Two-dimensional phase structure of SU(2) gauge-Higgs model

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@Lattice2014

Overview

- Introduction

 - Fradkin-Shenker-Osterwalder-Seiler theorem

 - 4 D phase structure with SU(2) Higgs model

- 2D phase structure with SU(2) Higgs-Kibble model from lattice simulations

Fradkin-Shenker-Osterwalder-Seiler theorem

SU(2) Higgs-Kibble model with frozen length of the Higgs fields

$$S = \frac{\beta}{2}\sum \mathrm{Tr}U_{\mu\nu} + \frac{\gamma}{2}\sum \mathrm{Tr}\left[\phi^\dagger(x)U_\mu(x)\phi(x+\hat{\mu})\right] \qquad \phi \in \mathrm{SU}(2)$$

E.H.Fradkin, S.H.Shenker (1979)

In the β-γ phase diagram, gauge-invariant quantities may be continued analytically between two regions (β,γ)<<1 and (β,γ)>>1

4D Schematic picture of SU(2) Higgs-Kibble model with frozen length of the Higgs fields

C. Lang, C. Rebbi and M. Virasoro (1981)
J. Jersak, C.B. Lang, T. Neuhaus, and G. Vones (1985)
W. Langguth, I. Montvey and P. Weisz (1986)
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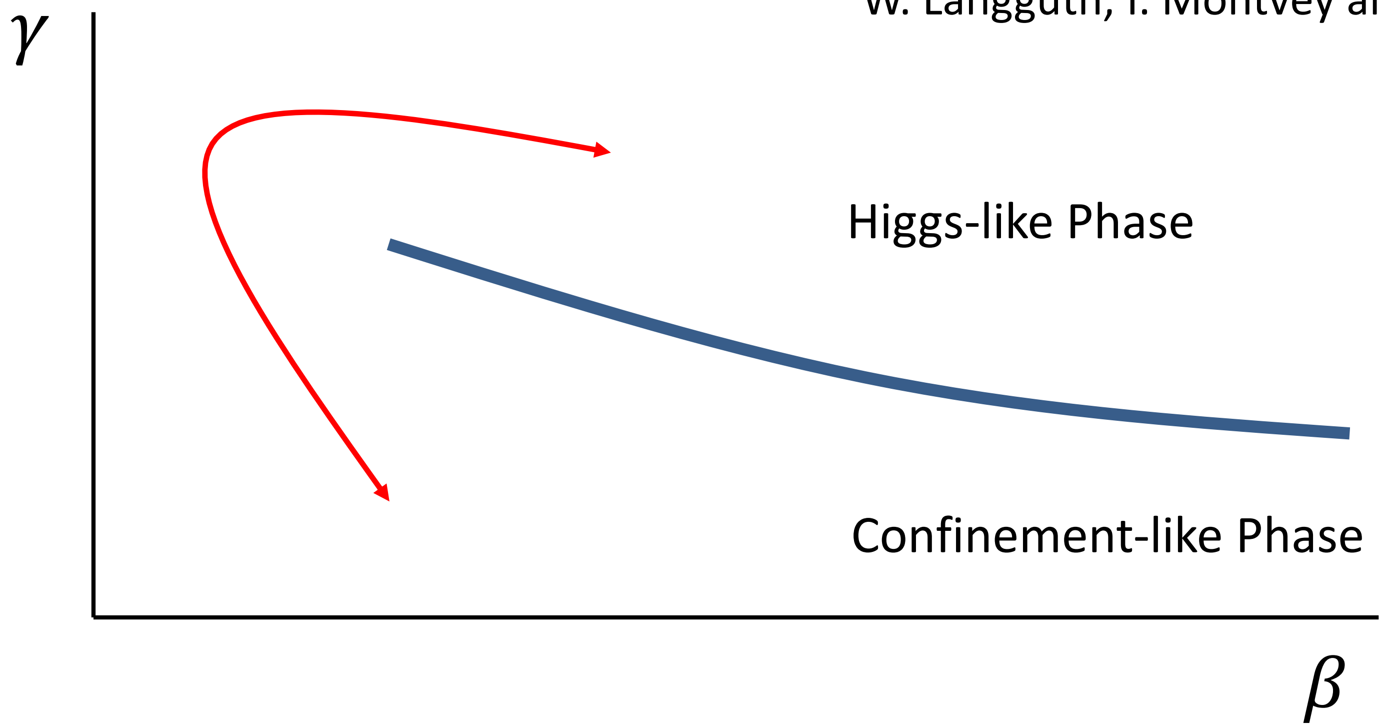
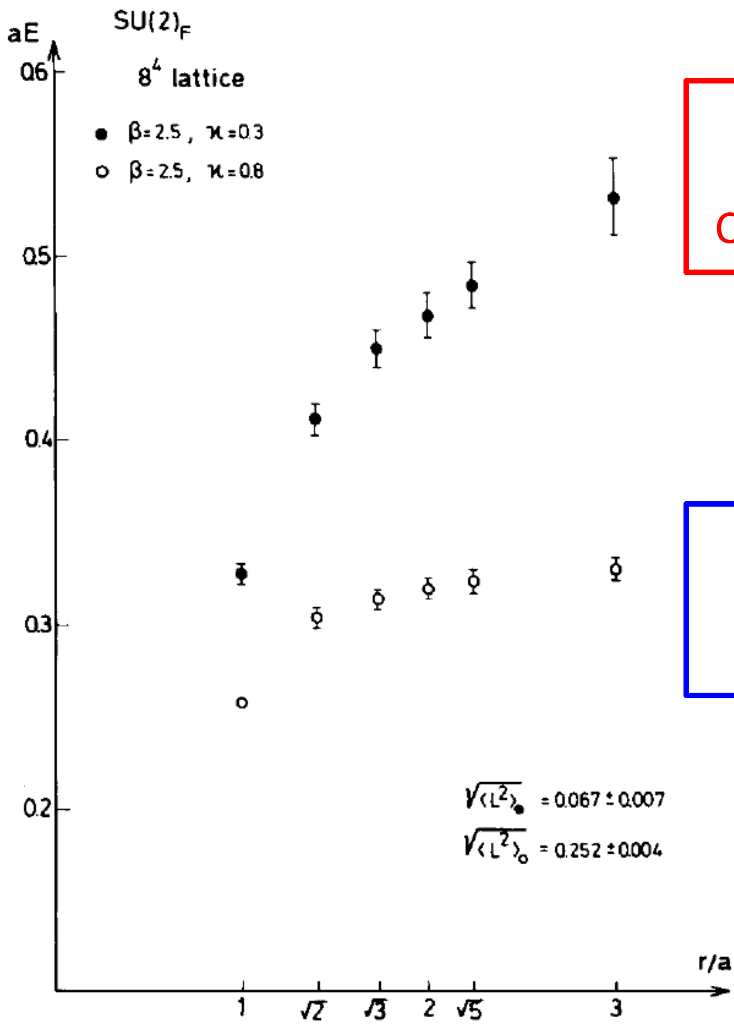


Numerical studies show that Higgs-like and confinement-like phases appear with a line of first order transitions (or crossover) which ends around (β, γ)=(2,1).

4D Higgs-Confinement Phase transition

Static potential

I. Montvay (1985)



$$V \sim -\frac{A}{r} + \sigma r$$

Confinement phase

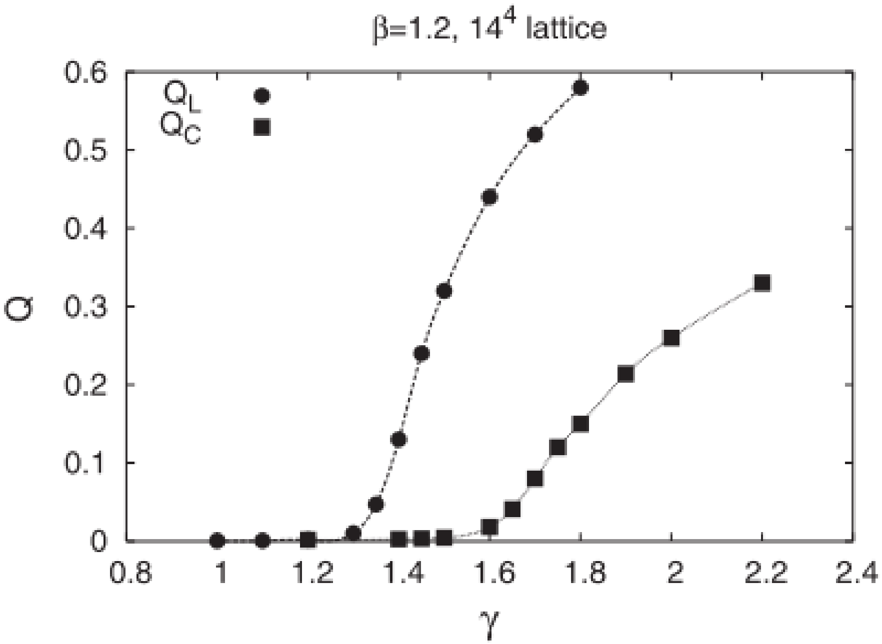
$$V \sim -\frac{B}{r} e^{-mr}$$

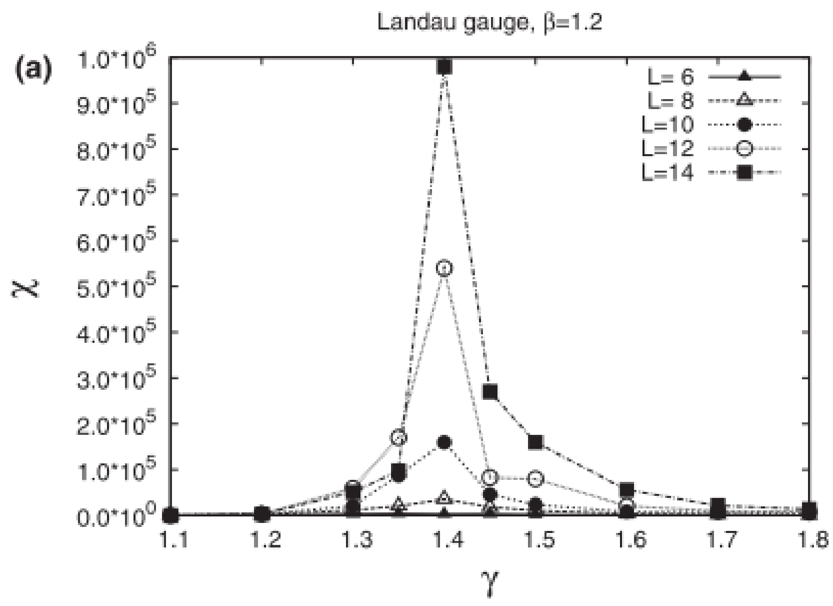
Higgs phase

Order parameter of the Phase transition

The gauge symmetry is not always broken down. (Elitzur theorem)

W. Caudy, J. Greensite (2008)





The phase transition is characterized by the remnant symmetry breaking in the Landau gauge.

There are many lattice studies for Higgs models

- E. H. Fradkin and S. H. Shenker(1979)
- C.B. Lang, C. Rebbi and M. Virasoro (1981), J. Jersak, C.B. Lang, T. Neuhaus and G. Vones (1985)
- I. Montvey(1985,1986), W. Langguth, I. Montvey and P. Weisz(1986)
- I. Campos (1998)
- ALPHA Collaboration(1998, 2000)
- J. Greensite, S. Olejnik and D. Zwanziger (2004), W. Caudy and J. Greensite(2008)
- A. Maas (2011), A. Maas and T. Mufti (2013)

…

2D phase structure of SU(2) gauge Higgs model from lattice simulations

Coleman theorem (Hohenberg-Mermin-Wagner theorem)

In 2D, continuous spontaneous symmetry breaking does not occur (No NG boson).

In Higgs case, there is no NG boson as a physical particle.

Higgs-like phase and confinement-like phase, which corresponds to a single phase, are not characterized by the order parameter gauge-invariantly due to the Eltizur theorem.

Thus the Coleman theorem is evaded and Higgs-like phase may appear, where Higgs mechanism occurs.

SU(2) Higgs-Kibble model with frozen length of the Higgs fields in two dimensions

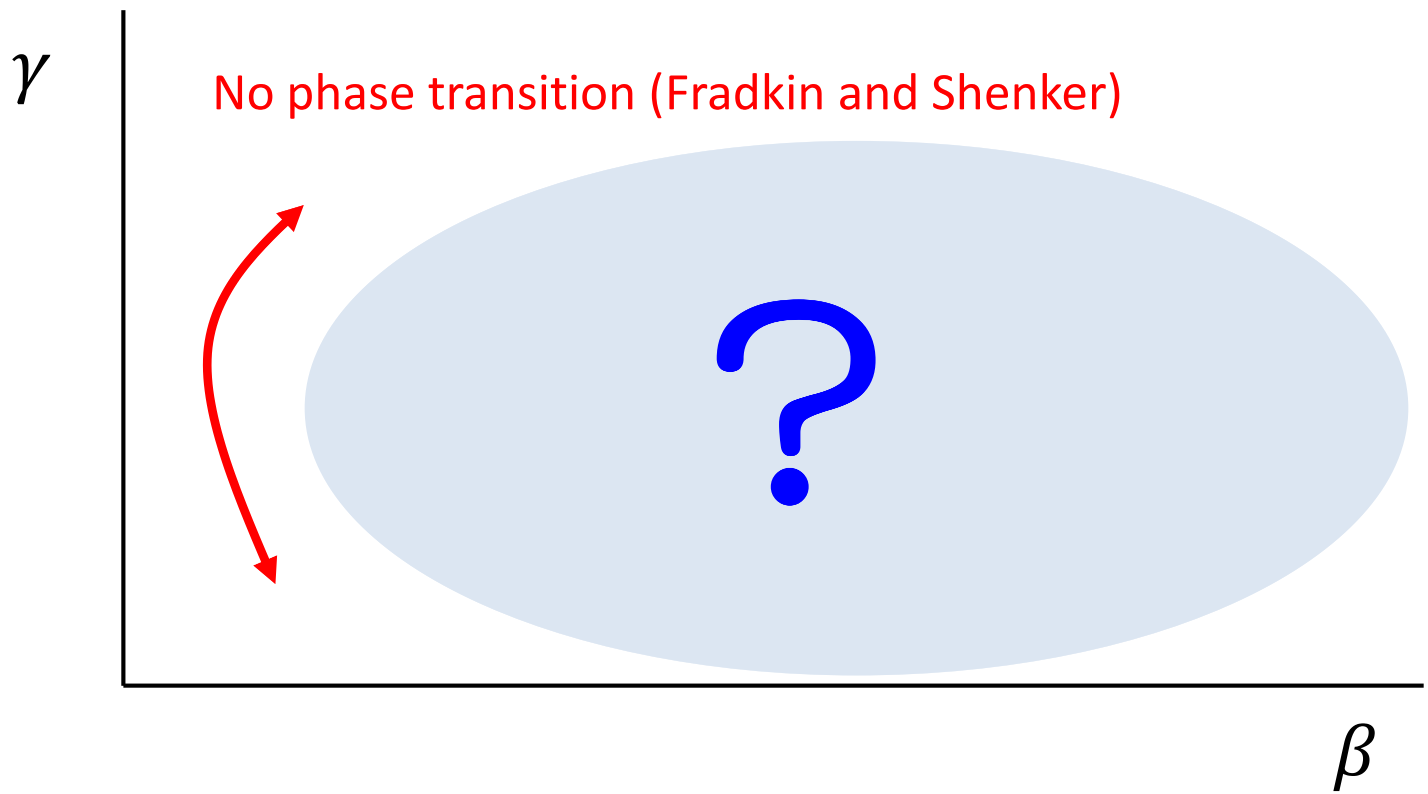
$$S = \frac{\beta}{2}\sum \mathrm{Tr}U_{\mu\nu} + \frac{\gamma}{2}\sum \mathrm{Tr}\left[\phi^\dagger(x)U_\mu(x)\phi(x+\hat{\mu})\right], \quad \phi(\mathrm{x}) \in \mathrm{SU}(2)$$

In unitary gauge, $\phi(x) = 1$,

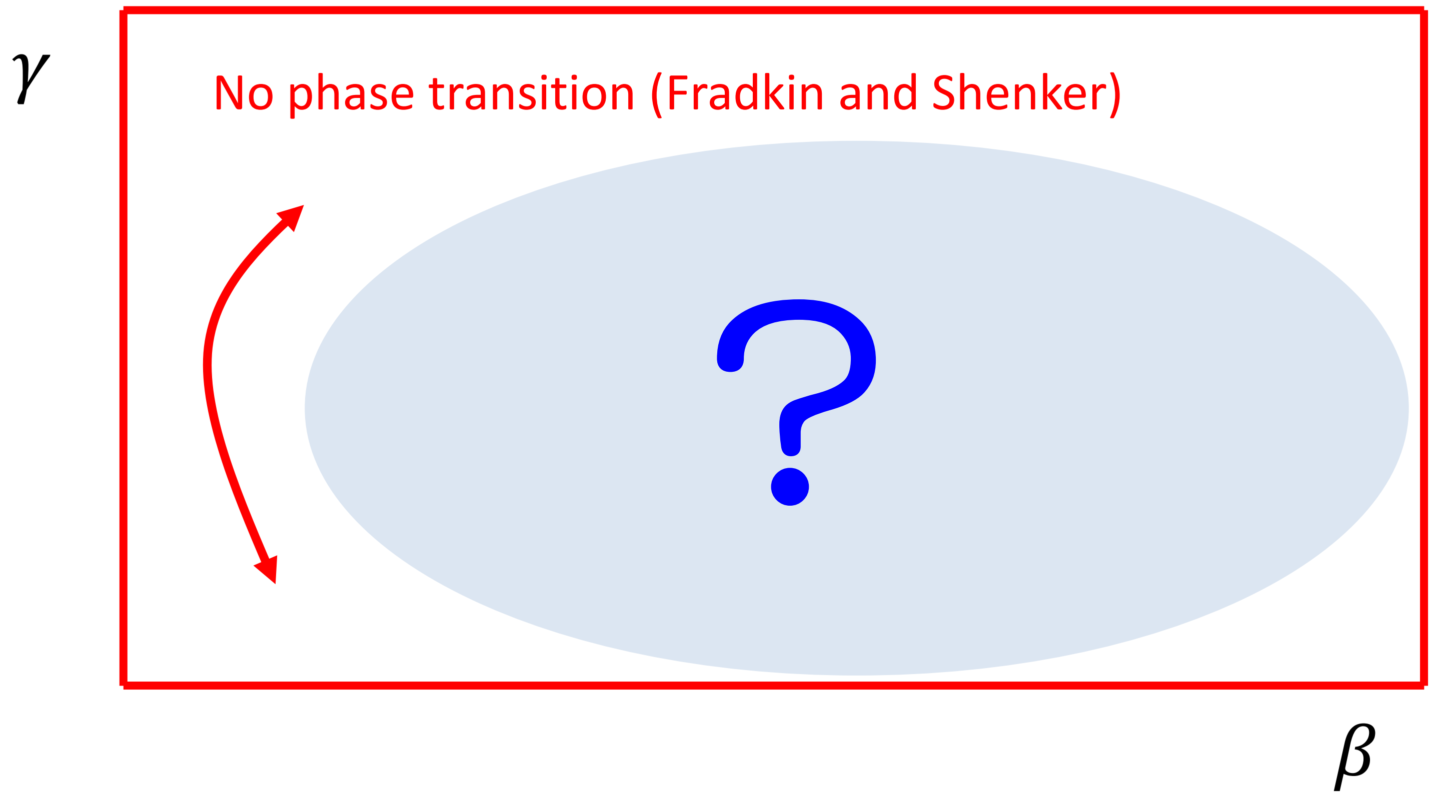
$$S = \frac{\beta}{2}\sum \mathrm{Tr}U_{\mu\nu} + \frac{\gamma}{2}\sum \mathrm{Tr}\left[U_\mu(x)\right]$$

- At β=∞ $U_{\mu\nu}$=1 and, in some gauge, the action corresponds to an O(4) Heisenberg model, which has no transition in two dimensions.
- At β=0, the action in unitary gauge corresponds to a complete disorderd system.
- At γ=∞, U =1
- At γ=0, this corresponds to pure gauge theory.

2D Schematic picture of the phase structure with SU(2) Higgs-Kibble model



2D Schematic picture of the phase structure with SU(2) Higgs-Kibble model



There is no transition in the boundary.

Lattice simulations with SU(2) gauge Higgs model in unitary gauge

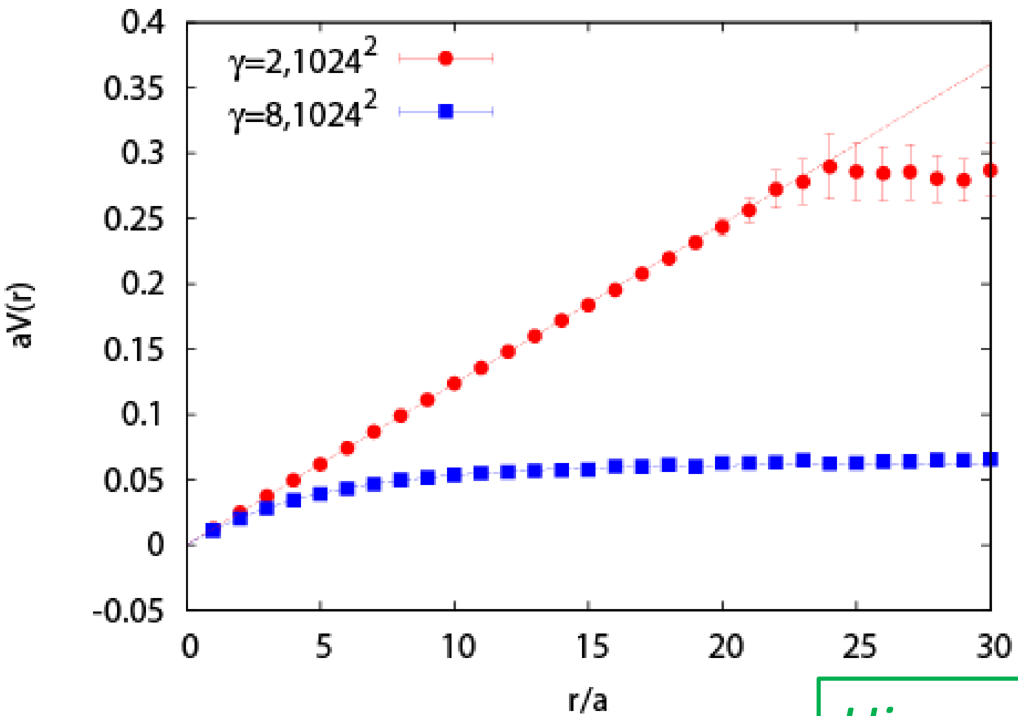
$$S = \frac{\beta}{2}\sum \mathrm{Tr}U_{\mu\nu} + \frac{\gamma}{2}\sum \mathrm{Tr}\left[U_\mu(x)\right]$$

- In unitary gauge, $\phi(x) = 1$, the Higgs fields disappear from the action and the link-variables are generated by modifying the program of the quenched calculation slightly.

- The Higgs fields are obtained by gauge transformations after generating the link-variables in unitary gauge.

$$\phi(x) = g^\dagger(x)$$

The static potential



$1024^2, \beta = 120$

$V \sim \sigma r +$ string breaking
Confinement-like phase

$V \sim -e^{-mr}$ 2D Yukawa pot.
Higgs-like Phase

Higgs mechanism occurs even in 2D!

- In confinement-like phase, the potential shows the linear potential and the string breaking at r/a ≒20.
- In Higgs-like phase, the gauge boson becomes a massive vector boson and the potential behaves like $V(r) \sim -\int dp \, \frac{e^{-ipr}}{p^2+m^2} \sim -e^{-mr}$

W propagator

$$D^0_{\mu\nu}(t) = \frac{1}{3V}\sum_{x,y,a}\left\langle W_\mu^a(x,t) W_\nu^a(y,0)\right\rangle$$

$$W_\mu^a(x,t) = \frac{1}{i}\mathrm{Tr}\left[\tau^a \phi^\dagger(x) U_\mu(x) \phi(x+\hat{\mu})\right]$$ *gauge invariant*

W propagator

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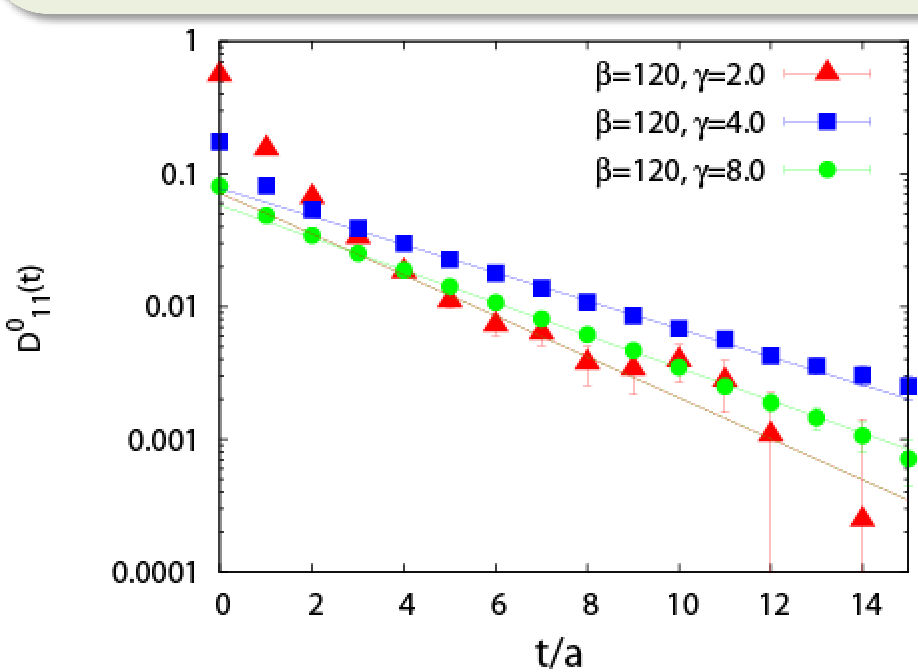
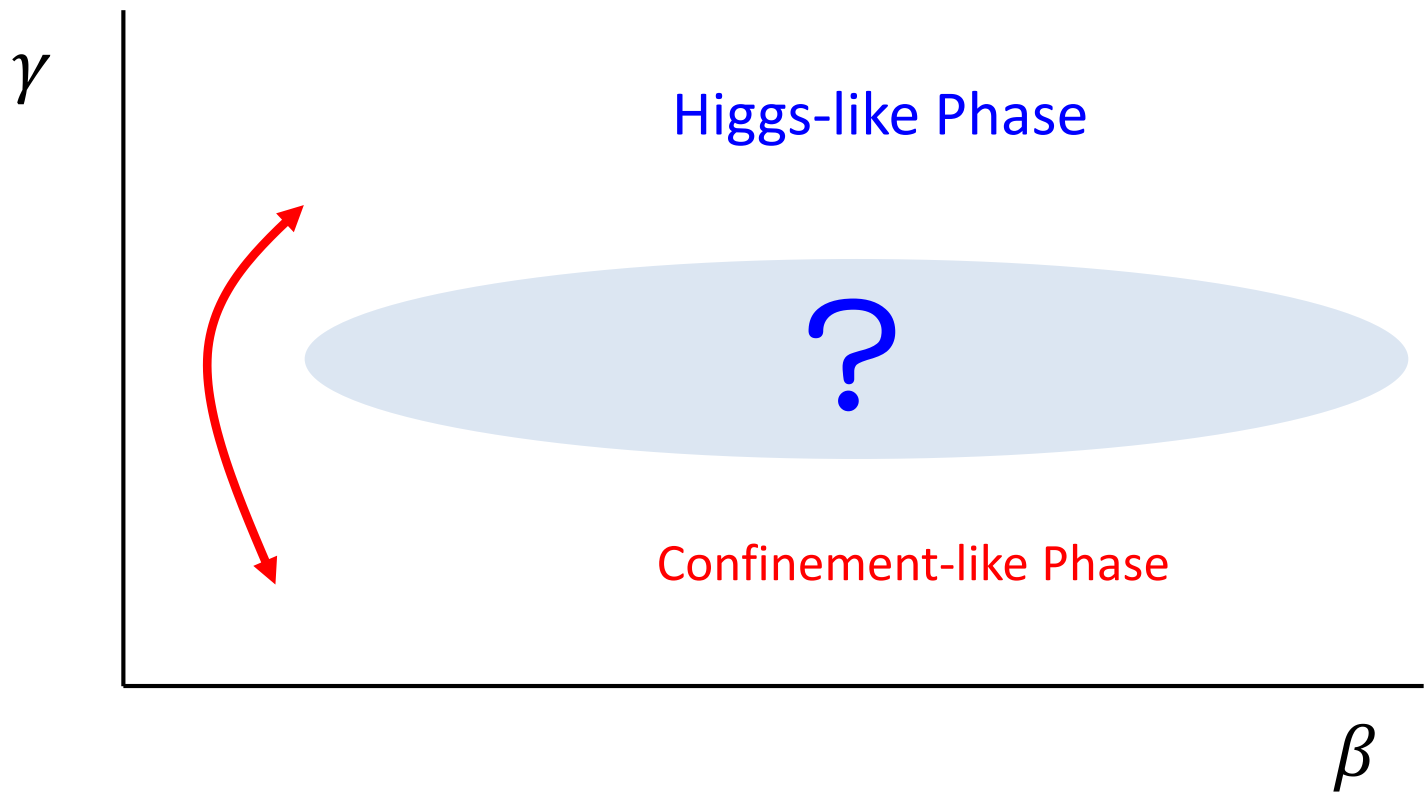


γ	m
2	0.36(3)
4	0.243(4)
8	0.282(1)

The effective mass is minimum in the region of γ = 2-8
, which may indicate a phase transition or cross-over.

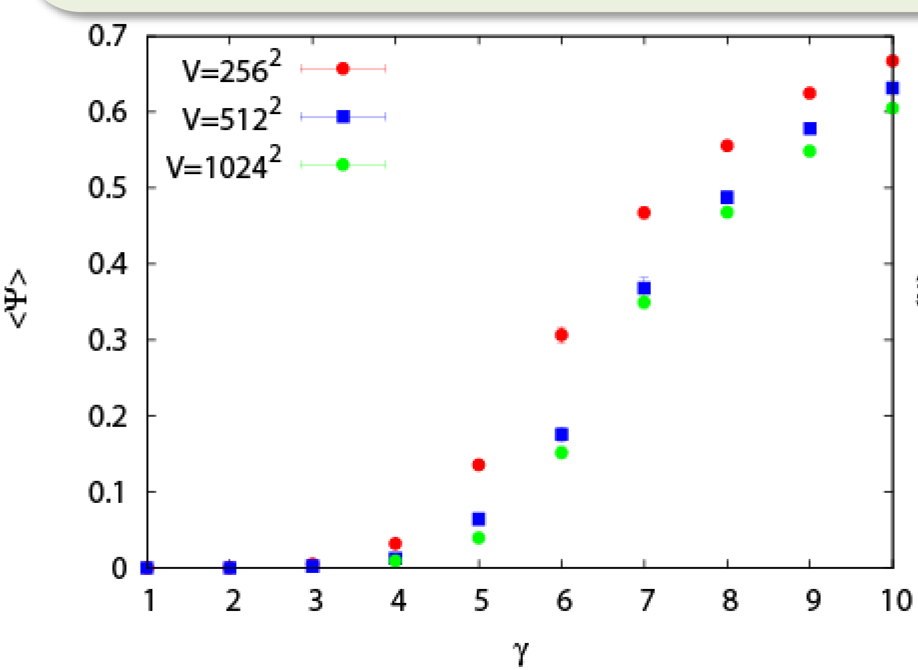
2D Schematic picture of the phase structure with SU(2) Higgs-Kibble model

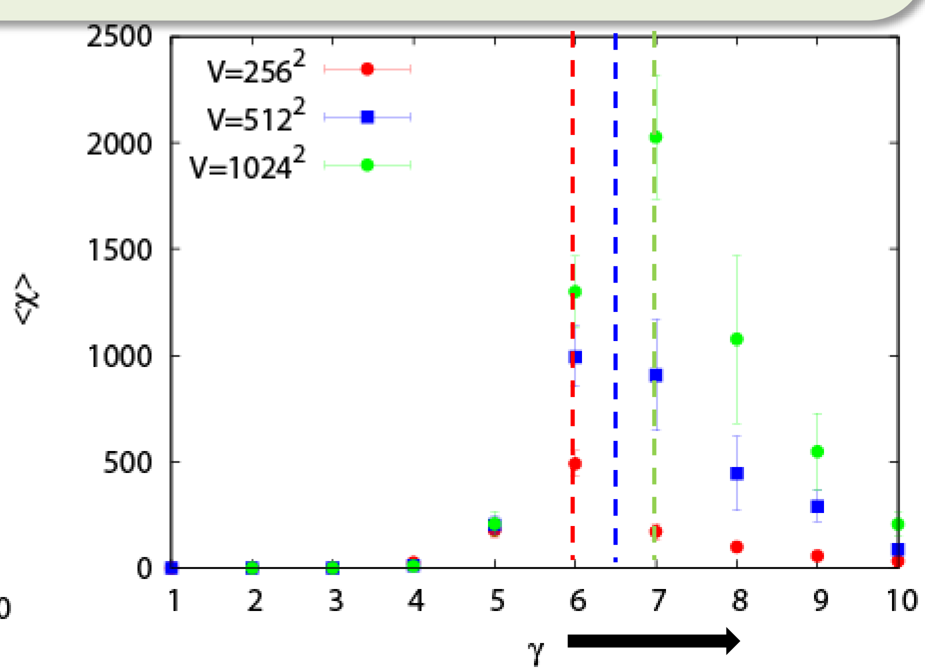


The order parameter in the Landau gauge

$$\Psi = \mathrm{Tr}\left[\Phi^{\dagger}\Phi\right] \qquad \Phi = \sum_{x} \phi(x)$$

This corresponds to the order parameter of the global (gauge) symmetry breaking in the Landau gauge.





The maximum point of susceptibility moves as V becomes large.
The behavior may indicate the *Kosterlitz-Thouless Phase*.

Summary

The behavior of the potential and the W propagator indicates that the Higgs-type mechanism occurs and there is a Higgs-like phase even in two dimensions (though the Higgs-like phase and confinement-like phase might be connected).

Thank you for your attention.