Calculation of disconnected contributions to nucleon form factors using hierarchical probing

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lattice 14

Noise method for trace of matrix inverse

[e.g. W. Wilcox, hep-lat/9911013]

Let ξ be a random vector with components $|\xi_i| = 1$. Then

$$\begin{aligned} \sum_{i,\,j} \xi_i^* \, (A^{-1})_{ij} \, \xi_j &= \mathrm{Tr}[A^{-1}] + \sum_{i \neq j} \xi_i^* \, (A^{-1})_{ij} \, \xi_j \\ \left\langle \sum_{i,\,j} \xi_i^* \, (A^{-1})_{ij} \, \xi_j \right\rangle_\xi &= \mathrm{Tr}[A^{-1}] \end{aligned}$$

The variance comes only from the off-diagonal elements of A^{-1}.

Dilution

[e.g. W. Wilcox, hep-lat/9911013]

Replace

$$\sum_{i,\,j} \xi_i^* \, (A^{-1})_{ij} \, \xi_j \quad \rightarrow \quad \sum_{p=1}^{P} \sum_{i,j} \xi_i^{(p)*} \, (A^{-1})_{ij} \, \xi_j^{(p)}$$

where $\xi_i^{(p)} = 0$ for $i \notin$ (partition p).

Requires a factor of P more inversions, but removes variance from elements with i, j in different partitions.

In lattice QCD, i labels space, color, and spin:

- spatial dilution
- color dilution
- spin dilution

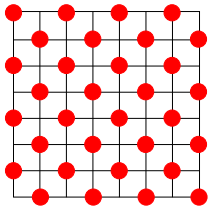
Hierarchical probing

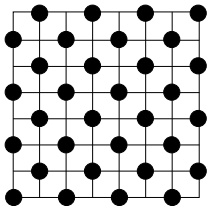
[A. Stathopoulos, J. Laeuchli, K. Orginos, arXiv:1302.4018]

- A spatial dilution method where the "level of dilution" is increased gradually using a sequence of Hadamard vectors
- The full data analysis can be performed at any stage
- The "level of dilution" can be increased without having to discard previous results

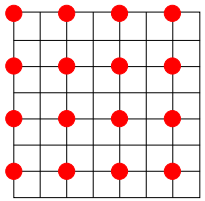
Hierarchical "coloring" in 2 dimensions

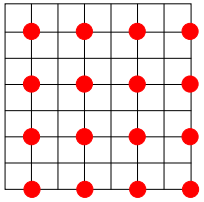
Even-odd coloring

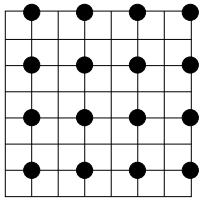


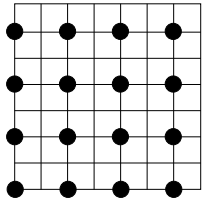


Split each color into 2^{d-1} regular sublattices

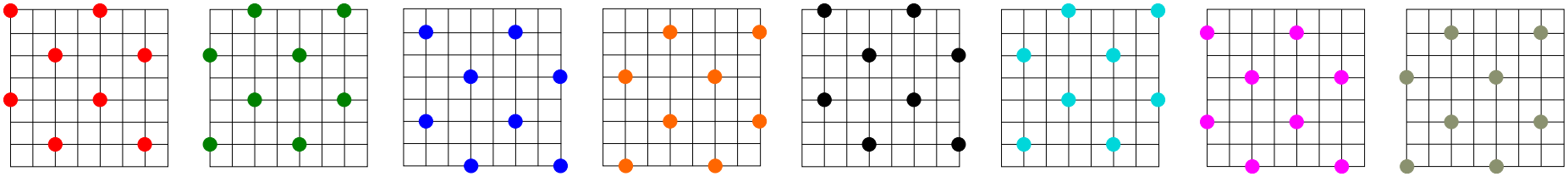








Repeat even-odd coloring for each sublattice



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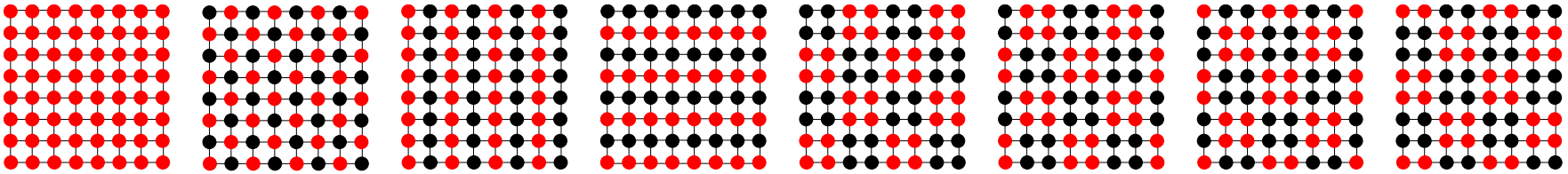
Hierarchical probing

Hadamard vectors in 2 dimensions:



$z^{(1)}$ $z^{(2)}$ $z^{(3)}$ $z^{(4)}$ $z^{(5)}$ $z^{(6)}$ $z^{(7)}$ $z^{(8)}$ ...

$\bullet = +1$
$\bullet = -1$

Hierarchical probing: replace

$$\sum_{i,j} \xi_i^* \, (A^{-1})_{ij} \, \xi_j \quad \rightarrow \quad \frac{1}{N_{\text{Hadamard}}} \sum_{n=1}^{N_{\text{Hadamard}}} \sum_{i,j} \xi_i^{(n)*} (A^{-1})_{ij} \, \xi_j^{(n)}$$

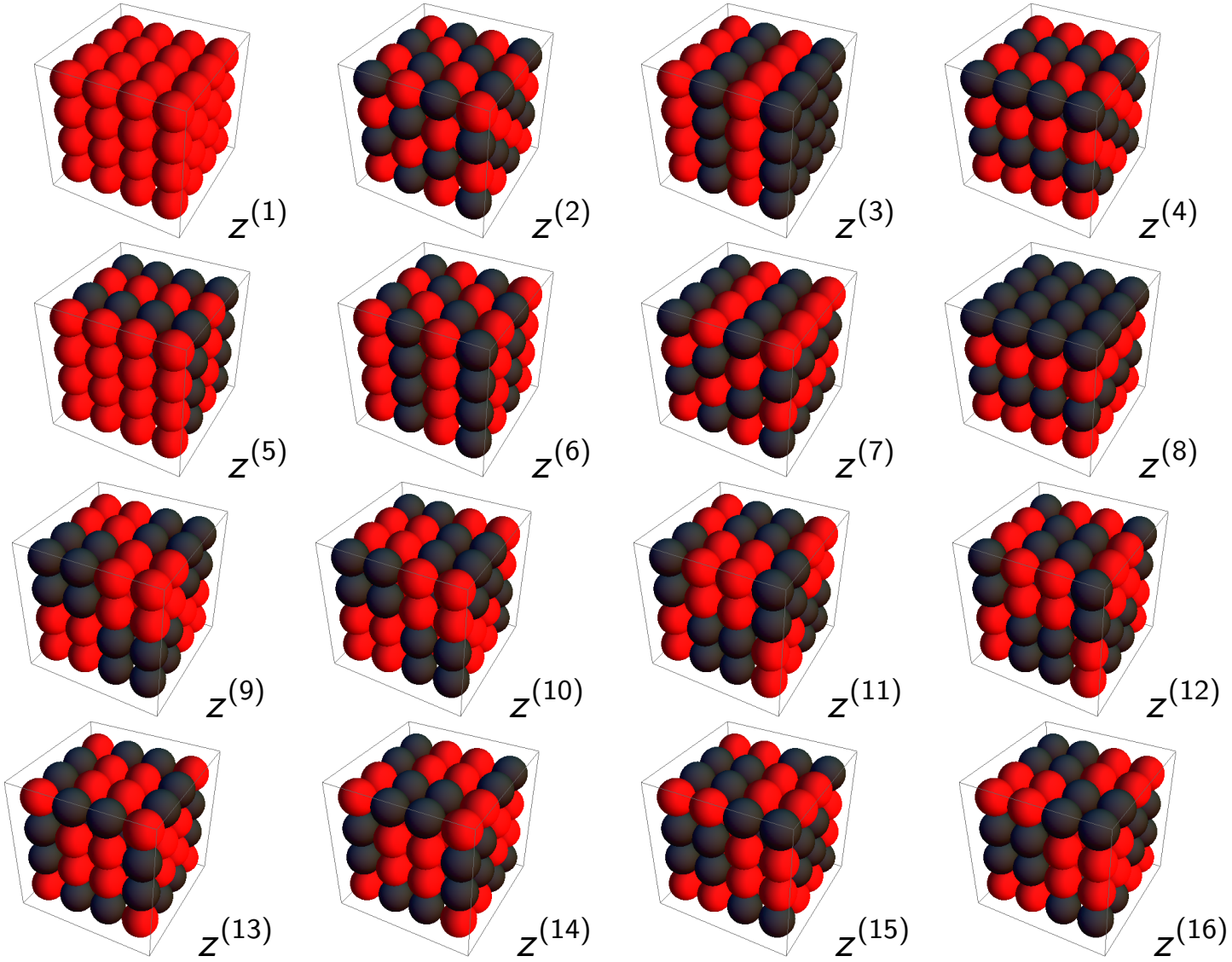
with

$$\xi^{(n)} = z^{(n)} \odot \xi$$

where ξ is a standard noise vector (possibly with color and spin dilution)

Hierarchical probing

Hadamard vectors in 3 dimensions:



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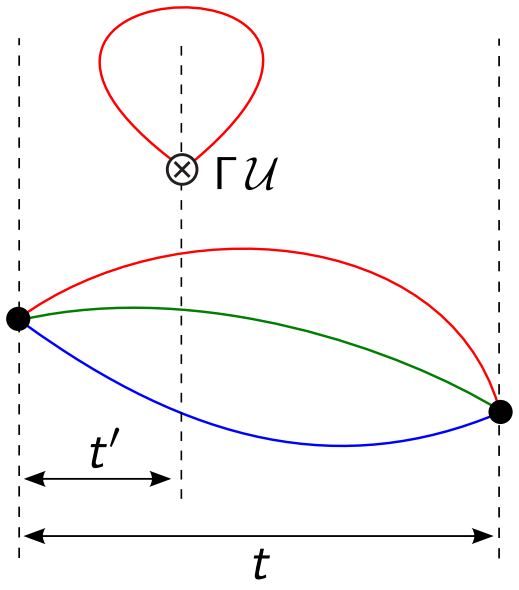
Disconnected three-point functions with hierarchical probing

Disconnected three-point function:

$$\left\langle C_{3\mathrm{pt}}^{(\mathrm{dis})} \right\rangle = \left\langle \left(T - \langle T \rangle\right) \left(C_{2\mathrm{pt}} - \langle C_{2\mathrm{pt}} \rangle\right) \right\rangle$$

(averages subtracted to reduce variance)



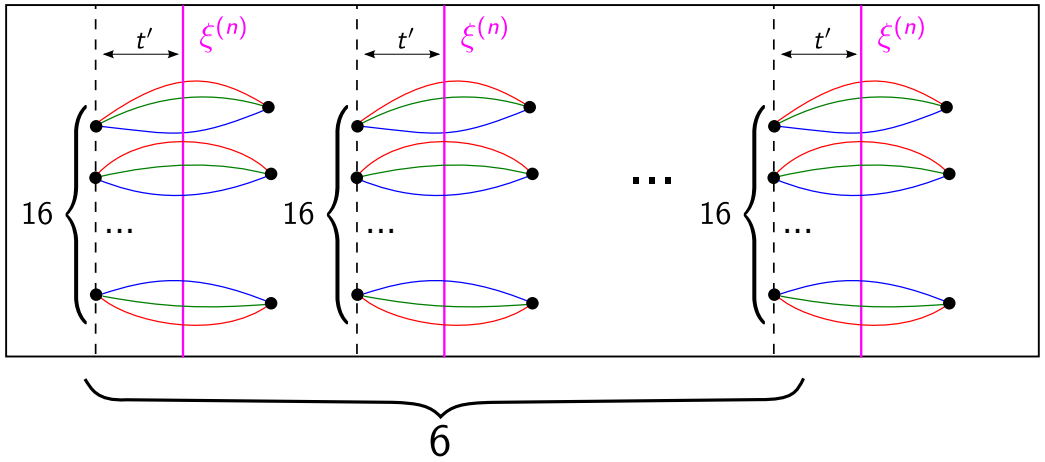
Disconnected quark loop:

$$T(\Gamma, \mathcal{U}, \mathbf{q}, t') = -\frac{1}{N_{\mathrm{Hadamard}}} \sum_{n=1}^{N_{\mathrm{Hadamard}}} \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}}\, \xi^{(n)\dagger}(\mathbf{y}) \sum_{\mathbf{z}} \Gamma\mathcal{U}G\left(\mathbf{y}, t', \mathbf{z}, t'\right) \xi^{(n)}(\mathbf{z})$$

We use full time dilution (noise source localized on time slice t' only) and perform 3-dimensional hierarchical probing.

Parameters of our calculation

- Symanzik glue, $N_f = 2 + 1$ clover (one level of stout smearing), $V = 32^3 \times 96$, $a \approx 0.114$ fm, $m_\pi \approx 320$ MeV
- $\mathbb{Z}_2 \times i\mathbb{Z}_2$ noise, color and spin dilution + hierarchical probing
- All 16 gamma matrices, 0- and 1-link displacement in the current
- ≈ 1000 configurations. On each configuration, we compute:



- Done on GPUs at JLab using QUDA
- Here only $\boxed{t'/a = 5}$. Additional separations in progress
- We already have the corresponding **connected** three-point functions for five source-sink separations [J. Green et al., Lattice 2013, arXiv:1310.7043]

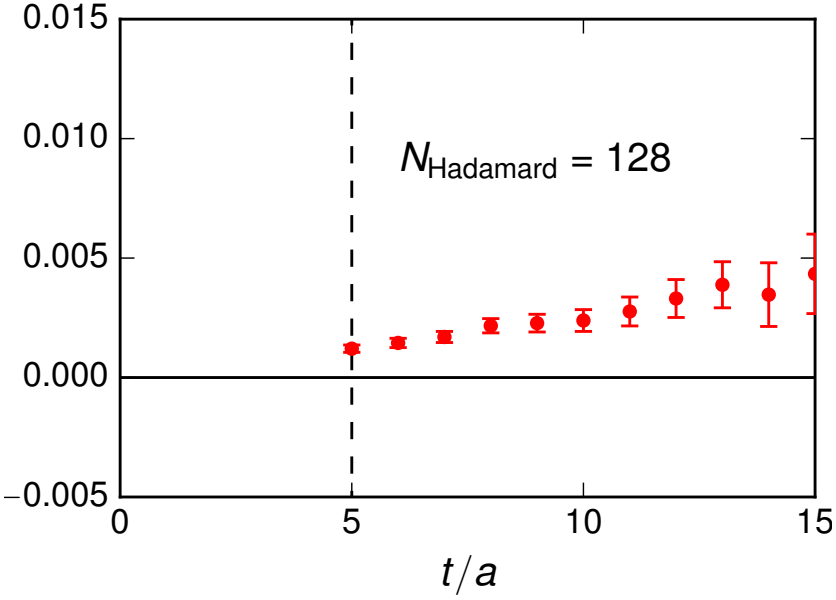
1 Methods

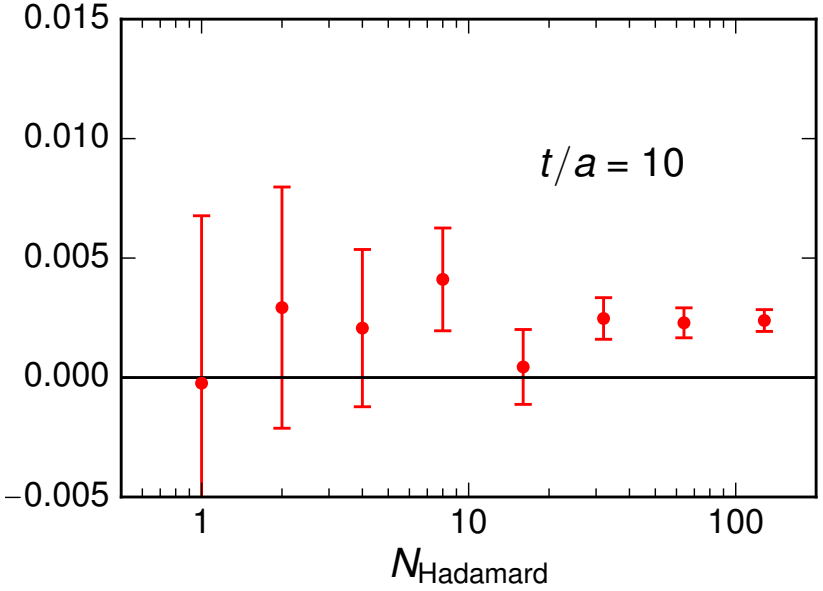
2 Preliminary Results

- Dependence on t and N_{Hadamard}
- Hierarchical probing vs. standard noise method
- Comparison of disconnected and connected contributions

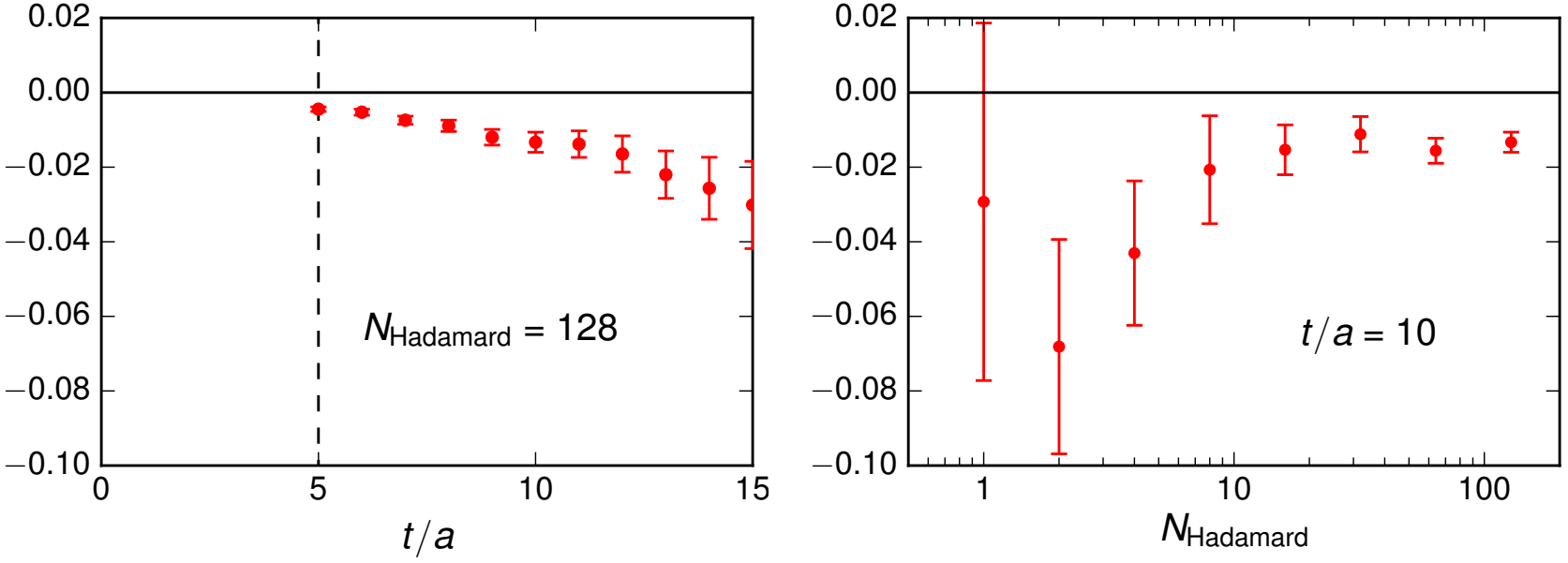
3 Conclusions

$G_E^{(\frac{2}{3}u-\frac{1}{3}d)}$ $(Q^2 \approx 0.11\ \mathrm{GeV}^2)$ (disconnected)

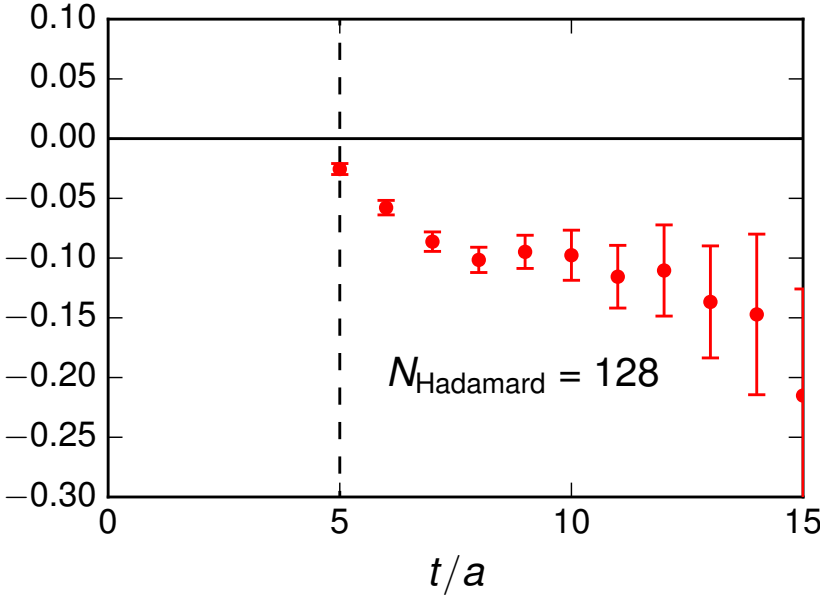


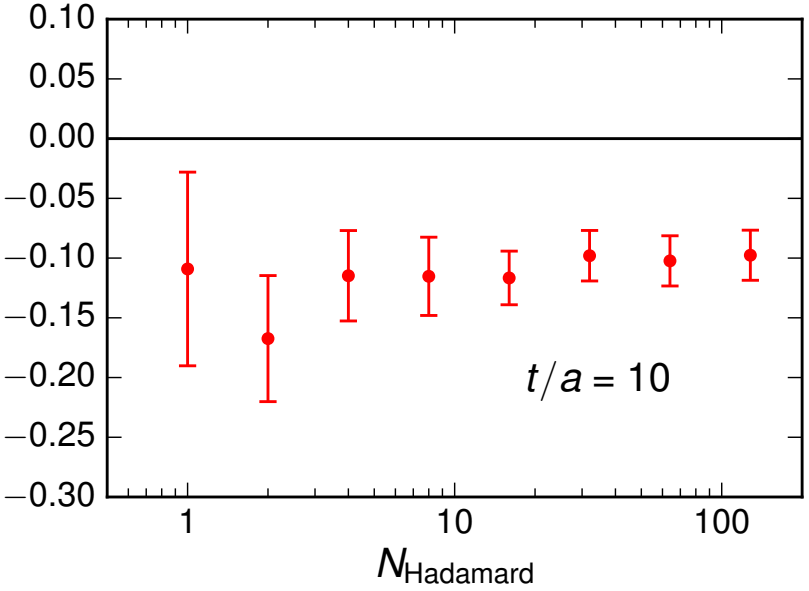


$G_M^{(\frac{2}{3}u-\frac{1}{3}d)}$ $(Q^2 \approx 0.11\ \mathrm{GeV}^2)$ (disconnected)

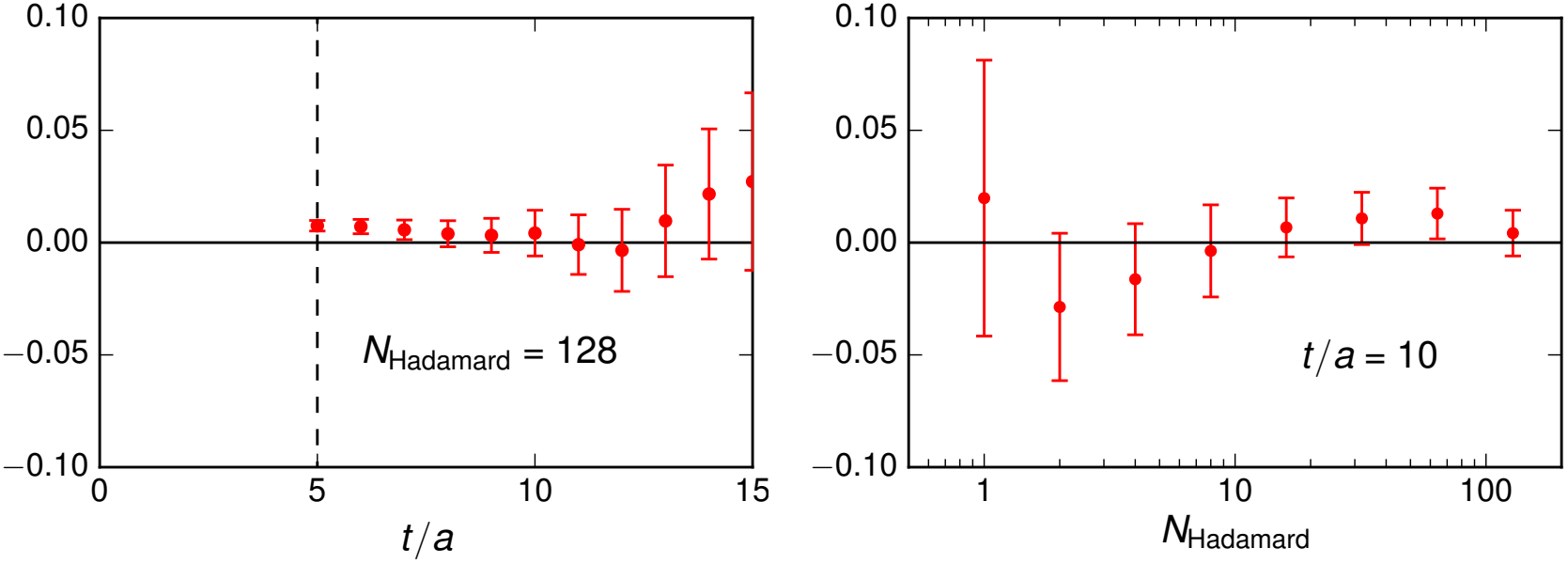


$g_A^{(u+d)}$ (disconnected, bare)

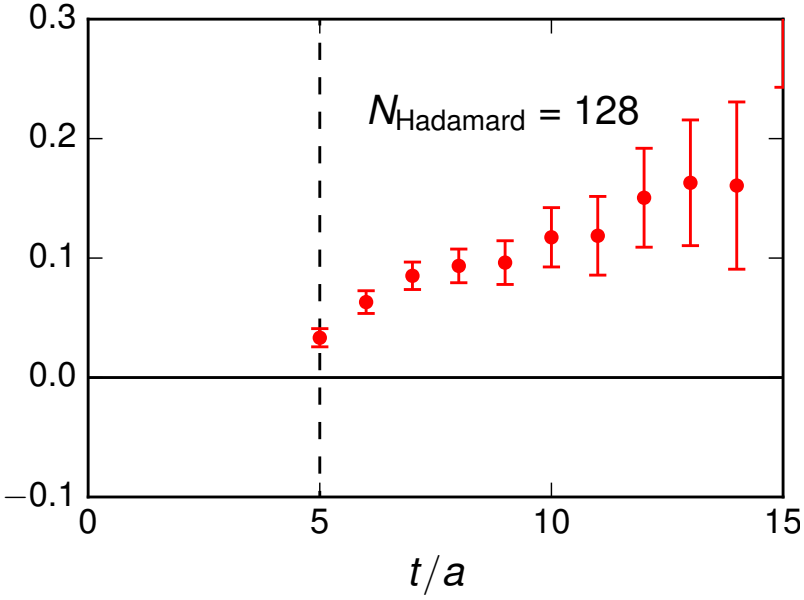


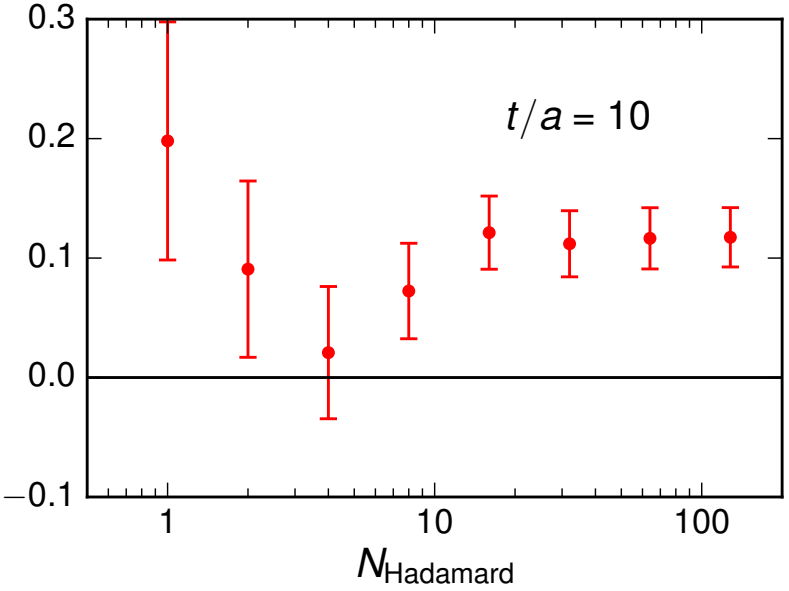


$g_T^{(u+d)}$ (disconnected, bare)

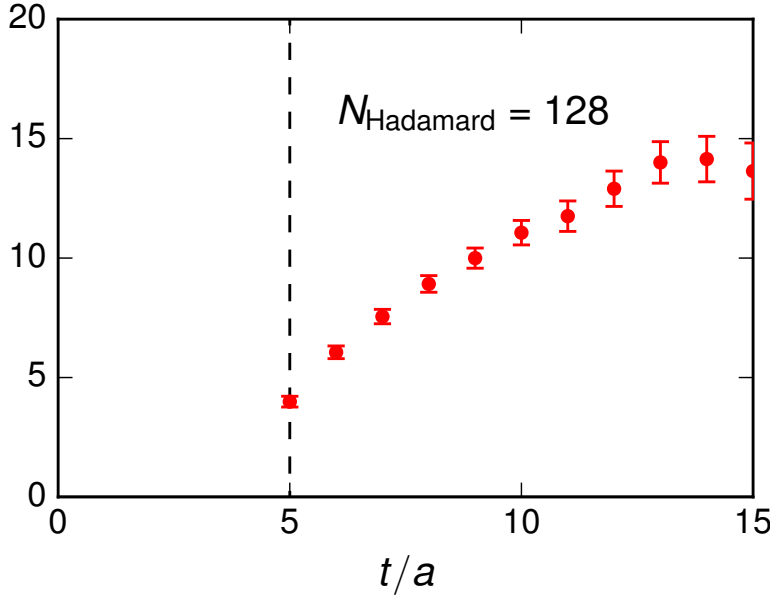


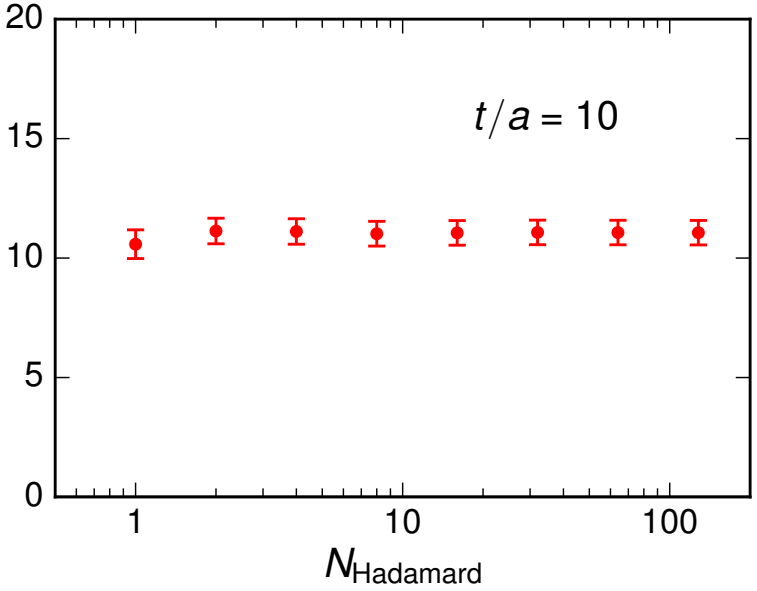
$\langle x \rangle^{(u+d)}$ (disconnected, bare)





$g_S^{(u+d)}$ (disconnected, bare)

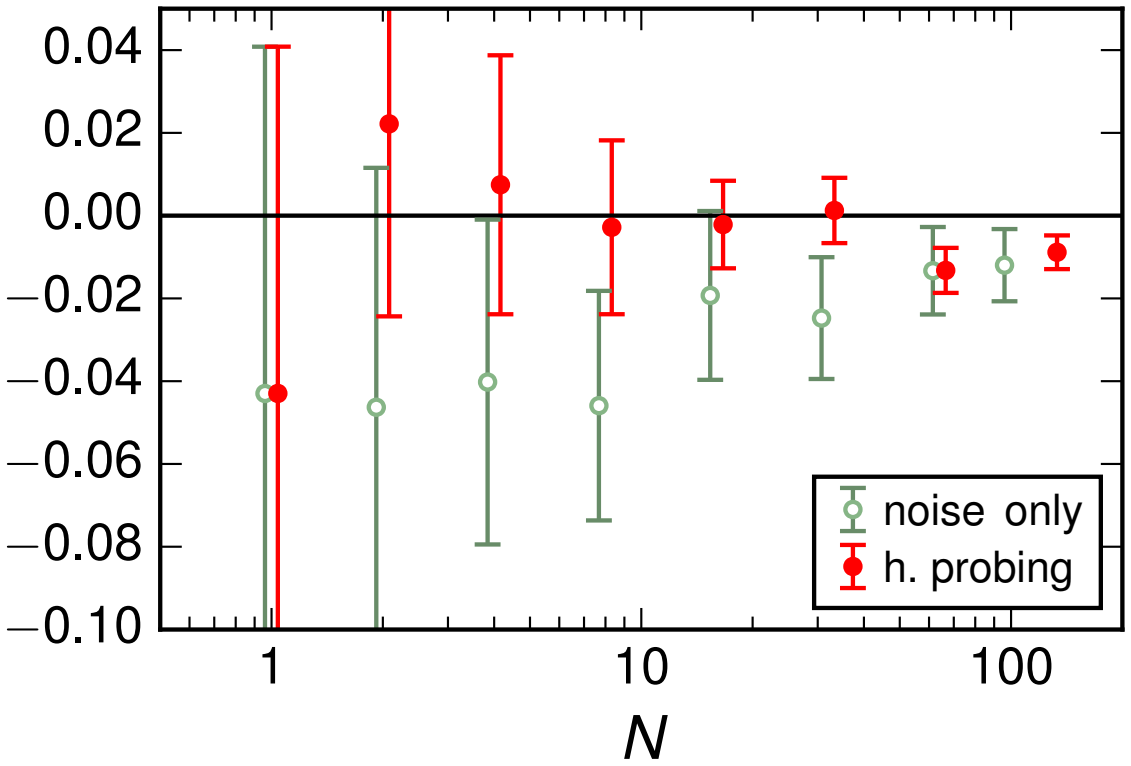




Hierarchical probing vs. standard noise method

Only 1/3 of configurations used here.

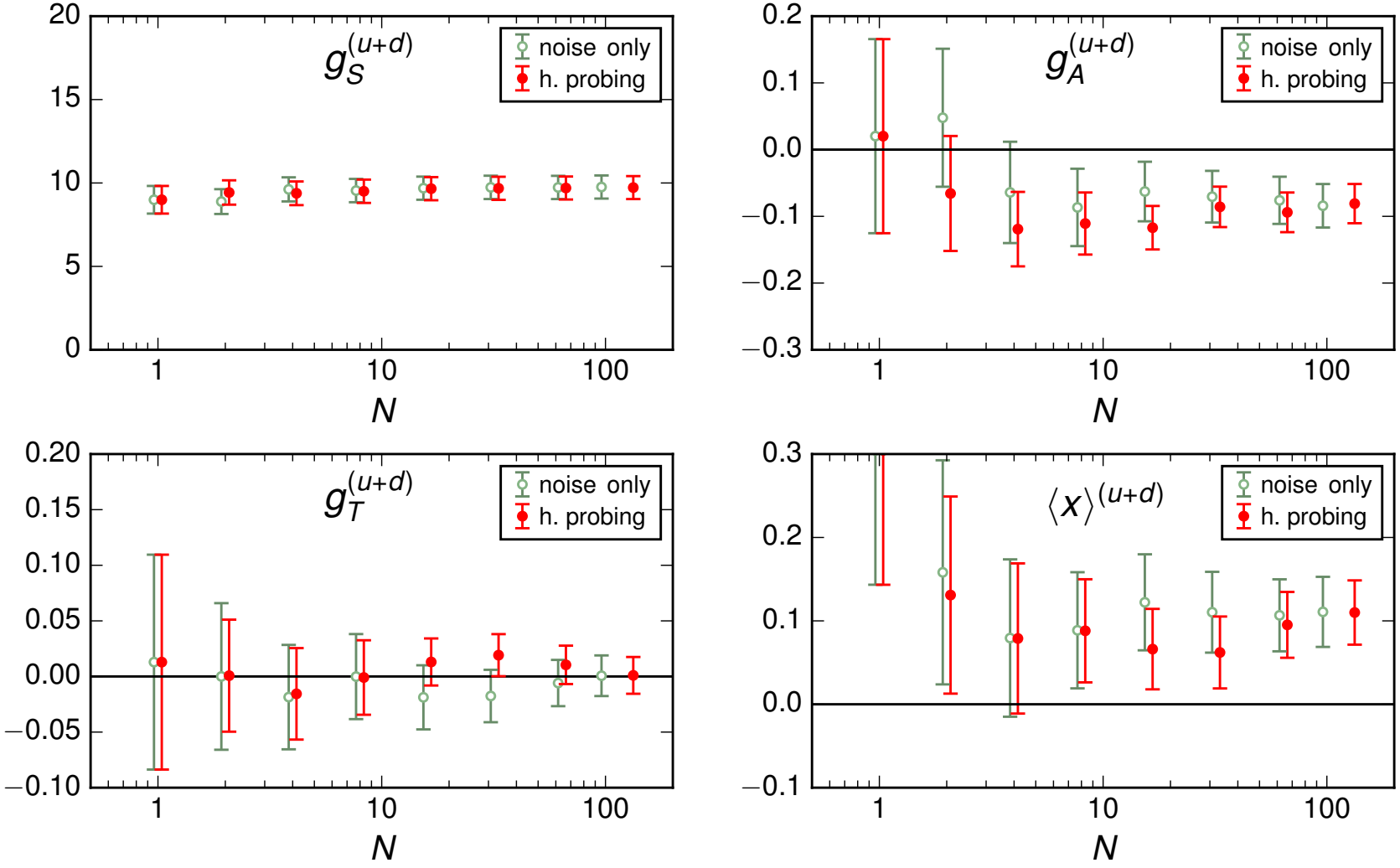
$G_M^{(\frac{2}{3}u-\frac{1}{3}d)}$ $(Q^2 \approx 0.11\ \mathrm{GeV}^2)$ (disconnected)



Equal cost at same N ($= N_{\mathrm{Hadamard}}$ or N_{noise}). Points offset horizontally.

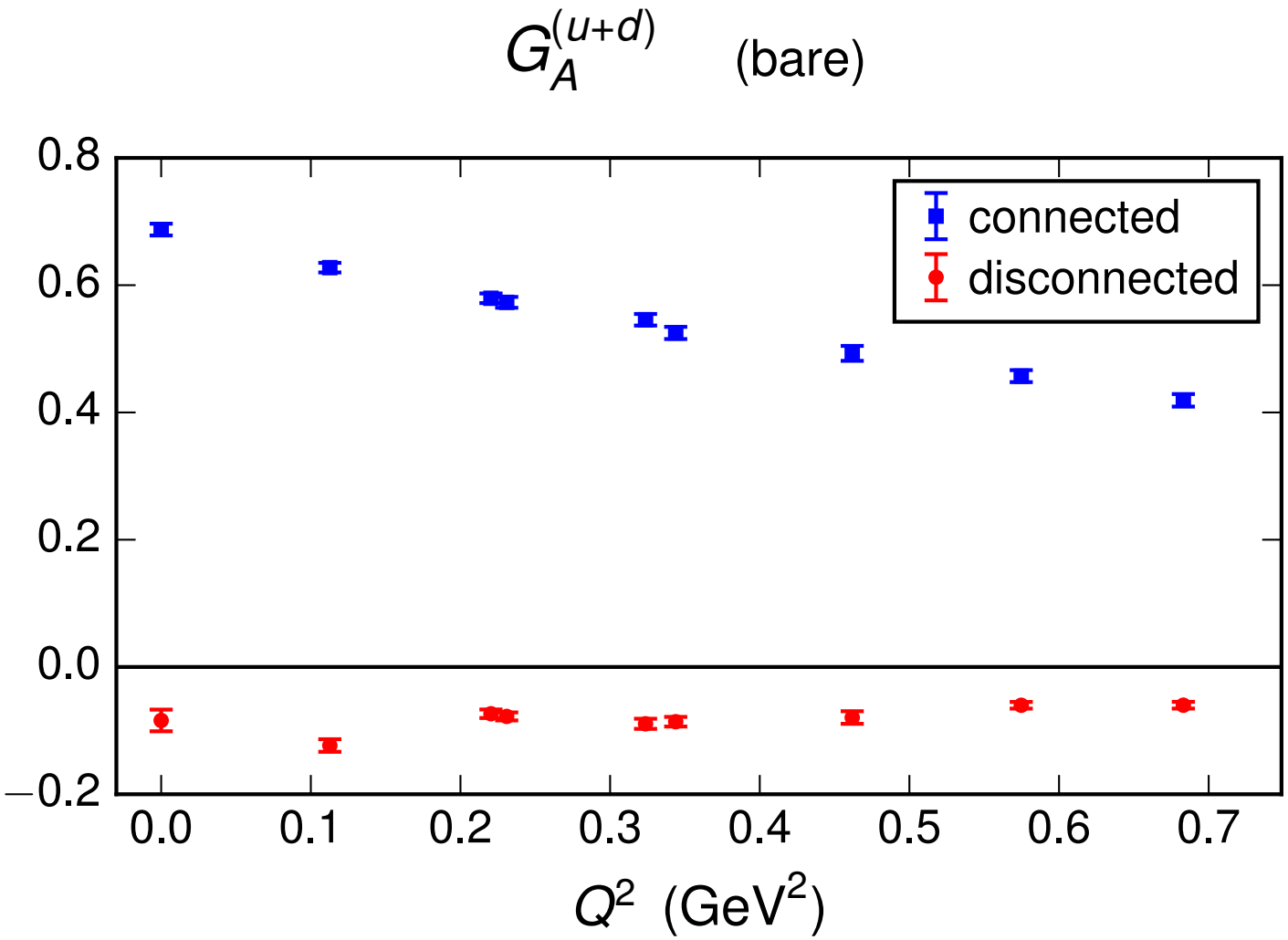
Hierarchical probing vs. standard noise method

Only 1/3 of configurations used here.

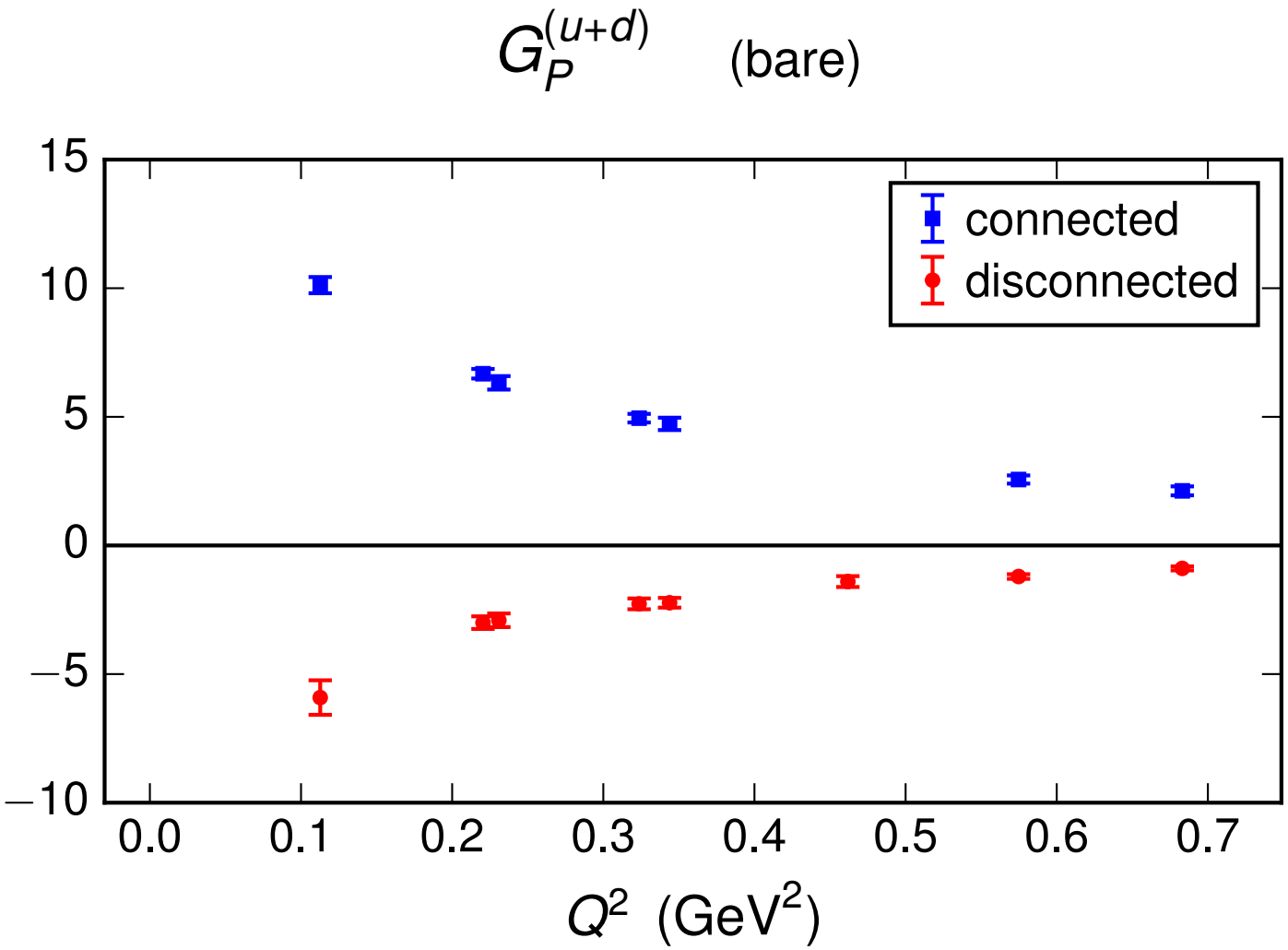


Equal cost at same N ($= N_{\text{Hadamard}}$ or N_{noise}). Points offset horizontally.

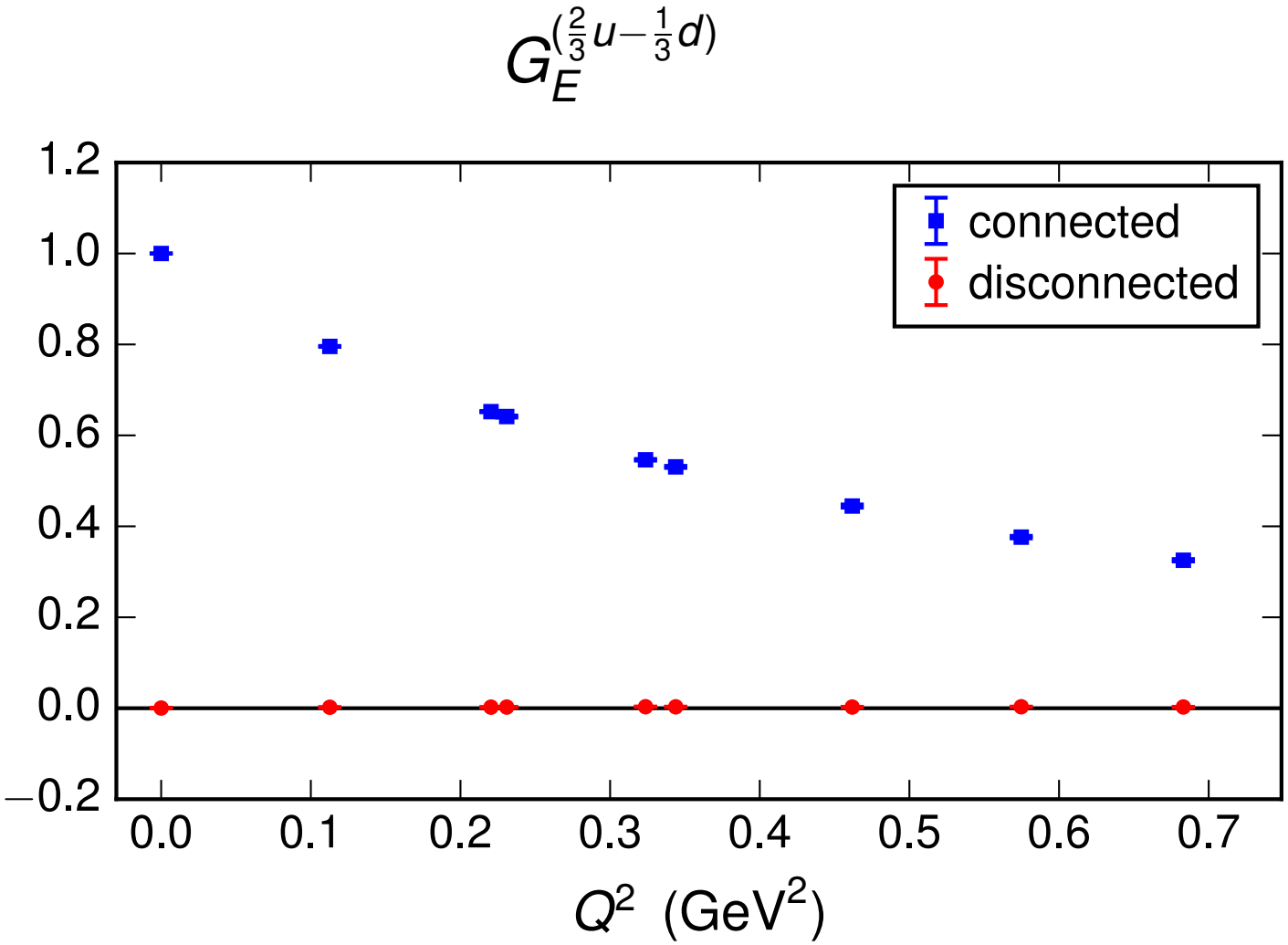
Axial form factor



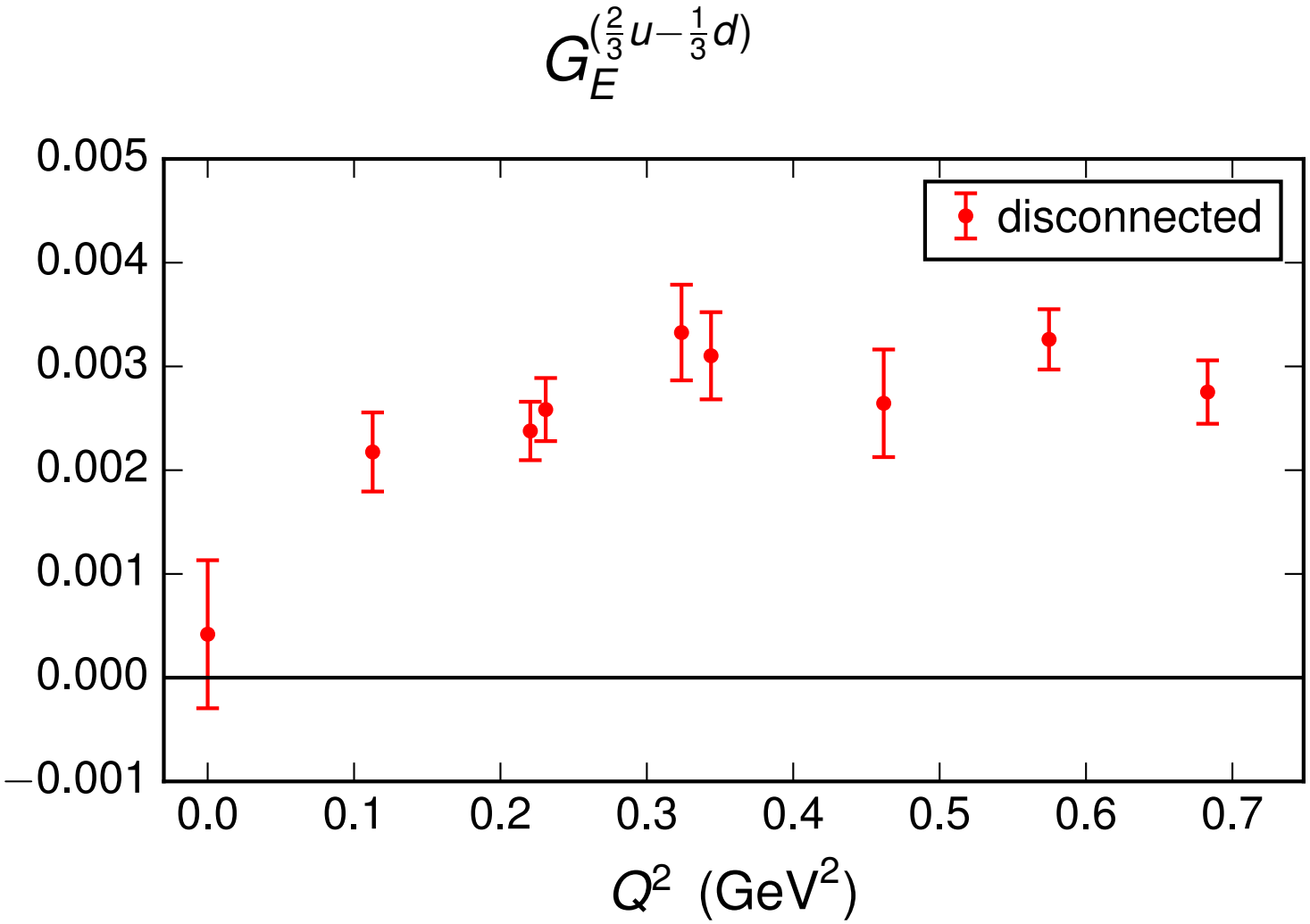
Induced pseudoscalar form factor



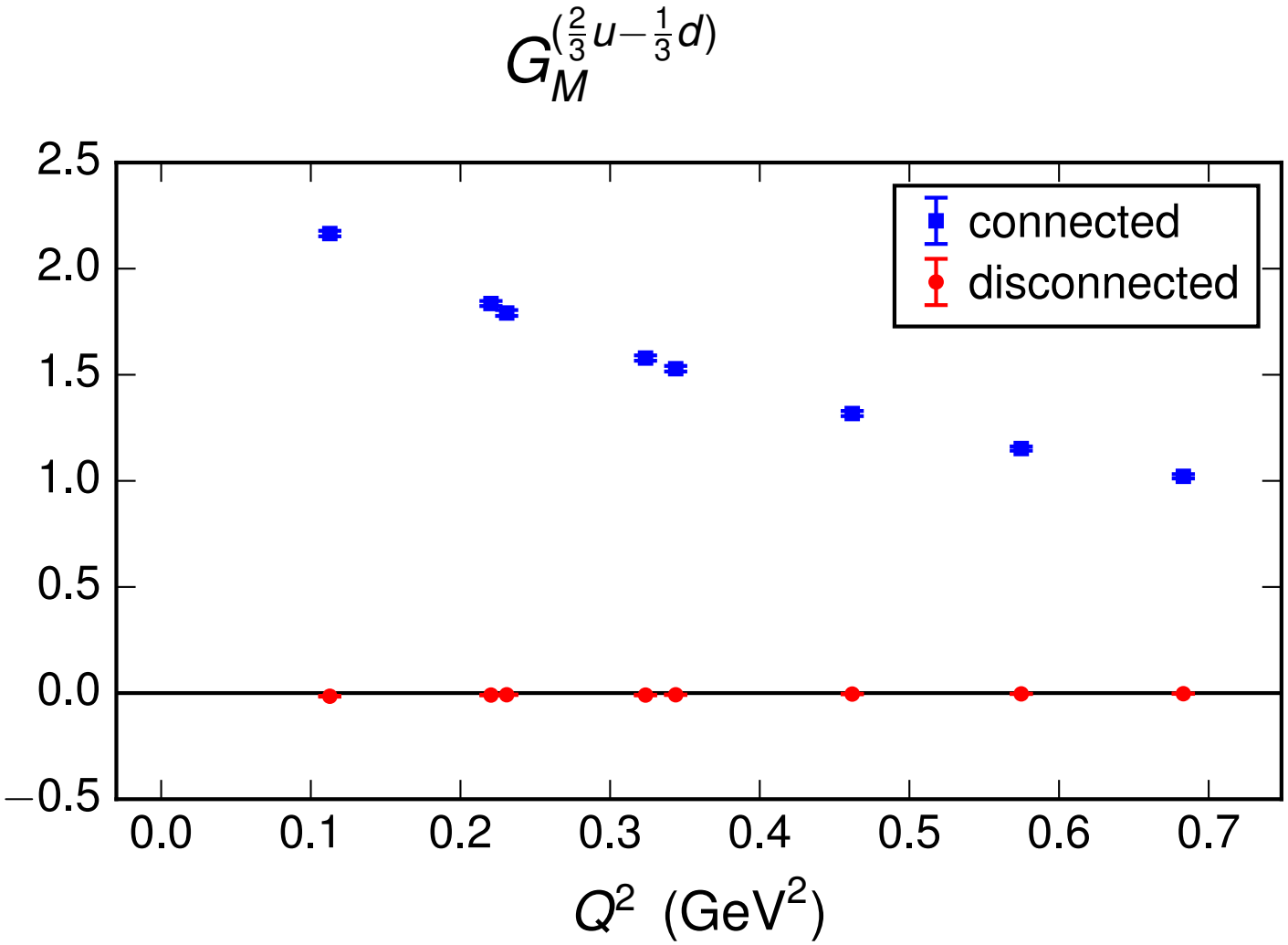
Electric form factor



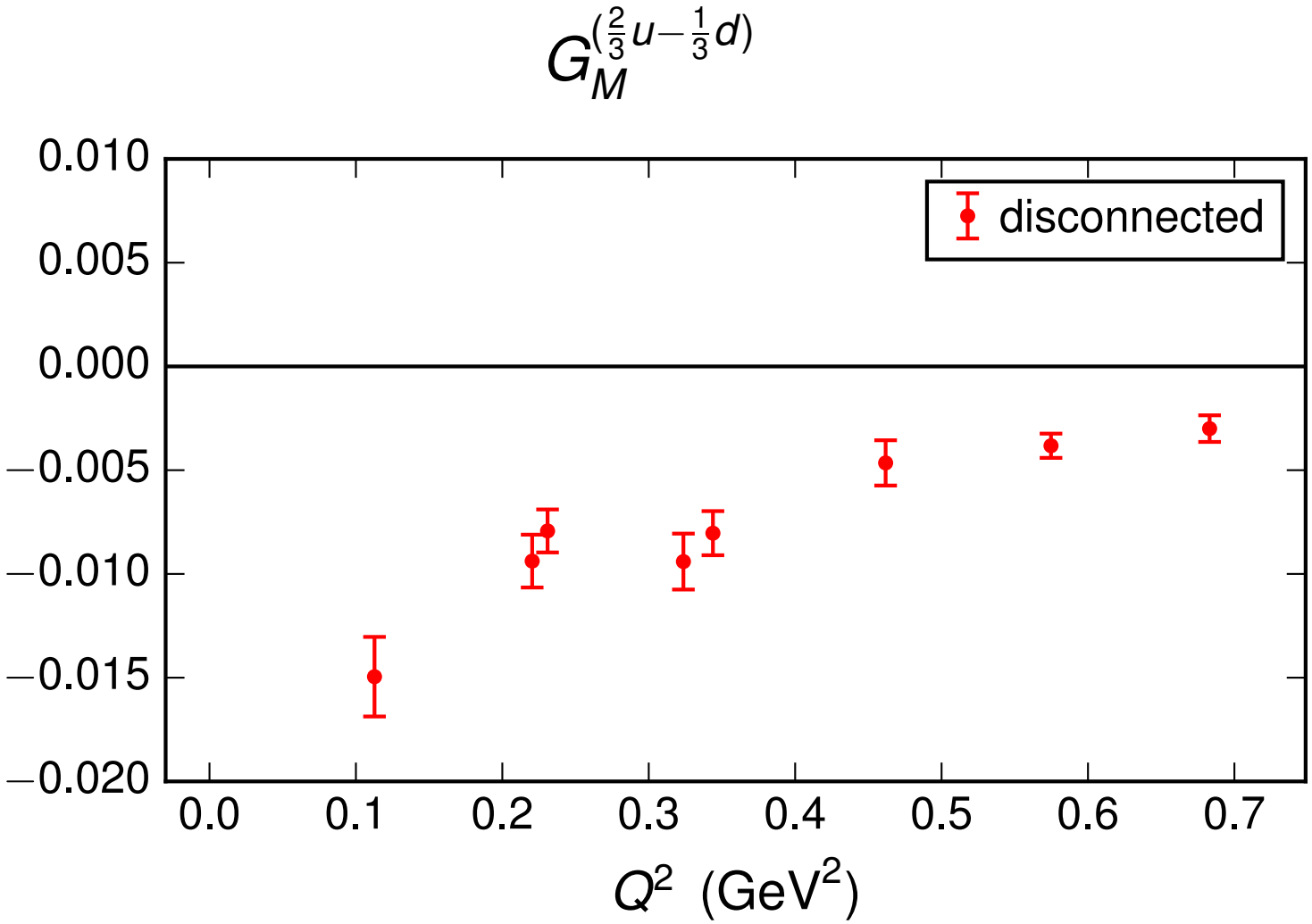
Electric form factor



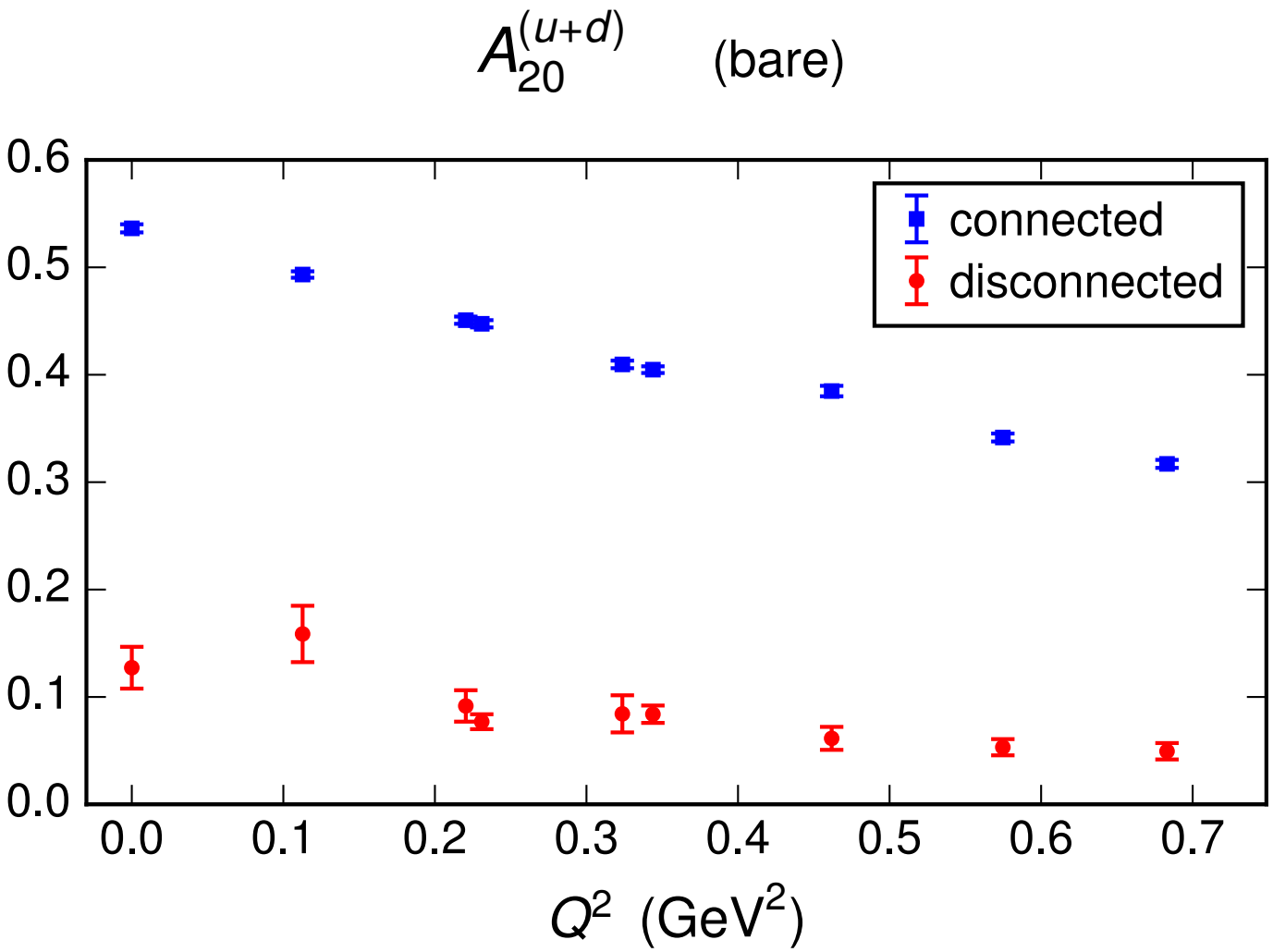
Magnetic form factor



Magnetic form factor



Generalized form factor A_{20}



Conclusions

Hierarchical probing:

- Always performs better than standard noise method
- Large reduction in uncertainty for vector current
- Uncertainties for some observables dominated by gauge noise

Nucleon structure:

- Relative size of disconnected and connected contributions varies widely:

Observable	$\left\lvert \frac{\text{disconnected}}{\text{connected}} \right\rvert$
$G_{E,M}^{(\frac{2}{3}u-\frac{1}{3}d)}$	~ 0.005
$G_A^{(u+d)}$	~ 0.15
$\langle x \rangle^{(u+d)}$	~ 0.2
$G_P^{(u+d)}$	~ 0.5
$G_S^{(u+d)}$	~ 2

To do:

- More source-current separations
- Renormalization