty
- **no sign problem**
  real and positive for single flavor: $\mathrm{SU}(2) \to \mathrm{U}(1)_\mathrm{B}$
  2 Goldstone bosons: scalar (anti)diquarks



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G_2 Gauge Theory

- **breaks down to QCD**

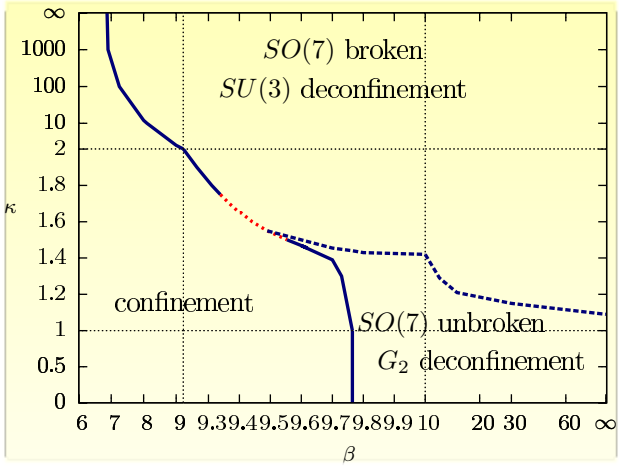
$$G_2 \xrightarrow{\text{Higgs}} SU(3)$$

coset:

$$G_2/SU(3) \sim SO(7)/SO(6) \sim S^6$$

$$\{7\} \to \{3\} \oplus \{\bar{3}\} \oplus \{1\}$$

$$\{14\} \to \{3\} \oplus \{\bar{3}\} \oplus \{8\}$$



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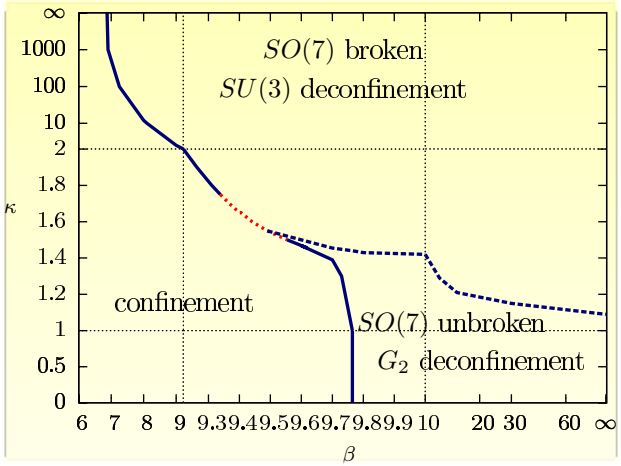
massive Higgs

gluons

heavy gauge



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G_2 Gauge Theory

- **breaks down to QCD**

Higgs

$$G_2 \longrightarrow SU(3)$$

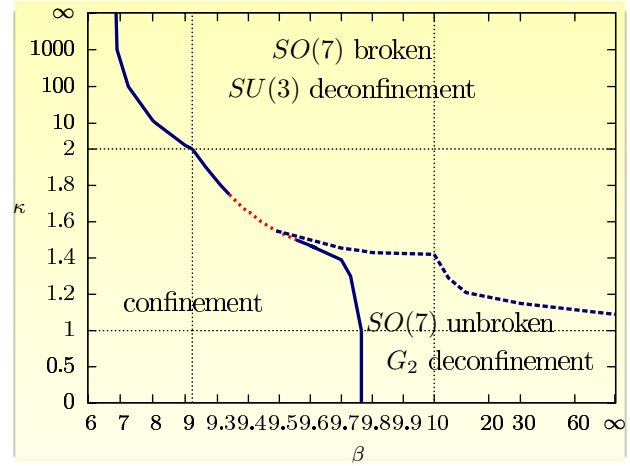
coset:

$$G_2/SU(3) \sim SO(7)/SO(6) \sim S^6$$

$$\{7\} \to \{3\} \oplus \{\bar{3}\} \oplus \{1\}$$ ← massive Higgs

$$\{14\} \to \{3\} \oplus \{\bar{3}\} \oplus \{8\}$$ ← gluons

heavy gauge



Wellegehausen, Wipf & Wozar, PRD 83 (2011) 114502

- **G_2 glueball sectrum, Casimir scaling & string breaking**

  Pepe & Wiese, NPB 768 (2007) 21

  Wellegehausen *et al.*, PRD 83 (2011) 016001

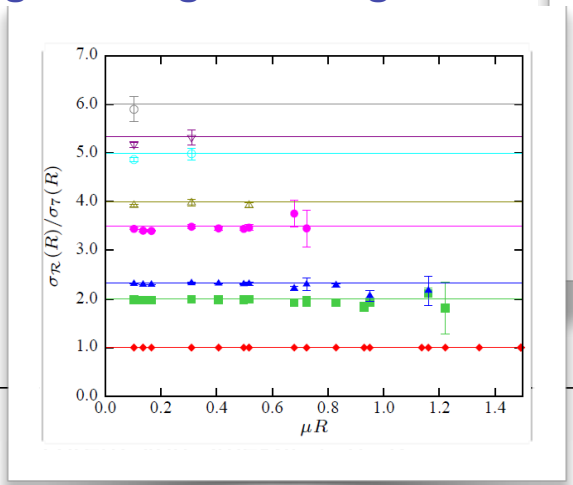
  Lacroix *et al.*, PRD 87 (2013) 054025

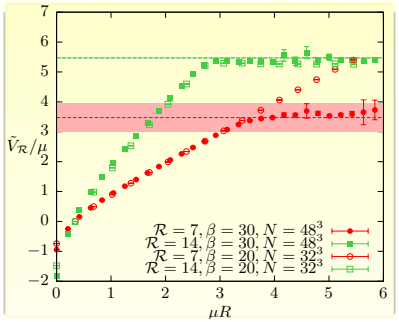
- **vortices, monopoles, instantons...**

  Greensite *et al.*, PRD 75 (2007) 034501

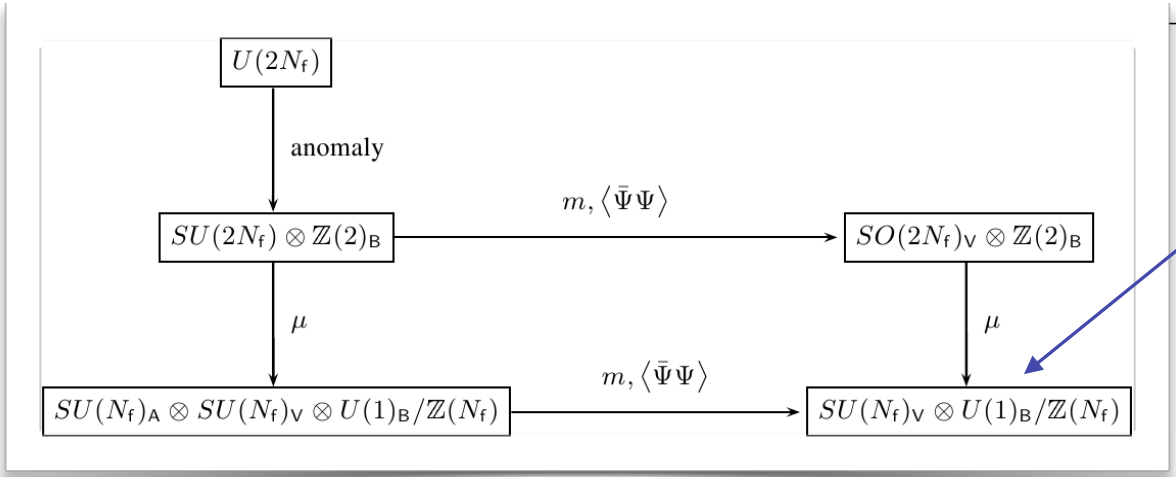
  Di Giacomo *et al.*, JHEP 10 (2008) 096

  Ilgenfritz & Maas, PRD 86 (2012) 114508



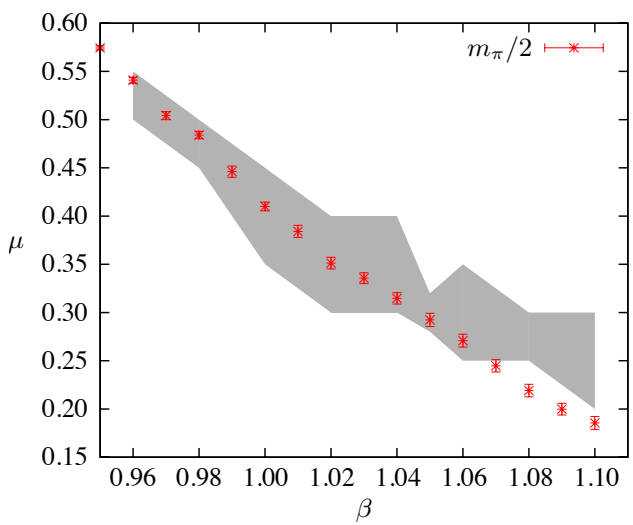


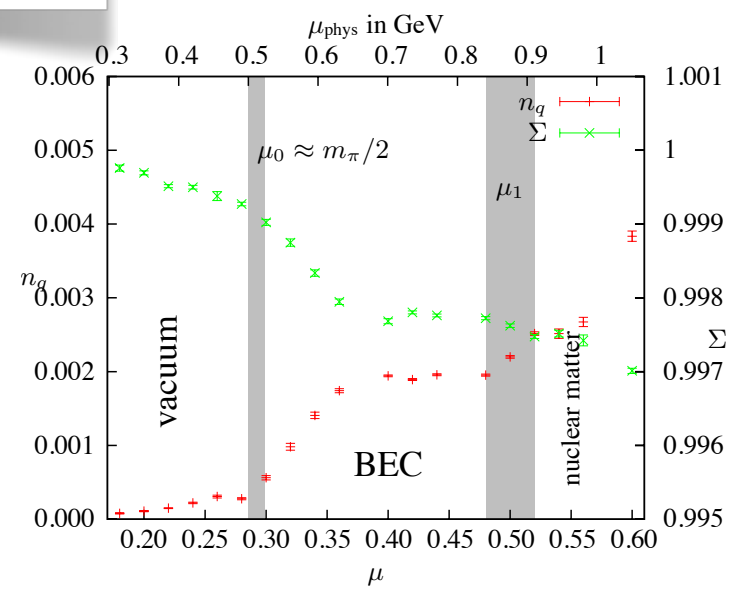
G_2-QCD at Finite Density



$U(1)_B$ breaks spontaneously at $\mu_B = m_{d_0^+} = m_\pi$

- diquark condensation as in QC_2D



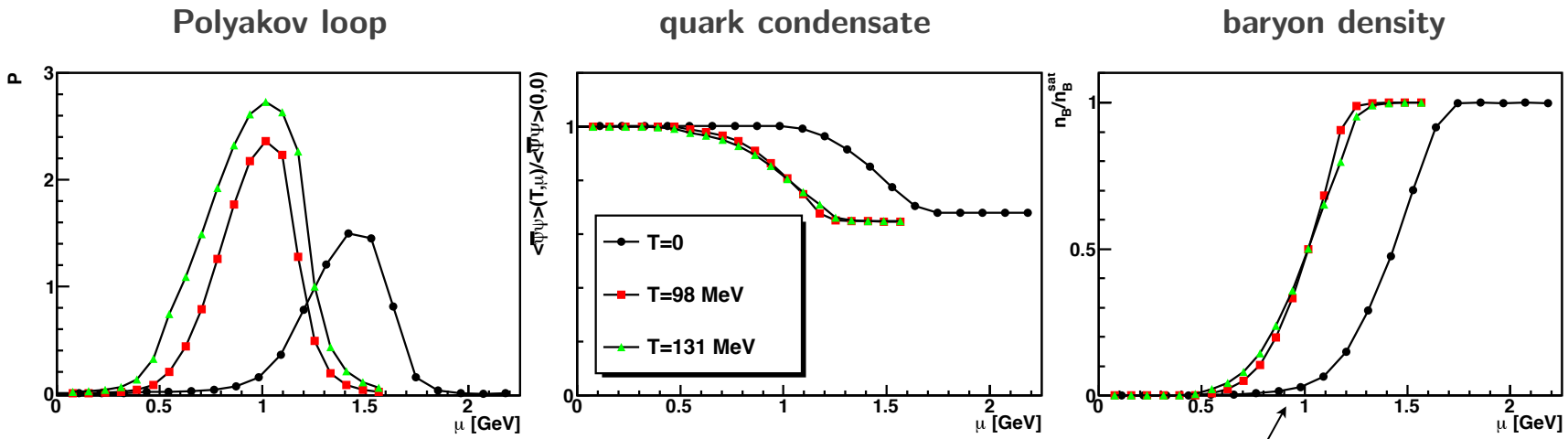


Bjoern Wellegehausen, PhD thesis, Jena 2012

G_2 Gauge Theory at Finite Density

- **but has fermionic baryons also**
  (as adjoint QCD, in principle)

- **finite baryon density (bosonic and fermionic)**

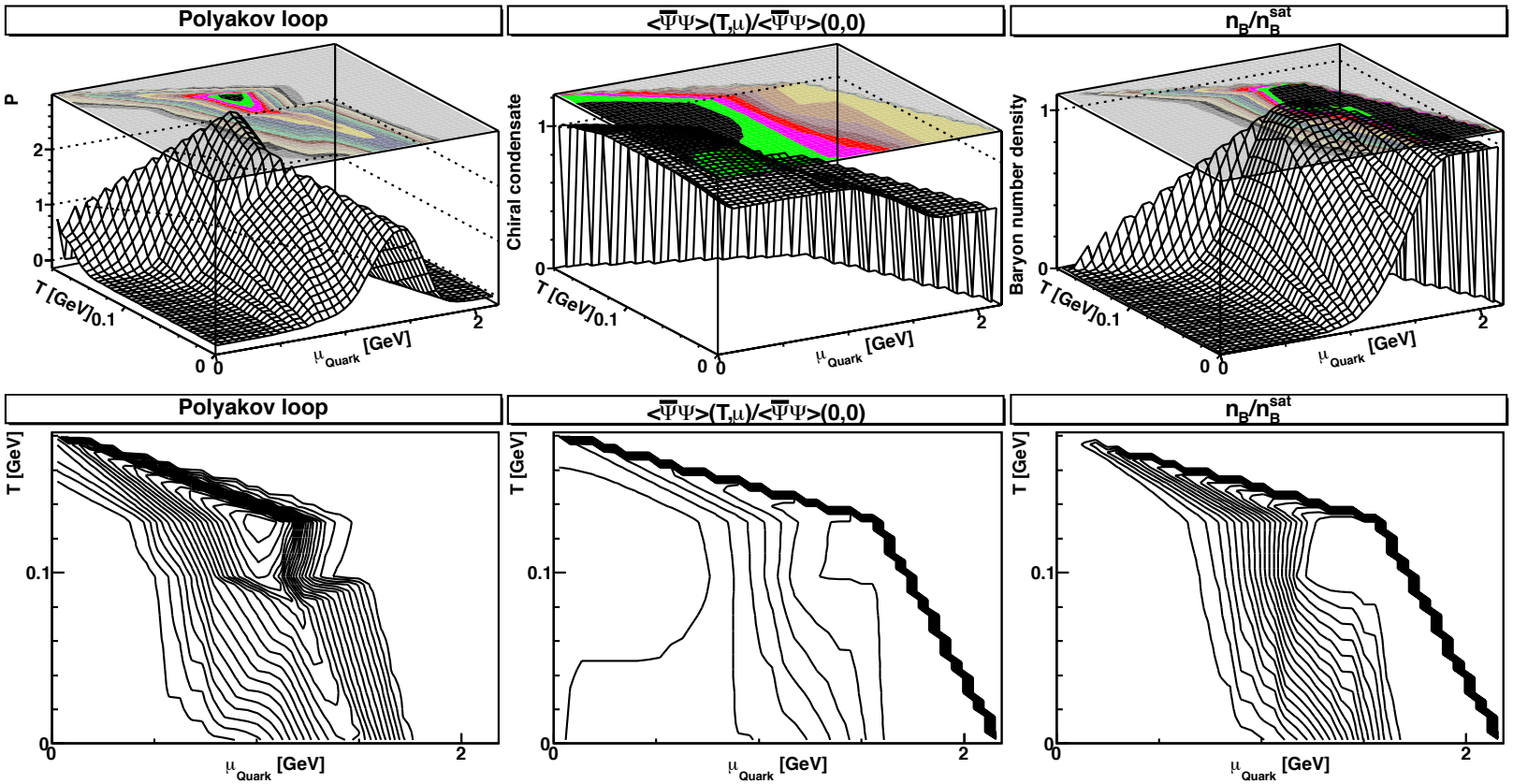


G_2 nuclear matter?

Maas, LvS, Wellegehausen & Wipf, Phys. Rev. D 86 (2012) 111901R

G_2-QCD Phase Diagram

- 1 flavor dynamcial Wilson



Maas, LvS, Wellegehausen & Wipf, Phys. Rev. D 86 (2012) 111901R

G_2 Spectroscopy

$$\{7\} \otimes \{7\} = \{1\} \oplus \{7\} \oplus \{14\} \oplus \{27\}$$
$$\{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus 4 \cdot \{7\} \oplus 2 \cdot \{14\} \oplus \ldots$$
$$\{14\} \otimes \{14\} = \{1\} \oplus \{14\} \oplus \{27\} \oplus \ldots,$$
$$\{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \{7\} \oplus 5 \cdot \{14\} \oplus \ldots,$$
$$\{7\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \ldots$$

T: (x, s, C, F)

mesons (baryon number 0)

Name	$\mathcal{O}$	T	J	P	C
π	$\bar{u}\gamma_5 d$	SASS	0	-	+
η	$\bar{u}\gamma_5 u$	SASS	0	-	+
a	$\bar{u}d$	SASS	0	+	+
f	$\bar{u}u$	SASS	0	+	+
ρ	$\bar{u}\gamma_\mu d$	SSSA	1	-	+
ω	$\bar{u}\gamma_\mu u$	SSSA	1	-	+
b	$\bar{u}\gamma_5\gamma_\mu d$	SSSA	1	+	+
h	$\bar{u}\gamma_5\gamma_\mu u$	SSSA	1	+	+

diquarks (baryon number 2)

Name	$\mathcal{O}$	T	J	P	C
$d(0^{++})$	$\bar{u}^C\gamma_5 u + c.c.$	SASS	0	+	+
$d(0^{+-})$	$\bar{u}^C\gamma_5 u - c.c.$	SASS	0	+	-
$d(0^{-+})$	$\bar{u}^C u + c.c.$	SASS	0	-	+
$d(0^{--})$	$\bar{u}^C u - c.c.$	SASS	0	-	-
$d(1^{++})$	$\bar{u}^C\gamma_\mu d - \bar{d}^C\gamma_\mu u + c.c.$	SSSA	1	+	+
$d(1^{+-})$	$\bar{u}^C\gamma_\mu d - \bar{d}^C\gamma_\mu u - c.c.$	SSSA	1	+	-
$d(1^{-+})$	$\bar{u}^C\gamma_5\gamma_\mu d - \bar{d}^C\gamma_5\gamma_\mu u + c.c.$	SSSA	1	-	+
$d(1^{--})$	$\bar{u}^C\gamma_5\gamma_\mu d - \bar{d}^C\gamma_5\gamma_\mu u - c.c.$	SSSA	1	-	-

exotic particles (baryon number 1)

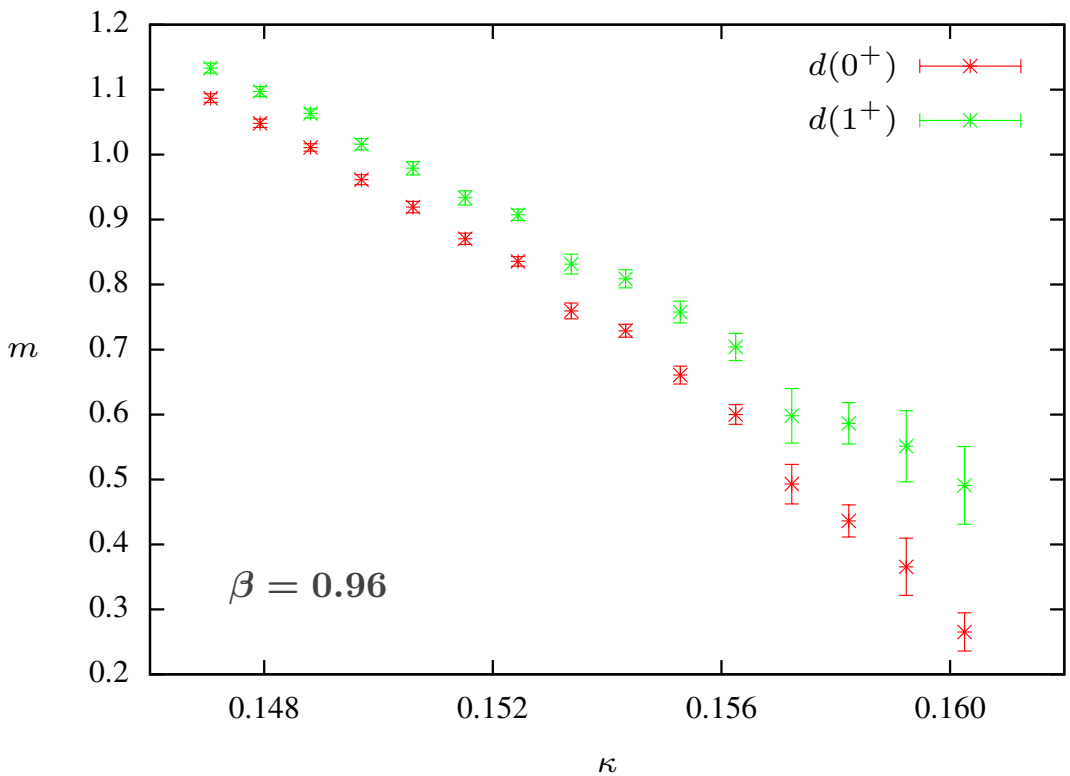
Name	$\mathcal{O}$	T	J	P	C
N'	$T^{abc}(\bar{u}_a\gamma_5 d_b)u_c$	SAAA	1/2	$\pm$	$\pm$
Δ'	$T^{abc}(\bar{u}_a\gamma_\mu u_b)u_c$	SSAS	3/2	$\pm$	$\pm$
Hybrid	$\epsilon_{abcdefg} u^a F^{bc}_{\mu\nu} F^{de}_{\mu\nu} F^{fg}_{\mu\nu}$	SSSS	1/2	$\pm$	$\pm$

baryons (baryon number 3)

Name	$\mathcal{O}$	T	J	P	C
N	$T^{abc}(\bar{u}^C_a\gamma_5 d_b)u_c$	SAAA	1/2	$\pm$	$\pm$
Δ	$T^{abc}(\bar{u}^C_a\gamma_\mu u_b)u_c$	SSAS	3/2	$\pm$	$\pm$

G_2 Spectroscopy

- ***N_f = 1:*** real and positive for single flavor: $SU(2) \to U_B(1)$
  2 Goldstone bosons: scalar (anti)diquarks



- ***N_f = 2:***
  exact mass relations

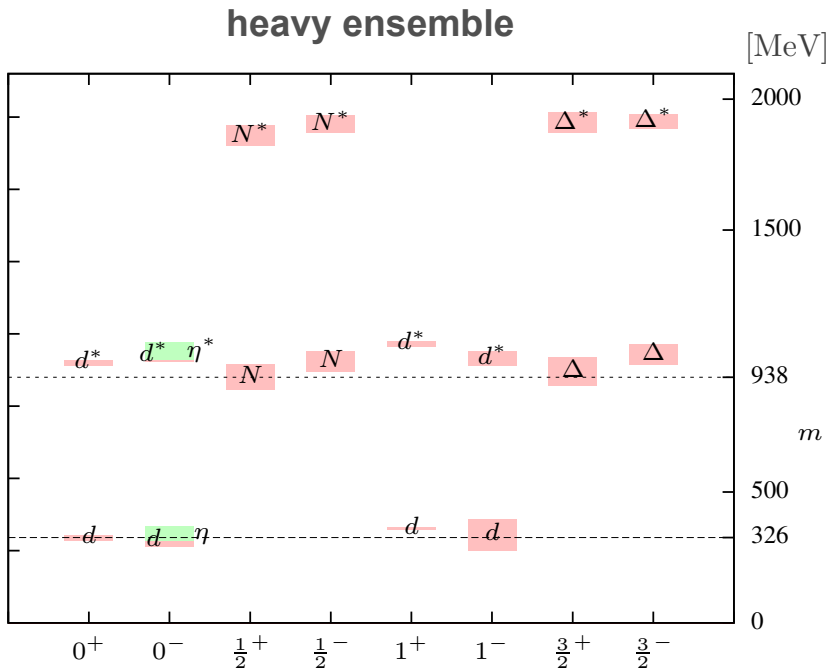
$$m_{d(0^+)} = m_{\pi(0^-)}$$

$$m_{d(1^+)} = m_{\rho(1^-)}$$

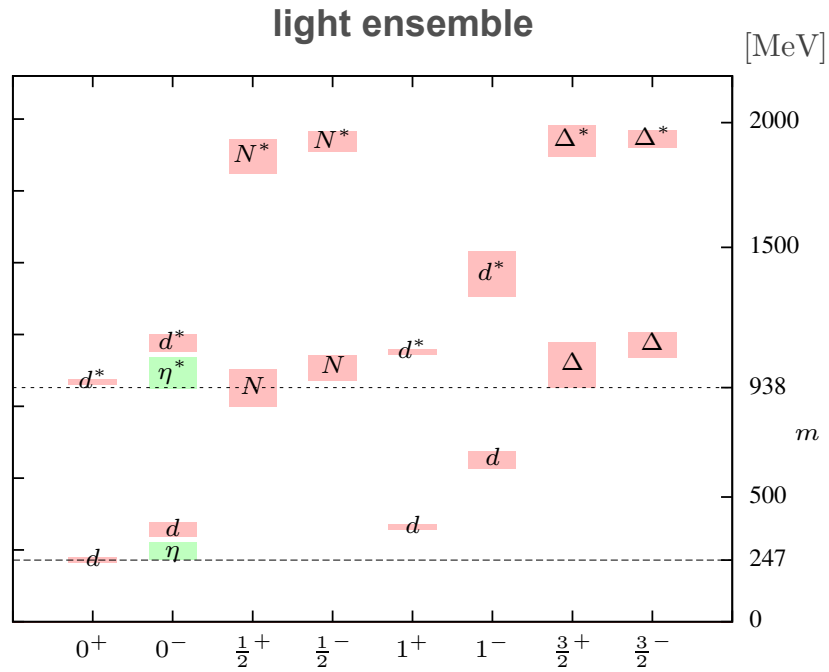
Name	β	κ	$m_{d(0^+)}$
Heavy ensemble	1.05	0.147	326 MeV
Light ensemble	0.96	0.15924	247 MeV

Wellegehausen, Maas, Wipf & LvS, PRD 89 (2014) 056007

G_2 Spectroscopy



$\beta = 1.05,\ \kappa = 0.147$

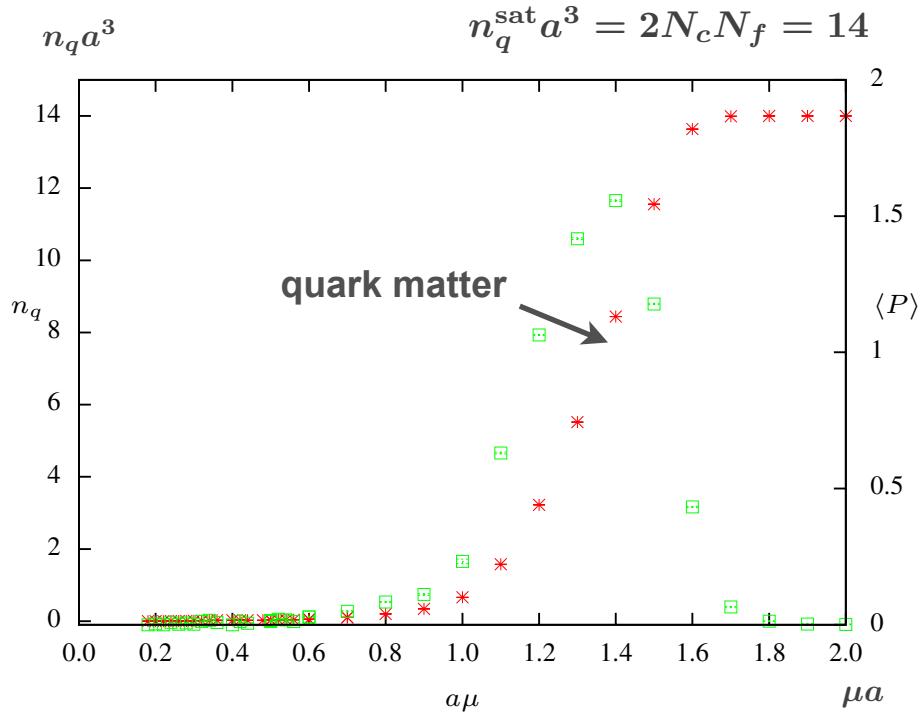


$\beta = 0.96,\ \kappa = 0.15924$

Wellegehausen, Maas, Wipf & LvS, PRD 89 (2014) 056007

Finite Baryon Density

heavy ensemble



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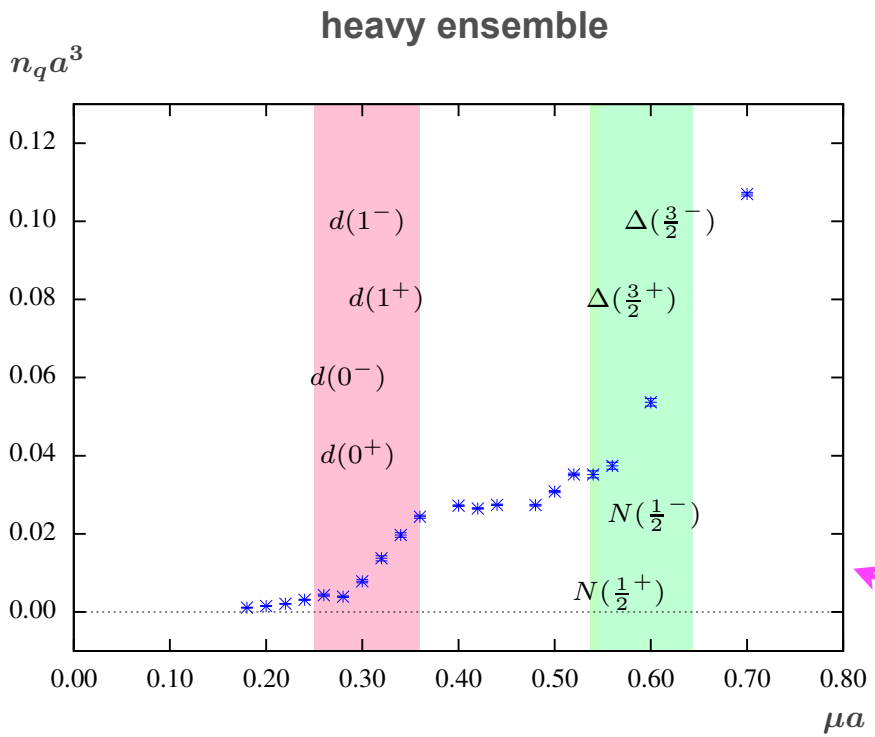

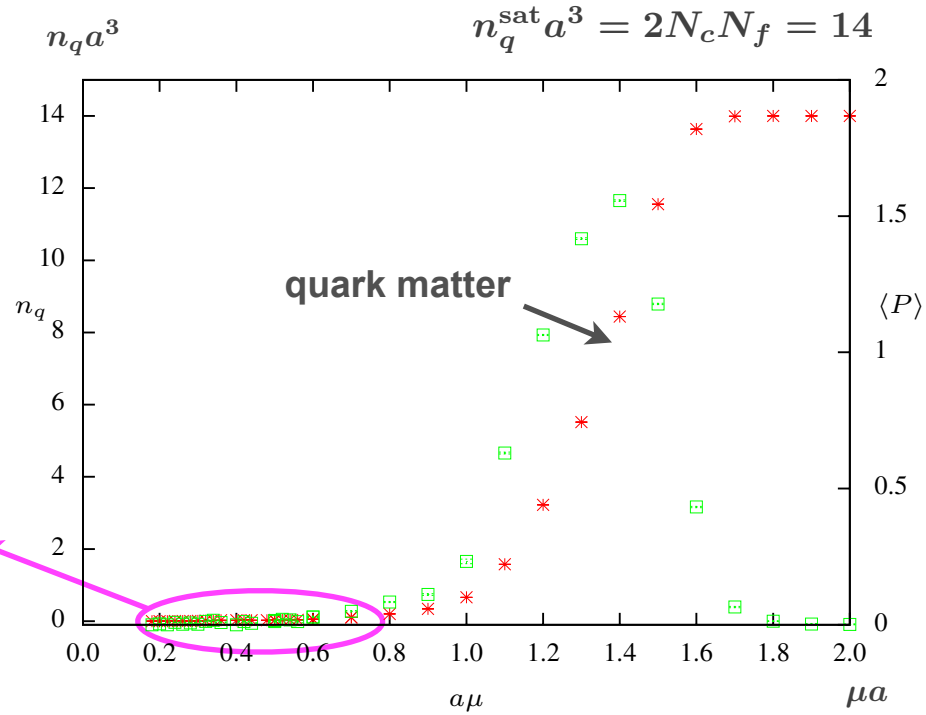

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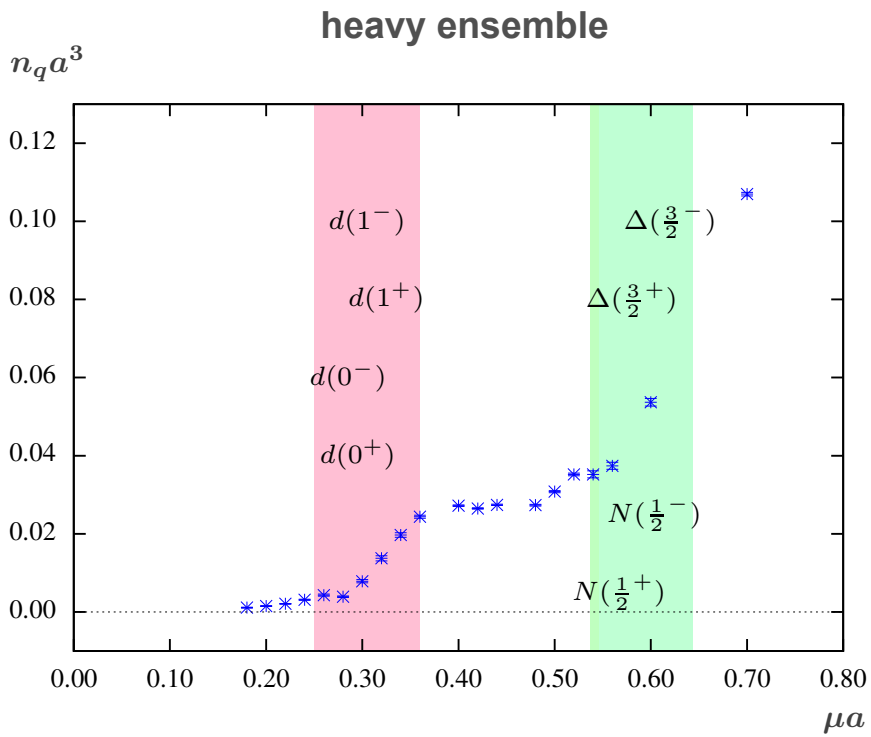

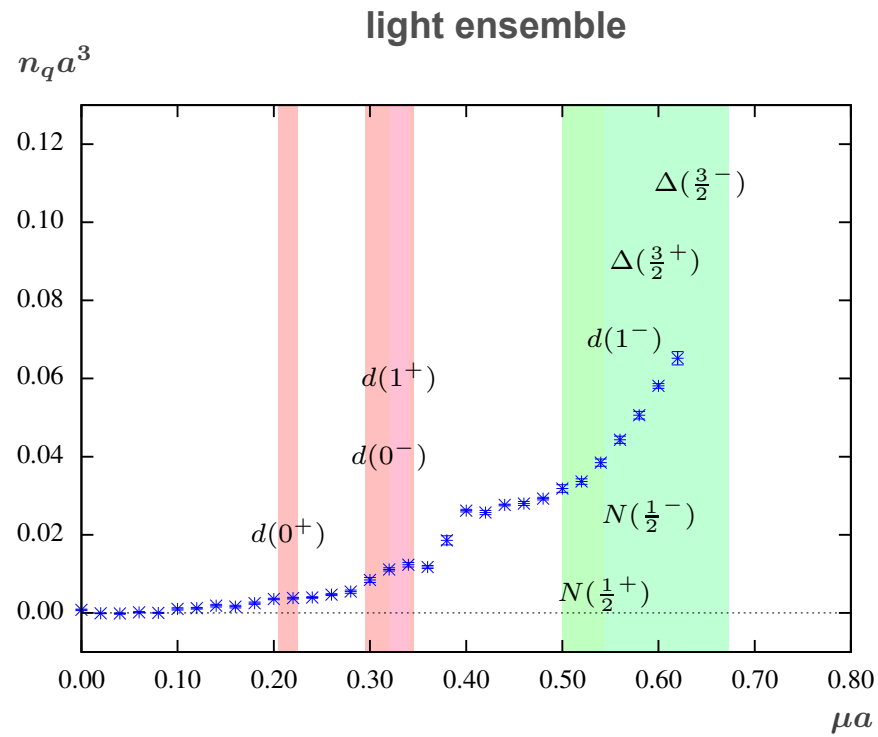

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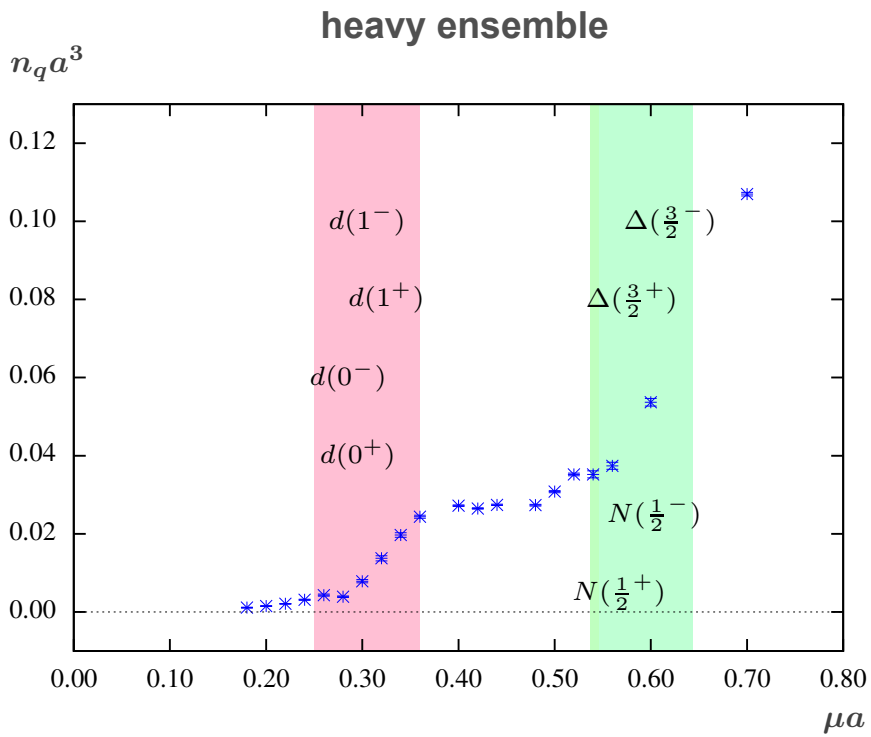
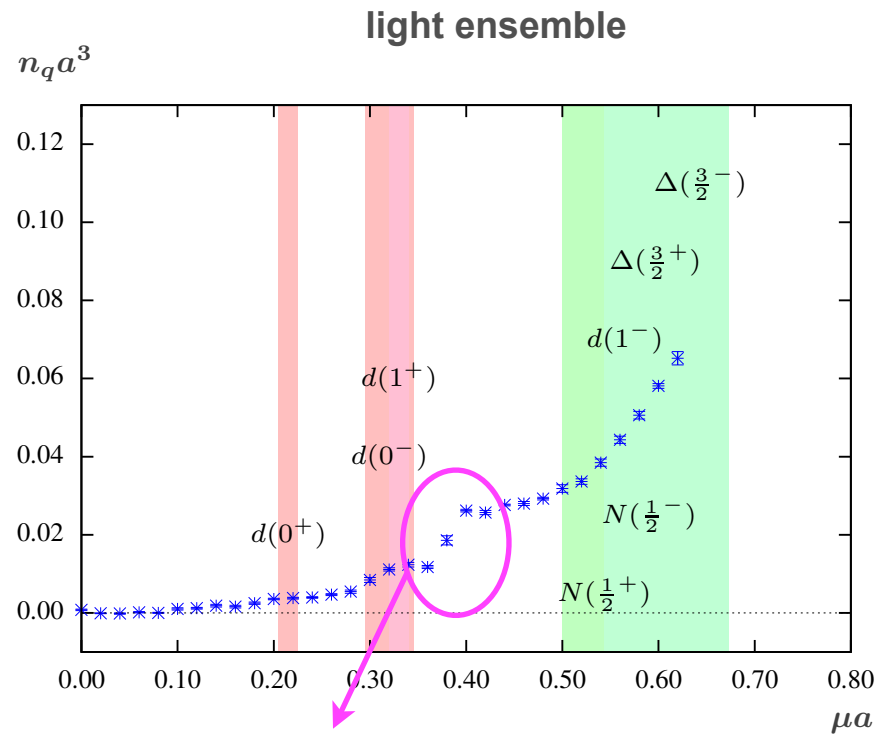




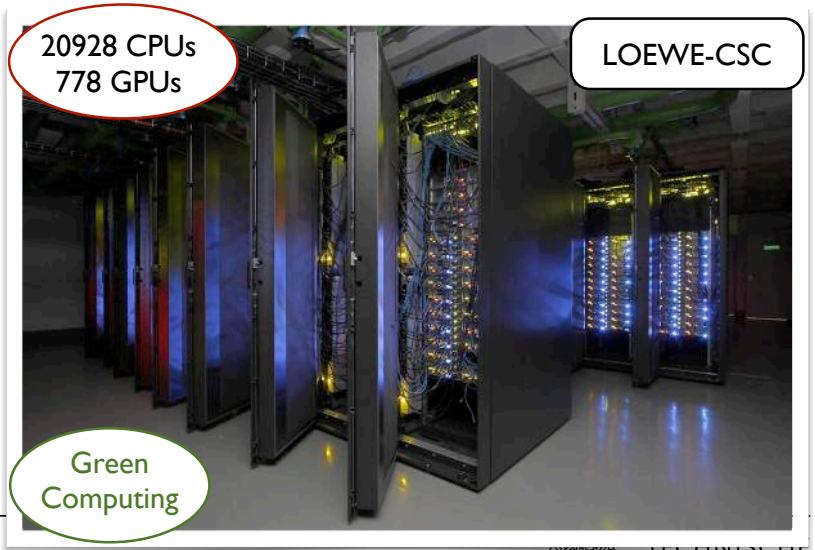
1st order?
liquid-gas transition of G2 nuclear matter?

Wellegehausen, Maas, Wipf & LvS, PRD 89 (2014) 056007

Summary & Outlook

- **G_2-QCD, a useful laboratory for finite density studies**
  - no sign problem, most QCD-like
  - finite baryon density region of phase diagram with MC simulations
  - refine functional methods and models
  - test effective lattice theories for heavy quarks
- **spectroscopy & baryon density**
  - physics of bosonic baryons as in two-color QCD
  - fermionic baryons dominate above G_2-nuclear matter transition
  - further clarify nature of cold and dense phases

[to be continued in Bjoern Wellegehausen's talk next]



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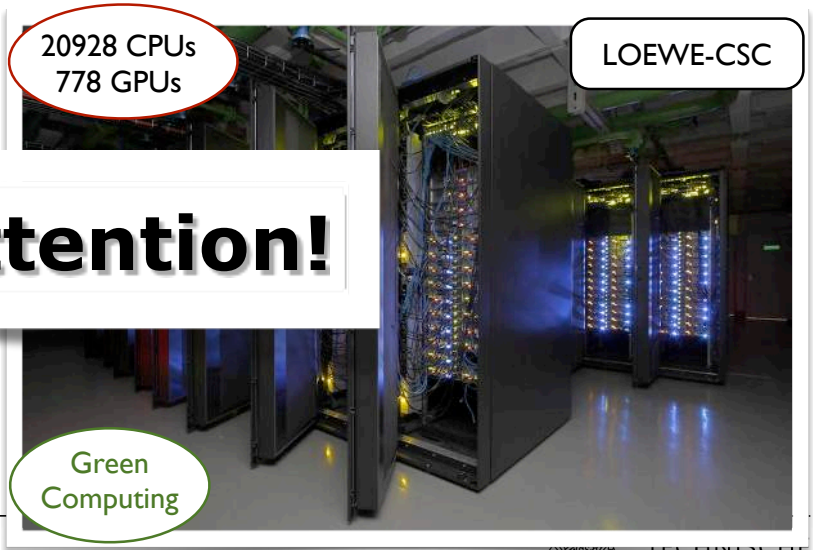

Thank You for Your Attention!

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