Electromagnetic matrix elements for excited Nucleons

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Outline

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- Calculate set of cross-correlation functions

$$\mathcal{G}_{ij}(t, \vec{p}; \Gamma) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \mathrm{tr}(\Gamma \langle \Omega | \chi_i(x) \bar{\chi}_j(0) | \Omega \rangle)$$

$$= \sum_{\alpha} e^{-E_\alpha(\vec{p})\, t} Z_i^\alpha(\vec{p}) \bar{Z}_j^\alpha(\vec{p}) \mathrm{tr}\left(\frac{\Gamma(\not{p} + m_\alpha)}{2E_\alpha(\vec{p})} \right)$$

where Z_i^α, $\bar{Z}_j^\alpha$ are the couplings of sink operator (χ_i) and source operator $(\bar{\chi}_j)$ to the state α

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- use our basis of operators to construct these new operators

$$\left.\begin{aligned} \bar{\phi}_\alpha(x, \vec{p}) &= \sum_{i=1}^{N} u_i^\alpha(\vec{p}) \bar{\chi}_i(x) \\ \phi_\alpha(x, \vec{p}) &= \sum_{i=1}^{N} v_i^\alpha(\vec{p}) \chi_i(x) \end{aligned}\right\} \quad \text{optimal coupling to state } |\, M_\alpha, p, s \,\rangle$$

CM Analysis (cont)

- Knowledge of the time dependence provides the recurrence relation

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CM Eigenvalue Equation

$$\begin{aligned}
[\,\mathcal{G}^{-1}(t_0,\vec{p};\Gamma)\,\mathcal{G}(t_0+\delta t,\vec{p};\Gamma)\,]_{ij}\,u_j^\alpha(\vec{p}) &= e^{-E_\alpha(\vec{p})\delta t}\,u_j^\alpha(\vec{p}) \\
v_i^\alpha(\vec{p})\,[\,\mathcal{G}(t_0+\delta t,\vec{p};\Gamma)\,\mathcal{G}^{-1}(t_0,\vec{p};\Gamma)\,]_{ij} &= e^{-E_\alpha(\vec{p})\delta t}\,v_i^\alpha(\vec{p})
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- Using $v_i^\alpha(\vec{p})$, $u_j^\alpha(\vec{p})$ we are able to project out the correlation function for the state $|\,M_\alpha,p,s\,\rangle$

$$\mathcal{G}_\alpha(t,\vec{p};\Gamma) = v_i^\alpha(\vec{p})\mathcal{G}_{ij}(t,\vec{p};\Gamma)u_j^\alpha(\vec{p})$$

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$$\mathcal{G}^{\alpha}(\vec{p}', \vec{p};\, t_2, t_1;\, \Gamma') = v_i^{\alpha}(\vec{p}')\, \mathcal{G}_{ij}(\vec{p}', \vec{p};\, t_2, t_1;\, \Gamma')\, u_j^{\alpha}(\vec{p})\,.$$

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- In this work we have used the following ratio,

$$R^{\alpha}(\vec{p}', \vec{p};\, \Gamma', \Gamma) = \sqrt{\frac{\mathcal{G}^{\alpha}(\vec{p}', \vec{p};\, t_2, t_1;\, \Gamma')\, \mathcal{G}^{\alpha}(\vec{p}, \vec{p}';\, t_2, t_1;\, \Gamma')}{\mathcal{G}^{\alpha}(\vec{p}, t_2;\, \Gamma)\, \mathcal{G}^{\alpha}(\vec{p}', t_2;\, \Gamma)}}\,.$$

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- Use of varying widths allows us to separate radial excitations
- Multi-particle states couple poorly, but through mixing of eigenstates they are still present in the correlator
- In particular, we use 4 levels of gauge invariant Gaussian smearing at the source and sink with smearing fraction $\alpha = 0.7$.[1]

Table : The rms radii for the various levels of smearing considered in this work.[1]

Sweeps of smearing	rms radius (fm)
16	0.216
35	0.319
100	0.539
200	0.778

Our operator basis (cont)

- We use both χ_1 and χ_2

$$\chi_1(x) = \epsilon^{abc}(u^{T\,a}(x)\, C\gamma_5\, d^b(x))\, u^c(x)$$
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[1] *M. S. Mahbub et al., Phys. Lett. B.* ***707***, *(2012) 389*
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$$\Gamma_4^+ = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

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- For negative parity states we use the projector[2]:

$$\Gamma_4^- = -\gamma_5 \Gamma_4^+ \gamma_5$$

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- In doing this, we are able to construct orthonormal eigenvectors w_j^α, related to our u_i^α through

$$w_j^\alpha(\vec{p}) = \mathcal{G}_{ij}^{1/2}(t_0, \vec{p}; \Gamma) u_j^\alpha(\vec{p})$$

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- We can identify corresponding eigenvectors across momenta as those with

$$w^\alpha(\vec{p}) \cdot w^\beta(0) \approx \delta^{\alpha\beta}$$

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Nucleon Matrix Elements

- Both positive and negative parity nucleon electromagnetic matrix elements can be decomposed into the standard Pauli-Dirac form

$$\langle N, p', s' | J^\mu | N, p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2) \right] u(p, s)$$

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- Sachs Form Factors are related to these via

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

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- We use a conserved vector current, with $\vec{q} = \frac{2\pi}{L}\hat{x}$
- We evaluate the three-point functions with $\vec{p} = 0$ and $\vec{p}' = \vec{q}$
- The ratios used to extract the form factors G_E and G_M are

$$G_E(Q^2) = \left(\frac{2E_q}{E_q + M}\right)^{1/2} R(\vec{q}, 0; \Gamma_4^{\pm}, \Gamma_4^{\pm}; \mu = 4)$$

$$G_M(Q^2) = \frac{E_q + M}{|\vec{q}|}\left(\frac{2E_q}{E_q + M}\right)^{1/2} R(\vec{q}, 0; \Gamma_2^{\pm}, \Gamma_4^{\pm}; \mu = 3)$$

where

$$\Gamma_i^+ = \begin{pmatrix} \sigma^i & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \Gamma_i^- = -\gamma_5 \Gamma_i^+ \gamma_5$$

Ensemble Details

- For this calculation we are working with the PACS-CS (2+1)-flavour Full QCD ensembles[1] made available through the ILDG

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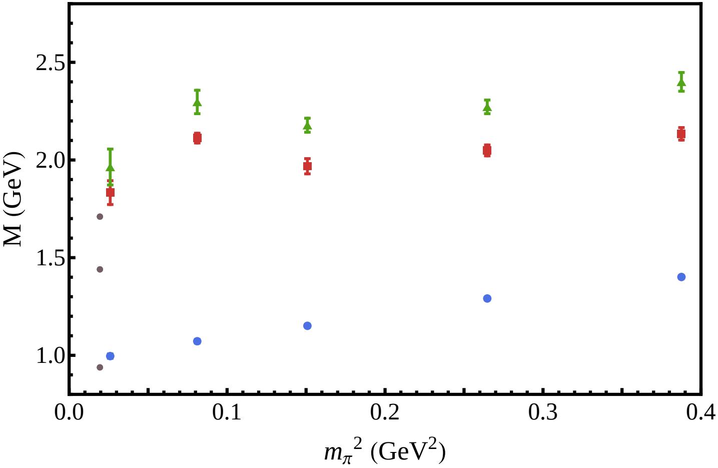
- There are five light quark-masses

Table : Ensemble details

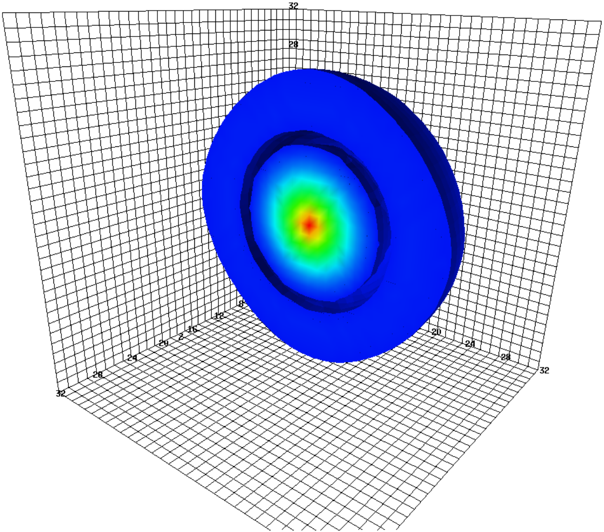
m_π (MeV)	n_{cfgs}	$n_{\mathrm{srcs/cfg}}$	n_{srcs}
702	350	2	700
570	350	2	700
411	350	2	700
296	350	2	700
156	200	6	1200

[1] *S. Aoki et al., Phys. Rev. D* **79**, *034503 (2009)*

Positive Parity Spectrum

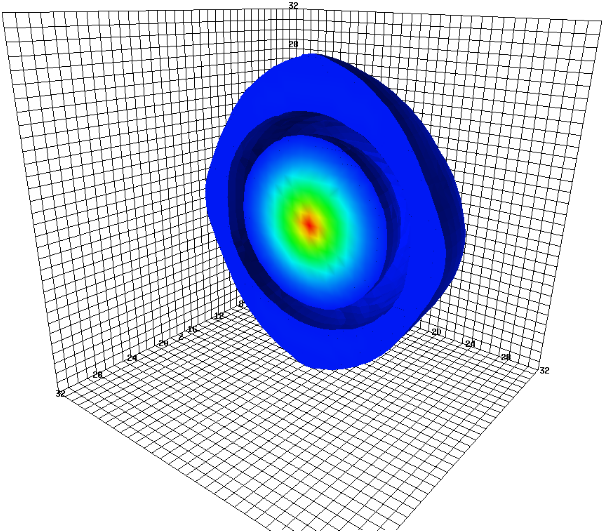


$N^*(1/2^+)$ wave function[1] – $m_\pi = 570$ MeV



[1] *D. Roberts et al., Phys. Rev. D* **89**, *074501 (2014)*

$N^*(1/2^+)$ wave function[1] – $m_\pi = 156$ MeV



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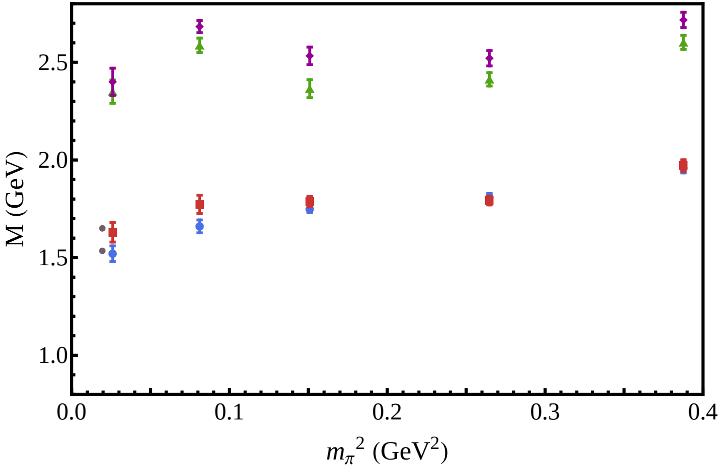
Negative Parity Spectrum



LogG

- Multi-particle states couple weakly to our choice of interpolators and so their contribution is only significant in the tail of the correlator.[1]

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- We consider logG of our projected 2pt-correlators to identify regions where multi-particle contributions are suppressed relative to the nucleon excitation
- On going work will broaden our basis to include multi-particle operators

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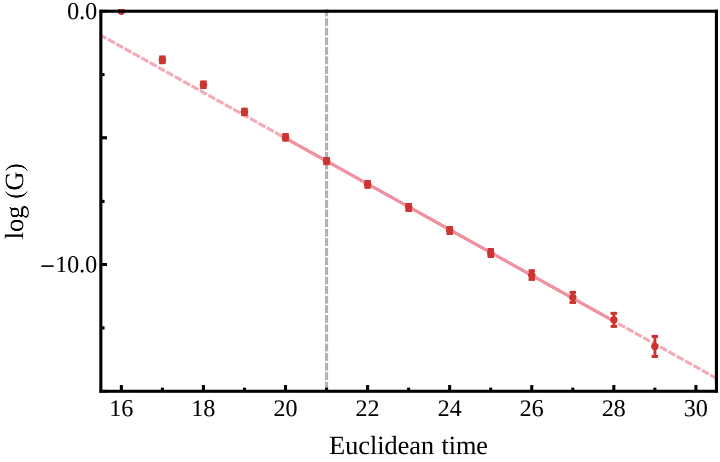
Projected Correlator for the second $^1/_2{}^-$ eigenstate: $m_\pi = 570$ MeV

Want linear behaviour in logG around and after $t_s = 21$



Projected Correlator for the second $^1/_2^-$ eigenstate: $m_\pi = 296$ MeV

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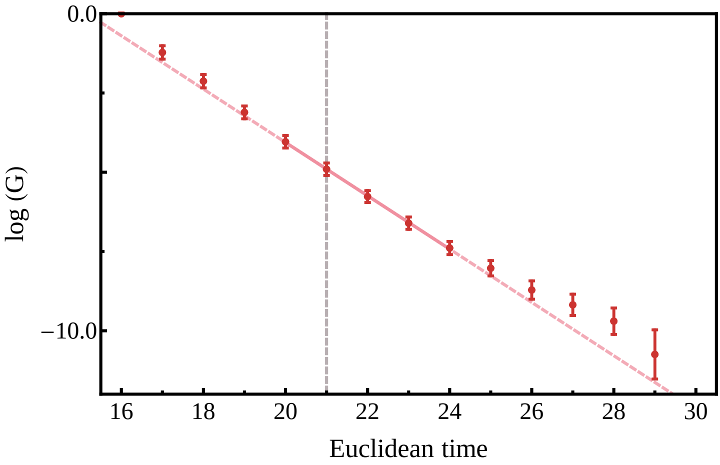
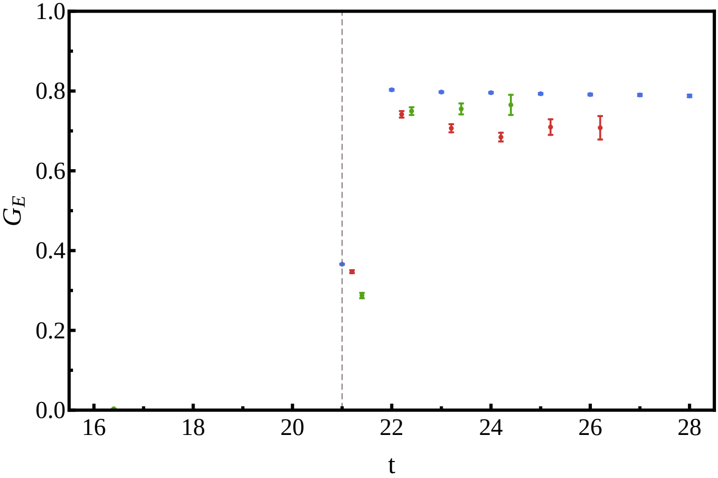


Quark Sector Results: GE, u in p (Positive Parity) $m_\pi = 570$ MeV



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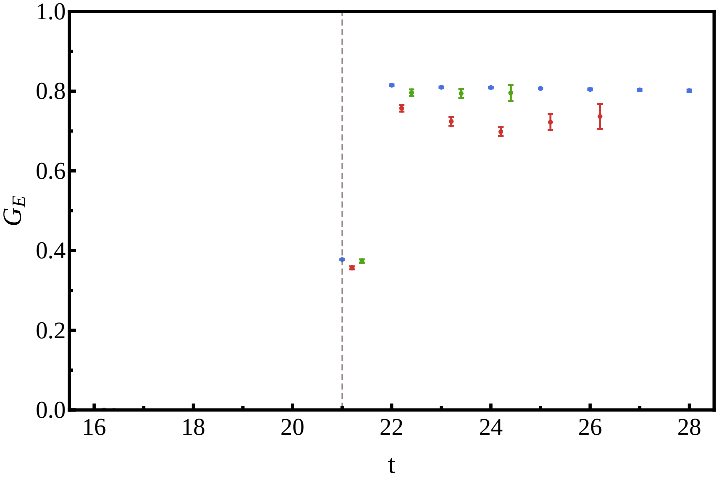

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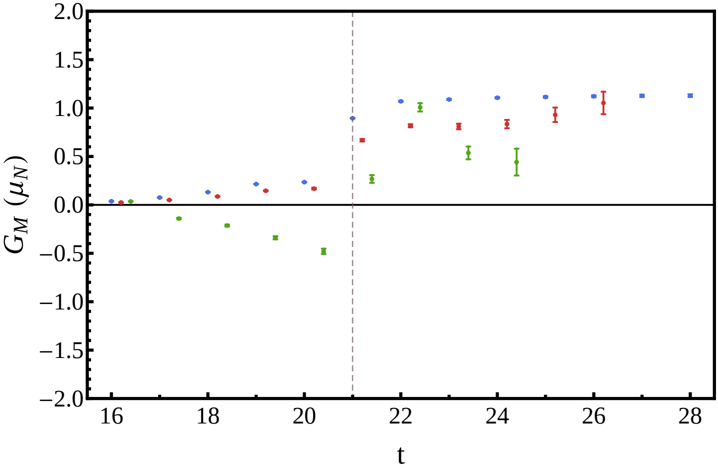

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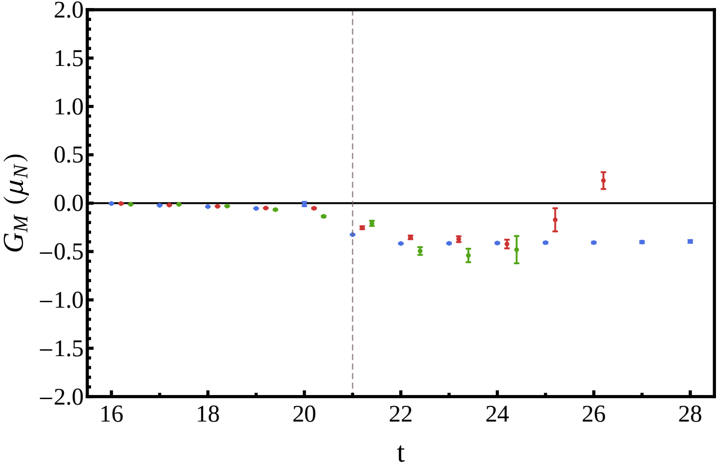
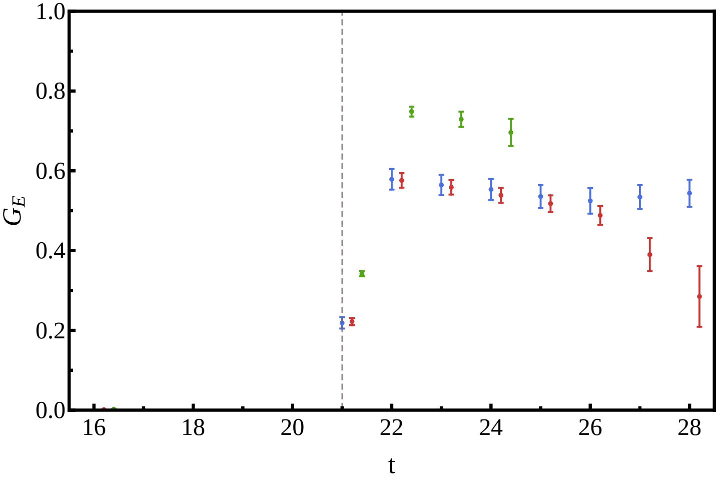


Quark Sector Results: GE, u in p (Negative Parity) $m_\pi = 570$ MeV



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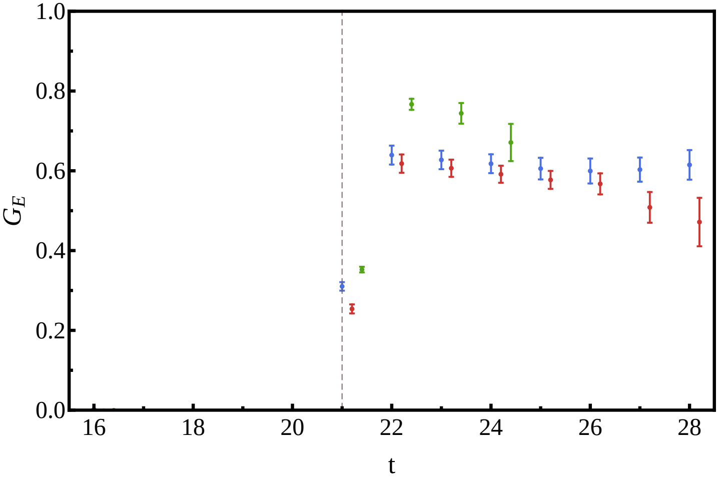

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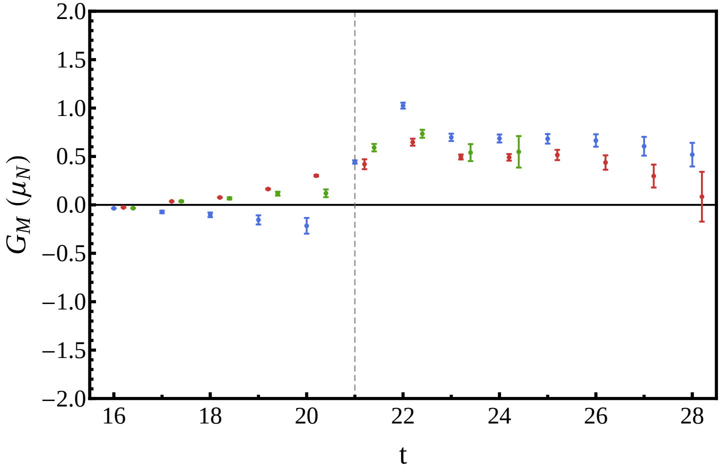

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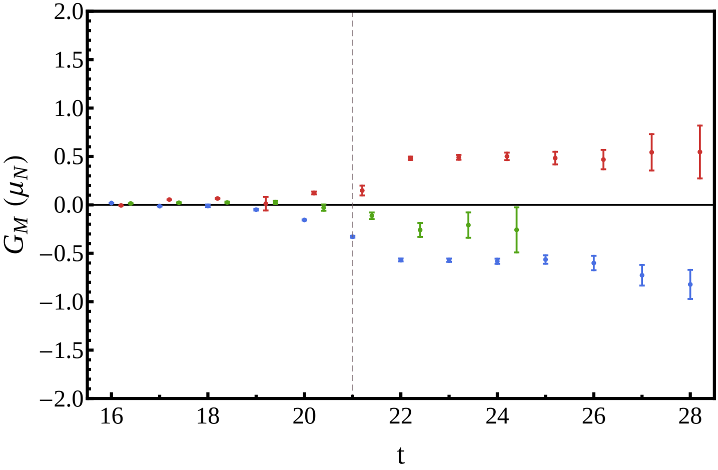

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$$G_i(Q^2) = \left(\frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^2 G_i(0)$$

to perform a small shift in Q^2

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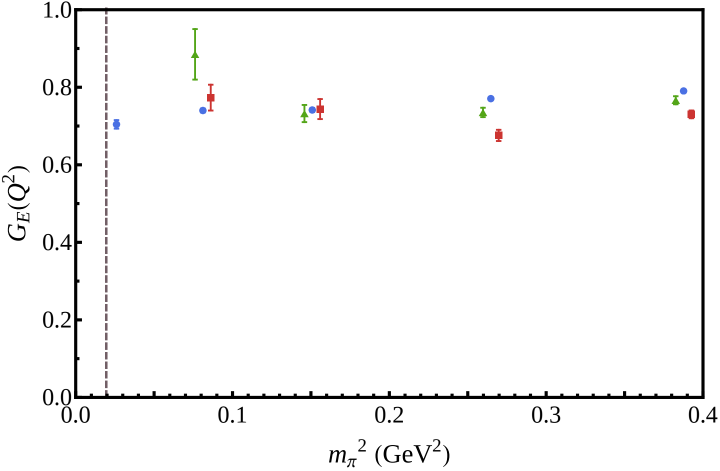
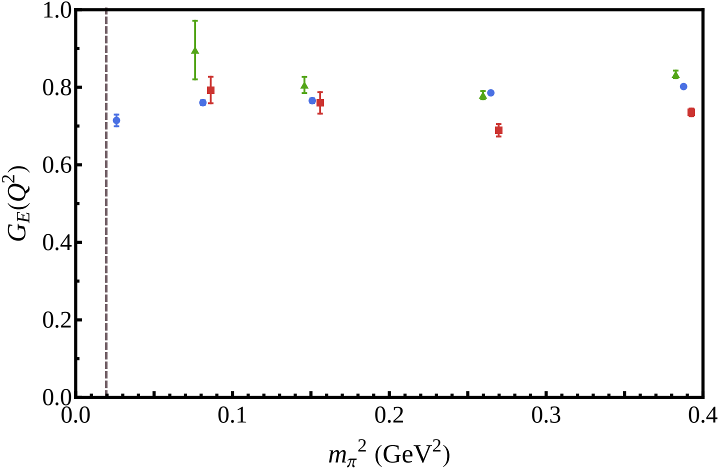
- For this ensemble, we choose to shift all our extracted form factors to the common value of $Q^2 = 0.16$ GeV^2

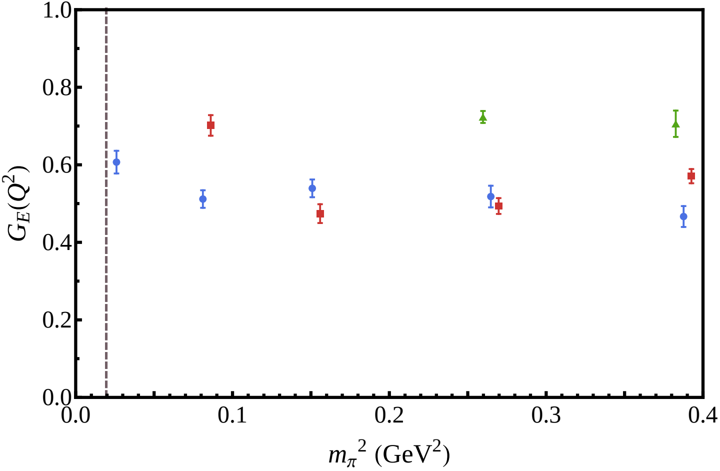
Quark Sector Results: GE, u in p (Positive Parity)



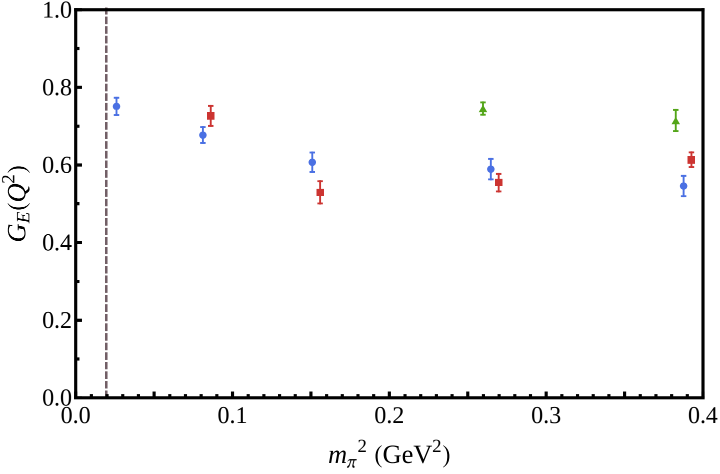
Quark Sector Results: GE, d in p (Positive Parity)



Quark Sector Results: GE, u in p (Negative Parity)



Quark Sector Results: GE, d in p (Negative Parity)



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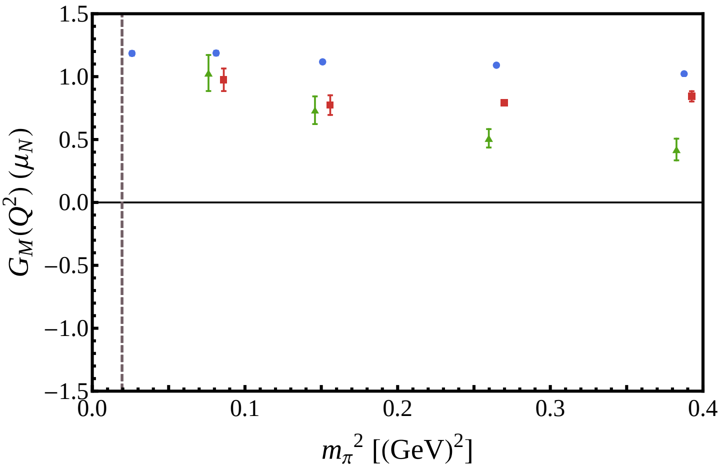
  - An interesting possibility is that we have important Δ^{++}, π^- dressings

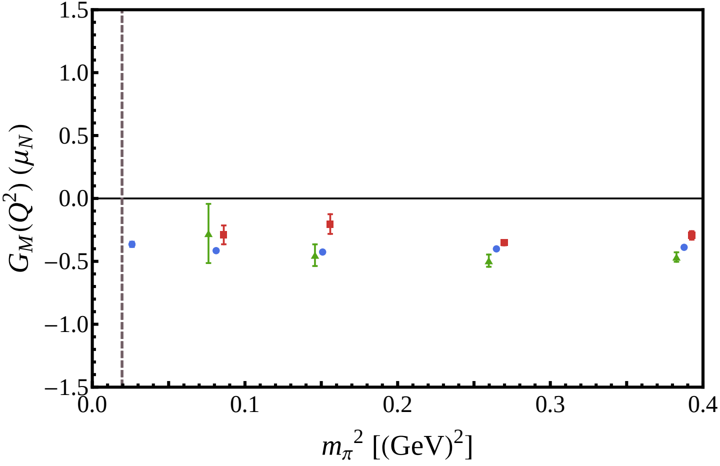
$$\frac{1}{\sqrt{2}}|\Delta^{++}\pi^-\rangle - \frac{1}{\sqrt{3}}|\Delta^{+}\pi^0\rangle + \frac{1}{\sqrt{6}}|\Delta^{0}\pi^+\rangle$$

    which would lead to accumulation of positive charge at the origin

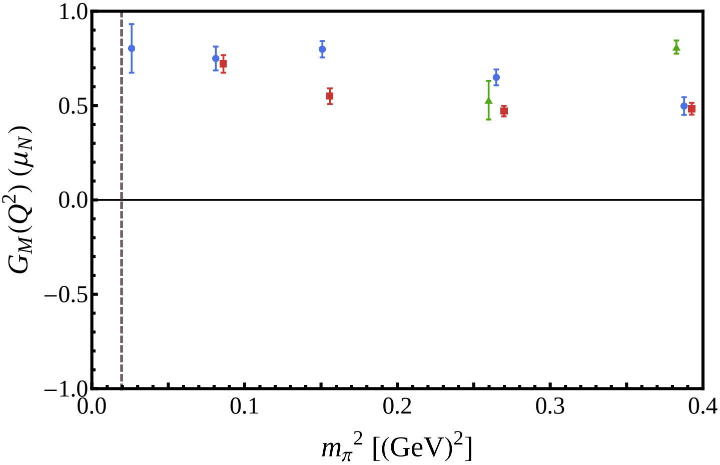
Quark Sector Results: GM, u in p (Positive Parity)



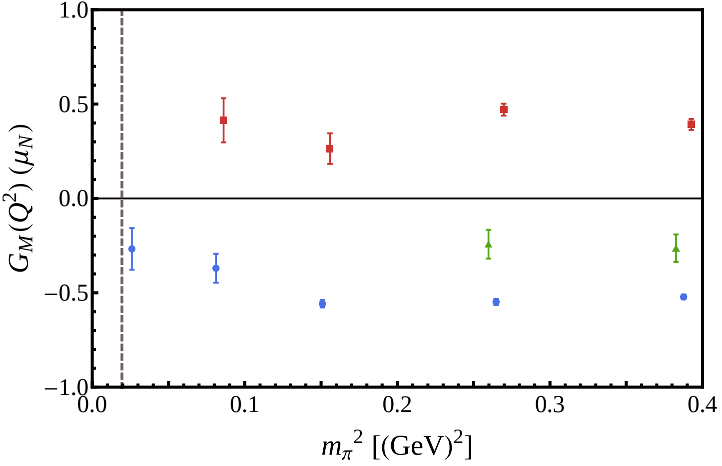
Quark Sector Results: GM, d in p (Positive Parity)



Quark Sector Results: GM, u in p (Negative Parity)



Quark Sector Results: GM, d in p (Negative Parity)



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- Examine the transition amplitudes for ground state nucleon to both positive and negative parity excitations