Electromagnetic matrix elements for excited Nucleons

Benjamin Owen

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Outline

1. Correlation Matrix Techniques
2. Calculation Details
3. Results
   - Excited State Spectrum
   - Form Factor extraction
   - Quark Sector Results
A systematic framework for generating ideal operators for Hamiltonian Eigenstates
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Require a basis of operators: \( \{ \chi_i \} ; \ i \in [1, N] \)
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Calculate set of cross-correlation functions

\[
G_{ij}(t, \vec{p}; \Gamma) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \text{tr}(\Gamma \langle \Omega | \chi_i(x) \chi_j(0) | \Omega \rangle)
\]

\[
= \sum_{\alpha} e^{-E_\alpha(\vec{p}) t} Z_i^\alpha(\vec{p}) \bar{Z}_j^\alpha(\vec{p}) \text{tr} \left( \frac{\Gamma (\vec{p} + m_\alpha)}{2E_\alpha(\vec{p})} \right)
\]

where \( Z_i^\alpha \), \( \bar{Z}_j^\alpha \) are the couplings of sink operator \( (\chi_i) \) and source operator \( (\bar{\chi}_j) \) to the state \( \alpha \)
CM Analysis (cont)

- Desire $N$ optimised sink ($\phi_\alpha$) and source ($\bar{\phi}_\alpha$) operators
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Ideally, we want these operators to satisfy

$$\langle \Omega | \phi_\beta | M_\alpha, p, s \rangle = \delta_{\alpha\beta} \mathcal{Z}^\alpha(p) \sqrt{\frac{M_\alpha}{E_\alpha(p)}} u(p, s)$$
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$$\langle \Omega | \phi_\beta | M_\alpha, p, s \rangle = \delta_{\alpha \beta} Z^\alpha(\vec{p}) \sqrt{\frac{M_\alpha}{E_\alpha(\vec{p})}} u(p, s)$$

use our basis of operators to construct these new operators

$$\bar{\phi}_\alpha(x, \vec{p}) = \sum_{i=1}^{N} u_i^\alpha(\vec{p}) \bar{\chi}_i(x)$$

$$\phi_\alpha(x, \vec{p}) = \sum_{i=1}^{N} v_i^\alpha(\vec{p}) \chi_i(x)$$

optimal coupling to state $| M_\alpha, p, s \rangle$
Knowledge of the time dependence provides the recurrence relation

\[ G_{ij}(t + \delta t, \vec{p}; \Gamma) u_j^\alpha = e^{-E_\alpha(\vec{p}) \delta t} G_{ij}(t, \vec{p}; \Gamma) u_j^\alpha \]
Knowledge of the time dependence provides the recurrence relation

\[ G_{ij}(t + \delta t, \vec{p}'; \Gamma) u_j^\alpha = e^{-E_\alpha(\vec{p}) \delta t} G_{ij}(t, \vec{p}; \Gamma) u_j^\alpha \]

Thus, the desired values for \( u_j^\alpha \) and \( v_i^\alpha \) are given by
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CM Eigenvalue Equation

\[
\begin{align*}
\left[ G^{-1}(t_0, \vec{p}; \Gamma) G(t_0 + \delta t, \vec{p}; \Gamma) \right]_{ij} u_j^\alpha(\vec{p}) &= e^{-E_{\alpha}(\vec{p}) \delta t} u_j^\alpha(\vec{p}) \\
v_i^\alpha(\vec{p}) \left[ G(t_0 + \delta t, \vec{p}; \Gamma) G^{-1}(t_0, \vec{p}; \Gamma) \right]_{ij} &= e^{-E_{\alpha}(\vec{p}) \delta t} v_i^\alpha(\vec{p})
\end{align*}
\]
Knowledge of the time dependence provides the recurrence relation

\[ G_{ij}(t + \delta t, \vec{p}^*; \Gamma) u_j^\alpha = e^{-E^\alpha(\vec{p})\delta t} G_{ij}(t, \vec{p}^*; \Gamma) u_j^\alpha \]

Thus, the desired values for \( u_j^\alpha \) and \( v_i^\alpha \) are given by

\[
\begin{align*}
[ G^{-1}(t_0, \vec{p}; \Gamma) G(t_0 + \delta t, \vec{p}; \Gamma) ]_{ij} u_j^\alpha(\vec{p}) &= e^{-E^\alpha(\vec{p})\delta t} u_j^\alpha(\vec{p}) \\
v_i^\alpha(\vec{p}) [ G(t_0 + \delta t, \vec{p}; \Gamma) G^{-1}(t_0, \vec{p}; \Gamma) ]_{ij} &= e^{-E^\alpha(\vec{p})\delta t} v_i^\alpha(\vec{p})
\end{align*}
\]

Using \( v_i^\alpha(\vec{p}) \), \( u_j^\alpha(\vec{p}) \) we are able to project out the correlation function for the state \( | M_\alpha, p, s \rangle \)

\[ G_\alpha(t, \vec{p}; \Gamma) = v_i^\alpha(\vec{p}) G_{ij}(t, \vec{p}; \Gamma) u_j^\alpha(\vec{p}) \]
CM Analysis for 3pt-functions

- The eigenvectors derived from the two-point analysis can be used to project out the three-point function
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\[ G^\alpha(\vec{p}', \vec{p}; t_2, t_1; \Gamma') = v_i^\alpha(\vec{p}') G_{ij}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') u_j^\alpha(\vec{p}) . \]
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- With the desired state now isolated, one simply uses the projected correlation functions in the ratio to extract the matrix element.
CM Analysis for 3pt-functions

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\[ G^\alpha(\vec{p}', \vec{p}; t_2, t_1; \Gamma') = v^\alpha_i(\vec{p}') G_{ij}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') u^\alpha_j(\vec{p}) \, . \]

- With the desired state now isolated, one simply uses the projected correlation functions in the ratio to extract the matrix element.
- In this work we have used the following ratio,

\[ R^\alpha(\vec{p}', \vec{p}; \Gamma', \Gamma) = \sqrt{\frac{G^\alpha(\vec{p}', \vec{p}; t_2, t_1; \Gamma') G^\alpha(\vec{p}, \vec{p}'; t_2, t_1; \Gamma')}{G^\alpha(\vec{p}, t_2; \Gamma') G^\alpha(\vec{p}', t_2; \Gamma')}} \, . \]
Our operator basis

- It is important to use a basis that has good overlap with the states of interest

Table: The rms radii for the various levels of smearing considered in this work.

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\[ \alpha = 0.7 \]

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- In particular, we use 4 levels of gauge invariant Gaussian smearing at the source and sink with smearing fraction $\alpha = 0.7$.\(^1\)

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Our operator basis (cont)

- We use both $\chi_1$ and $\chi_2$

$$\chi_1(x) = \epsilon^{abc}(u^T a(x) C \gamma_5 d^b(x)) u^c(x)$$
$$\chi_2(x) = \epsilon^{abc}(u^T a(x) C d^b(x)) \gamma_5 u^c(x)$$

This gives us 8 operators resulting in an $8 \times 8$ Correlation Matrix

We perform a single CM analysis and use these eigenvectors to project out the eigenstate correlators for all times slices

For our variational parameters, we use $t_0 = 18$ and $\delta t = 2$.

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- For positive parity states we use the projector:

$$\Gamma_4^+ = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$


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- For negative parity states we use the projector\(^2\):

$$
\Gamma_4^- = -\gamma_5 \Gamma_4^+ \gamma_5
$$


Tracking eigenstates

- Between momenta, it is important to ensure that we order eigenvectors consistently.

\[ w_{\alpha j}(\vec{p}) = \frac{G_{ij}(t_0, \vec{p}; \Gamma)}{2} u_{\alpha j}(\vec{p}) \]

We can identify corresponding eigenvectors across momenta as those with
\[ w_{\alpha}(\vec{p}) \cdot w_{\beta}(0) \approx \delta_{\alpha\beta} \]

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\(^1\) M. S. Mahbub et al., Phys. Rev. D 87, 094506 (2013)
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- In doing this, we are able to construct orthonormal eigenvectors $w^\alpha_j$, related to our $u^\alpha_i$ through

$$w^\alpha_j(\vec{p}) = G_{ij}^{1/2}(t_0, \vec{p}; \Gamma) u^\alpha_j(\vec{p})$$

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Nucleon Matrix Elements

- Both positive and negative parity nucleon electromagnetic matrix elements can be decomposed into the standard Pauli-Dirac form

\[
\langle N, p', s' | J^\mu | N, p, s \rangle = \bar{u}(p', s') \left[ \gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2) \right] u(p, s)
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\]

- Sachs Form Factors are related to these via

\[
G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)
\]
\[
G_M(Q^2) = F_1(Q^2) + F_2(Q^2)
\]
Nucleon Matrix Elements (cont)

- SST-propagators are evaluated with the inversion done through the current

\[
\vec{q} = 2\pi \hat{x}
\]

We evaluate the three-point functions with \( \vec{p} = 0 \) and \( \vec{p}' = \vec{q} \)

The ratios used to extract the form factors \( G_E \) and \( G_M \) are

\[
G_E(Q^2) = \left( \frac{2E_q + M}{2E_q + M + 2\gamma_{\pm}^2} \right)^{1/2} R(\vec{q}, 0; \Gamma_{\pm}, \Gamma_{\pm}; \mu = 4)
\]

\[
G_M(Q^2) = \left( \frac{2E_q + M}{2E_q + M + 2\gamma_{\pm}^2} \right)^{1/2} R(\vec{q}, 0; \Gamma_{\pm}, \Gamma_{\pm}; \mu = 3)
\]

where \( \Gamma_{\pm} = \sigma_{i000} \) and \( \Gamma_{\pm} = -\gamma_5 \Gamma_{\pm} \gamma_5 \)
Nucleon Matrix Elements (cont)

- SST-propagators are evaluated with the inversion done through the current
- We use a conserved vector current, with $\mathbf{q} = \frac{2\pi}{L} \hat{x}$
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- $G_M(Q^2) = E_q + M |\vec{q}| \left( \frac{2E_q + M}{2E_q} \right)^{1/2} R(\vec{q}, 0; \Gamma^\pm 2, \Gamma^\pm 4; \mu = 3)$
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\[
G_E(Q^2) = \left( \frac{2E_q}{E_q + M} \right)^{1/2} R(\vec{q}, 0; \Gamma_4^\pm, \Gamma_4^\pm; \mu = 4)
\]

\[
G_M(Q^2) = \frac{E_q + M}{|\vec{q}|} \left( \frac{2E_q}{E_q + M} \right)^{1/2} R(\vec{q}, 0; \Gamma_2^\pm, \Gamma_4^\pm; \mu = 3)
\]

where

\[
\Gamma_i^+ = \begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \Gamma_i^- = -\gamma_5 \Gamma_i^+ \gamma_5
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Ensemble Details

- For this calculation we are working with the PACS-CS (2+1)-flavour Full QCD ensembles\(^1\) made available through the ILDG

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There are five light quark-masses

\[
\begin{array}{cccc}
\hline
m_\pi\ (\text{MeV}) & n_{\text{cfgs}} & n_{\text{srcs/cfg}} & n_{\text{srcs}} \\
\hline
702 & 350 & 2 & 700 \\
570 & 350 & 2 & 700 \\
411 & 350 & 2 & 700 \\
296 & 350 & 2 & 700 \\
156 & 200 & 6 & 1200 \\
\hline
\end{array}
\]

Positive Parity Spectrum

\[ M \text{ (GeV)} \]

\[ m_{\pi}^2 \text{ (GeV}^2) \]
$N^*(1/2^+) \text{ wave function} - m_\pi = 570 \text{ MeV}$

$N^*(1/2^+) \text{ wave function} - m_\pi = 156 \text{ MeV}$

$^1D. \text{ Roberts et al., Phys. Rev. D 89, 074501 (2014)}$
Negative Parity Spectrum
Multi-particle states couple weakly to our choice of interpolators and so their contribution is only significant in the tail of the correlator.\textsuperscript{1}

\textsuperscript{1}M. S. Mahbub et al., Annals Phys. \textbf{342} (2014) 270-282
Multi-particle states couple weakly to our choice of interpolators and so their contribution is only significant in the tail of the correlator.\textsuperscript{1}

We consider logG of our projected 2pt-correlators to identify regions where multi-particle contributions are suppressed relative to the nucleon excitation.

\textsuperscript{1}M. S. Mahbub et al., Annals Phys. \textbf{342} (2014) 270-282
LogG

- Multi-particle states couple weakly to our choice of interpolators and so their contribution is only significant in the tail of the correlator.\(^1\)
- We consider logG of our projected 2pt-correlators to identify regions where multi-particle contributions are suppressed relative to the nucleon excitation.
- On going work will broaden our basis to include multi-particle operators.

\(^1\)M. S. Mahbub et al., Annals Phys. 342 (2014) 270-282
Projected Correlator for the second $\frac{1}{2}^-$ eigenstate:

$m_\pi = 570$ MeV

Want linear behaviour in logG around and after $t_s = 21$
Projected Correlator for the second $^{1/2^-}$ eigenstate:

$m_\pi = 296$ MeV

Want linear behaviour in logG around and after $t_s = 21$
Quark Sector Results: GE, $u$ in $p$ (Positive Parity)

$m_\pi = 570$ MeV

![Graph showing GE as a function of t]
Quark Sector Results: GE, $d$ in $p$ (Positive Parity)

$m_\pi = 570$ MeV
Quark Sector Results: GM, $u$ in $p$ (Positive Parity)

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Comparison across $m_\pi^2$

- In comparing between states and different values of $m_\pi$, we need to take into account the small difference in $Q^2$
Comparison across $m^2_\pi$

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- To facilitate a comparison, we make use of a dipole Ansatz

$$G_i(Q^2) = \left( \frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^2 G_i(0)$$

where $\Lambda$ is a characteristic scale and $G_i(0)$ is the electric form factor.

To perform a small shift in $Q^2$, we choose to shift all our extracted form factors to the common value of $Q^2 = 0.16$ GeV$^2$. 
Comparison across $m^2_\pi$

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to perform a small shift in $Q^2$.
- As we are using a conserved current, we are to extract $\Lambda^2$ from the Electric form factor where $G_E(0) = 1$. 
Comparison across $m_π^2$

- In comparing between states and different values of $m_π$, we need to take into account the small difference in $Q^2$.
- To facilitate a comparison, we make use of a dipole Ansatz

$$G_i(Q^2) = \left( \frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^2 G_i(0)$$

to perform a small shift in $Q^2$.
- As we are using a conserved current, we are to extract $\Lambda^2$ from the Electric form factor where $G_E(0) = 1$.
- For this ensemble, we choose to shift all our extracted form factors to the common value of $Q^2 = 0.16 \text{ GeV}^2$. 

Quark Sector Results: GE, \( u \) in \( p \) (Positive Parity)
Quark Sector Results: GE, $d$ in $p$ (Positive Parity)
Quark Sector Results: GE, \( u \) in \( p \) (Negative Parity)
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$G_E(Q^2)$ vs $m_{\pi}^2$ (GeV$^2$)

- Blue points: Data
- Red points: Theory
- Green points: Experiment
In the positive parity sector, at the heavier masses, $G_E$ for the first excited state is smaller than the ground state consistent with the expectation that the state is larger.
$G_E$ summary

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- An interesting possibility is that we have important $\Delta^{++}, \pi^-$ dressings

\[
\frac{1}{\sqrt{2}} |\Delta^{++}\pi^-\rangle - \frac{1}{\sqrt{3}} |\Delta^+\pi^0\rangle + \frac{1}{\sqrt{6}} |\Delta^0\pi^+\rangle
\]

which would lead to accumulation of positive charge at the origin.
Quark Sector Results: GM, $u$ in $p$ (Positive Parity)
Quark Sector Results: GM, $d$ in $p$ (Positive Parity)
Quark Sector Results: $G_M, u \ in \ p$ (Negative Parity)

\[
G_M(Q^2) (\mu_N)
\]

\[
m_{\pi}^2 \ [\text{(GeV)}^2]
\]
Quark Sector Results: GM, $d$ in $p$ (Negative Parity)
In the negative parity sector, we observe the first and second excitations have differing signs for the single quark sector.
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- First excitation appears consistent with $s = \frac{1}{2}$, $l = 1$ to give $j = \frac{1}{2}$.

- Second excitation appears consistent with $s = \frac{3}{2}$, $l = 1$ to give $j = \frac{1}{2}$.
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- This is much like the difference in quark sectors observed between the $p$ and $\Delta^+$.
- This is consistent with the states having differing spin configurations:
  - First excitation appears consistent with $s = \frac{1}{2}, l = 1$ to give $j = \frac{1}{2}$.
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Conclusions and Future Work

- Demonstrated how correlation matrix methods allow us to probe excited state structure

- Obtained quality plateaus in both the positive parity and negative parity sectors

- Observed interesting enhancement in electric form factor at lighter masses

- Observed qualitative difference between the quark sectors of the first and second negative parity excitations

- Attempt to access smaller values of $Q^2$ by using boosts

- Examine the transition amplitudes for ground state nucleon to both positive and negative parity excitations
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