Low lying charmonium states at the physical point

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For the Fermilab Lattice and MILC collaborations
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Low-lying charmonium: A precision benchmark

- Well understood from potential models
- Well determined in experiment
- Spin-dependent mass splittings extremely sensitive to the charm-quark mass and heavy-quark discretization

meson	mass	width
η_c	2983.7(7)	32.0(9)MeV
J/Ψ	3096.916(11)	92.9(2.8)keV
χ_{c0}	3414.75(31)	10.3(6)MeV
χ_{c1}	3510.66(3)	0.86(5)MeV
χ_{c2}	3556.20(9)	1.97(11)MeV
h_c	3525.38(11)	0.7(4)MeV

Charmonium mass splittings

We use the *Fermilab method* for the charm quark

El-Khadra et al., PRD 55, 3933 (1997)

- m_c tuned by demanding the D_s meson kinetic mass to be physical
- We quote splittings among charmonium states

$$\begin{aligned}
\Delta M_{HF} &= M_{n^3L} - M_{n^1L} \\
\Delta M_{1P1S} &= M_{\overline{1P}} - M_{\overline{1S}} \\
\Delta M_{Spin-Orbit} &= (5M_{\chi_{c2}} - 3M_{\chi_{c1}} - 2M_{\chi_{c0}})/9 \\
\Delta M_{Tensor} &= (3M_{\chi_{c1}} - M_{\chi_{c2}} - 2M_{\chi_{c0}})/9
\end{aligned}$$

with

$$\begin{aligned}
M_{\overline{1P}} &= (M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}})/9 \\
M_{\overline{1S}} &= (M_{\eta_c} + 3M_{J/\psi})/4
\end{aligned}$$

MILC asqtad ensembles used for charmonium

$\approx$ a [fm]	m_l/m_h	size	# of sources	κ_c	κ_{sim}
0.14	0.2	$16^3 \times 48$	2524	0.12237	0.1221
0.14	0.1	$20^3 \times 48$	2416	0.12423	0.12423
0.114	0.2	$20^3 \times 64$	4800	0.12722	0.12722
0.114	0.1	$24^3 \times 64$	3328	0.12960	0.1298
0.082	0.2	$28^3 \times 96$	1904	0.130921	0.1310
0.082	0.1	$40^3 \times 96$	4060	0.12231	0.1221
0.058	0.2	$48^3 \times 144$	2604	0.12423	0.1245
0.058	0.1	$64^3 \times 144$	1984	0.12714	0.12714
0.043	0.2	$64^3 \times 192$	3204	0.12955	0.1296

- 5 lattice spacings with two different light sea-quark masses
  $\rightarrow$ Controlled extrapolation to the chiral-continuum limit
- 4 source time slices per gauge configuration
- Follows previous efforts

  T. Burch et al. PRD 81 034508, 2010
- Disconnected contributions are omitted

Interpolator basis and fit methodology

- Large correlator matrix from smeared stochastic wall sources
- Variational method

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$
$$\lambda^{(k)}(t) \propto \mathrm{e}^{-tE_k}\left(1 + \mathcal{O}\left(\mathrm{e}^{-t\Delta E_k}\right)\right)$$

Michael Nucl. Phys. B259, 58 (1985)
Lüscher and Wolff Nucl. Phys. B339, 222 (1990)
Blossier et al. JHEP 04, 094 (2009)

- (multi)exponential fits to the eigenvalues in $[t_{min}, t_{max}]$
- Keep t_0 and t_{min} approximately constant in fm across ensembles
- Pick t_{max} such that eigenvectors remain stable
- Correct data for unphysical κ_c where needed

Chiral and continuum fits

- Clear sea-quark mass dependence for some observables
- We need to take into account unphysical sea-quark masses
- Fit model

$$M = M_0 + c_1(2x_l + x_h) + c_2 f_1(a) + c_3 f_2(a) + \dots$$
$$x_l = \frac{m_{ud,sea} - m_{ud,phys}}{m_{s,phys}}$$
$$x_h = \frac{m_{s,sea} - m_{s,phys}}{m_{s,phys}}$$

For the lattice spacing dependence

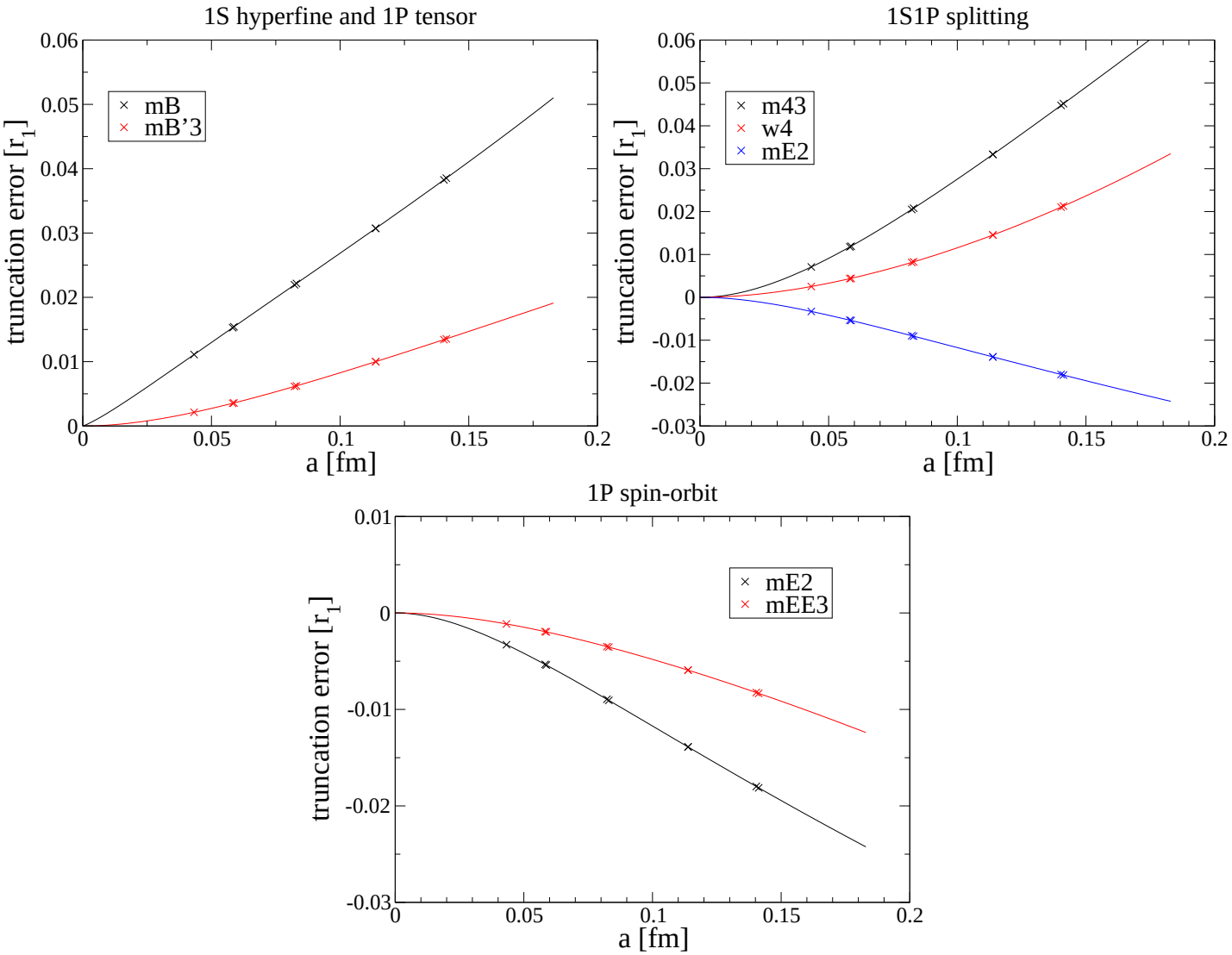
- 2-3 functions with shape estimated from expected cutoff effects
- Set fit priors to twice the expected magnitude centered around 0
- For comparison: Fits with just the leading shape and no prior

Discretization uncertainties I

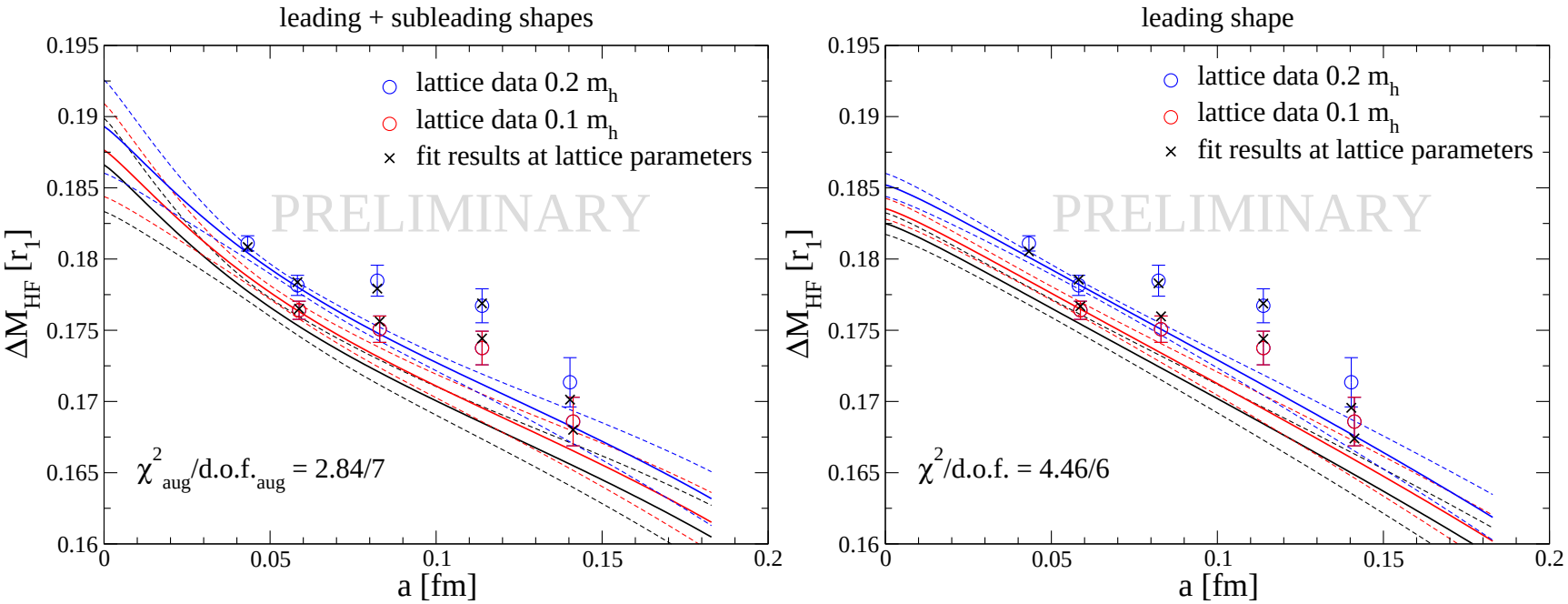
- Qualitatively, we expect errors of order a^2 and $\alpha_s a$ for quarkonium
- Most of these are worked out in `Oktay and Kronfeld PRD 78 014504, 2008` using NRQCD power counting
- Use $v^2 = 0.3$ and $mv^2 \approx 420\mathrm{MeV} \approx 1\mathrm{P1S}$ splitting for charm
- We consider the leading two contributions arising at either v^4 or v^6
- Shapes take into account tree level mismatches as derived in OK-action paper
- For the $1P1S$-splitting, allow for a term from rotational symmetry breaking (w_4 term)

Discretization uncertainties II

Expected shape of discretization uncertainties ($c_i = 1 \forall i$)



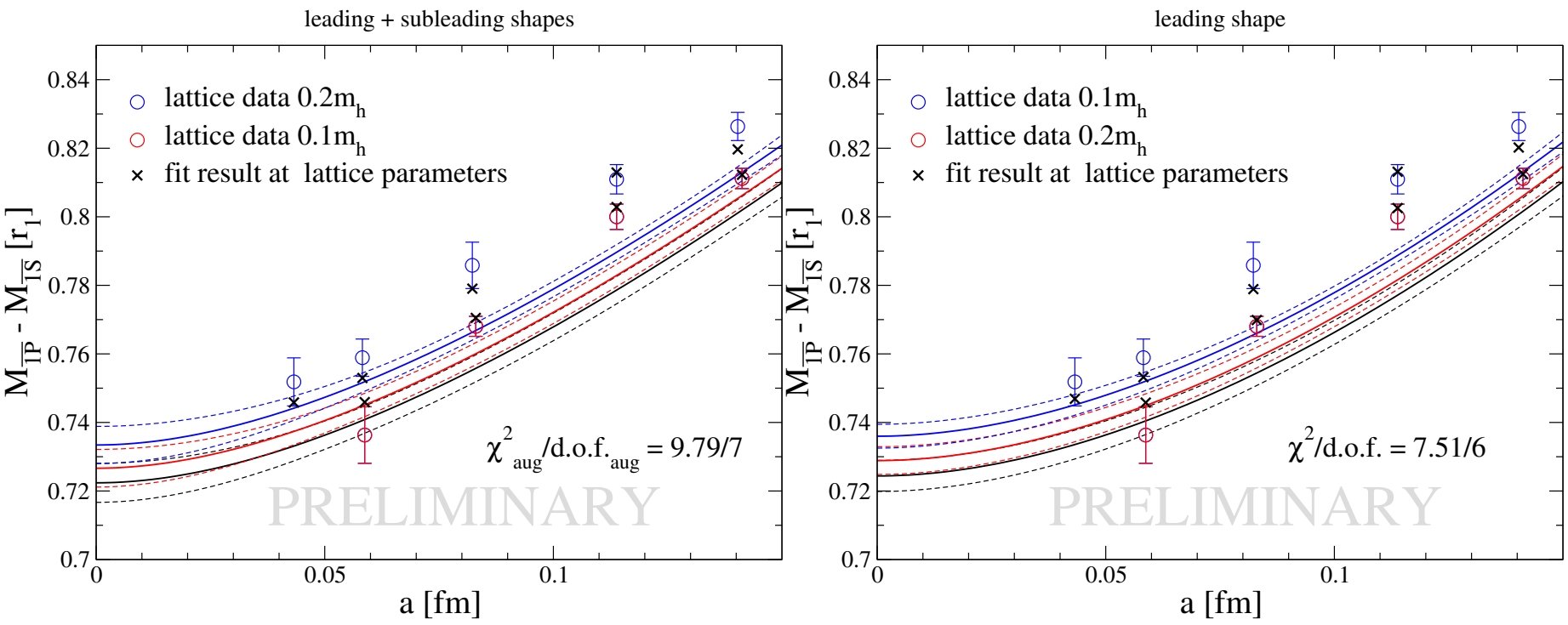
The 1S hyperfine splitting



- Curves for physical (black), $0.1m_s$, and $0.2m_s$ light-quark masses
- Including subleading effects significantly enlarges the uncertainty
- Significant autocorrelatations on some ensembles
- Significant contribution from disconnected diagrams expected

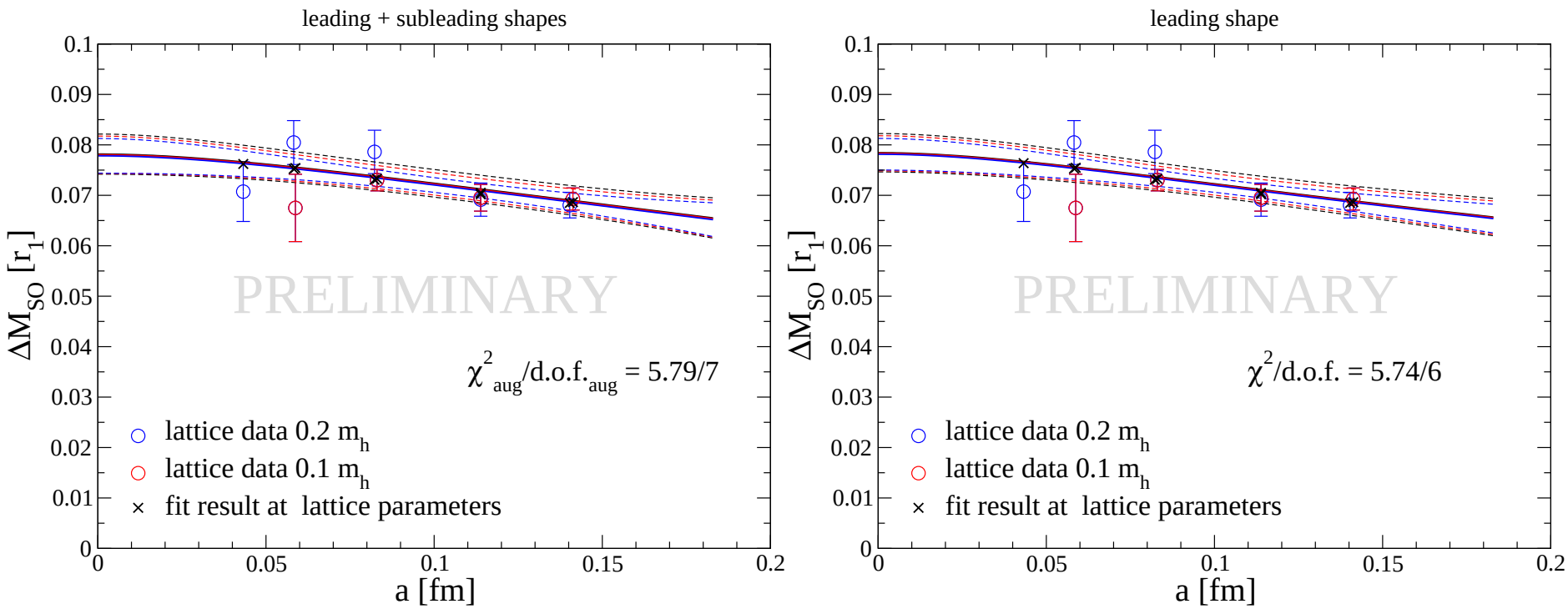
Levkova and DeTar, PRD 83 074504, 2011

The S-wave P-wave splitting



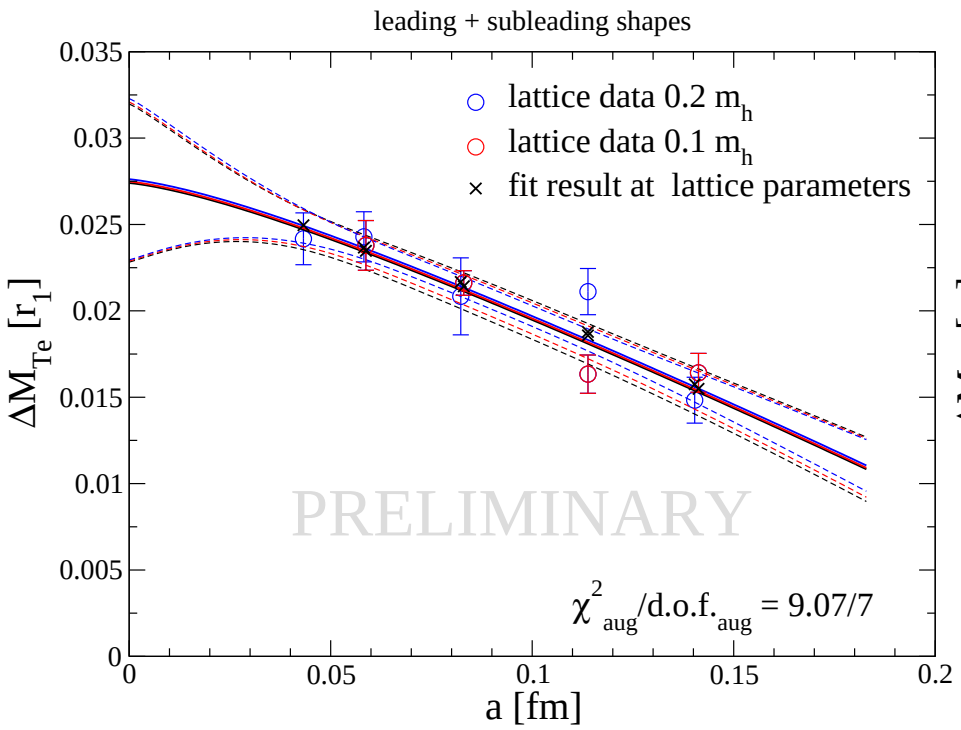
- Effects from unphysical strange quarks clearly visible
- Fits are stable with regard to the number of shapes

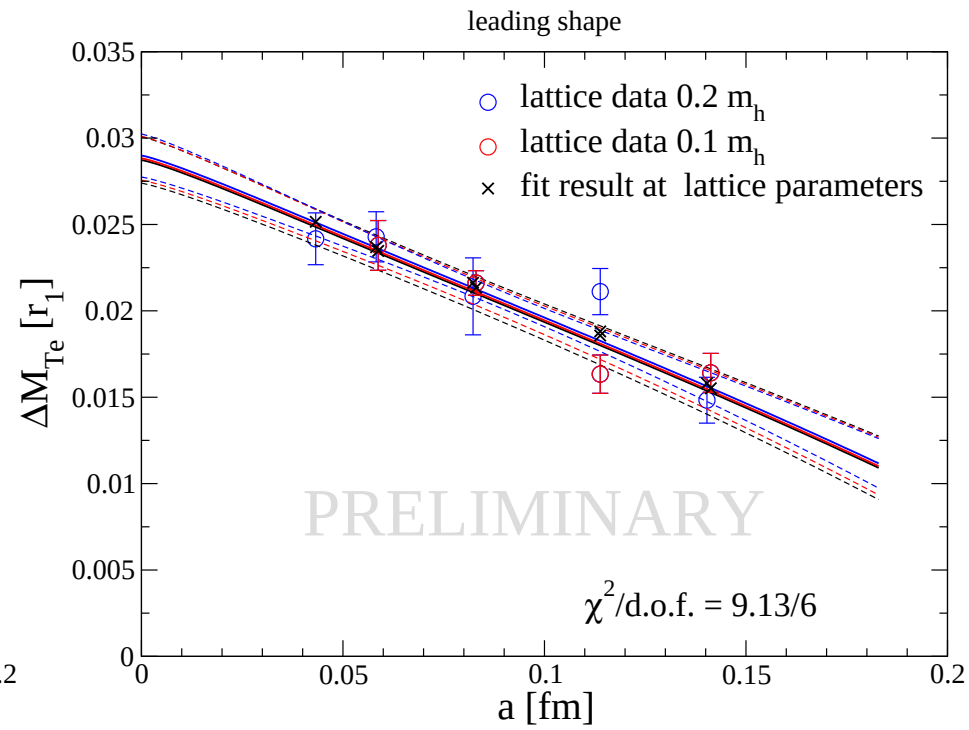
The P-wave spin-orbit splitting



- Small discretization uncertainties

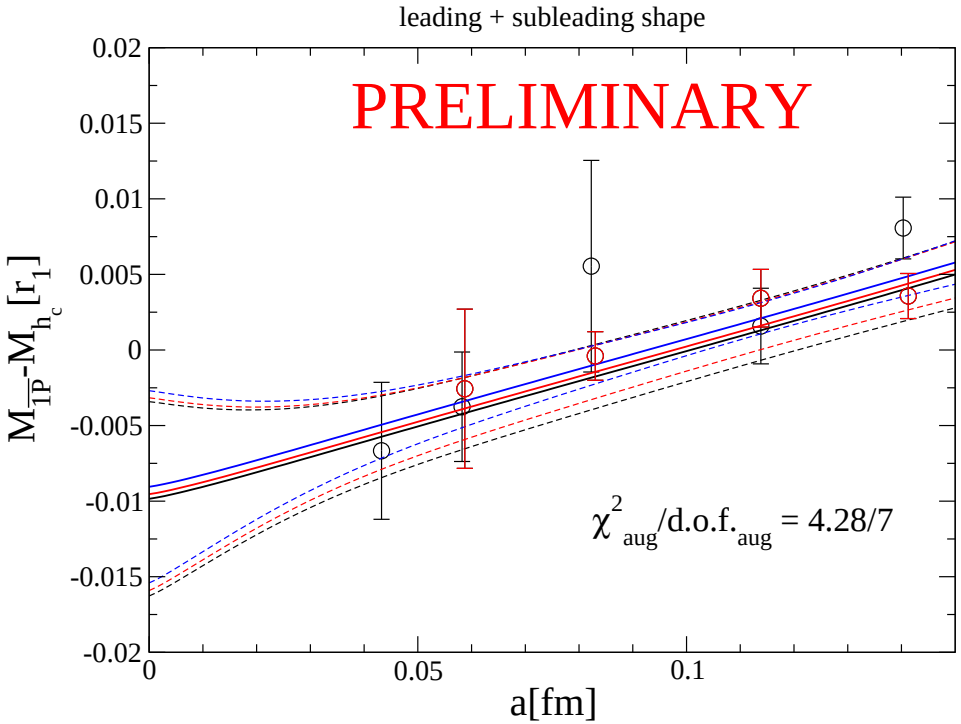
The P-wave Tensor splitting





- Significant uncertainty from the choice of fit model

The P-wave hyperfine splitting



- Expected to be very small
- Data still preliminary!

Preliminary comparison to experiment

- Errors include statistics, chiral and continuum extrapolations
- Not yet included: Scale-setting uncertainty (significant for ΔM_{HF} and ΔM_{1P1S})
- Second uncertainty on the 1S hyperfine splitting is best-estimate for disconnected contributions
- Volume effects are expected to be negligible

Mass difference	This analysis [MeV]	Experiment [MeV]
1P1S	457.3 ± 3.6	457.5 ± 0.3
1S hyperfine	$118.1 \pm 2.1^{-1.5}_{-4.0}$	113.2 ± 0.7
1P spin-orbit	49.5 ± 2.5	46.6 ± 0.1
1P tensor	17.3 ± 2.9	16.25 ± 0.07
1P hyperfine	-6.2 ± 4.1	-0.10 ± 0.22

Thank you!