Charmed and Strange Pseudoscalar Meson Decay Constants from HISQ Simulations

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Fermilab Lattice and MILC Collaborations
$f_{D^+}$, $f_{D_s}$ and $f_{K^+}$, together with experimental leptonic decay rate determinations, provide precise determinations of $|V_{cd}|$, $|V_{cs}|$ and $|V_{us}|$.

For higher precision than available with the asqtad action, we have moved to the HISQ action.

Advantage of HISQ is that charm may be treated with same action as light quarks.

MILC’s HISQ ensembles include ones with physical value of quark masses — reducing or eliminating errors from chiral extrapolation.
After determination of meson masses and amplitudes for each pair of valence-quark masses, analysis follows two stages:

1. The physical-mass ensembles allow for a simple analysis without ChPT:
   - used to compute quark-mass ratios and \( f_{K^+}/f_{\pi^+} \).
   - also compute intermediate scale setting quantity \( F_{p4s} \) (the decay constant when valence masses are \( 0.4m_s^{phys} \) and the sea masses are physical) and corresponding meson mass \( M_{p4s} \).

2. Analyze heavy-light decay constants on all ensembles (physical and unphysical mass) using ChPT:
   - reduces statistical error.
   - more control of continuum extrapolation.
   - uses quark mass ratios, \( F_{p4s} \) and \( F_{p4s}/M_{p4s} \) from stage one.
21 ensembles are used.

14 ensembles have $m_s$ tuned close to its physical value.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$m'_1/m'_s$</th>
<th>size</th>
<th>$N_{lats}$</th>
<th>$\approx a$ (fm)</th>
<th>$L$ (fm)</th>
<th>$M_\pi L$</th>
<th>$M_\pi (MeV)$</th>
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<td>$16^3 \times 48$</td>
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<td>0.06</td>
<td>5.44</td>
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Valence Masses for Charmed Decay Constant

- For light quark, about 10 masses ranging from $m'_l$ to $m'_s$
- For heavy quark, 2 masses close to the charm quark mass

<table>
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<tr>
<th>$\beta$</th>
<th>$m'_l/m'_s$</th>
<th>light masses ($m_v/m'_s$)</th>
<th>heavy masses ($m_Q/m'_c$)</th>
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<td>0.9,1.0</td>
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Illustration of the lattice spacing and valence quark mass tuning

Upper panels: $f_\pi$, $M_\pi$ and $M_K$ are used to fix $a$, $m_l$ and $m_s$

Similarly $m_c$ is fixed from $M_{D_s}$

$m_u/m_d$ comes from $M_{K^+}$ and $M_{K^0}$ (with inputs from our EM project)

Lower Panel: determinations of $f_{K^+}$, $f_{D^+}$ and $f_{D_s}$.

Doing the whole procedure inside a jackknife resampling gives the statistical errors
Black and red solid lines are from fits that only include physical mass ensembles.

Solid blue line is from fit that also includes blue squares for the physical light quark mass.

Dotted blue line is from same fit, but for heavier sea-quark mass corresponding to blue squares. \( \frac{m'_l}{m'_s} = 0.1 \).
Results of Stage One (Preliminary)

- Results for $f_{K^+}/f_{\pi^+}$ and quark mass ratios:

$$f_{K^+}/f_{\pi^+} = 1.1956(10)_{\text{stat}}^{+23}_{-14}a^2_{\text{extrap}}(10)_{\text{FV}}(5)_{\text{EM}}$$

$$m_s/m_l = 27.352(51)_{\text{stat}}^{+80}_{-20}a^2_{\text{extrap}}(39)_{\text{FV}}(55)_{\text{EM}}$$

$$m_c/m_s = 11.747(19)_{\text{stat}}^{+52}_{-32}a^2_{\text{extrap}}(6)_{\text{FV}}(27)_{\text{EM}}$$

- And intermediate scale setting quantities:

$$F_{p4s} = 153.9(9)_{\text{stat}}^{+14}_{-23}a^2_{\text{extrap}}(15)_{\text{FV}}(5)_{\text{EM}}$$

$$M_{p4s} = 433.24(17)_{\text{stat}}^{+1}_{-33}a^2_{\text{extrap}}(2)_{\text{FV}}(43)_{\text{EM}}$$

$$R_{p4s} \equiv F_{p4s}/M_{p4s} = 0.35527(24)_{\text{stat}}^{+52}_{-15}a^2_{\text{extrap}}(30)_{\text{FV}}(24)_{\text{EM}}$$

- Finite Volume error comes from the FV error in $f_\pi$ which in turn comes from ChPT. [A. Bazavov et al. PRD 110, 172003 (2013)]

- EM error is from the residual error in our tuned values of quark masses. Uses input $\epsilon = 0.84(21)$ ($\epsilon$ characterizes violations of Dashen’s theorem) from MILC EM project. (See talk by C. Bernard, Friday, 2:35 P.M., Pupin 428.)
Relative lattice scales are determined by $F_{pA}$.

Has very small statistical errors, so the mistuning in sea quark masses can be important.

So we adjust the data for mistunings in order to have a precise calculation of $aF_{pA}$ and $am_{pA}$.

Absolute scale comes from value of $F_{pA}$ in physical units (from stage one) which comes ultimately from $f_\pi$. 
Light sea masses of the ensembles available at \( a = 0.12 \) fm

Three ensembles inside the red ellipse are used to calculate \( m_l \) derivatives

Five ensembles inside the blue ellipse are used to calculate \( m_s \) derivatives

We calculate \( m_c \) derivatives by taking advantage of the ensembles mistuned by \( \sim 10\% \) in their sea charm masses

These derivatives are used to adjust data for mistunings before calculating \( aF_{p4s} \) and \( am_{p4s} \)

Using the decoupling theorem, we can calculate effects of mistuning in the charm mass analytically; result is in reasonable agreement with our numerical procedure.
Charmed Decay Constant Data

Vertical Axes:
\[ \Phi_D = f_D \sqrt{M_D} \]

Horizontal axes:
light valence mass

Two heavy valence masses: \( m'_c \) and 0.9\( m'_c \)

For each color, higher points: \( m'_c \), lower points: 0.9\( m'_c \)

Data for unphysical \( m_s \) ensembles (and multiple volumes) not shown, but included in fits. [366 data points total.]
Fit to all 366 data points.

- 27 parameters.
- $\chi^2$/dof = 347/339
- $p = 0.36$

Fit to chiral form for all-staggered heavy-light mesons worked out in C. Bernard and J. K., PRD 88, 094017 (2013)
Central Chiral Fit

Orange band: valence and sea light masses are equal, up to small difference between $m_d$ and $m_l$.

Black burst shows Cont. Extrap. result for $\Phi_{D+}$

Error in continuum band is statistical (from fit only).

Our final statistical error is obtained using jackknife analysis, i.e., includes the statistical error coming from inputs to the fit.
Discussion of Systematic Errors

For continuum extrapolation/chiral interpolation errors, use two methods:

- By straightforward comparison with various continuum extrapolations of physical-mass ensemble results.

- Self-contained error analysis:
  - Have 18 acceptable chiral fits \((p > 0.1)\), which:
    - keep or drop \(a = 0.15\) fm ensembles.
    - add or drop higher order terms. (Number of fit parameters ranging from 23 to 28.)
    - constrain higher order chiral terms and/or discretization terms with priors, or leave them unconstrained.
    - determine relative value of the strong coupling \(\alpha_S\) for discretization terms from measured light-light pseudoscalar taste splittings, or determine it from plaquette.
    - various ways of fixing or determining the LECs \(f, g_\pi\) and \(B\).
  - Have 6 versions of inputs (quark masses, \(F_{p4s}\) in physical units from \(f_\pi\) and \(R_{p4s}\)) from physical-mass ensemble results.
  - Histogram results of 108 composite analyses.
The central fit is chosen deliberately to be near the centers of the histograms.

We include the results of physical-mass ensemble analyses, performed in stage one; red bars.

Conservatively we take the full difference as the systematic error of extrapolation to the continuum.
Results:

\[ f_{D^+} = 212.6 \pm 0.4_{\text{stat}}^{+0.9}_{-1.1} |a^2_{\text{extrap}} \pm 0.3_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.3_{f_\pi \text{ PDG}} \text{ MeV} \]

\[ f_{D_s} = 249.0 \pm 0.3_{\text{stat}}^{+1.0}_{-1.4} |a^2_{\text{extrap}} \pm 0.2_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.4_{f_\pi \text{ PDG}} \text{ MeV} \]

\[ f_{D_s}/f_{D^+} = 1.1712(10)_{\text{stat}}^{+28}_{-31} |a^2_{\text{extrap}}(3)_{\text{FV}}(6)_{\text{EM}} \]

Repeating the result for \( f_{K^+}/f_{\pi^+} \) and quark mass ratios from the physical-ensemble analysis (stage one):

\[ f_{K^+}/f_{\pi^+} = 1.1956(10)_{\text{stat}}^{+23}_{-14} |a^2_{\text{extrap}}(10)_{\text{FV}}(5)_{\text{EM}} \]

\[ m_s/m_l = 27.352(51)_{\text{stat}}^{+80}_{-20} |a^2_{\text{extrap}}(39)_{\text{FV}}(55)_{\text{EM}} \]

\[ m_c/m_s = 11.747(19)_{\text{stat}}^{+52}_{-32} |a^2_{\text{extrap}}(6)_{\text{FV}}(27)_{\text{EM}} \]
Comparison to Previous Work

\[ N_f = 2 \]

\[ N_f = 2+1 \]

\[ N_f = 2+1+1 \]

\[ f_D \]

\[ f_{D_s} \]

ETM 09
ETM 11
ETM 13
ALPHA Lat’13

FNAL/MILC 05
HPQCD 07
HPQCD 10
FNAL/MILC 11
HPQCD 12
\( \chi_{QCD} \) Lat’13

FNAL/MILC Lat’12
ETM Lat’13

This work

\[ f_{D_s}/f_D \]

ETM 09
ETM 11
ETM 13
ALPHA Lat’13

FNAL/MILC 05
HPQCD 07
FNAL/MILC 11
HPQCD 12

FNAL/MILC Lat’12
ETM Lat’13

This work
\[ \frac{|V_{us}|}{|V_{ud}|} = 0.23081(29)\text{BR}(K\ell_2)(21)\text{EM}(52)\text{LQCD} \]

Taking \(|V_{ud}|\) from nuclear \(\beta\) decay, we obtain

\[ |V_{us}| = 0.22487(29)\text{BR}(K\ell_2)(20)\text{EM}(51)\text{LQCD}(5)\text{V}_{ud} \]

\[ |V_{cd}| = 0.217(5)_{\text{expt}}(1)\text{LQCD}(1)\text{EM} \]
\[ |V_{cs}| = 1.010(18)_{\text{expt}}(5)\text{LQCD}(6)\text{EM} \]

Errors for \(|V_{cd}|\) and \(|V_{cs}|\) are mostly from experiment.

EM errors are from hadronic structure-dependent EM effects (from matching of QCD+QED to QCD).
Thank You for Your Attention!
Back-up Slides

\( f_{D^+}, f_{D_s} \text{ and } f_{K^+} \)
Finite Volume Effects

\[ f_{D^+}, f_{D_s^0} \text{ and } f_{K^+} \]

Lattice 2014, New York, USA