Calculation of disconnected contributions to nucleon form factors using hierarchical probing

Stefan Meinel,

Jeremy Green, Michael Engelhardt, Stefan Krieg, Jesse Laeuchli, John Negele, Kostas Orginos, Andrew Pochinsky, Andreas Stathopoulos, Sergey Syritsyn
1 Methods

2 Preliminary Results

3 Conclusions
Noise method for trace of matrix inverse

[e.g. W. Wilcox, hep-lat/9911013]

Let $\xi$ be a random vector with components $|\xi_i| = 1$. Then

$$\sum_{i, j} \xi_i^* (A^{-1})_{ij} \xi_j = \text{Tr}[A^{-1}] + \sum_{i \neq j} \xi_i^* (A^{-1})_{ij} \xi_j$$

$$\langle \sum_{i, j} \xi_i^* (A^{-1})_{ij} \xi_j \rangle_{\xi} = \text{Tr}[A^{-1}]$$

The variance comes only from the off-diagonal elements of $A^{-1}$. 
Dilution
[e.g. W. Wilcox, hep-lat/9911013]

Replace

$$\sum_{i,j} \xi_i^* (A^{-1})_{ij} \xi_j \rightarrow \sum_{p=1}^{P} \sum_{i,j} \xi_i^{(p)*} (A^{-1})_{ij} \xi_j^{(p)}$$

where $\xi_i^{(p)} = 0$ for $i \notin$ (partition $p$).

Requires a factor of $P$ more inversions, but removes variance from elements with $i,j$ in different partitions.

In lattice QCD, $i$ labels space, color, and spin:

- spatial dilution
- color dilution
- spin dilution
Hierarchical probing


- A spatial dilution method where the “level of dilution” is increased gradually using a sequence of Hadamard vectors
- The full data analysis can be performed at any stage
- The “level of dilution” can be increased without having to discard previous results
Hierarchical “coloring” in 2 dimensions

Even-odd coloring

Split each color into $2^{d-1}$ regular sublattices

Repeat even-odd coloring for each sublattice

...
Hierarchical probing

Hadamard vectors in 2 dimensions:

\[ \sum_{i,j} \xi_i^* (A^{-1})_{ij} \xi_j \rightarrow \frac{1}{N_{\text{Hadamard}}} \sum_{n=1}^{N_{\text{Hadamard}}} \sum_{i,j} \xi_i^{(n)*} (A^{-1})_{ij} \xi_j^{(n)} \]

with

\[ \xi^{(n)} = z^{(n)} \odot \xi \]

where \( \xi \) is a standard noise vector (possibly with color and spin dilution)
Hierarchical probing

Hadamard vectors in 3 dimensions:
Disconnected three-point functions with hierarchical probing

Disconnected three-point function:

\[ \langle C_{3pt}^{(\text{dis})} \rangle = \langle (T - \langle T \rangle) (C_{2pt} - \langle C_{2pt} \rangle) \rangle \]

(averages subtracted to reduce variance)

Disconnected quark loop:

\[ T(\Gamma, U, q, t') = -\frac{1}{N_{\text{Hadamard}}} \sum_{n=1}^{N_{\text{Hadamard}}} \sum_{y} e^{iq \cdot y} \xi^{(n)\dagger}(y) \sum_{z} \Gamma U G(y, t', z, t') \xi^{(n)}(z) \]

We use full time dilution (noise source localized on time slice \( t' \) only) and perform 3-dimensional hierarchical probing.
Parameters of our calculation

- Symanzik glue, $N_f = 2 + 1$ clover (one level of stout smearing), $V = 32^3 \times 96$, $a \approx 0.114$ fm, $m_\pi \approx 320$ MeV
- $\mathbb{Z}_2 \times i\mathbb{Z}_2$ noise, color and spin dilution + hierarchical probing
- All 16 gamma matrices, 0- and 1-link displacement in the current
- $\approx 1000$ configurations. On each configuration, we compute:

- Done on GPUs at JLab using QUDA
- Here only $t'/a = 5$. Additional separations in progress
- We already have the corresponding connected three-point functions for five source-sink separations [J. Green et al., Lattice 2013, arXiv:1310.7043]
1 Methods

2 Preliminary Results

- Dependence on $t$ and $N_{\text{Hadamard}}$
- Hierarchical probing vs. standard noise method
- Comparison of disconnected and connected contributions

3 Conclusions
$G_E^{(\frac{2}{3}u - \frac{1}{3}d)}$ 

$(Q^2 \approx 0.11 \text{ GeV}^2)$ (disconnected)

![Graphs showing $t/a$ and $N_{\text{Hadamard}}$]
$G_M^{(2/3 u - 1/3 d)}$ 
($Q^2 \approx 0.11 \text{ GeV}^2$) (disconnected)

$N_{\text{Hadamard}} = 128$

$t/a = 10$
$g_A^{(u+d)}$ (disconnected, bare)

- $N_{\text{Hadamard}} = 128$
- $t/a = 10$
$g_T^{(u+d)}$ (disconnected, bare)

- $N_{\text{Hadamard}} = 128$
- $t/a = 10$
\( \langle x \rangle^{(u+d)} \) (disconnected, bare)
$g_S^{(u+d)}$ (disconnected, bare)

- $N_{\text{Hadamard}} = 128$
- $t/a = 10$
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Hierarchical probing vs. standard noise method

Only 1/3 of configurations used here.

\[ G_M^{(\frac{2}{3}u - \frac{1}{3}d)} \quad (Q^2 \approx 0.11 \text{ GeV}^2) \quad (\text{disconnected}) \]

Equal cost at same \( N \) (\( = N_{\text{Hadamard}} \) or \( N_{\text{noise}} \)). Points offset horizontally.
Hierarchical probing vs. standard noise method

Only 1/3 of configurations used here.

Equal cost at same $N$ (= $N_{\text{Hadamard}}$ or $N_{\text{noise}}$). Points offset horizontally.
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Axial form factor

\[ G_A^{(u+d)} \ (\text{bare}) \]
Induced pseudoscalar form factor

$G_P^{(u+d)}$ (bare)

$Q^2$ (GeV$^2$)
Electric form factor

\[ G_{E}^{(\frac{2}{3}u-\frac{1}{3}d)} \]
Electric form factor

\[ G_E^{\left( \frac{2}{3}u - \frac{1}{3}d \right) \text{ disconnected}} \]
Magnetic form factor

\[ G_M^{(\frac{2}{3}u - \frac{1}{3}d)} \]

\[ Q^2 \text{ (GeV}^2) \]

connected

disconnected
Magnetic form factor

\[ G_M^{(2/3u - 1/3d)} \]
Generalized form factor $A_{20}$

$A_{20}^{(u+d)}$ (bare)

$Q^2$ (GeV$^2$)

connected

disconnected

$A_{20}$

$A_{20}^{(u+d)}$ (bare)
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Conclusions

Hierarchical probing:
• Always performs better than standard noise method
• Large reduction in uncertainty for vector current
• Uncertainties for some observables dominated by gauge noise

Nucleon structure:
• Relative size of disconnected and connected contributions varies widely:

<table>
<thead>
<tr>
<th>Observable</th>
<th>disconnected ( \text{connected} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_{E,M}^{\left(\frac{2}{3}u - \frac{1}{3}d\right)} )</td>
<td>( \sim 0.005 )</td>
</tr>
<tr>
<td>( G_{A}^{(u+d)} )</td>
<td>( \sim 0.15 )</td>
</tr>
<tr>
<td>( \langle x \rangle^{(u+d)} )</td>
<td>( \sim 0.2 )</td>
</tr>
<tr>
<td>( G_{P}^{(u+d)} )</td>
<td>( \sim 0.5 )</td>
</tr>
<tr>
<td>( G_{S}^{(u+d)} )</td>
<td>( \sim 2 )</td>
</tr>
</tbody>
</table>

To do:
• More source-current separations
• Renormalization