Charmed and Strange Pseudoscalar Meson Decay Constants from HISQ Simulations

Presenter: Javad Komijani

Washington University
St. Louis, USA

Fermilab Lattice and MILC Colaborations

A. Bazavov, C. Bernard, C. Bouchard, C. DeTar, D. Du, A.X. El-Khadra, J. Foley, E.D. Freeland, E. Gamiz, Steven Gottlieb, U.M. Heller, J. Kim, J. Komijani, A.S. Kronfeld, J. Laiho, L. Levkova, P.B. Mackenzie, E.T. Neil, J.N. Simone, R.L. Sugar, D. Toussaint, R.S. Van de Water, R. Zhou

Introduction

- f_{D^+}, f_{D_s} and f_{K^+}, together with experimental leptonic decay rate determinations, provide precise determinations of $|V_{cd}|$, $|V_{cs}|$ and $|V_{us}|$.
- For higher precision than available with the asqtad action, we have moved to the HISQ action.
- Advantage of HISQ is that charm may be treated with same action as light quarks.
- MILC's HISQ ensembles include ones with physical value of quark masses — reducing or eliminating errors from chiral extrapolation.

Introduction

- After determination of meson masses and amplitudes for each pair of valence-quark masses, analysis follows two stages:
  1. The physical-mass ensembles allow for a simple analysis without ChPT
     - used to compute quark-mass ratios and f_{K^+}/f_{π^+}.
     - also compute intermediate scale setting quantity F_{p4s} (the decay constant when valence masses are $0.4m_s^{phys}$ and the sea masses are physical) and corresponding meson mass M_{p4s}.
  2. Analyze heavy-light decay constants on all ensembles (physical and unphysical mass) using ChPT.
     - reduces statistical error.
     - more control of continuum extrapolation.
     - uses quark mass ratios, F_{p4s} and F_{p4s}/M_{p4s} from stage one.

Ensembles Used

- 21 ensembles are used.
- 14 ensembles have m_s tuned close to its physical value.

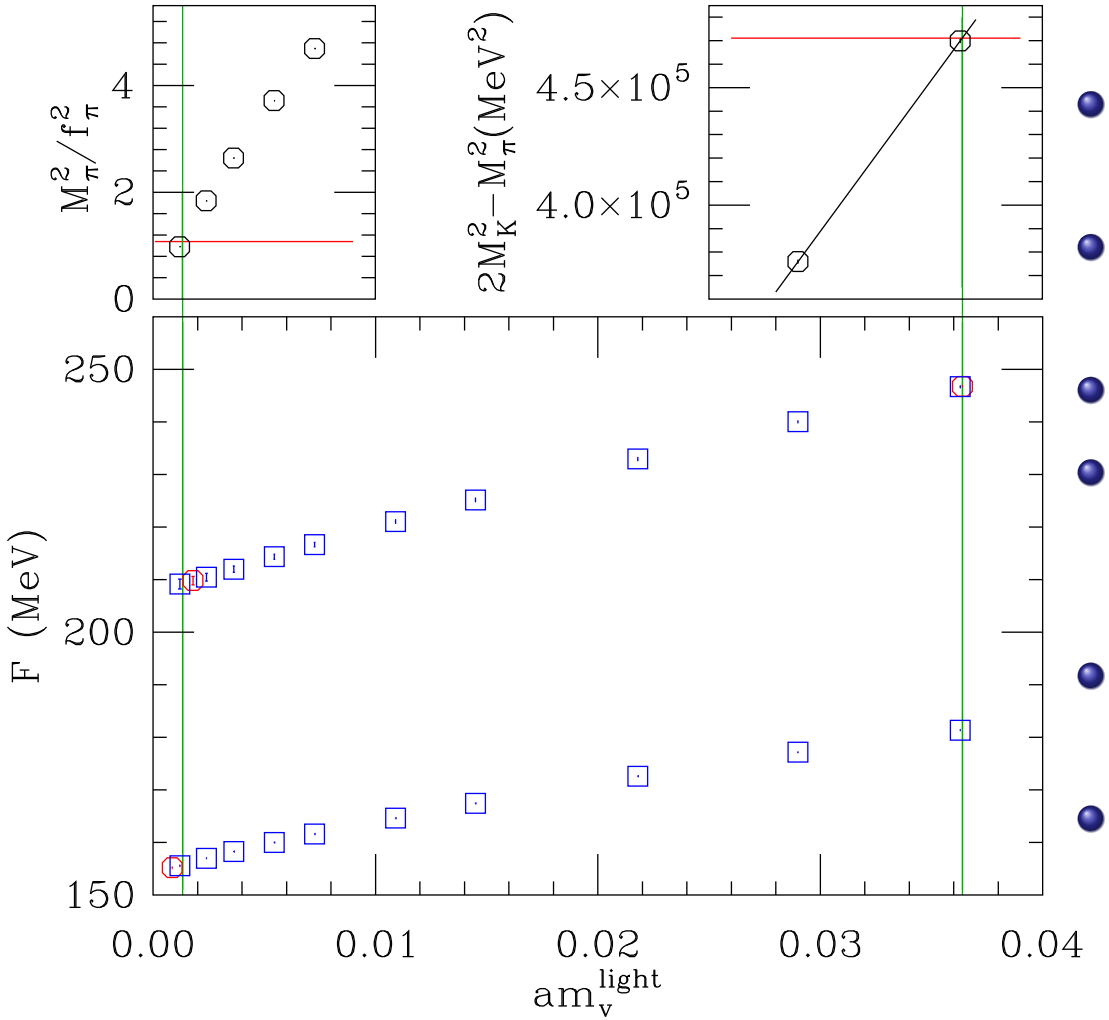
β	m_l'/m_s'	size	N_{lats}	$\approx a$ (fm)	L (fm)	$M_\pi L$	$M_\pi(MeV)$
5.80	1/5	$16^3 \times 48$	1020	0.15	2.38	3.8	314
5.80	1/10	$24^3 \times 48$	1000	0.15	3.67	4.0	214
5.80	1/27	$32^3 \times 48$	1000	0.15	4.83	3.2	130
6.00	1/5	$24^3 \times 64$	1040	0.12	3.00	4.5	299
6.00	1/10	$24^3 \times 64$	1020	0.12	2.89	3.2	221
6.00	1/10	$32^3 \times 64$	1000	0.12	3.93	4.3	216
6.00	1/10	$40^3 \times 64$	1028	0.12	4.95	5.4	214
6.00	1/27	$48^3 \times 64$	999	0.12	5.82	3.9	133
6.30	1/5	$32^3 \times 96$	1011	0.09	2.95	4.5	301
6.30	1/10	$48^3 \times 96$	1000	0.09	4.33	4.7	215
6.30	1/27	$64^3 \times 96$	1031	0.09	5.62	3.7	130
6.72	1/5	$48^3 \times 144$	1016	0.06	2.94	4.5	304
6.72	1/10	$64^3 \times 144$	1166	0.06	3.79	4.3	224
6.72	1/27	$96^3 \times 192$	583*	0.06	5.44	3.7	135

Valence Masses for Charmed Decay Constant

- For light quark, about 10 masses ranging from m_l' to m_s'
- For heavy quark, 2 masses close to the charm quark mass

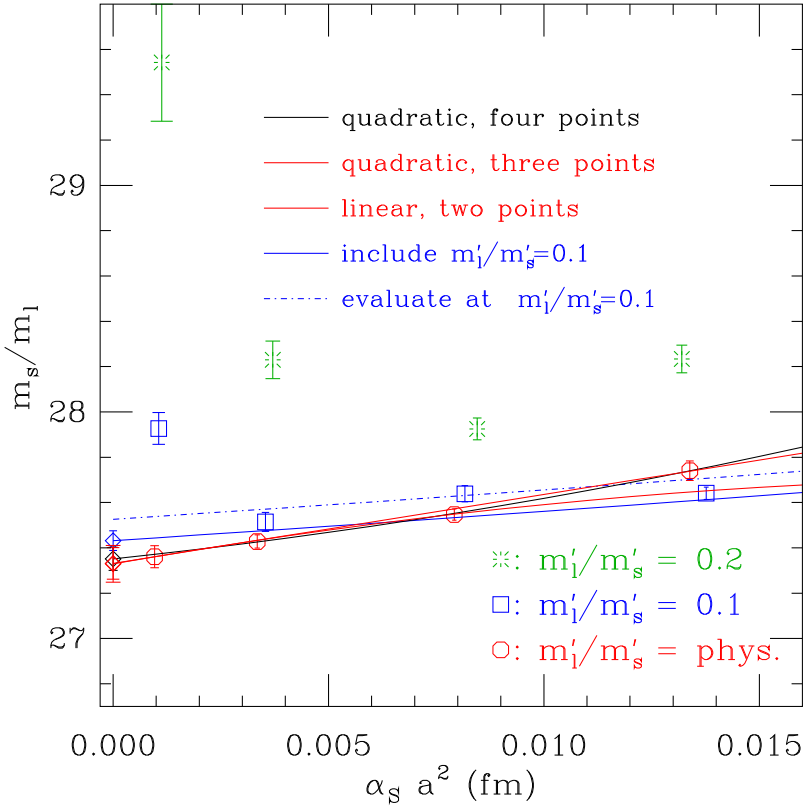
β	m_l'/m_s'	light masses ($m_{\rm v}/m_s'$)	heavy masses (m_Q/m_c')
5.80	1/5	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
5.80	1/10	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
5.80	1/27	0.036,0.07,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	1/5	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	1/10	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	1/10	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	1/10	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.00	1/27	0.036,0.073,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	1/5	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	1/10	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.30	1/27	0.033,0.066,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	1/5	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	1/10	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0
6.72	1/27	0.036,0.068,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0

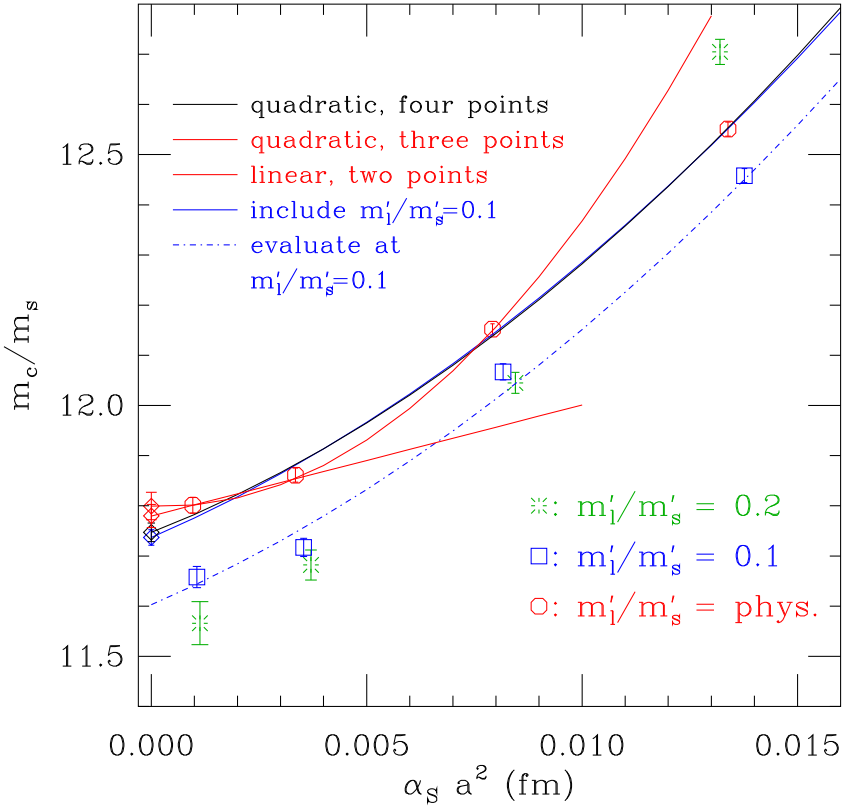
Lattice Spacing and Valence Quark Mass Tuning



- Illustration of the lattice spacing and valence quark mass tuning
- Upper panels: f_π, M_π and M_K are used to fix a, m_l and m_s
- Similarly m_c is fixed from M_{D_s}
- m_u/m_d comes from M_{K^+} and M_{K^0} (with inputs from our EM project)
- Lower Panel: determinations of f_{K^+}, f_{D^+} and f_{D_s}.
- Doing the whole procedure inside a jackknife resampling gives the statistical errors

Extrapolation to Continuum





- Black and red solid lines are from fits that only include physical mass ensembles.
- Solid blue line is from fit that also includes blue squares for the physical light quark mass.
- Dotted blue line is from same fit, but for heavier sea-quark mass corresponding to blue squares. $(m_l'/m_s' = 0.1.)$

Results of Stage One (Preliminary)

- Results for f_{K^+}/f_{π^+} and quark mass ratios:

$$\begin{aligned} f_{K^+}/f_{\pi^+} &= 1.1956(10)_{\rm stat}\,^{+23}_{-14}|_{a^2\,\rm extrap}(10)_{\rm FV}(5)_{\rm EM} \\ m_s/m_l &= 27.352(51)_{\rm stat}\,^{+80}_{-20}|_{a^2\,\rm extrap}(39)_{\rm FV}(55)_{\rm EM} \\ m_c/m_s &= 11.747(19)_{\rm stat}\,^{+52}_{-32}|_{a^2\,\rm extrap}(6)_{\rm FV}(27)_{\rm EM} \end{aligned}$$

- And intermediate scale setting quantities:

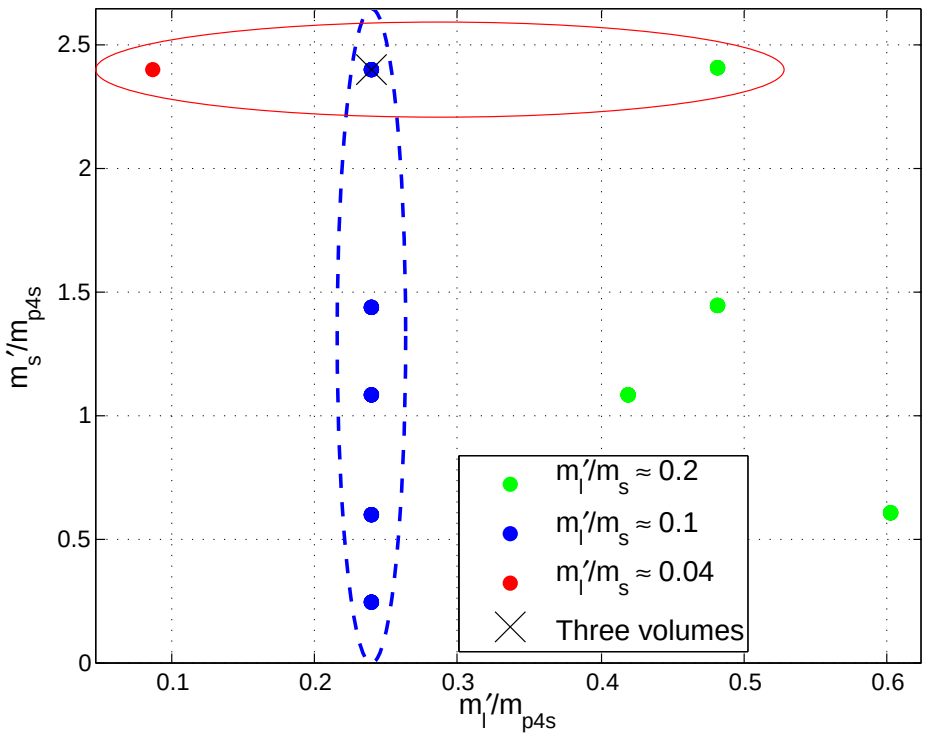
$$\begin{aligned} F_{p4s} &= 153.9(9)_{\rm stat}\,^{+14}_{-23}|_{a^2\,\rm extrap}(15)_{\rm FV}(5)_{\rm EM} \\ M_{p4s} &= 433.24(17)_{\rm stat}\,^{+1}_{-33}|_{a^2\,\rm extrap}(2)_{\rm FV}(43)_{\rm EM} \\ R_{p4s} &\equiv F_{p4s}/M_{p4s} = 0.35527(24)_{\rm stat}\,^{+52}_{-15}|_{a^2\,\rm extrap}(30)_{\rm FV}(24)_{\rm EM} \end{aligned}$$

- Finite Volume error comes from the FV error in f_π which in turn comes from ChPT. [*A. Bazavov et al. PRD* **110**, *172003 (2013)*]
- EM error is from the residual error in our tuned values of quark masses. Uses input $\epsilon = 0.84(21)$ (ϵ characterizes violations of Dashen's theorem) from MILC EM project. (See talk by C. Bernard, Friday, 2:35 P.M., Pupin 428.)

Scale Setting for Chiral Analysis

- Relative lattice scales are determined by F_{p4s}
- Has very small statistical errors, so the mistuning in sea quark masses can be important.
- So we adjust the data for mistunings in order to have a precise calculation of aF_{p4s} and am_{p4s}.
- Absolute scale comes from value of F_{p4s} in physical units (from stage one) which comes ultimately from f_π.

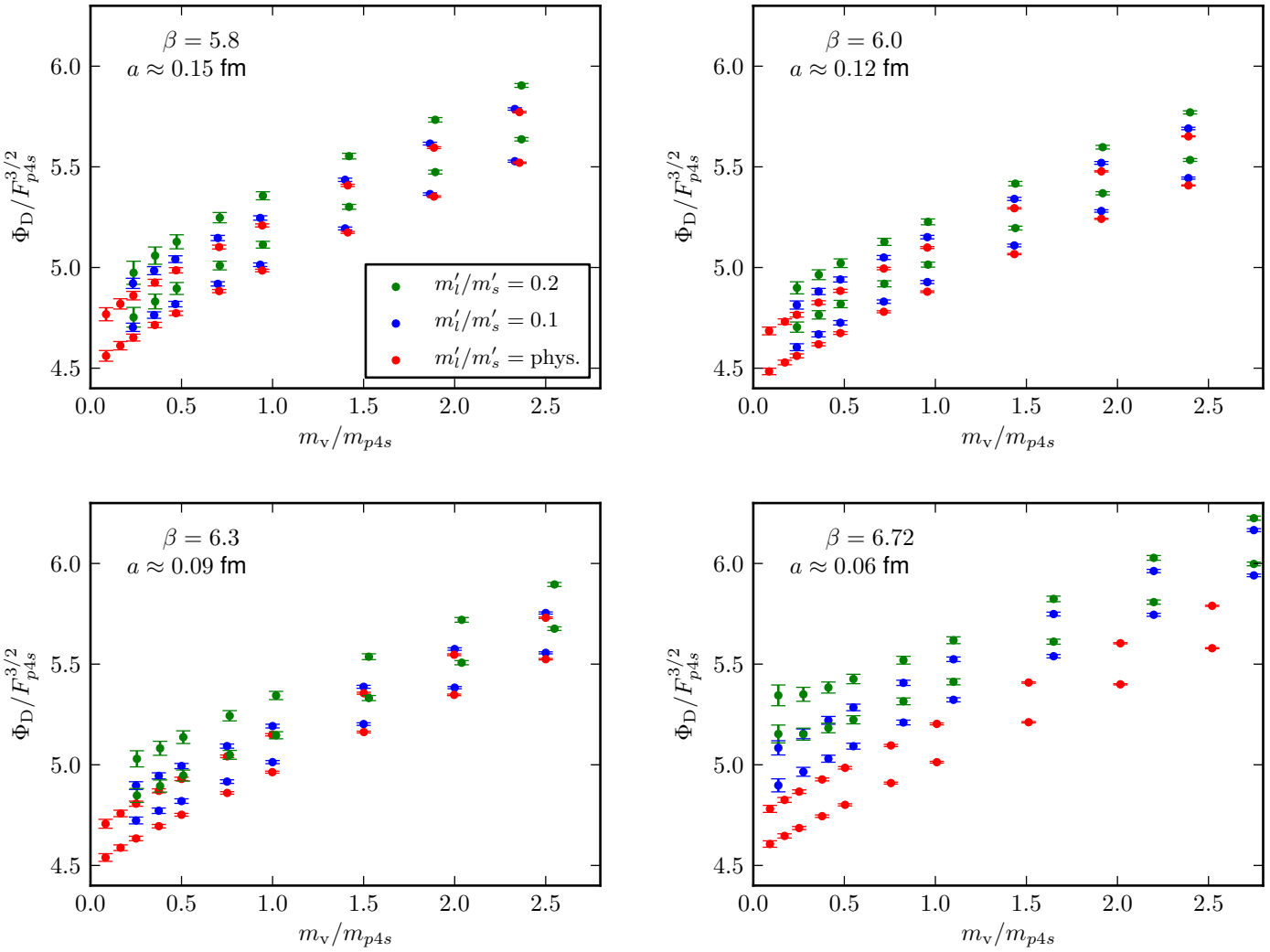
Adjustment for Mistuning in Sea Quark Masses



- Light sea masses of the ensembles available at $a = 0.12$ fm
- Three ensembles inside the red ellipse are used to calculate m_l derivatives
- Five ensembles inside the blue ellipse are used to calculate m_s derivatives
- We calculate m_c derivatives by taking advantage of the ensembles mistuned by $\sim 10\%$ in their sea charm masses
- These derivatives are used to adjust data for mistunings before calculating aF_{p4s} and am_{p4s}

Using the decoupling theorem, we can calculate effects of mistuning in the charm mass analytically; result is in reasonable agreement with our numerical procedure.

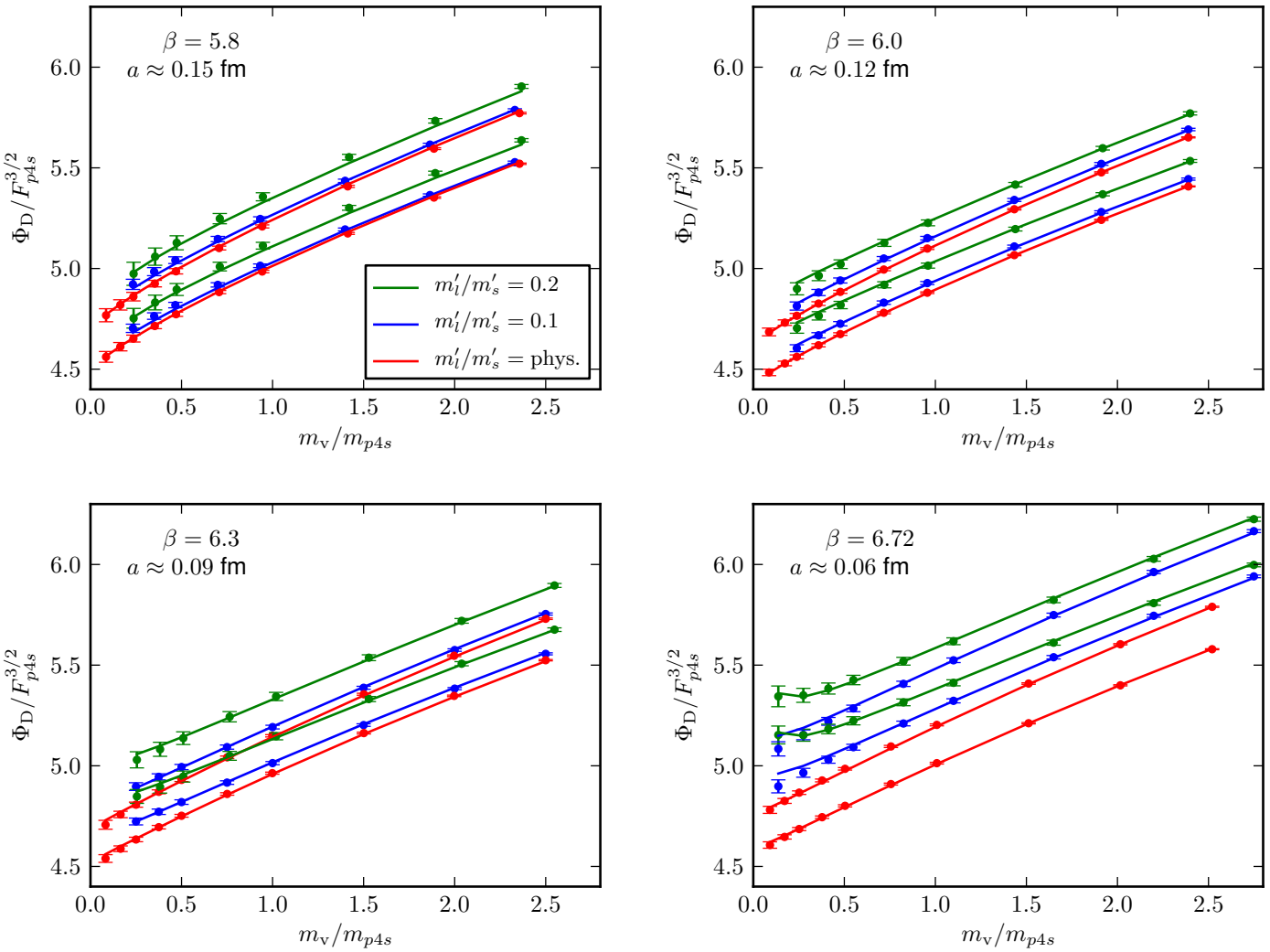
Charmed Decay Constant Data



- Vertical Axes: $\Phi_D = f_D\sqrt{M_D}$
- Horizontal axes: light valence mass
- Two heavy valence masses: m'_c and $0.9m'_c$
- For each color, higher points: m'_c, lower points: $0.9m'_c$

Data for unphysical m_s ensembles (and multiple volumes) not shown, but included in fits. [366 data points total.]

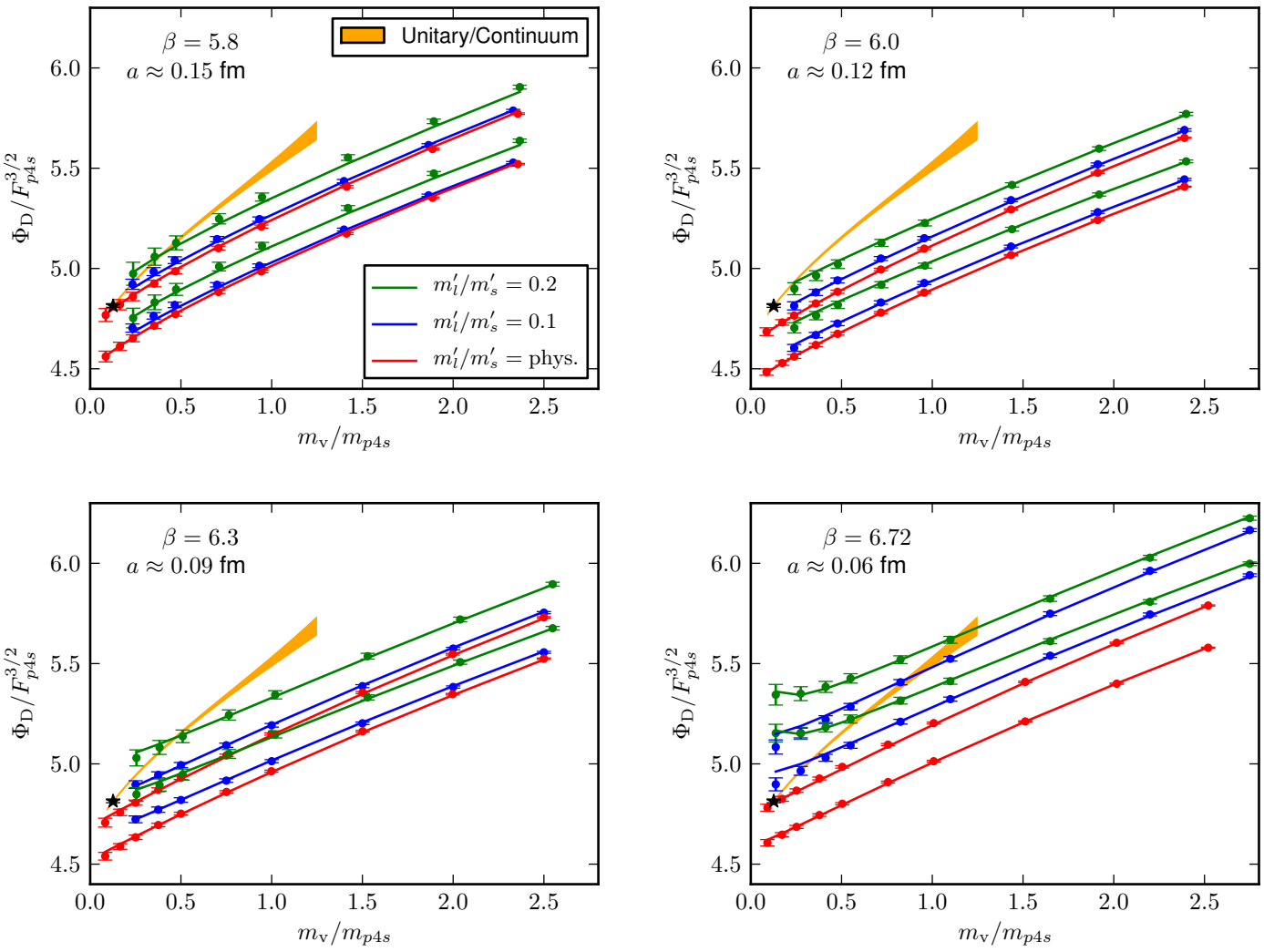
Central Chiral Fit



- Fit to all 366 data points.
- 27 parameters.
- $\chi^2/\text{dof} = 347/339$
- $p = 0.36$

Fit to chiral form for all-staggered heavy-light mesons worked out in *C. Bernard and J. K., PRD* **88**, *094017 (2013)*

Central Chiral Fit



- Orange band: valence and sea light masses are equal, up to small difference between m_d and m_l.
- Black burst shows Cont. Extrap. result for Φ_{D^+}
- Error in continuum band is statistical (from fit only).

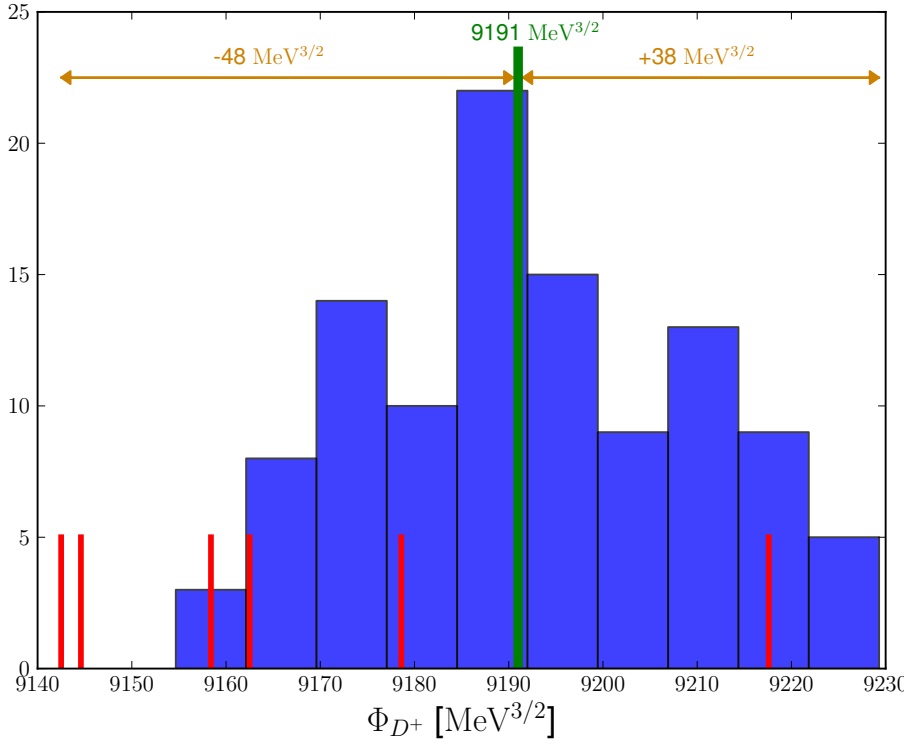
Our final statistical error is obtained using jackknife analysis, *i.e.*, includes the statistical error coming from inputs to the fit.

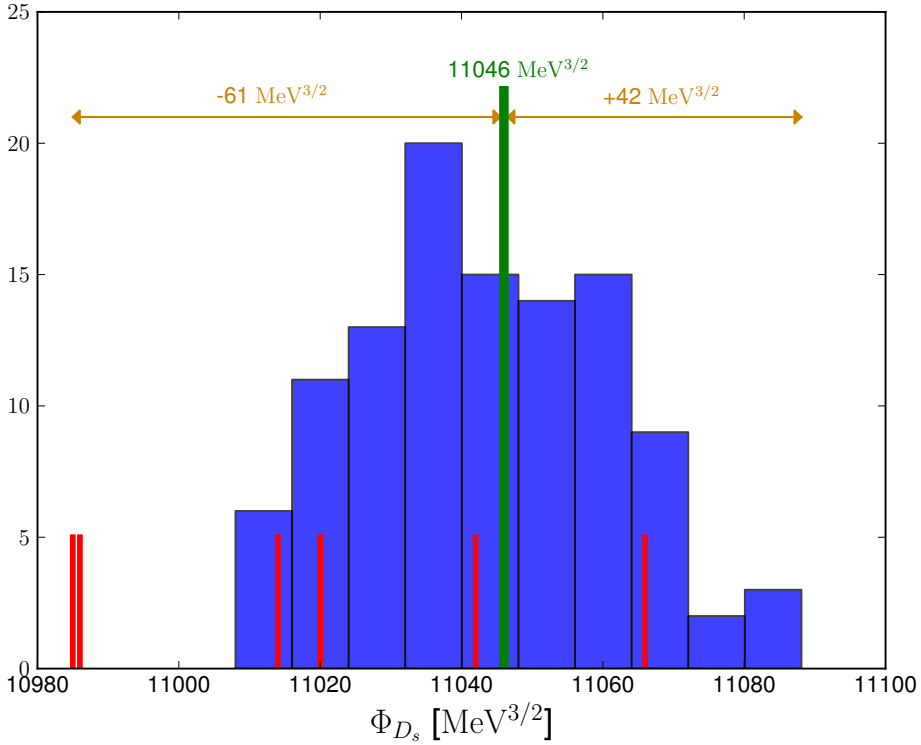
Discussion of Systematic Errors

For continuum extrapolation/chiral interpolation errors, use two methods:

- By straightforward comparison with various continuum extrapolations of physical-mass ensemble results.
- Self-contained error analysis:
  - Have 18 acceptable chiral fits $(p > 0.1)$, which:
    - keep or drop $a = 0.15$ fm ensembles.
    - add or drop higher order terms. (Number of fit parameters ranging from 23 to 28.)
    - constrain higher order chiral terms and/or discretization terms with priors, or leave them unconstrained.
    - determine relative value of the strong coupling α_S for discretization terms from measured light-light pseudoscalar taste splittings, or determine it from plaquette.
    - various ways of fixing or determining the LECs f, g_π and B.
  - Have 6 versions of inputs (quark masses, F_{p4s} in physical units from f_π and R_{p4s}) from physical-mass ensemble results.
  - Histogram results of 108 composite analyses.

Histogram





- The central fit is chosen deliberately to be near the centers of the histograms.
- We include the results of physical-mass ensemble analyses, performed in stage one; red bars.
- Conservatively we take the full difference as the systematic error of extrapolation to the continuum.

Results (Preliminary)

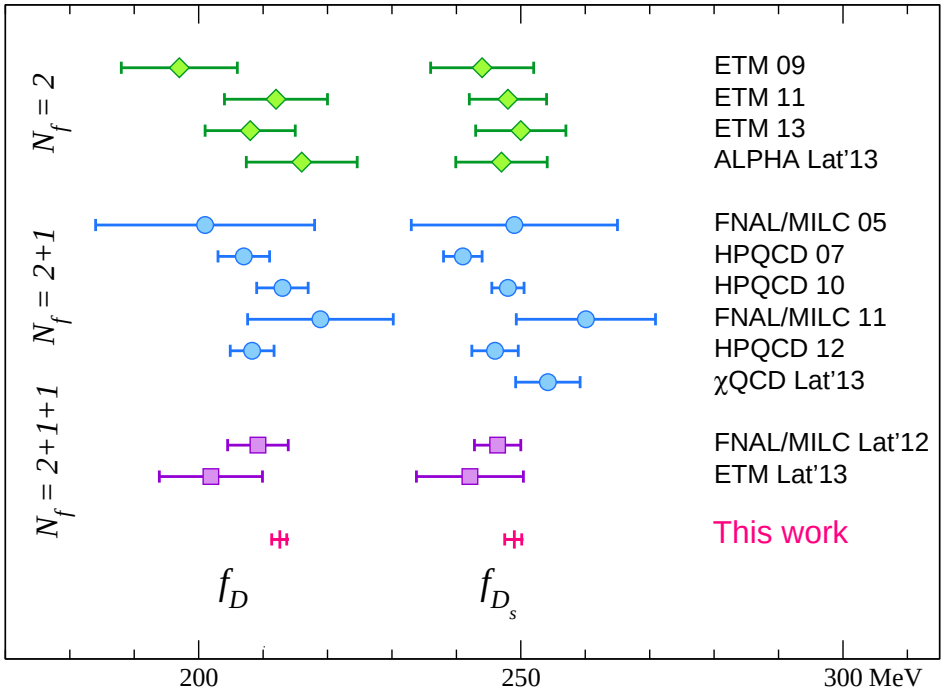
- Results:

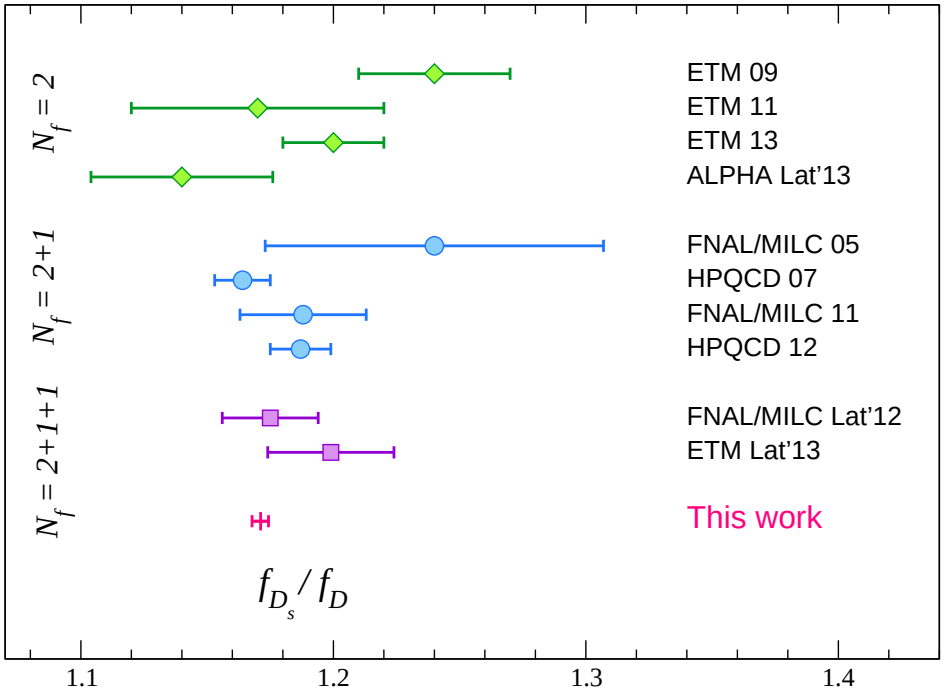
$$
\begin{aligned}
f_{D^+} &= 212.6 \pm 0.4_{\text{stat}}\ {}^{+0.9}_{-1.1}|_{a^2\ \text{extrap}} \pm 0.3_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.3_{f_\pi\ \text{PDG}}\ \text{MeV} \\
f_{D_s} &= 249.0 \pm 0.3_{\text{stat}}\ {}^{+1.0}_{-1.4}|_{a^2\ \text{extrap}} \pm 0.2_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.4_{f_\pi\ \text{PDG}}\ \text{MeV} \\
f_{D_s}/f_{D^+} &= 1.1712(10)_{\text{stat}}({}^{+28}_{-31})_{a^2\ \text{extrap}}(3)_{\text{FV}}(6)_{\text{EM}}
\end{aligned}
$$

- Repeating the result for f_{K^+}/f_{π^+} and quark mass ratios from the physical-ensemble analysis (stage one):

$$
\begin{aligned}
f_{K^+}/f_{\pi^+} &= 1.1956(10)_{\text{stat}}\ {}^{+23}_{-14}|_{a^2\ \text{extrap}}(10)_{\text{FV}}(5)_{\text{EM}} \\
m_s/m_l &= 27.352(51)_{\text{stat}}\ {}^{+80}_{-20}|_{a^2\ \text{extrap}}(39)_{\text{FV}}(55)_{\text{EM}} \\
m_c/m_s &= 11.747(19)_{\text{stat}}\ {}^{+52}_{-32}|_{a^2\ \text{extrap}}(6)_{\text{FV}}(27)_{\text{EM}}
\end{aligned}
$$

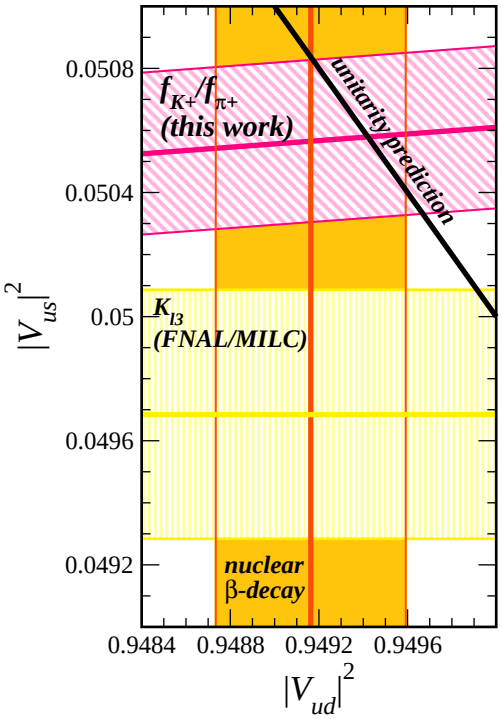
Comparison to Previous Work



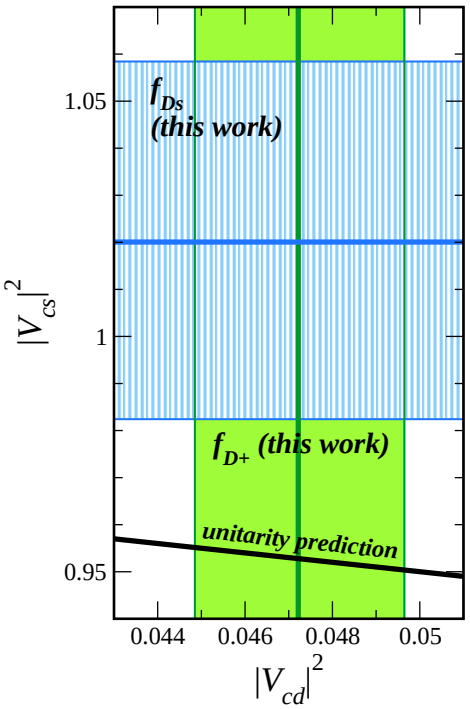


Unitarity Tests of the CKM Matrix (Preliminary)

First row



Second row



$$\frac{|V_{us}|}{|V_{ud}|} = 0.23081(29)_{\mathrm{BR}(K_{\ell 2})}(21)_{\mathrm{EM}}(52)_{\mathrm{LQCD}}$$

Taking $|V_{ud}|$ from nuclear β decay, we obtain

$$|V_{us}| = 0.22487(29)_{\mathrm{BR}(K_{\ell 2})}(20)_{\mathrm{EM}}(51)_{\mathrm{LQCD}}(5)_{V_{ud}}$$

$$\begin{aligned} |V_{cd}| &= 0.217(5)_{\mathrm{expt}}(1)_{\mathrm{LQCD}}(1)_{\mathrm{EM}} \\ |V_{cs}| &= 1.010(18)_{\mathrm{expt}}(5)_{\mathrm{LQCD}}(6)_{\mathrm{EM}} \end{aligned}$$

Errors for $|V_{cd}|$ and $|V_{cs}|$ are mostly from experiment

EM errors are from hadronic structure-dependent EM effects (from matching of QCD+QED to QCD).

Thank You for Your Attention!

Back-up Slides

Finite Volume Effects